Raymond M. Smullyan First-order logic Corrected republication of XL 237. Dover Publications, New York 1995, xii + 158 pp.

In reconciling the contrary viewpoints of syntax and semantics, Gödel's completeness theorem lies at the heart of mathematical logic. Over the years numerous approaches to proving completeness have been explored, the efforts justified by the theorem's primary importance. Smullyan's *First-order logic,* a corrected reprinting of the 1968 original (XL 237), explains many of these approaches in detail.

For example, the method of *analytic tableaux* runs roughly as follows. Let φ be a sentence of predicate logic, and suppose we want to prove that either φ is satisfiable or its negation is provable. Try to satisfy φ by building a tree: to satisfy $\theta \wedge \psi$ one needs to satisfy both θ and ψ; to satisfy $\theta \vee \psi$ one can branch and try to satisfy either one of the two; to satisfy $\exists x\, \theta(x)$ one tries to satisfy $\theta(c)$ for a suitable constant c; and so on. If every branch of the tree yields a contradiction at some finite stage, we have the desired proof of $\neg\varphi$. On the other hand, if the construction has been done carefully, an infinite branch yields what is called a *Hintikka set.* These sets, though not maximal, are saturated downwards (e.g. if $\theta \wedge \psi \in S$ then $\theta \in S$ and $\psi \in S$, though not necessarily conversely), and from such a set it is easy to build a model of φ.

Such constructions are the essence of Smullyan's book. Part I covers propositional logic and various proofs of compactness, including the tableaux method described above. Part II introduces predicate logic and its semantics, extends the tableaux method to that domain, and abstracts the general notion of an *analytic consistency property* from the resulting completeness proof. There is also a discussion of various Hilbert-style axiomatizations of predicate logic, and of approaches to proving completeness using witnessing constants in the style of Henkin and Hasenjaeger. Part III treats Gentzen-style proof systems and the cut-elimination theorem (Hauptsatz), Craig's interpolation lemma, and some interesting variants of Gentzen-style deduction.

Two of the book's most salient features are its thoroughness and its rich use of notation. In most cases varying approaches to proving a theorem are compared and contrasted, and their underlying ideas are abstracted and generalized. So, for example, variations of analytic tableaux using signed and unsigned formulas are considered; analytic (and synthetic) consistency properties are introduced to generalize the notion of consistency in a proof system without cut (and with cut, respectively); and the abstract notions of "regular" and "magic" sets are extracted from the Henkin–Hasenjaeger completeness proofs described above. This style has its downside: at times the abstraction and terminology are overwhelming and make casual reading difficult. But all in all the exposition is clear and organized, and the subject matter is pithy and well chosen.

I can see the book being useful in the following capacities:

1. *As a supplementary logic text.* The prose is friendly and readable, with helpful side-remarks and examples. With selective cutting and pruning, the chapters on propositional logic and tableaux would make interesting supple-

mentary reading for an introductory logic course, and consistency properties provide a uniform method of proving completeness, compactness, the interpolation lemma, Robinson's joint consistency theorem, and the like.

2. *As a general logic reference.* The book surveys a number of important syntactic constructions and methods, providing some historical perspective as well.

3. *As a proof theory reference.* Since analytic tableaux are essentially just Gentzen-style cut-free proofs turned upside-down (Chapter XI discusses the exact relationship), the book offers a transparent proof of the completeness of cut-free Gentzen systems. Proof theorists will further appreciate, for example, a complete Hilbert-style system that avoids the use of modus ponens, a discussion of the extended Hauptsatz (sometimes referred to as the midsequent lemma), and a constructive proof of the interpolation lemma.

4. *As a theoretical computer science reference.* Tableaux methods have made their way to the fields of automated theorem proving and proof complexity. Though the book does not cover resolution methods, for example, it provides good background to the study of such topics.

This Dover paperback edition is nicely bound and features the same attractive typesetting of the original edition. At a list price of about $8, it is well worth the cost.

Jeremy Avigad