1. $z=\sqrt{3}\,r \quad x=r\cos\theta \quad y=r\sin\theta.$

$r_r=\langle \cos\theta, \sin\theta, \sqrt{3}\rangle \quad r_\theta=\langle -r\sin\theta, r\cos\theta, 0\rangle$

$r_r \times r_\theta$:

$$\begin{vmatrix} i & j & k \\ \cos\theta & \sin\theta & \sqrt{3} \\ -r\sin\theta & r\cos\theta & 0 \end{vmatrix}$$

$-\sqrt{3}\,r\cos\theta\,\hat{i}-\sqrt{3}\,r\sin\theta\,\hat{j}+r\hat{k}.$

$\underline{3r^2+r^2}$

$|r_r \times r_\theta| = 2r.$

$$A=\int_0^{2\pi}\int_{\frac{3}{\sqrt{3}}}^{\frac{6}{\sqrt{3}}} 2r\,dr\,d\theta = 18\pi$$

(b)

$$\iint_S F\cdot d\vec{S} = \iint_S \underset{F}{F}\cdot \underset{\hat{n}}{\hat{n}}\,dS = \iint_D -\sqrt{3}\,r^2-3\sqrt{3}\,r^4\,dA = \int_0^{2\pi}\int_{\sqrt{3}}^{\frac{6}{\sqrt{3}}} -\sqrt{3}\,r^2-3\sqrt{3}\,r^4\,dr\,d\theta = \frac{-5232}{5}\pi$$

$\langle -r\cos\theta, -r\sin\theta, 3\sqrt{3}\,r^3\rangle\cdot\langle \sqrt{3}\,r\cos\theta, \sqrt{3}\,r\sin\theta, -r\rangle$

$-\sqrt{3}\,r^2\cos^2\theta-\sqrt{3}\,r^2\sin^2\theta-3\sqrt{3}\,r^4 = -\sqrt{3}\,r^2(\cos^2\theta+\sin^2\theta)-3\sqrt{3}\,r^4$

2.

S' as the disc $x^2+z^2=9$

lying in the plane $y=1$

By stoke's thm

$$\iint_S \nabla\times F\cdot dS = \iint_{S'} \nabla\times F\,dS$$

$$\nabla\times F = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2xyz+5z & e^x\cos(yz) & x^2y \end{vmatrix} = \langle x^2-e^x y\sin(yz), 5, e^x\cos(yz)-2xz\rangle$$

$$\iint_{S'} \langle x^2-e^x y\sin(yz), 5, e^x\cos(yz)-2xz\rangle\cdot\langle 0,1,0\rangle\,dA$$

$$=\iint_{S'} 5\,dA = 5\times\pi\times 3^2 = 45\pi$$