1. $x=t \quad y=-2t^2+2$

$r(t)=\langle t, -2t^2+2\rangle$

$r'(t)=\langle 1, -4t\rangle$

$\int_{C_1} F\cdot d\vec{r} = \int_0^1 \langle t+2t^2-2, -2t^3+2t\rangle \cdot \langle 1,-4t\rangle \, dt$

$= -1.9$

$t+2t^2-2+8t^4-8t^2$

$8t^4-6t^2+t-2$

$\frac{8}{5}t^5-\frac{6}{3}t^3+\frac{1}{2}t^2-2t$

$\frac{8}{5}-2+\frac{1}{2}-2$

$x=t \; y=0$

$\int_{C_2} F\cdot d\vec{r} = \int_1^2 \langle t, 0\rangle\cdot\langle 1,0\rangle \, dt = \frac{3}{2}$

$\frac{1}{2}t^2\Big|_1^2$

$\frac{1}{2}\times 4-\frac{1}{2}$

$\int_C F\cdot d\vec{r} = \int_{C_1} F\cdot d\vec{r} + \int_{C_2} F\cdot d\vec{r} = -1.9+1.5 = -0.4$

2. $\frac{\partial Q}{\partial x} = e^{xy} + yxe^{xy} - 4\sin(4x+y)$

$\frac{\partial P}{\partial y} = e^{xy} + yxe^{xy} - 4\sin(4x+y)$

$\frac{\partial Q}{\partial x} = \frac{\partial P}{\partial y}$ conservative field, independent of path

$F=\nabla f \quad f=\int ye^{xy}+4\cos(4x+y)\,dx + h(y)$

$= e^{xy}+\sin(4x+y)+h(y)$

$\frac{\partial f}{\partial y} = xe^{xy}+\cos(4x+y)+h'(y) \quad h'(y)=0$

$f(\sqrt{2},\sqrt{2})-f(0,0)=e^2+\sin(4\sqrt{2}+\sqrt{2})$

$= e^2+\sin(5\sqrt{2})-1$

3. $y = r\sin t$

$x = r\cos t$

$$\oint_C \vec{F}\cdot d\vec{r} = \oint_C \tan^{-1}\left(\frac{y}{x}\right)dx + \ln(x^2+y^2)\,dy$$

$$= \iint_D \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\, dA$$

$$= \iint \frac{2x}{x^2+y^2} - \frac{x}{x^2+y^2}\, dx\, dy$$

$$= \int_0^{\frac{\pi}{2}}\int_1^2 \frac{\cancel{r}\cos\theta}{r^{\cancel{2}}}\cdot \cancel{r}\; dr\, d\theta$$

$$= 1$$

4. $\because y^2+z^2=16 \quad r=4.$

using x and θ; according to cylindrical coordinate

$$\vec{r}(x,\theta) = \langle x, 4\cos\theta, 4\sin\theta \rangle$$

on domain $0 \le x \le 5 \ \wedge \ 0 \le \theta \le 2\pi$