Hybrid Optimization for Time-Dependent Sequencing

Willem-Jan van Hoeve

Tepper School of Business
Carnegie Mellon University
www.andrew.cmu.edu/user/vanhoeve/

Joris Kinable

Robotics Institute & Tepper School of Business
Carnegie Mellon University

Andre A. Cire

Rotman School of Management & U Toronto Scarborough
University of Toronto

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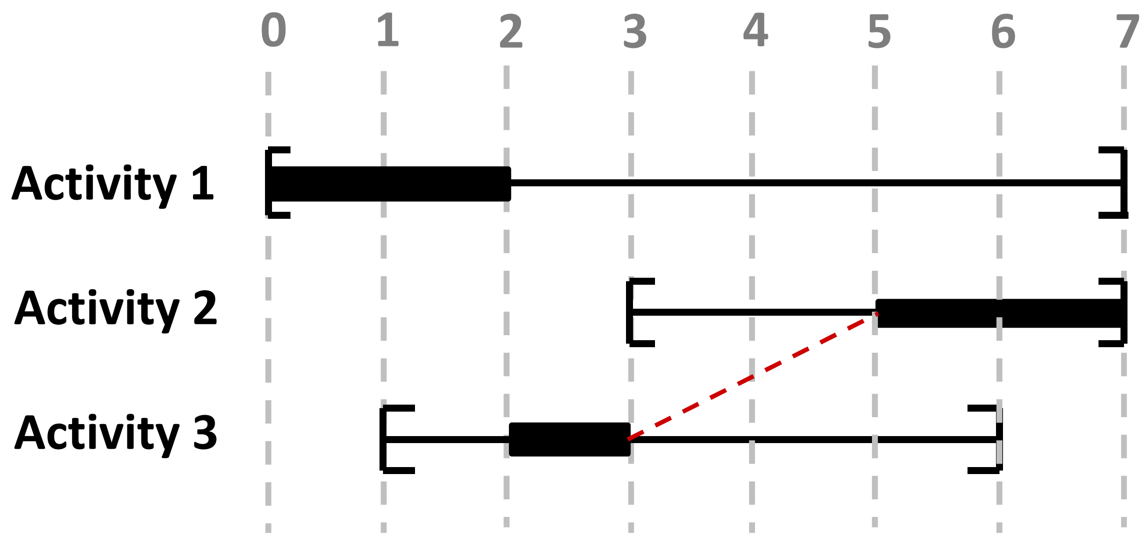
Motivation

- Time-dependent sequencing
  - machine scheduling, routing
- Challenging problem
  - best results with dedicated methods
  - not easy to extend with side constraints
- Utilize constraint programming framework?
  - strengthened constraint propagation with MDDs
  - improved bounds via additive bounding with LP
  - evaluate on TD-TSP and TD-SOP

Time-Dependent Sequencing

- Activities
  - processing time p_i
  - released date r_i
  - deadline d_i



- Resource
  - non-preemptive
  - process one activity at a time
  - sequence-dependent setup times: also depend on position!

  $\delta_{i,j}^{t}$ = setup time between i and j if i is at position t

Extensions

- Precedence relations between activities
- Various objective functions
  - Sum of setup times
  - Makespan
  - (Weighted) sum of completion times
  - (Weighted) tardiness
  - number of late jobs
  - …

Existing Approaches

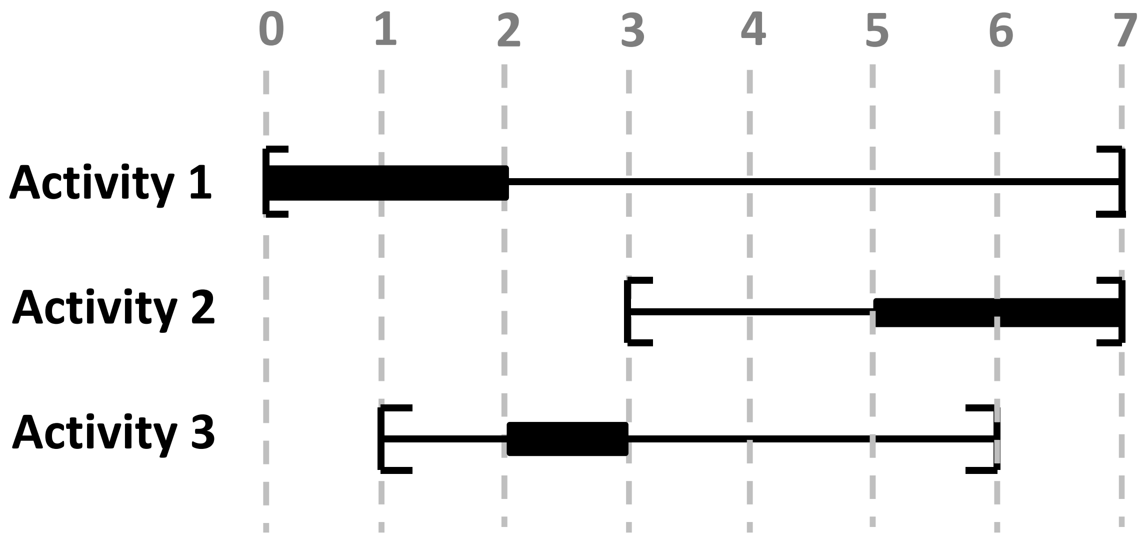
- MIP-based
  - Picard and Queyranne (1978)
  - Gouveia and Voss (1995)
  - Abeledo et al. (2013)
  - …

- Many more approaches to *time*-dependent TSP
  - Ichoua et al. (2003)
  - Cordeau et al. (2014)
  - Melgarejo et al. (2015)
  - …

Constraint Programming Model

- Every solution can be written as a permutation $\boldsymbol{\pi}$

$\pi_1, \pi_2, \ldots, \pi_n$: activity sequencing



$\pi_1 = 1, \pi_2 = 3, \pi_3 = 2$

Constraint Programming Model

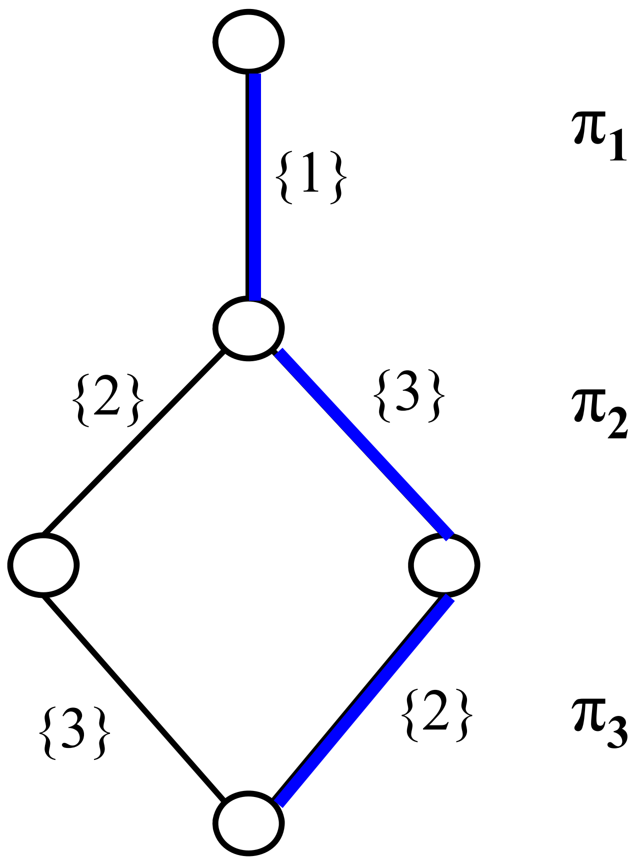
- Variables π_i : label of i^th activity in the sequence

  L_i : position of activity i in the sequence

$$
\begin{aligned}
\min \quad & \sum_{i=0}^{n} \delta^{i}_{\pi_i,\pi_{i+1}} & \\
\text{s.t.} \quad & \text{AllDiff}(\pi_1,\ldots,\pi_n) & \\
& L_{\pi_i} = i & \forall i = 1,\ldots,n \\
& L_i < L_j & \forall (i \ll j) \in P \\
& L_i \in \{1,\ldots,n\} & \forall i = 1,\ldots,n \\
& \pi_i \in \{1,\ldots,n\} & \forall i = 1,\ldots,n
\end{aligned}
$$

- Weak model: objective and AllDiff are decoupled

MDD Representation



Act	r_i	p_i	d_i
1	0	2	3
2	4	2	9
3	3	3	8

Path {1} – {3} – {2} :

$0 \leq start_1 \leq 1$

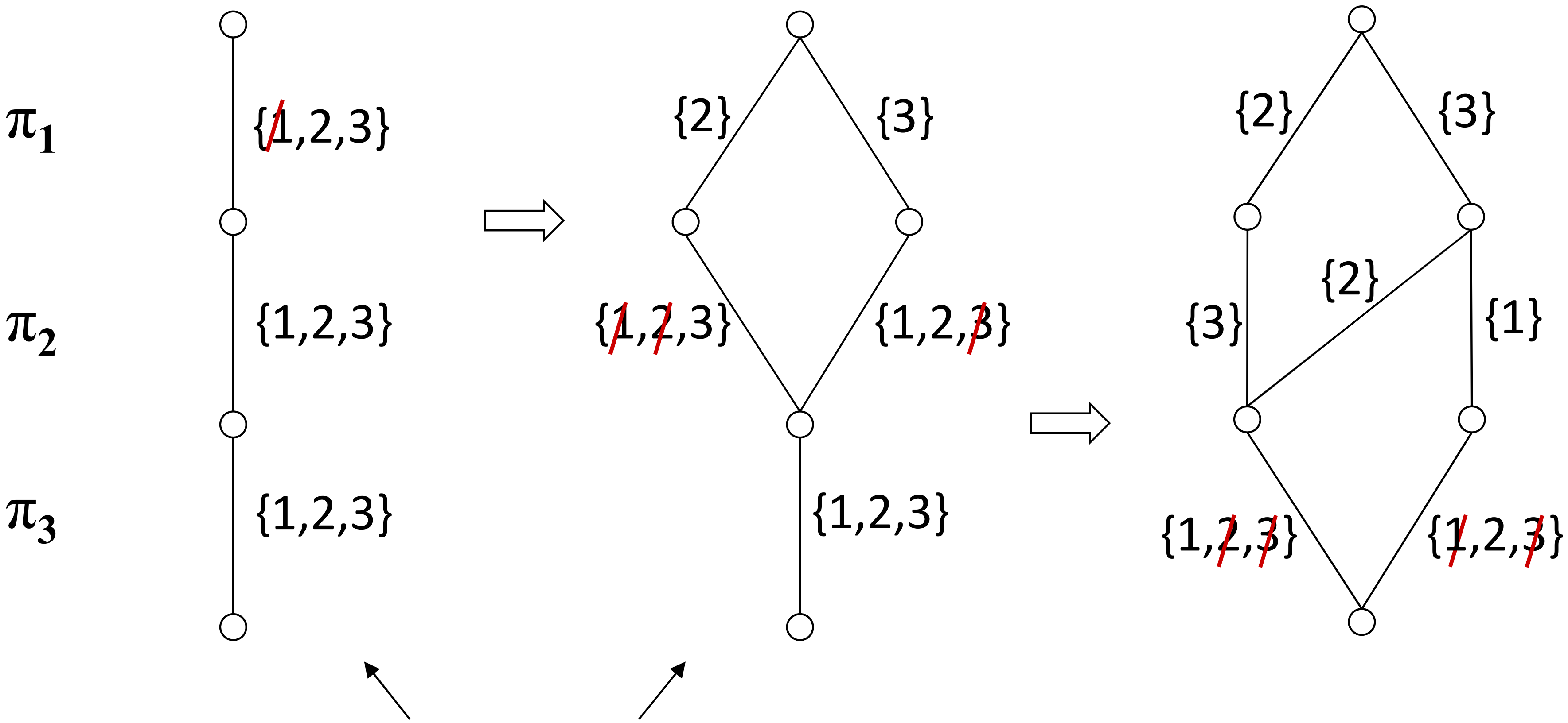
$6 \leq start_2 \leq 7$

$3 \leq start_3 \leq 5$

Cire and v.H. (2013)

Top-down MDD compilation

precedence: 3 << 1



relaxed MDDs
(strength is controlled by maximum *width*)

exact MDD

MDD-based propagation

Propagation: remove infeasible arcs from the MDD

We can utilize several structures/constraints:

- *Alldifferent* for the permutation structure
- Precedence relations
- Earliest start time and latest end time

Propagating MDDs rather than variable domains can yield orders of magnitude speedup

Andersen et al. (2007), Hoda et al. (2010), Cire&v.H. (2013), Bergman et al. (2015)

MDD-based propagation

Propagation: remove infeasible arcs from the MDD

We can utilize several structures/constraints:

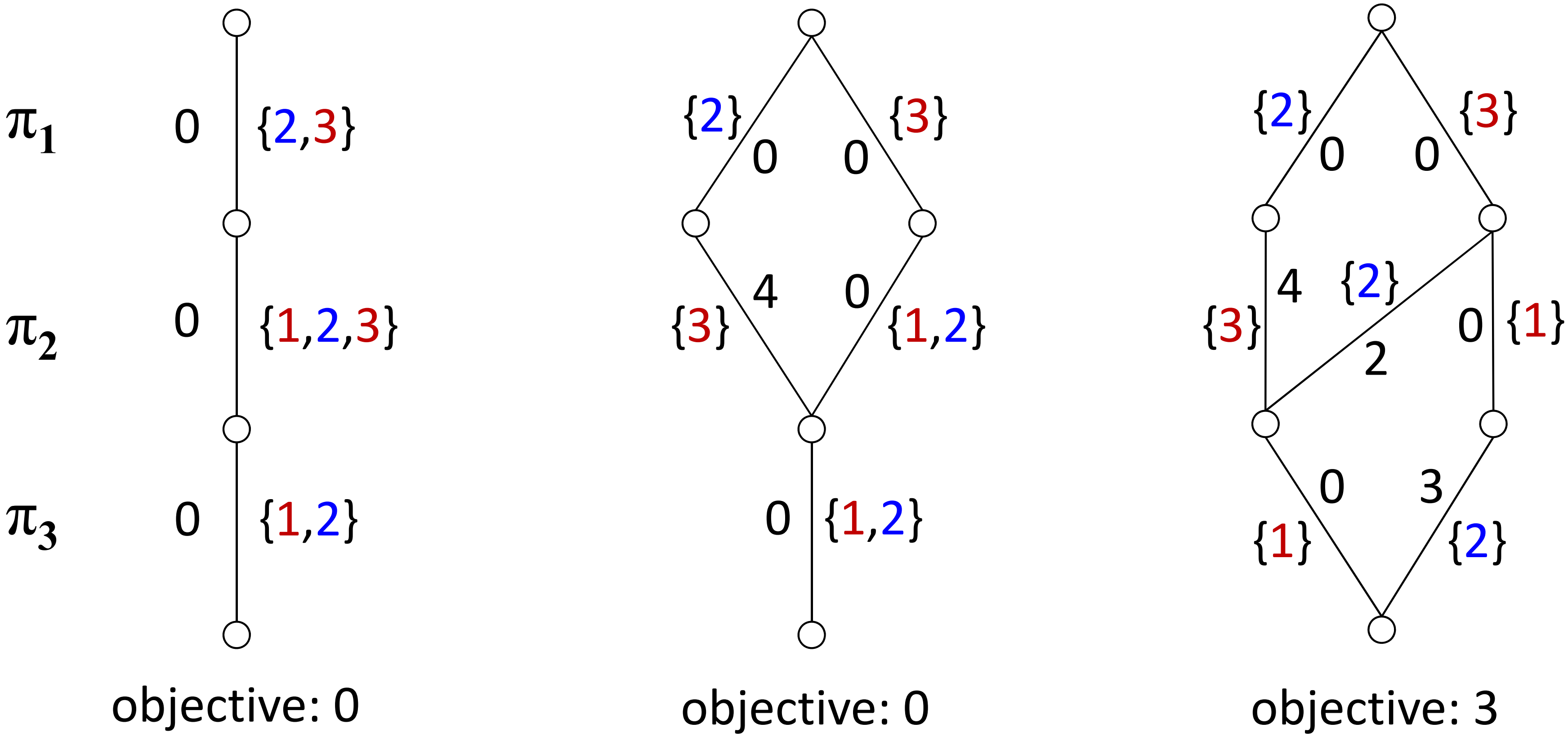
- *Alldifferent* for the permutation structure
- Precedence relations
- Earliest start time and latest end time
  - adapt rule: $\delta_{i,j}$ becomes $\delta_{i,j}^{t}$
- Also needed for objective
  - minimize sum of setup times

Updated CP Model

$$
\begin{aligned}
\min \quad & z \\
\text{s.t.} \quad & \text{AllDiff}(\pi_1, \ldots, \pi_n) \\
& \text{MDDconstr}(\pi_1, \ldots, \pi_n, W, z, \delta^t, P) \\
& L_{\pi_i} = i && \forall i = 1, \ldots, n \\
& L_i < L_j && \forall (i \ll j) \in P \\
& L_i \in \{1, \ldots, n\} && \forall i = 1, \ldots, n \\
& \pi_i \in \{1, \ldots, n\} && \forall i = 1, \ldots, n \\
& z \in \{0, \ldots, \infty\}
\end{aligned}
$$

Stronger model: objective handled within MDD constraint

Bounds from relaxed MDDs

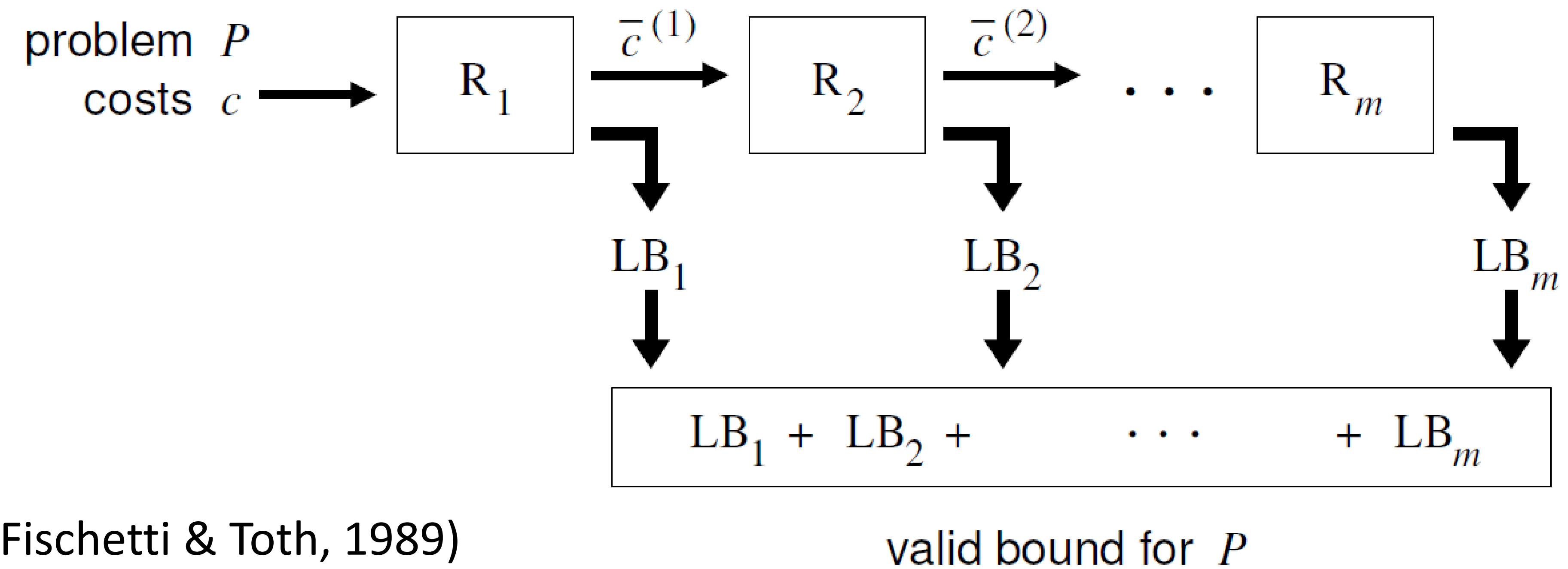


act 1: red
act 2: blue
act 3: red

$\delta^1_{\text{red,blue}} = 2$ $\delta^1_{\text{blue,red}} = 4$

$\delta^2_{\text{red,blue}} = 3$ $\delta^2_{\text{blue,red}} = 6$

Additive Bounding

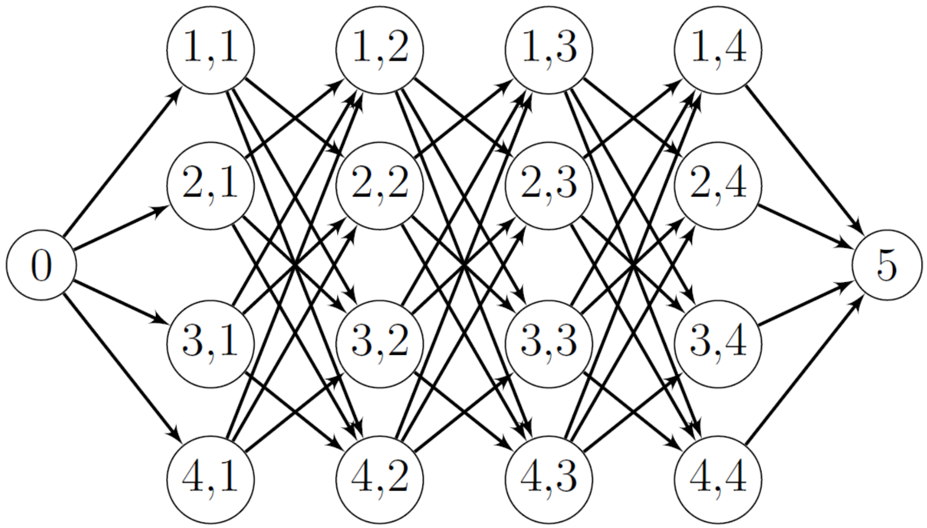


(Fischetti & Toth, 1989)

Add LP reduced costs to MDD relaxation

- Continuous LP relaxation ‘discretized’ through MDD
- Stronger bounds
- Improved cost-based filtering

MIP and LP relaxation



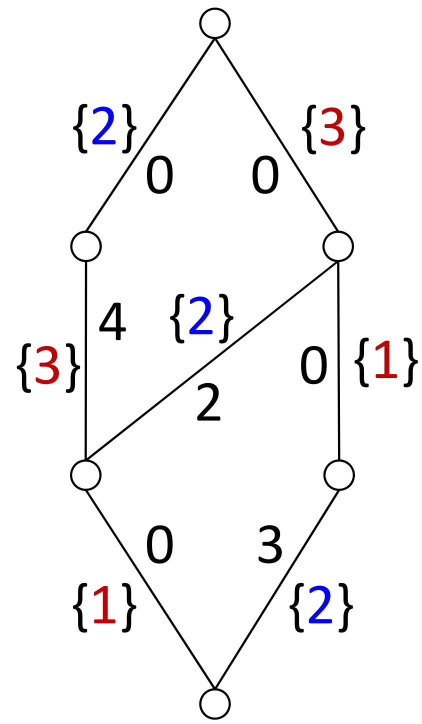
- Time-space network model (Picard & Queyranne, 1978)
- Variables

$$x_{i,j}^{t} = \begin{cases} 1 & \text{if i is performed at t and followed by j} \\ 0 & \text{otherwise} \end{cases}$$

- Constraints: flow conservation; perform each activity

Embedding reduced costs in MDD

- Approach
  - solve LP relaxation
  - in MDD, replace $\delta^t_{i,j}$ with $\overline{c}^t_{i,j}$
- Since MDD is relaxation, shortest path is valid bound



Experiments

- Time-dependent TSP benchmark
  - 38 instances from TSPLIB (14-107 jobs)
  - $\delta_{i,j}^{t}$ = (n-t)*$\delta_{i,j}$
- Time limit: 30 minutes
- IBM ILOG CPLEX and CP Optimizer 12.4
- MDD added to CP Optimizer (Cire & v.H., 2013)
  - maximum width 512

Impact of additive bounding

- Compare root node bound improvement

	LP	MDD	MDD+LP	percentage improvement w.r.t. LP	percentage improvement w.r.t. MDD
berlin52	112,350.0	49,056.0	119,694.0	6.54%	143.99%
dTSP50.0	10,465.7	5,537.0	10,646.0	1.72%	92.27%
kroA100	693,870.0	223,039.0	719,992.0	3.76%	222.81%
pr76	2,496,050.0	2,116,910.0	2,679,143.0	7.34%	26.56%

Average improvement (38 instances): 5.96% 103.34%

Overall performance

	#Solved	Avg end gap*
(Picard & Queyrrane, 1978) (MIP)	6/38	33.37%
(Gouveia & Voss, 1995) (MIP)	6/38	31.64%
(Abeledo et al., 2013) (BPC**)	35/38	1.64%
Pure CP	0/38	45.02%
CP + MDD	7/38	7.42%
CP + MDD + Additive Bounding	12/38	6.49%

* Average end gap w.r.t. overall best bound

** Dedicated method; much longer time limit

Sequential Ordering Problem

	#Solved	Avg end gap
(Picard & Queyrrane, 1978) (MIP)	6/30	29.85%
(Gouveia & Voss, 1995) (MIP)	6/30	29.17%
Pure CP	5/30	25.68%
CP + MDD + Additive Bounding	10/30	21.22%

On average, additive MDD+LP bound improves

- LP root node bound by 51.41%
- MDD root node bound by 9.54%

Summary

- Hybrid optimization method for time-dependent sequencing
  - CP framework
  - MDD relaxation for improved propagation
  - Additive bounding with LP for stronger bounds
  - Side constraints are easily added
- Experiments
  - Competitive generic approach