MDD Propagation for Disjunctive Scheduling

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ISMP 2012

Motivation

- Constraint-based scheduling: *Exploit subproblem structure*
  - High-level, structured constraints (*disjunctive, cumulative…)*
  - Sophisticated inference techniques
  - Process constraints one at a time

- … but how to pool the results of constraint processing?
  - Constraint store - Shared data structure that accumulates *implications* of each constraint

Motivation

- In practice: constraint store is the domain store
  - Implications are of the form

$$x_i \leq v, \ x_i \geq v, \ \text{or} \ x_i \neq v \ \text{ for } \ v \in domain(x_i)$$

  - *Propagation*: reduce domains as much as possible

- Domain store is a natural relaxation, but may be too weak

Motivation

alldiff(x_1, x_2, x_3),

$x_1 + x_2 + x_3 \leq 12$,

$x_1, x_2, x_3 \in \{1,9,10\}$.

Problem is infeasible

1. Propagation of *alldiff*
   - No inference.
2. Propagation of *sum*
   - No inference.

Domains remain unchanged!

- Common solution: new global constraint
  - *cost-alldiff, cost-sum-weighted-alldiff, etc* …

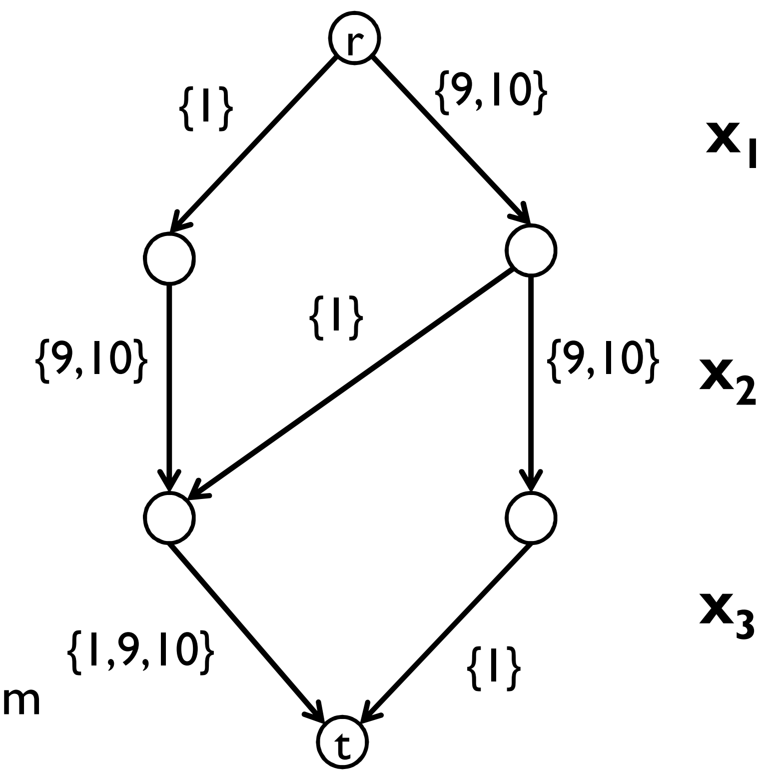
Motivation

- Other alternative: a richer constraint store
- Proposal: *Relaxed Multivalued Decision Diagrams* (MDDs)
  - Initial framework by Andersen et al (CP2007).
- Fundamental questions
  - How to effectively process MDDs for particular constraints?
  - When does it perform better than domain store?
  - …
- Our goal: application to constraint-based scheduling

Relaxed MDDs

alldiff(x_1, x_2, x_3),
$x_1 + x_2 + x_3 \leq 12$,
$x_1, x_2, x_3 \in \{1,9,10\}$.

- Compact representation of a search tree
- Ordered Acyclic Digraph
  - *Layers*: variables
  - *Arc labels*: variable assignments
- Paths from **r** to **t**: solutions to the problem



Relaxed MDDs

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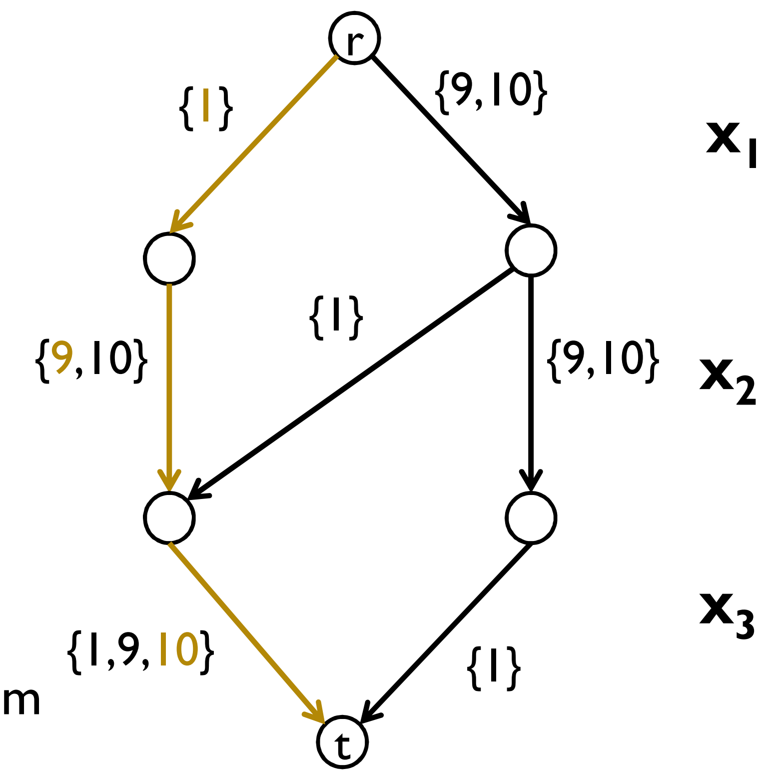
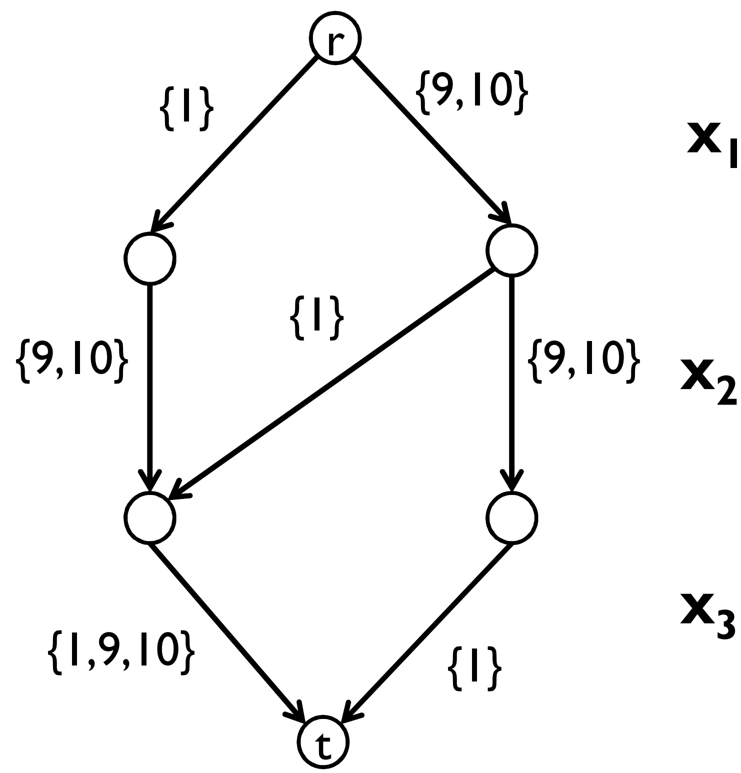
- Compact representation of a search tree
- Ordered Acyclic Digraph
  - *Layers*: variables
  - *Arc labels*: variable assignments
- Paths from **r** to **t**: solutions to the problem
  - Example: $\mathbf{x_1}=1$, $\mathbf{x_2}=9$, $\mathbf{x_3}=10$



Relaxed MDDs

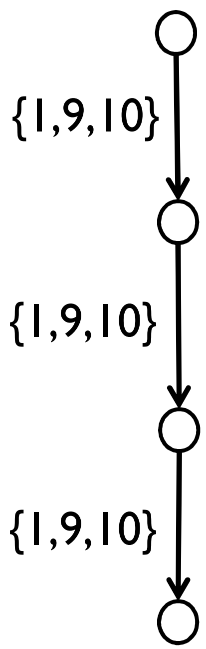
alldiff(x_1, x_2, x_3),
$x_1 + x_2 + x_3 \leq 12$,
$x_1, x_2, x_3 \in \{1,9,10\}$.

- *Relaxed*
  - It encodes *all* feasible solutions
  - It may encode *infeasible* solutions

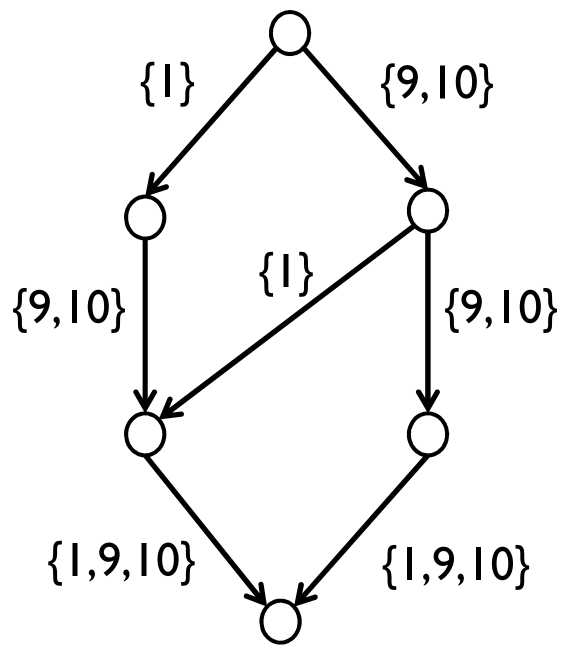


Relaxed MDDs

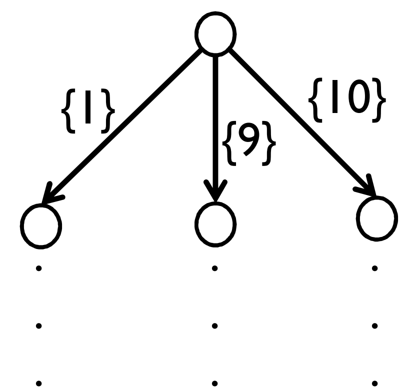
- Relaxation is adjustable
  - Controlled by the **width** of the graph



Width 1 = Domain Store



Width 2



Unlimited width: original problem

Relaxed MDDs

- Constraint processing
  - Refine the MDD representation by removing / adding arcs

alldiff(x_1, x_2, x_3),
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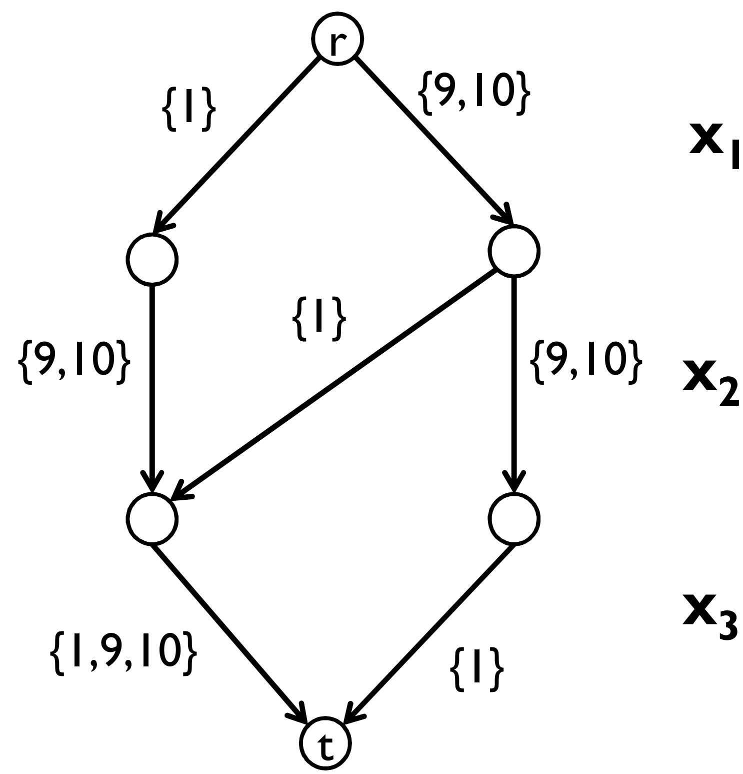
1. Propagation of *alldiff*



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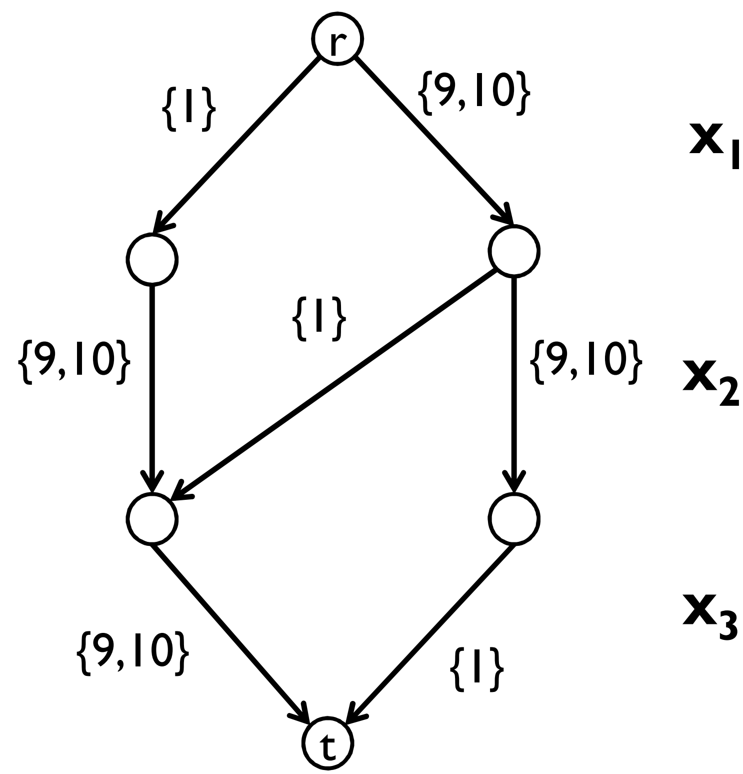

Relaxed MDDs

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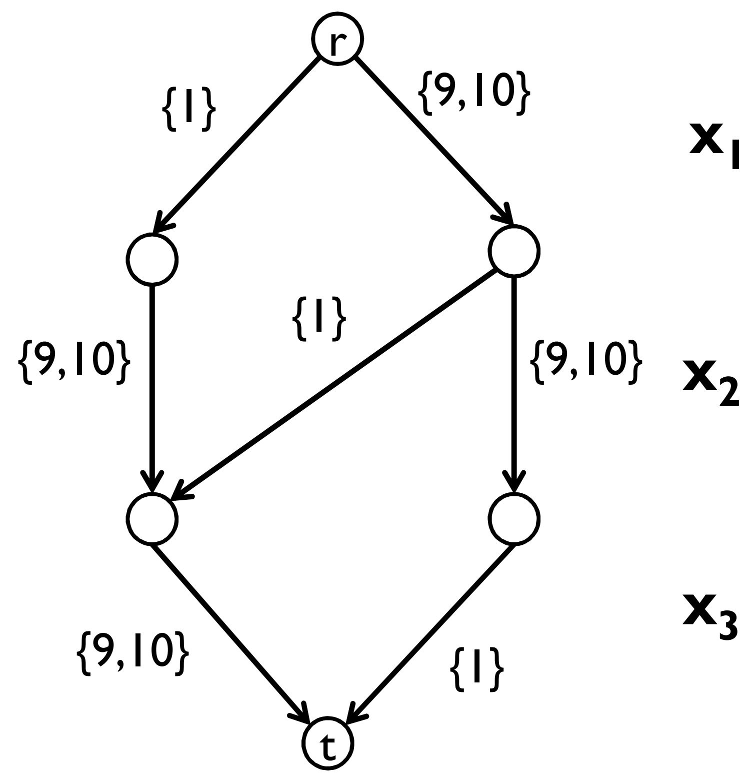
1. Propagation of *alldiff*

2. Propagation of *sum*
   - Detects infeasibility!



Relaxed MDDs and Scheduling

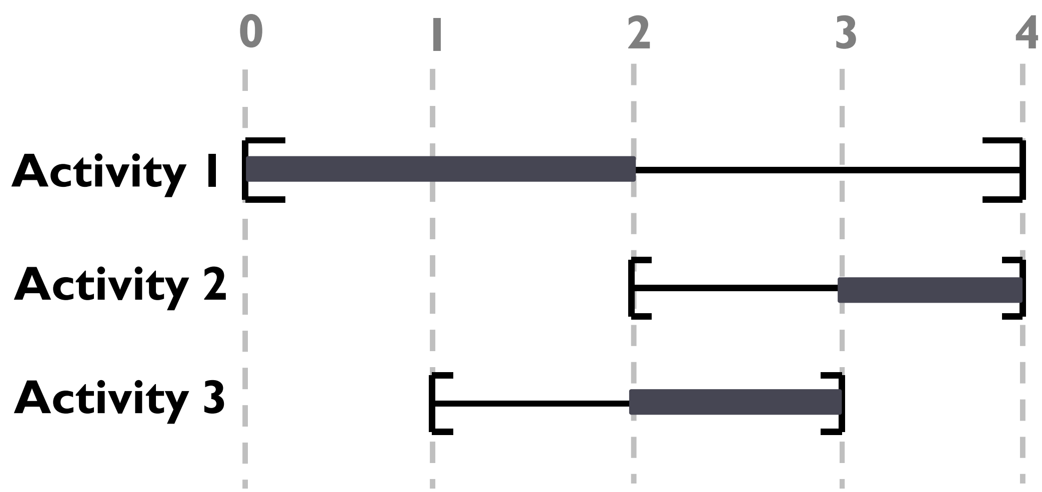
- Focus: disjunctive scheduling
  - Highlight of CP, widespread application
  - Still has particular deficiencies

- MDD constraint processing for disjunctive scheduling

Disjunctive Scheduling

- Sequencing and scheduling of activities on a resource

- *Activities*
  - Processing time: p_i
  - Release time: r_i
  - Deadline: d_i



- *Resource*
  - Nonpreemptive
  - Process one activity at a time

Common Side Constraints

- Precedence relations between activities
- Sequence-dependent setup times
- Induced by objective function
  - Makespan
  - Sum of setup times
  - Sum of completion times
  - Tardiness / number of late jobs
  - …

Inference

- Inference for disjunctive scheduling
  - Precedence relations
  - Time intervals that an activity can be processed

- Sophisticated techniques include:
  - *Edge-Finding*
  - *Not-first / not-last rules*

- Challenges arise in presence of
  - Sequence-dependent setup times
  - Complex objective functions

MDDs for Disjunctive Scheduling

Our two three main considerations:

- Representation
  - How to represent solutions of disjunctive scheduling in an MDD?
- Construction
  - How to construct this relaxed MDD?
- Inference techniques
  - What can we infer using the relaxed MDD?

MDD Representation

- Natural representation as MDDs

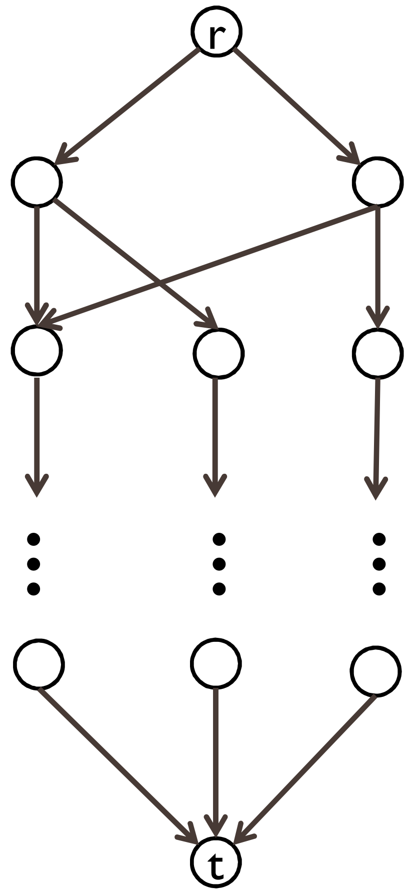
- Every solution can be written as a permutation $\boldsymbol{\pi}$

  $\pi_1, \pi_2, \pi_3, \ldots, \pi_n$: activity sequencing in the resource

- Schedule is *implied* by a sequence, e.g.:

$$start_{\pi_i} \geq start_{\pi_{i-1}} + p_{\pi_{i-1}} \qquad i = 2, \ldots, n$$

MDD Representation

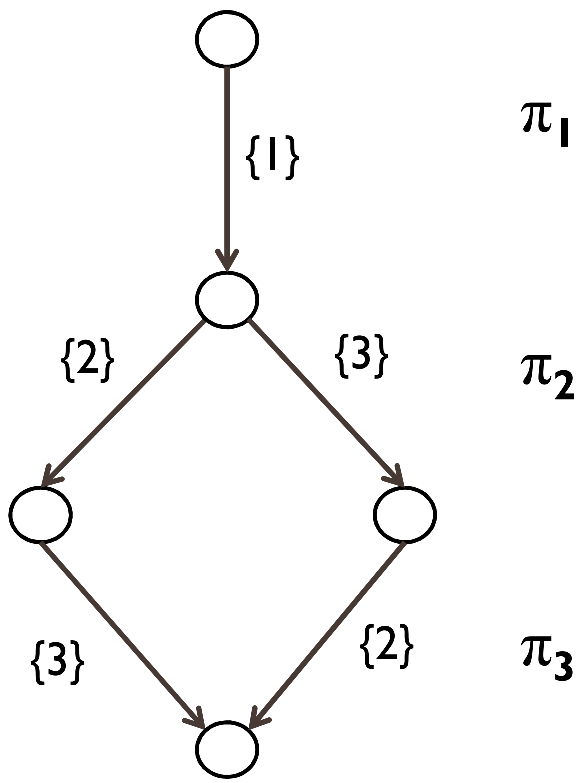


π_1 : first activity

π_2 : second activity

π_n : n-th activity

MDD Representation



Act	r_i	d_i	p_i
1	0	**3**	2
2	**4**	9	2
3	**3**	8	3

Path {1} – {3} – {2}

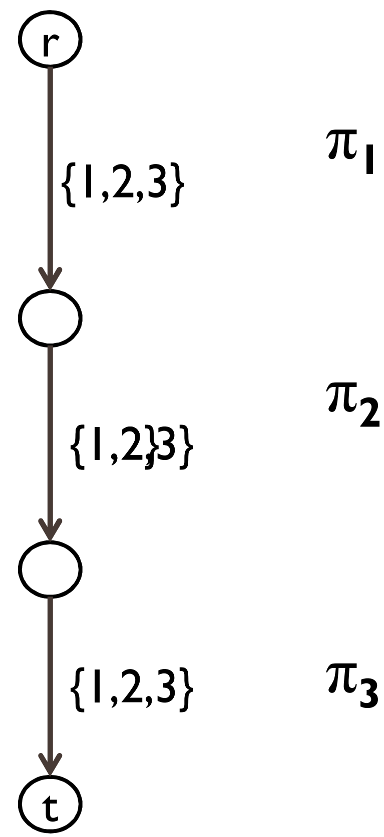
$0 \leq start_1 \leq 1$
$6 \leq start_2 \leq 7$
$3 \leq start_3 \leq 5$

MDD Construction

- In general, MDDs can grow exponentially
  - Polynomial-width for particular scheduling problems
- We fix a maximum width W
- Apply a variation of filter and refinement technique
  - Andersen et al. (CP2007), Hoda et al. (CP2010)

Filter and Refinement

- Start with a width-1 MDD
  - Straightforward MDD relaxation
- Filter infeasible arc values
  - Top-down/Bottom-up passes



Filter and Refinement

- Start with a width-1 MDD
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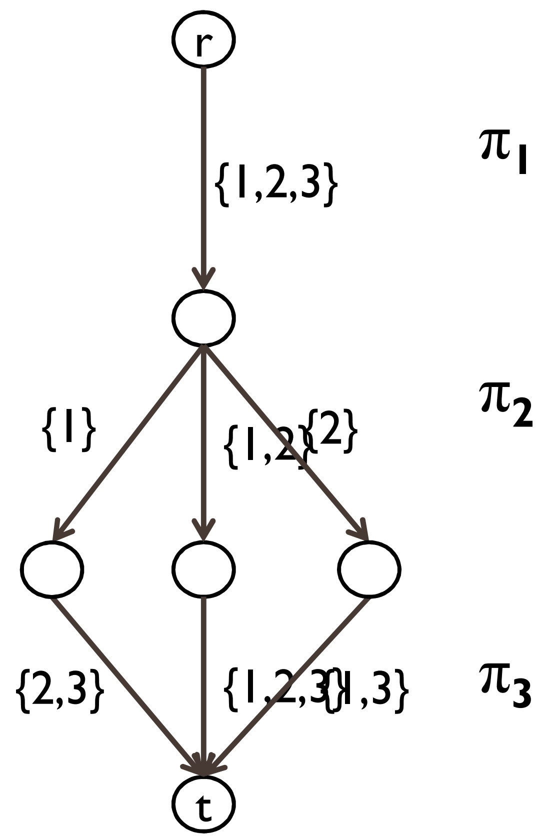
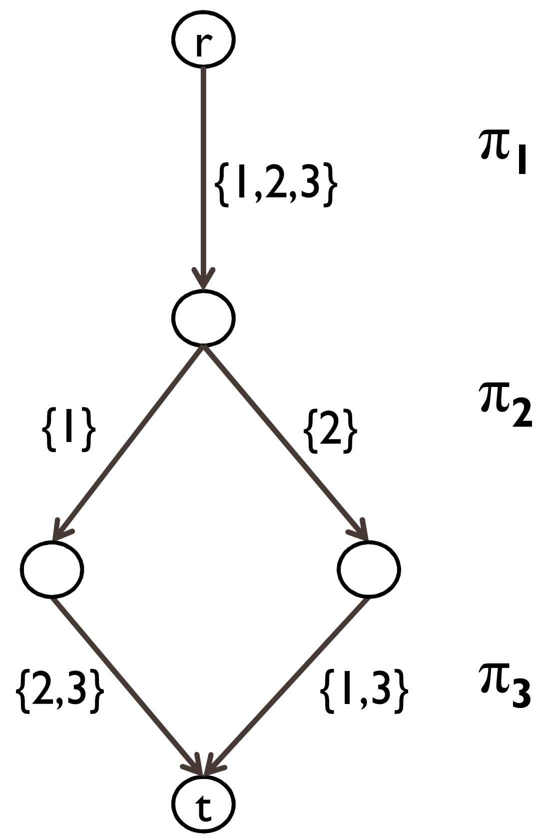
- Refinement
  - Add nodes to improve relaxation



Filter and Refinement

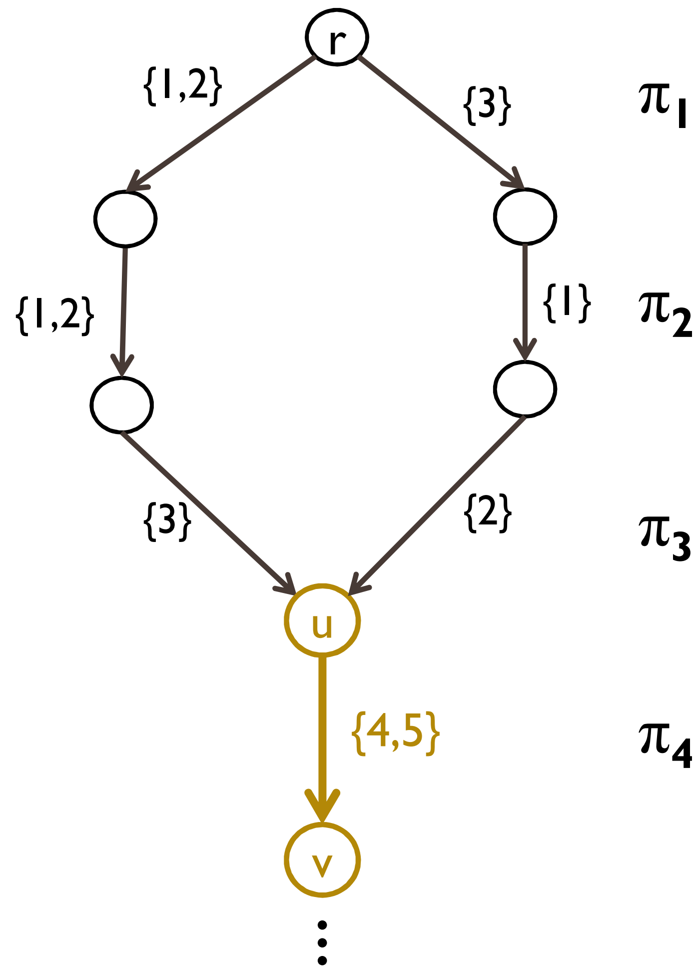
- Start with a width-1 MDD
  - Straightforward MDD relaxation
- Filter infeasible arc values
  - Top-down/Bottom-up passes
- Refinement
  - Add nodes to improve relaxation
- Repeat filtering/refinement until certain conditions are met



Filter: Top-Down Example

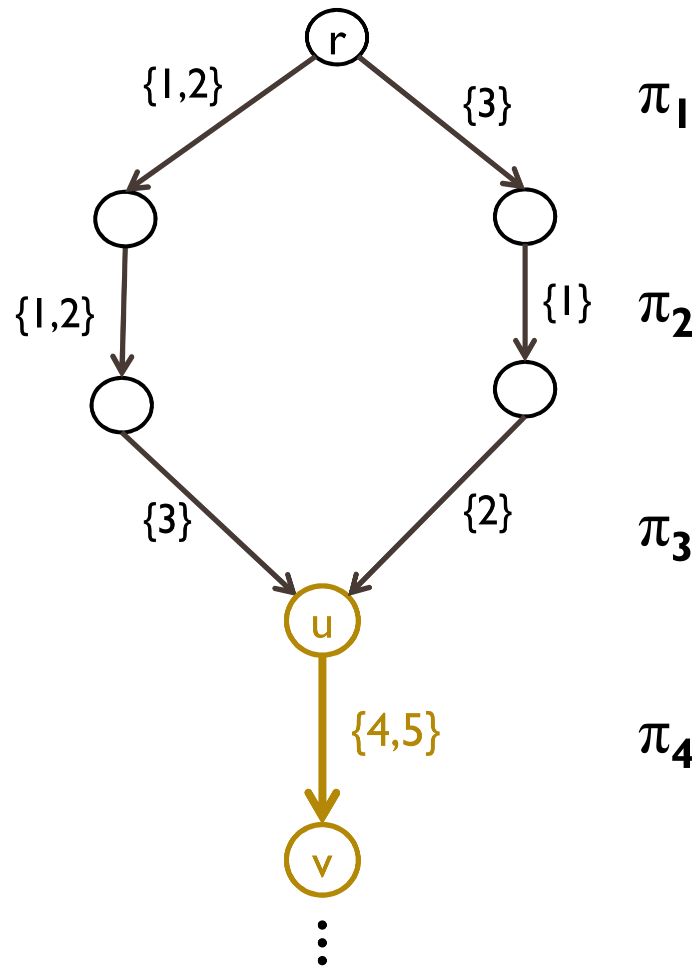
- Filter based on a *state* information at each node

- Example:
  Filtering arcs (u,v)



Filter: Top-Down Example

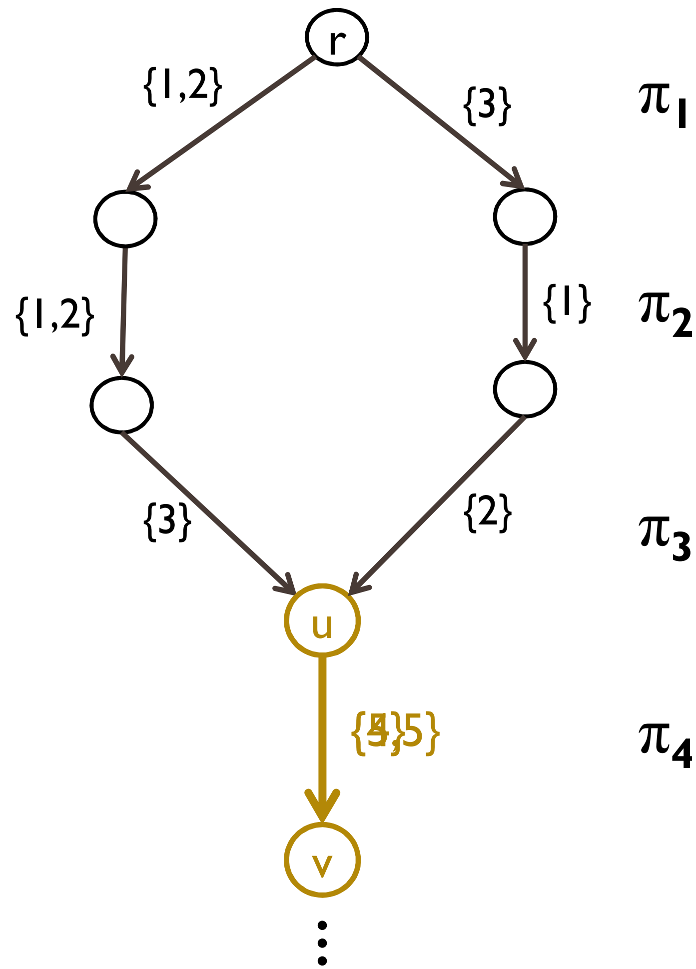
- *Earliest Completion Time: E_u*
  - *Minimum completion time of all partial sequences represented by paths from root to node u*
- *Similarly: Latest Completion Time*



Filter: Top-Down Example

Act	r_i	d_i	p_i
1	0	3	2
2	3	7	3
3	1	8	3
4	**5**	**6**	**1**
…	…	…	…

- $E_u = 7$
- *Eliminate 4 from (u,v)*



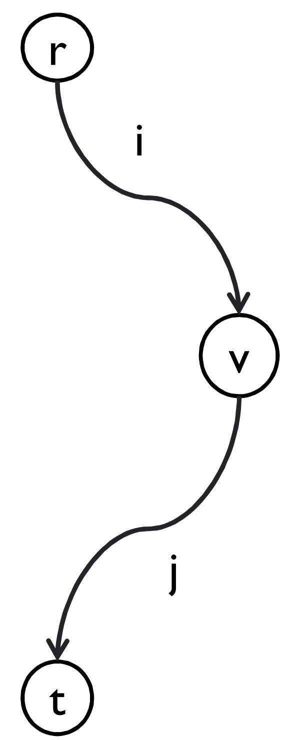
Filter and Refinement

- Other filters
  - *Node edge-finding, not-first/not-last rules*
  - *Precedence filtering*
  - Additional *alldiff* filters
  - …
- Refinement
  - Based on *earliest completion time* of a node

MDD Inference

Theorem: *Given the exact MDD M, we can deduce all implied activity precedences in polynomial time in the size of M*

- For a node *v*,
  - $A_v^{\downarrow}$: values in all paths from root to *v*
  - $A_v^{\uparrow}$: values in all paths from node v to terminal
- *Precedence relation* $i \ll j$ *holds if and only if* $(j \notin A_u^{\downarrow})$ or $(i \notin A_u^{\uparrow})$ *for all nodes u in M*
- Same technique applies to relaxed MDD



Communicate Precedence Relations

1. Provide precedences inferred from the MDD to solver
   - Update time variables
   - Other inference techniques may utilize them

2. We can filter the relaxed MDD using precedence relations inferred from other (CP) techniques

- Precedences deduced by this method might not be dominated by other techniques, even for small widths.

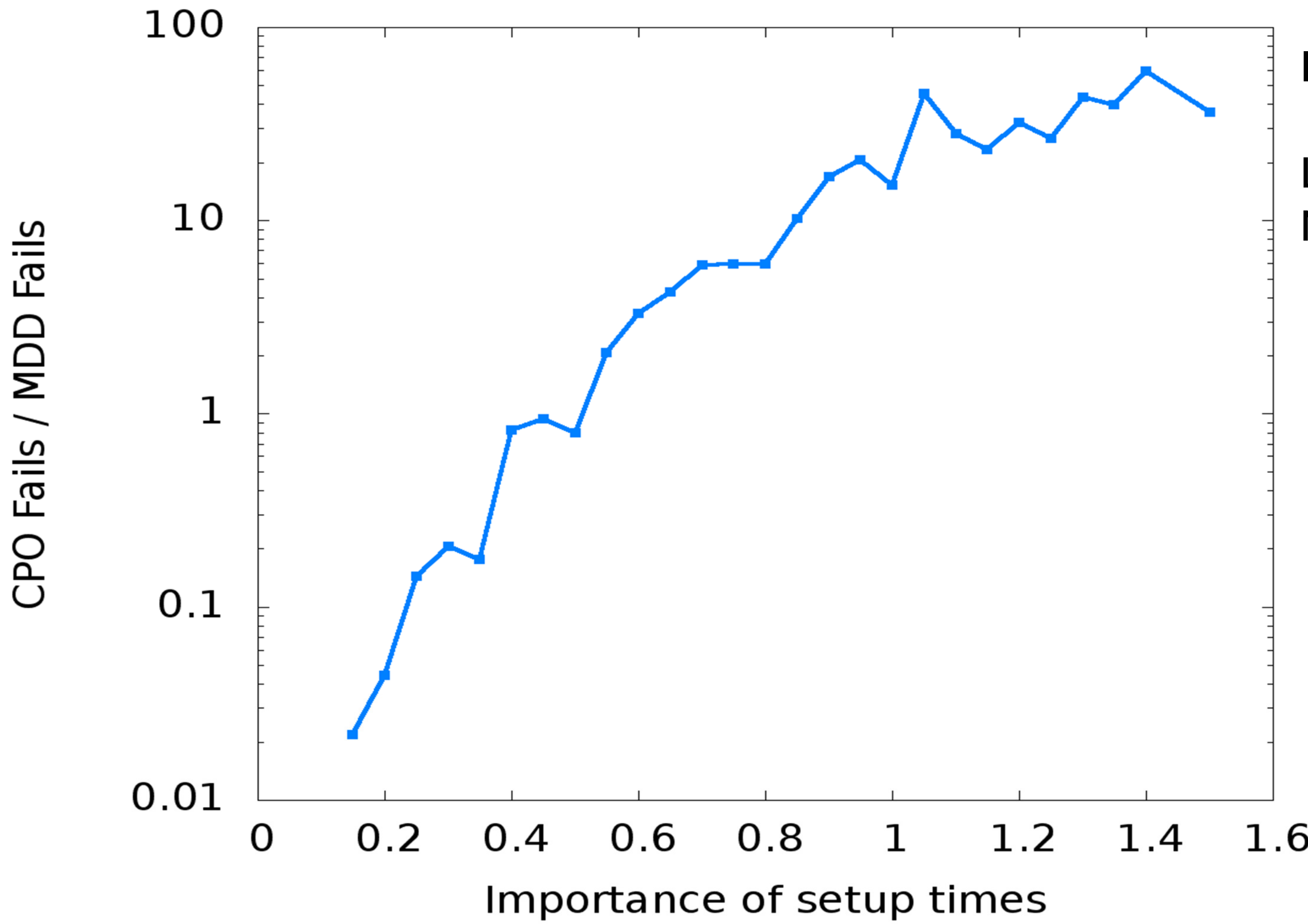
Experimental Results

- Implemented in *Ilog CP Optimizer (CPO)*
  - State-of-the-art constraint based scheduling solver
  - Uses a portfolio of inference techniques and LP relaxation

- Random and structured instances
  - Different classical objective functions

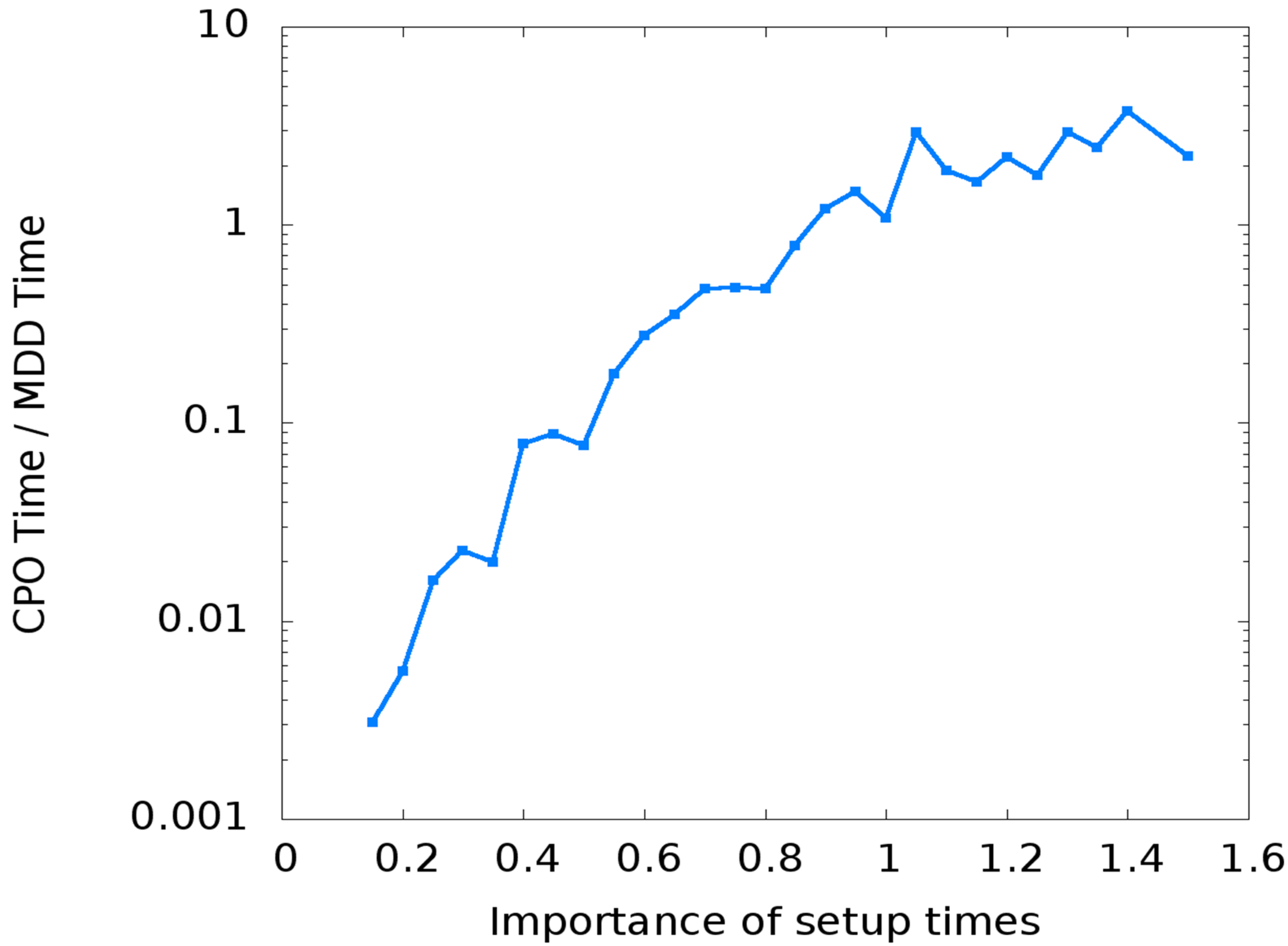
- Tested integration CPO+MDD

Makespan



Random instance
15 jobs
Lex search
MDD width: 16

Makespan



Random instance
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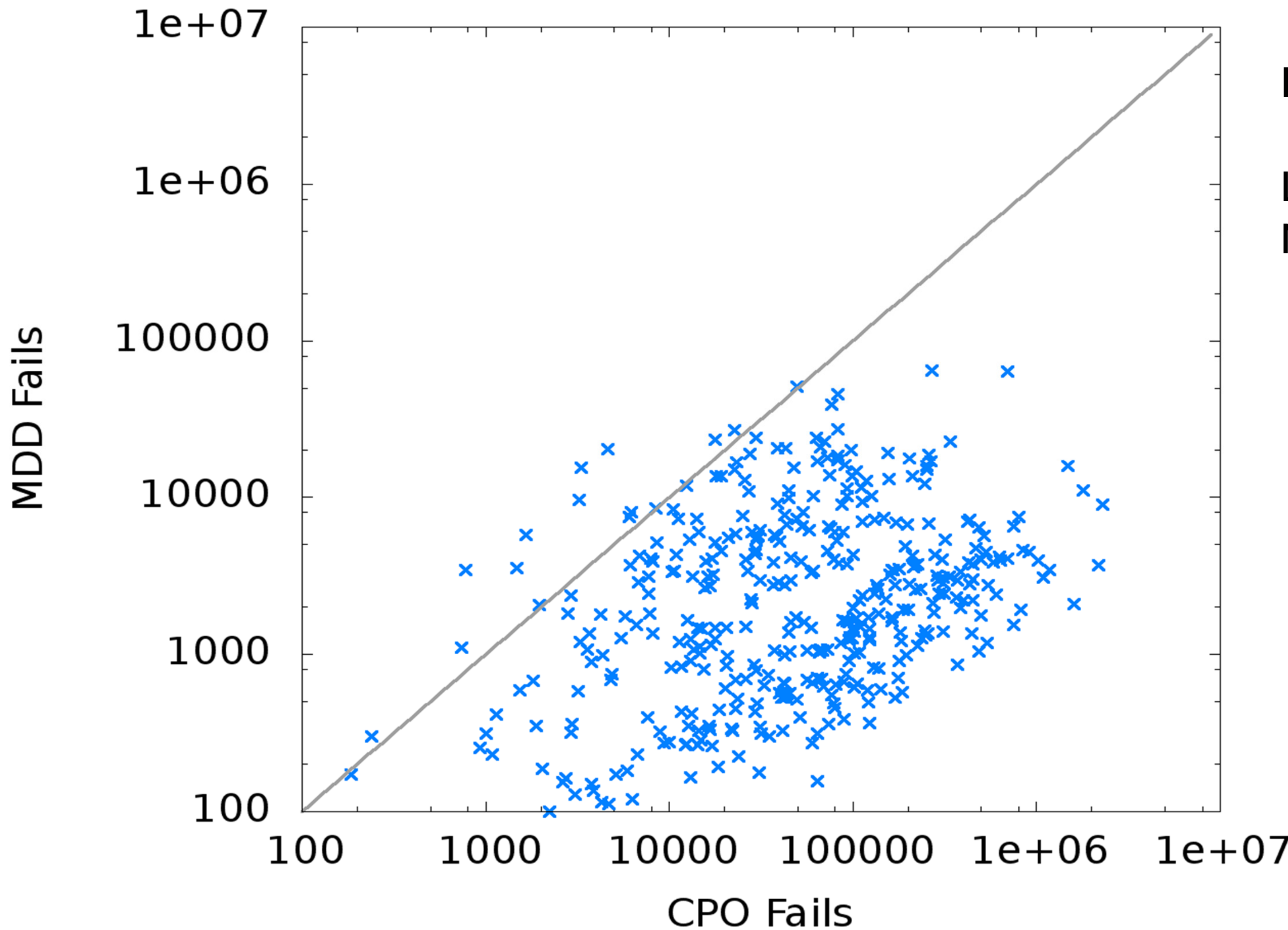
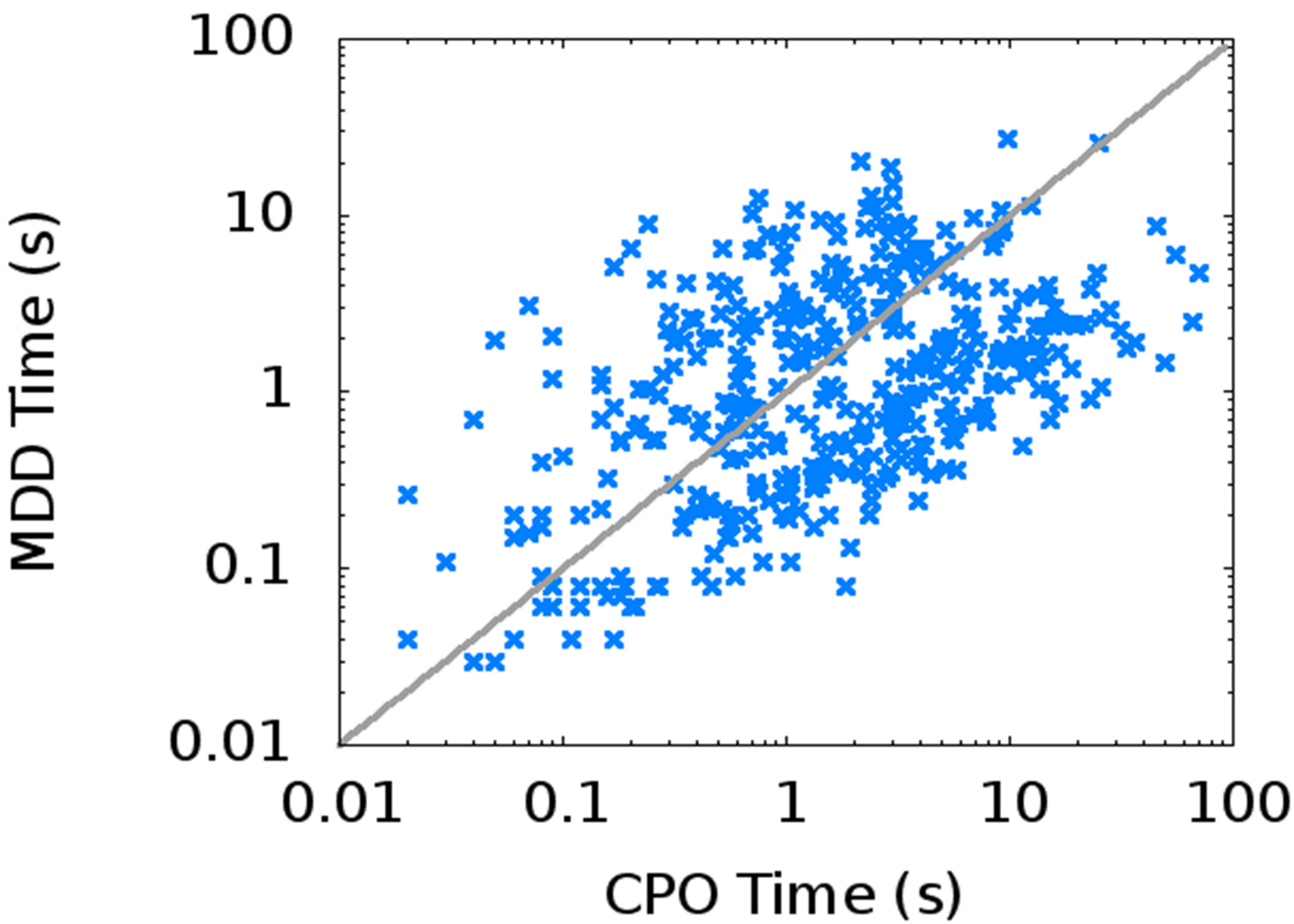
Sum of Completion Times



Random instances
12 jobs
Lex search
MDD width: 16

Sum of Completion Times



Random instances
12 jobs
Lex search
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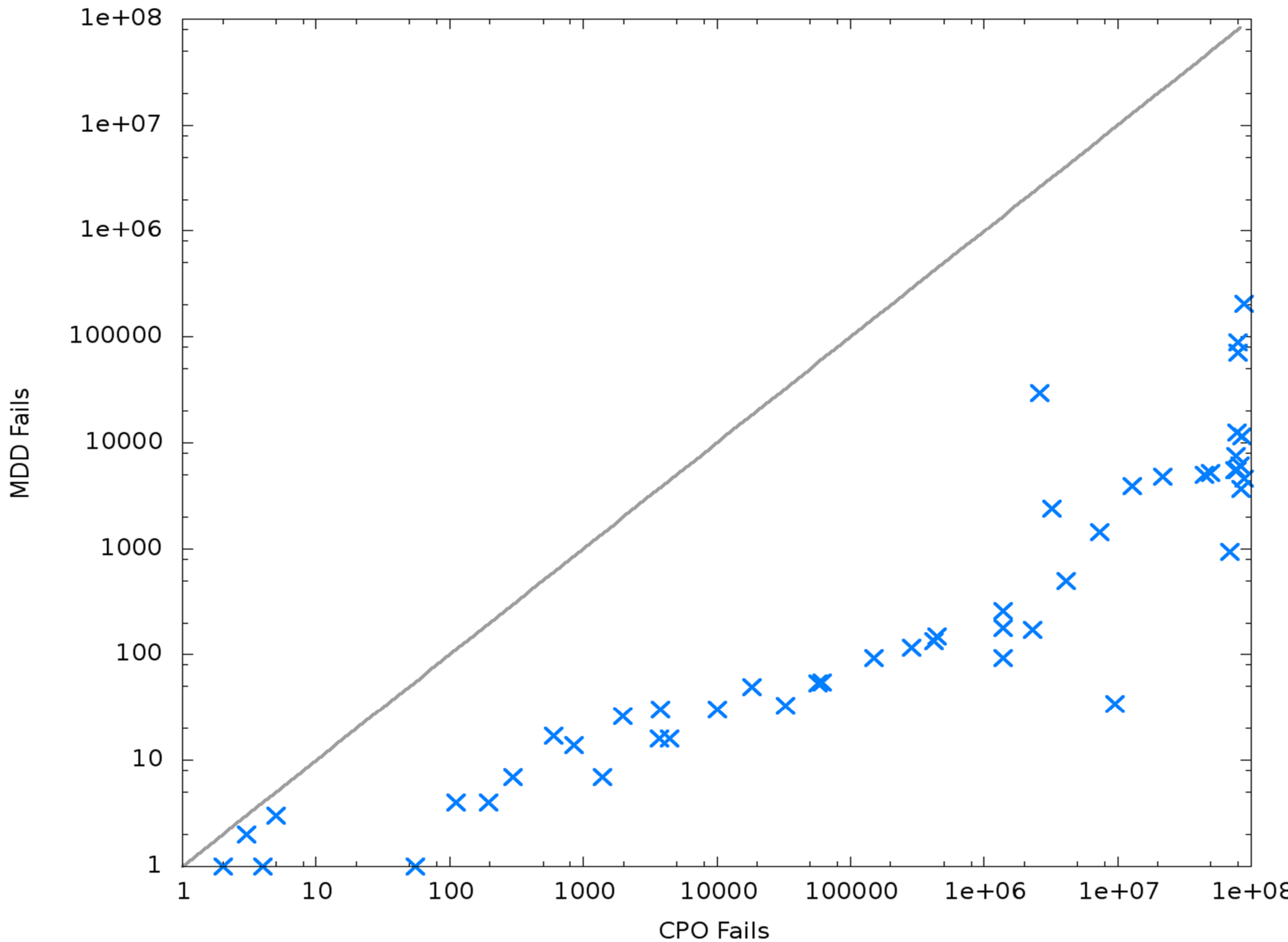
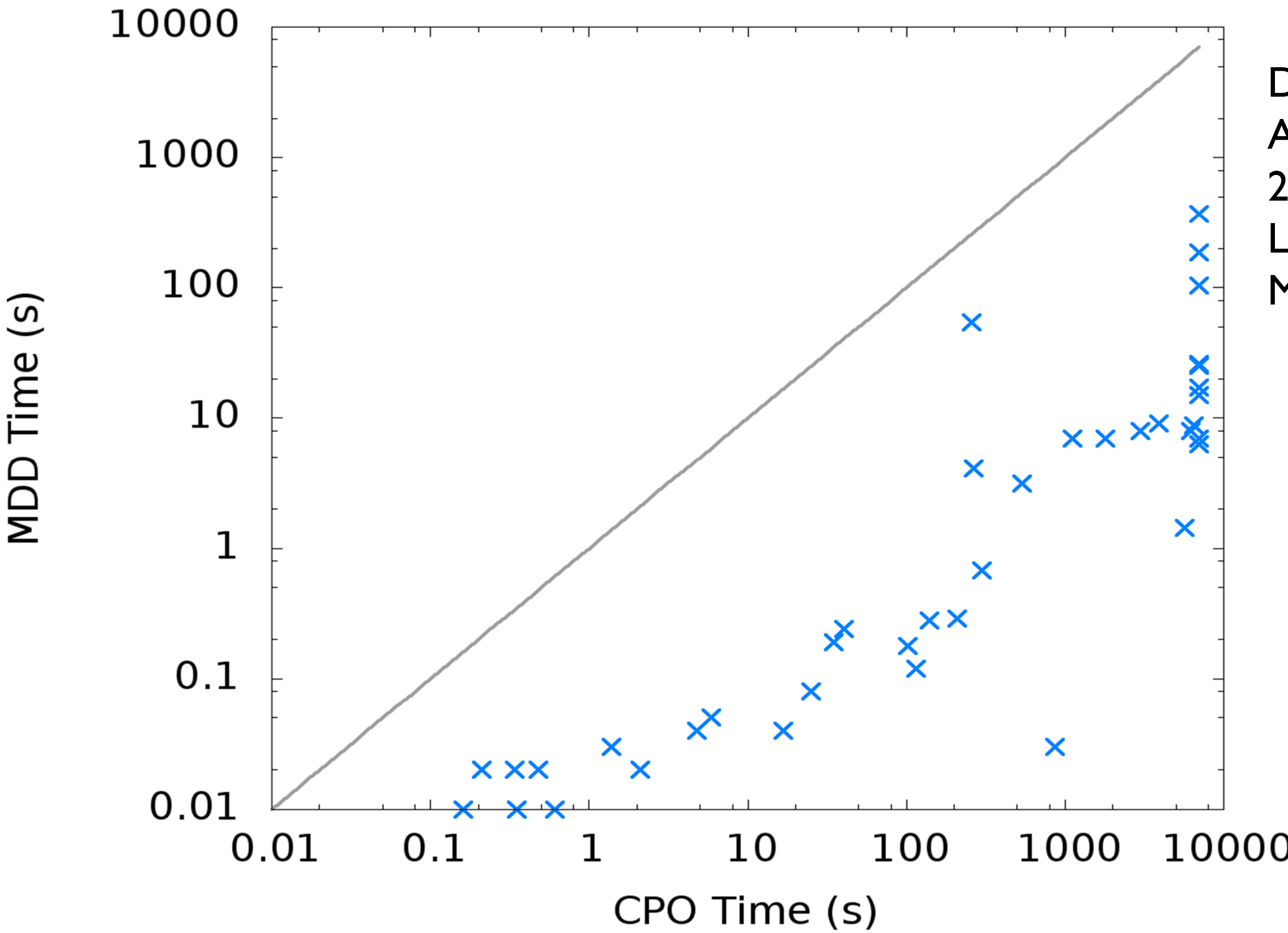
TSP with Time Windows



Dumas instances
Ascheuer instances
20-60 jobs
Lex search
MDD width: 16

TSP with Time Windows



Dumas instances
Ascheuer instances
20-60 jobs
Lex search
MDD width: 16

Instances Dumas (TSPTW)

		CPO		CPO+MDD	
Instance	**Cities**	**Backtracks**	**Time (s)**	**Backtracks**	**Time (s)**
n40w40.004	40	480,970	50.81	18	0.06
n60w20.001	60	908,606	199.26	50	0.22
n60w20.002	60	84,074	14.13	46	0.16
n60w20.003	60	> 22,296,012	> 3600	99	0.32
n60w20.004	60	2,685,255	408.34	97	0.24

minimize sum of setup times

MDDs have maximum width 16

Conclusions

- MDD for disjunctive constraints
  - Strong relation to precedence graph
  - High-level communication between MDD and other inference mechanisms
- Practical perspective
  - Current experiments suggest it is stronger for sums
    - Observed orders of magnitude improvement

Thank you!

Compilation

Theorem: *Constructing the exact MDD for a Disjunctive Instance is an NP-Hard problem*

Nevertheless, some interesting restrictions, e.g. (Balas [99]):

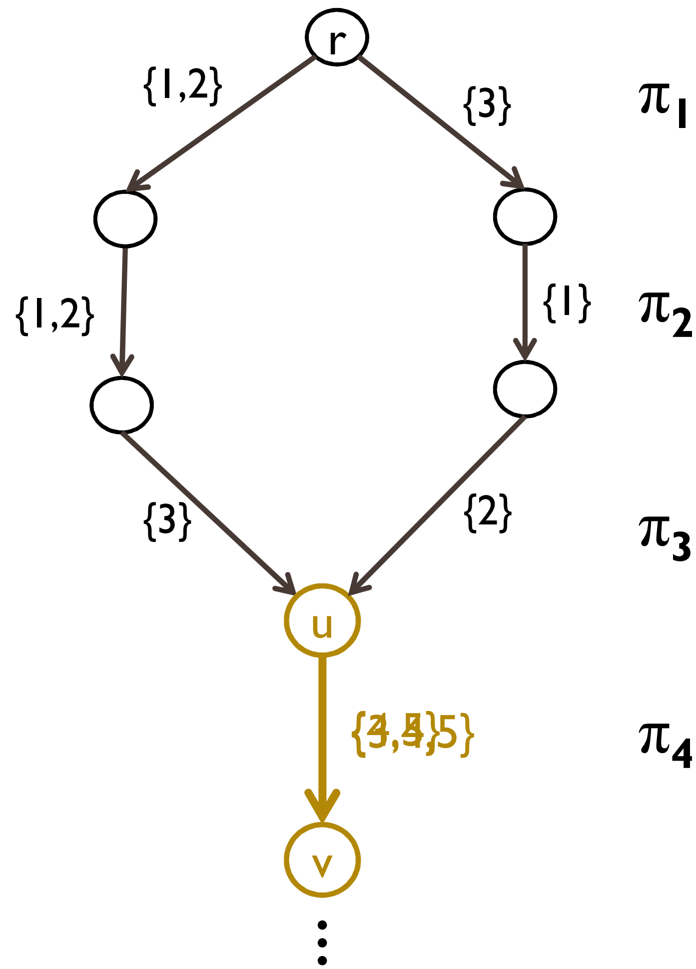
- TSP defined on a complete graph
- Given a fixed parameter **k**, we must satisfy

$$i \ll j \quad \text{if} \quad j - i \geq k \quad \text{for cities i, j}$$

Corollary: *The exact MDD for the TSP above has $O(n2^k)$ nodes*

Filter: Top-Down Example

- *All-paths state:* A_u
  - *Labels belonging to all paths from node r to node u*
  - $A_u = \{3\}$
  - *Thus eliminate {3} from (u,v)*



- Introduced for *Alldifferent* constraint in [Andersen et al 2007]

Outline

1. Disjunctive Scheduling
2. Multivalued Decision Diagram (MDD) Representation
3. Filtering and Precedence Relations
4. Experimental Results
5. Conclusion