12.1 Three Dimensional Coordinate System Notes

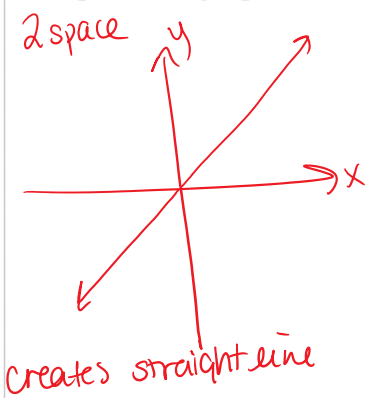
Thursday, September 20, 2018 12:37 PM

https://drive.google.com/file/d/1qQx0Doi4Ch5IjlB4nUHSdzYNifAck2kR/view

12.1 - Three Dimensional Coordinate System.notebook **September 20, 2018**

Section 12.1: Three Dimensional Coordinate System

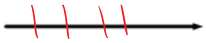
Compare the graph of the line y = x in both 2-space and 3-space.



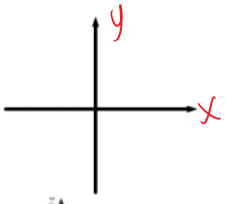
y = x
z can be anything
vertical
creates a plane
set of all pts
where x and y
are equal

Comparison of Coordinate Systems

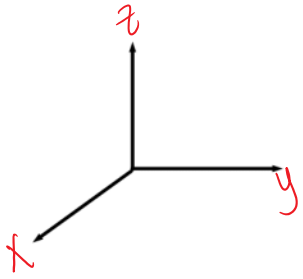
1-Dimensional

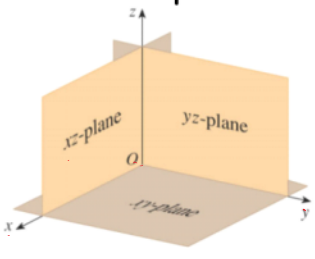


2-Dimensional



3-Dimensional

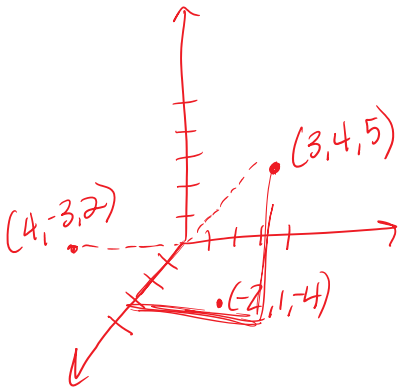


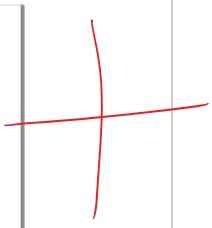


Plot the following 3-dimensional coordinates 8 octant

visualize in class

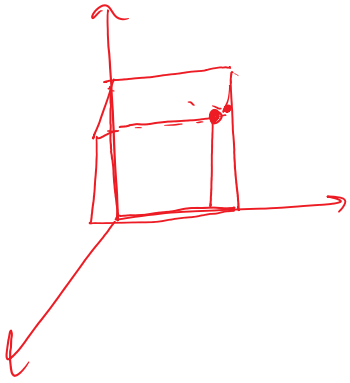
A(3, 4, 5)
B(-2, 1, -4)
C(4, -3, 2)

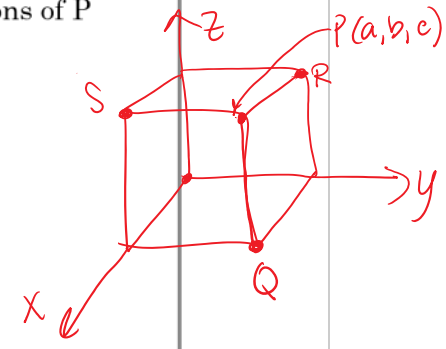




Point P(a,b,c) determines a rectangular box. If we drop a perpendicular from P to the xy-plane, we get a point Q with the coordinates (a, b, 0) called the **projection** of P onto the xy-plane.

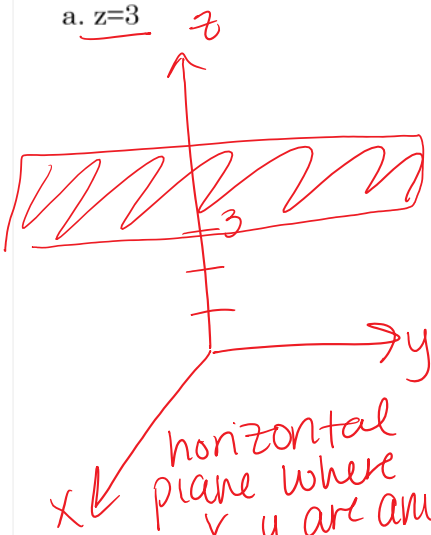
Similarly, R (0, b, c) and S (a, 0, c) are the projections of P onto the yz-plane and xz-plane.





What surfaces in R^3 are represented by the following equations?

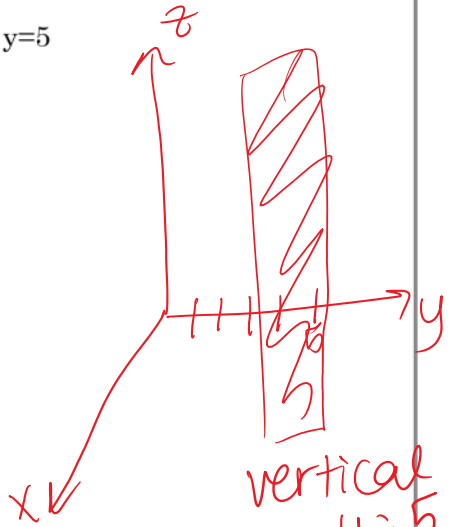
a. z=3



horizontal plane where x, y are anything

$z = 3$

b. y=5

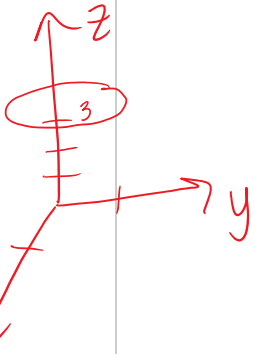


vertical plane

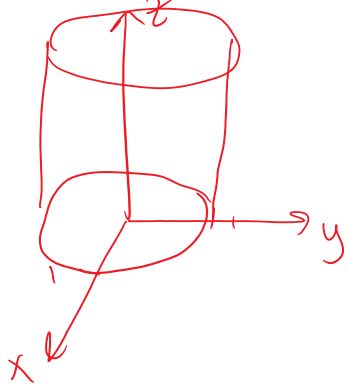
$y = 5$

Which points (x,y,z) satisfy the equations $x^2 + y^2 = 1$ and $z = 3$?

circle w/r=3



What does the equation $x^2 + y^2 = 1$ represent as a surface in R^3?



z missing - z can be anything

hollow cylinder

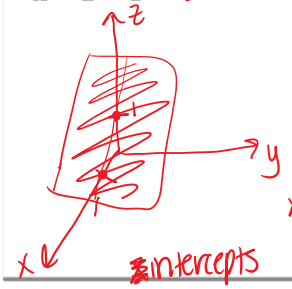
Describe the following surfaces in 3-space and sketch them.

$x = 0$ → yz plane

$y = x$

very in depth in class

$x + z = 1$ $z = -x + 1$



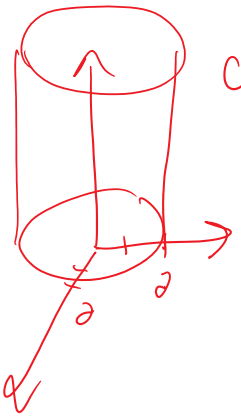
Side:

*intercepts

x int $x + z = 1$, $x = 1$

z int $x + z = 1$, $z = 1$

$x^2 + y^2 = 4$



hollow cylinder w/ r = 2

How do we find the distance between two points in 2-space?

Distance Formula

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Can we think of a way to extend this concept to 3-space?

Distance Formula:

The distance between two points $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ is

$$d = |P_1P_2| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Find |AB| and |AC|. Which is larger?

A(3, 4, 5)
B(-2, 1, -4)
C(4, -3, 2)

$|AB| = \sqrt{5^2+3^2+9^2} = \sqrt{115}$

$|AC| = \sqrt{1^2+7^2+3^2} = \sqrt{59}$

|AB| larger

$(x-h)^2+(y-k)^2=r^2$

Equation of a Sphere:

$(x-h)^2+(y-k)^2+(z-\ell)^2=r^2$

center @ (0, 0, 0)

$x^2+y^2+z^2=r^2$

Center: (h, k, ℓ)

Radius: r

Find the equation of the sphere with center at (0, 5, -3) with a radius of $\sqrt{2}$

$x^2+(y-5)^2+(z+3)^2=2$

Find the center and radius of the sphere that has the given equation:

$$4x^2+4y^2+4z^2-4x+8y-3=0$$

$4x^2-4x+4y^2+8y+4z^2=3$

$4(x^2-1x+\frac{1}{4})+4(y^2+2y+1)+4z^2=3+1+4$

$\frac{4(x-\frac{1}{2})^2}{4}+\frac{4(y+1)^2}{4}+\frac{4z^2}{4}=\frac{8}{4}$

$(x-\frac{1}{2})^2+(y+1)^2+z^2=2$

$C(\frac{1}{2},-1,0)$ Rad $\sqrt{2}$

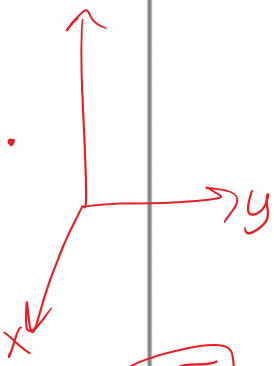
12. Find the distance from (4, -2, 6) to each of the following:

a. xy-plane
b. yz-plane
c. xz-plane
d. x-axis
e. y-axis
f. z-axis

a.) $z=0$ 6 units

b.) $x=0$ 4 units

c.) $y=0$ 2 units



d.) y+z are 0 (4,0,0)

$d=\sqrt{0+2^2+6^2}=\sqrt{40}=2\sqrt{10}$

e.) x+z 0 (0,-2,0)

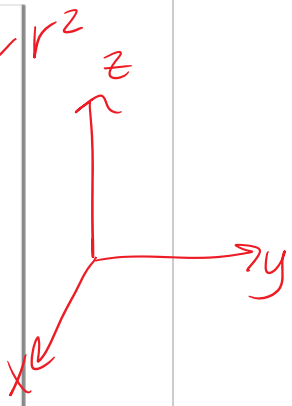
$d=\sqrt{4^2+0^2+6^2}=\sqrt{52}=2\sqrt{13}$

f.) x+y 0 (0,0,6)

$d=\sqrt{4^2+2^2+0^2}=\sqrt{20}=2\sqrt{5}$

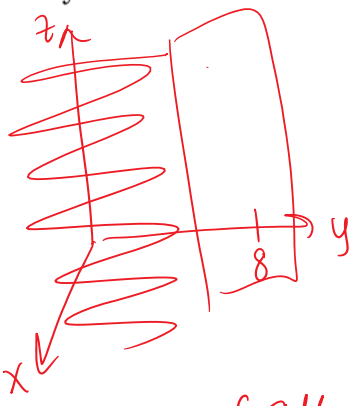
Find an equation of the largest sphere with center (5,4,9) that is contained in the first octant.

$(x-5)^2+(y-4)^2+(z-9)^2=16$



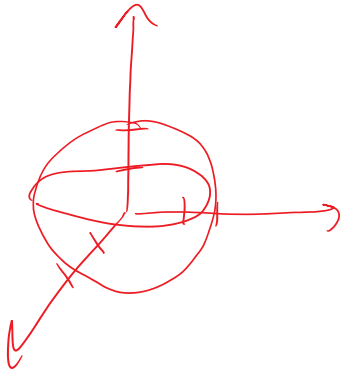
Describe the region of R^3 represented by the equation of inequality

27. $y < 8$



A half space of all pts to the left of plane $y=8$

34. $x^2+y^2+z^2 \le 4$



Filled in sphere all pts on or inside w/ $r=2$

45. Find an equation of the set of all points equidistant from points A(-1,5,3) and B(6,2,-2). Describe the set.

C(x, y, z) generic point

$$\left(\sqrt{(x+1)^2+(y-5)^2+(z-3)^2}\right)^2 = \left(\sqrt{(x-6)^2+(y-2)^2+(z+2)^2}\right)^2$$

$$x^2+2x+1+y^2-10y+25+z^2-6z+9 = x^2-12x+36+y^2-4y+4+z^2+4z+4$$

$$2x-10y-6z+35 = -12x-4y+4z+44$$

$$\boxed{14x-6y-10z=9}$$