An Articulation of the Concepts and Skills Which Underlie Engineering Statics

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Abstract - **Many instructional approaches are being developed with the goal of improving learning in Statics. This paper is aimed at providing guidance to such developments by articulating the conceptual basis for Statics. This paper recognizes the primary science prerequisite to Statics, freshman Newtonian mechanics, and addresses the essential ways in which Statics differs from freshman physics. A set of four concept clusters is proposed, together with a set of skills for implementing these concepts. Then, typical errors committed by students are presented. Examples of these errors are extracted from student solutions to Statics problems. These typical errors are then explained by appealing to the proposed concepts and skills. It is hoped that this paper can provide an impetus for mechanics educators to come to a community-wide agreement on a conceptual structure of this subject that can inform future instructional developments.**

Index Terms – concepts, errors, instruction, Statics

INTRODUCTION

Statics forms the essential pre-requisite to a number of follow-on courses, such as dynamics and mechanics of materials. These courses collectively constitute the engineering science backbone to mechanical design and much engineering practice. Instructors of these follow-on courses cite weaknesses in Statics as a significant source of difficulties for students. Likewise, instructors of engineering design feel that student understanding of Statics instruction is a major impediment to their success in design courses. Although there are a number of efforts to improve instruction in Statics, these are likely disadvantaged by the lack of a clear consensus on the primary ideas that instruction should emphasize. In the case of Newtonian mechanics in freshman physics, there have been efforts by the physics education community [1-3] to identify its basic concepts and associated misconceptions.

This paper seeks to take an initial step towards articulating the concepts of Statics. We begin focusing on what students should ideally know upon entering Statics. The most significant prerequisite for Statics is Newtonian mechanics as taught in freshman physics. From this, it is apparent why Statics may fail to get the attention it deserves, both from its students and instructors. The only scientific principle in Statics, the principle of equilibrium, which is captured by the net force equaling zero, is merely a subset of what is taught in physics. Surely, Statics must be a breeze for students who have passed physics!

This deceptive simplicity of Statics can lead, unfortunately, to instruction that is insufficiently sensitive to the subtleties of implementing the equilibrium principle, at least in the context of situations that are of interest in Statics. Admittedly, implementation of this principle is partially hamstrung by the increase in mathematical complexity. However, as pointed below, there are also significant conceptual challenges associated with the modeling steps that precede the implementation of equilibrium.

Although Physics and Statics are based on the same physical principles of Newton's laws, one can identify specific ways in which they differ. Physics problems often involve single bodies, while Statics problems feature multiple bodies that must be dismembered. In Physics problems usually only translation is of primary concern, while in Statics problems bodies have finite dimensions, and so rotations are also of concern. In Physics problems there is usually just a single force acting between two bodies, while in Statics problems there can be multiple forces and distributed forces between bodies, and directions of forces are often to be determined. As explained below, these complications of Statics over physics demand a significantly enhanced understanding of the same primary concepts of force and equilibrium.

CONTENT OF STATICS

Students confront a variety of types of problems in Statics. However, since we view Statics as essential preparation for engineering design, it seems useful to study the concepts and skills that are relevant to problems comprising interconnected parts. Such problems illustrate the prime differences between freshman physics and Statics; in Statics textbooks, these problems are designated as "frames and machines." Such problems typically involve one or more

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applied loads, with the various forces between the parts to be determined. An example of such a problem is the following:

The pliers shown, which consists of several parts, is used to grip the bolt in its jaws. Determine the forces exerted by the various parts on each other, and the forces squeezing the bolt. Neglect the weight of the parts.

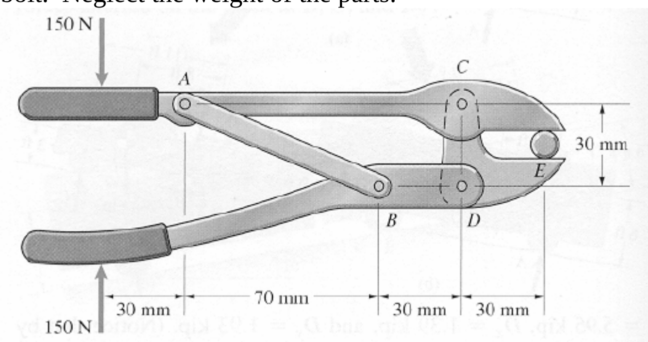

FIGURE 1
TYPICAL MULTI-BODY PROBLEM IN STATICS (BEDFORD AND FOWLER, ENGINEERING MECHANICS: STATICS, 3RD ED., PRENTICE-HALL, 2002)

Experts (e.g., instructors) solving this class of problems are likely to do so following these typical steps:

Parsing the system
Here one inspects the system, recognizes the distinct parts, where they are attached to one another and precisely the geometry of their connections.

Preliminary reasoning about forces between connecting parts
Here one uses information about the geometry of the connections between parts, together with the static equivalency principle and various approximate simplifications, to determine which forces or couples may be acting between parts. Of prime interest is the number of unknown forces at each connection.

Choosing a body on which to impose the equilibrium conditions
Here one considers various parts, combinations of parts, or portions of parts as candidate bodies to which the equilibrium principle may be applied. This choice is conditioned on which unknown forces are desired, and on whether the unknown forces acting on the chosen body can be determined with the limited number of equilibrium equations. For the given choice, one represents the forces exerted directly on that body in terms of vectors, variables and constants. At this stage the so-called free body diagrams are complete.

Applying the equilibrium principle to the chosen body to quantify forces
Here one derives the equations which capture the summation of forces and moments, for the body which was chosen. The unknown forces of interest are determined by solving these equations.

In Figure 2 we show the completed free body diagrams for the problem depicted in Figure 1. We have parsed the pliers into its major components and recognized they are connected by pins (at points A, B, C and D). A pin permits the members it connects to pivot about their shared pin, but not to translate relative to each other. In this particular problem, we draw a free body diagram for each part separately; often, it is necessary to draw a free body diagram for a collection of parts.

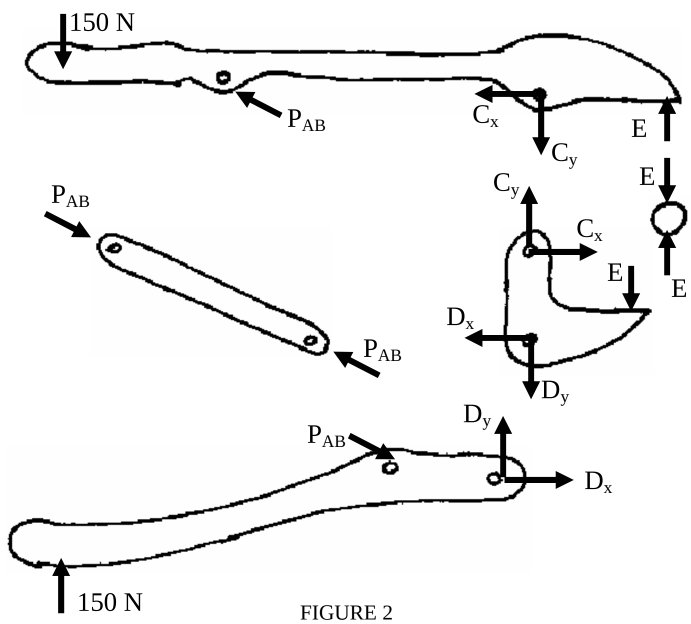

FIGURE 2
FREE BODY DIAGRAMS FOR PROBLEM IN FIGURE 1.

Since the parts are connected at points A, B, C and D by the same means (via pins), the types of interactions at these points are identical. The different representations for the unknown forces are explained below. The interaction between any two connected members can be expressed, in general, as a force and a couple, possibly acting in three-dimensions. A critical element of Statics is to reduce this general interaction to a more restricted one, if possible, through the use of various arguments. The tacit neglect of friction between the pin and each member implies the absence of a force along, and a couple about, the axis perpendicular to the plane of the problem. A major reduction in the forces here comes from the recognition that the problem is two-dimensional. In textbook problems, the nature of the drawing (and the chapter in the textbook!) often signal whether a problem is two-dimensional. However, when applying Statics to a real system (which is always three-dimensional), one looks for clues, such as whether the members are symmetrically located about a plane, and whether the dominant forces act parallel to this plane. In recognizing the two dimensionality of this problem, we take each pin to exert no couples about the x- and y-axes.

The conditions of equilibrium are often appealed to even in constructing the free body diagrams. The parts are connected at, for example, point C by a common pin. Since that pin has two forces acting on it, by equilibrium they must be equal and opposite. Thus, we dispense with showing the pin and only show the force components, C_x and C_y, acting equal and oppositely on the major parts. (This line of reasoning would need to be amended when a single pin connects three or more bodies.) The link AB has only two forces acting on it from the pins at A and B. The conditions of equilibrium on this body imply that these forces must act along the line joining A and B; this fact is registered in the diagram. Likewise, the bolt is pressed upon by two jaws, which could exert both vertical and horizontal (frictional) components. Equilibrium of the bolt implies, however, that the horizontal forces must be zero, and thus only vertical ones are drawn. It can be seen that the rationalizations behind the representation of forces, and the interweaving of equilibrium conditions while still drawing free body diagrams, are, indeed, complex.

CONCEPTUAL BASIS FOR STATICS

To make instruction effective and efficient, it is desirable to identify a minimal set of concepts that provide much of the basis for solving a significant set of Statics problems. Such a set of concepts can also be helpful when we study students' solutions to problems and we attempt to explain and remediate their difficulties at the most fundamental level. However, students make errors not only due to conceptual difficulties, but due to inadequate skills of implementation. We therefore, postulate a set of basic concepts and a primary set of skills that are necessary to implementing those concepts. While making a firm distinction between concepts and skills is difficult, we consider skills as actions which can be mastered with rote practice, while concepts demand much more careful explanation and deeper understanding. After proposing the set of concepts and skills, we set forth a set of typical errors that are based at least in part on conceptual lapses. For each error, an explanation of that error in terms of the concepts and skills is offered.

Concepts of Statics

C1. Forces act between bodies

Forces refer to the mechanical interactions between two pieces of matter that are in contact, or to the gravitational attraction between the earth and a body. Forces between two contacting bodies are equal and opposite and have a direction and magnitude.

C2. Combinations and/or distributions of forces acting on a body are statically equivalent to a force and couple

Two bodies often contact each other at more than a single point, in which case they will exert a combination or distribution of forces on each other. Such a combination or distribution of forces can be replaced by a single force and a couple provided they exert the same net force and moment.

C3. Conditions of contact between bodies or types of bodies imply simplification of forces

Although the forces of contact between two bodies are unknown, their direction or magnitude may be limited by considerations other than equilibrium. The positions of the bodies and loads may form a two dimensional system, the specific geometry of contact between the bodies, the neglect of friction or the presence of sliding between the bodies, or the nature of one of the bodies (e.g., cable), each may reduce the number of unknown forces and couples.

C4. Equilibrium conditions are imposed on a body

When a mechanical system or structure is in equilibrium (stationary or moving at constant speed), each portion of it is in equilibrium. For any portion (called body) to be in equilibrium, the combined effect of all external forces acting directly on that body must provide no net tendency to translate (zero net force) or to rotate (zero net moment).

Skills Needed for Implementing Concepts of Statics

Clearly, a number of mathematical skills are critical to solving Statics problems. These include the summation and decomposition of forces and the calculation of moments. In addition, a number of less recognized skills are also essential to Statics:

S1. Discern separate parts of an assembly and where each connects the others

S2. Discern the surfaces of contact between connected parts and/or the relative motions that are permitted between two connected parts

S3. Group separate parts of an assembly in various ways and discern external parts that contact a chosen group

S4. Translate the forces and couples which could be exerted at a connection (e.g., there is only a force in a known direction) into the variables, constants, and vectors that represent them

COMMON ERRORS IN STATICS PROBLEMS ASSOCIATED WITH CONCEPTUAL LAPSES

Errors found in student work are among the "field observations" which a conceptual framework for Statics should explain. We have attempted to reduce the wide range in errors committed to a modest number of types, and we have attempted to associate possible conceptual lapses with each type of error. These errors may also stem, however, not from a lack of conceptual understanding, but from inadequate skills. Specifically, the inability to visualize and parse the system being analyzed, in the sense of discerning its separate parts and how they are connected (S1-S3) can also produce many of these common errors. Common types of errors in Statics are now listed. After each error are potential explanations, beginning with SDRT (student does not realize that…).

<u>E1:</u> Failure to be clear as to which body is being considered for equilibrium:
SDRT the equilibrium conditions are always imposed on a specific body (C4).

<u>E2:</u> Failure to take advantage of the options of treating a collection of parts as a single body, dismembering a system into individual parts, or dividing a part into two:
SDRT the equilibrium conditions can be applied to any collection of material, or, if a system is in equilibrium, then any subset of it must be in equilibrium (C4, S3).

<u>E3:</u> Leaving a force off the free body diagram (FBD) which should be acting:
SDRT a FBD requires all external forces (C4), the body which contacts the isolated body is exerting a force (C1) or there is a body contacting (S1).

<u>E4:</u> Drawing a force as acting on the body of the FBD, even though that force is exerted by a part which is included in the body of the FBD:
SDRT a FBD requires only external forces (C4), or the force drawn is actually between two other bodies, both of which are included in the isolated body (C1, S1).

<u>E5:</u> Drawing a force as acting on the body of the FBD, even though that force does not act directly on the body Drawing a force as acting on the body of the FBD:
SDRT a FBD requires only external forces that act directly (C4), or the force drawn is actually between two other bodies, both of which are outside the isolated body (C1, S1).

<u>E6:</u> Failing to account for the mutual (equal and opposite) nature of forces between connected bodies that are separated for analysis:
SDRT a FBD requires all external forces (C4), the force on the first body was exerted by the second body (C1, S1), or all forces have an equal and opposite pair (C1).

<u>E7:</u> Ignoring a couple that could act between two bodies or falsely presuming its presence:
SDRT connected bodies which contact at multiple points may exert a couple on one another (C2), or the nature of the geometry of contact and/or neglect of friction imply that the combination of forces cannot produce a net couple (C3,S2).

<u>E8:</u> Not allowing for the full range of possible forces, or not sufficiently restricting the possible forces:
SDRT connected bodies can potentially exert forces in all directions (C2), the geometry of the bodies and loading, or geometry of contact and/or neglect of friction imply that certain force components are zero (C3, S2), certain bodies can sustain only certain forces or couples (C3), or the symbols and variables which are employed must represent all of, but only, the forces which can be present (S4).

<u>E9:</u> Presuming a friction force is at the slipping limit (μN), even though equilibrium is maintained with a friction force of lesser magnitude:
SDRT frictional forces have magnitude μN only when the bodies are sliding relative to each other; or two bodies in contact try to exert forces on each other to prevent their relative sliding motion, with the friction force preventing this motion having any magnitude from zero up to the maximum value μN (C3).

<u>E10:</u> Failure to impose balance of forces in all directions and moments about all axes:
SDRT equilibrium conditions require there to be no net force in any direction and no net moment about any axis (C4).

<u>E11:</u> Having a couple contribute to a force summation or not having a moment summation include a couple:
SDRT the couple is a combination of forces which produces a moment, but which adds no net force (C2).

EVIDENCE FOR COMMON ERRORS

The errors above were generated based on many years of observing student errors in Statics. Examples are given here based on the problem of Figure 1 that was recently given to students just entering a sophomore level Statics course in mechanical engineering. In addition to Physics, all students had taken a freshman mechanical engineering course with three weeks of Statics. While all students made errors, many appreciated the need to separate the bodies, to draw forces on bodies and to impose equilibrium conditions. Thus, students at this stage were viewed as offering some legitimate insight into the types of errors and misconceptions that need to be overcome in Statics. Examples of errors from these students are as follows:

<u>Student Solution 1 (Figure 3)</u>

- Forces are left off entirely at point A (E3)
- A force which presumably is exerted by one member on the other is shown acting at D, even though both bodies are in the diagram (E4)

- The forces at point C are not drawn as equal and opposite (E6)
- The direction of the force appears to be presumed at C, rather than having an unknown direction, say with components F_x and F_y (E8).

Student Solution 2 (Figure 4)

- Equilibrium (determining the force squeezing the bolt) appears to be imposed on a uncertain body (E1)
- Couples acting at pin connections (E7)

Notice that this problem does not offer scope for some of the errors (for example, that involving friction), and that students did not carry the solution far enough to have potentially committed additional errors, particularly those related to the imposition of equilibrium.

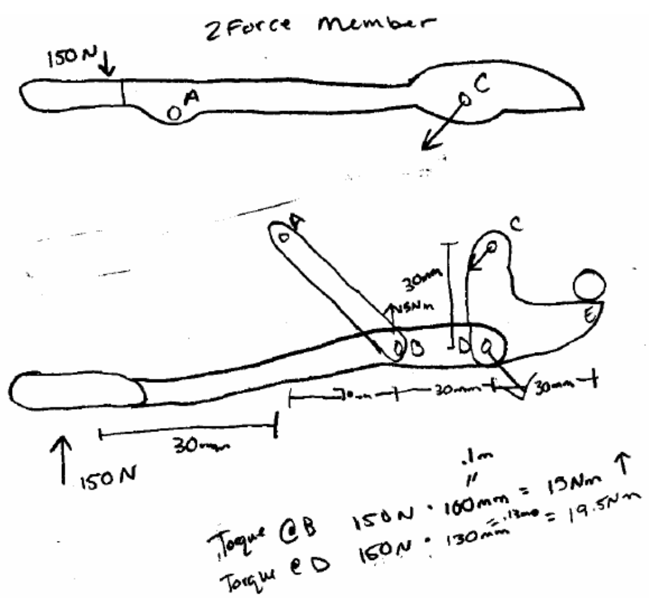

FIGURE 3
STUDENT SOLUTION 1 ILLUSTRATING ERRORS E3, E4, E6, E8.

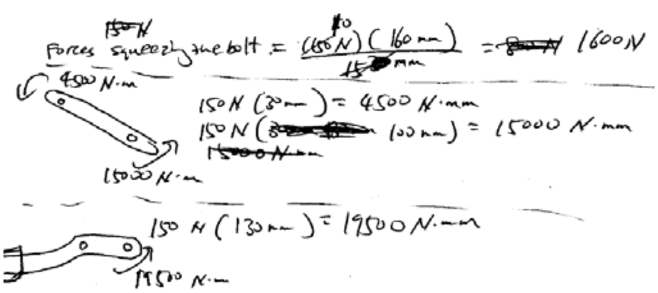

FIGURE 4
STUDENT SOLUTION 1 ILLUSTRATING ERRORS E1 AND E7.

RELATION OF CONCEPTS OF STATICS TO THOSE OF FRESHMAN PHYSICS

Since the conceptual framework of the prerequisite freshman physics course on Newtonian mechanics has been extensively studied, it is helpful to place the concepts of Statics in relation to those of Newtonian mechanics. In particular, we compare with the taxonomy put forth by Hestenes, Wells and Swackhammer [3], in which the FCI (Force Concept Inventory) is also described. In [3], the concepts are organized and the common misconceptions associated with those concepts are delineated. All of these concepts and misconceptions, some pertaining to forces and some pertaining to motion, are relevant to engineering Statics and/or Dynamics. Here, we call attention to those concepts and misconceptions identified in [3] which are most relevant to Statics, and we explain why subtle variations of these concepts and misconceptions are even more relevant to Statics.

A major misconception cited by the developers of the FCI, namely that only active agents exert forces (misconception AF1, in their notation), as well as its inverse misconception that rigid obstacles exert no forces (their misconception Ob), are a significant source of difficulties in Statics. Forces, as understood by most people, cannot be exerted by apparently rigid, unmoving bodies. This misconception is critical to Statics, since often the forces of interest are acting between parts that are stiff and unmoving, and thus students simply disregard these forces. Indeed, this difficulty may be accentuated by instructors who overemphasize the common, though unnecessary, assumption in Statics that bodies are rigid and do not move. In addition, in the typical presentation of problems, the presence of many forces of interest is not made explicit, but must be inferred from the fact that two bodies are in contact. Moreover, “contact” itself must be inferred from observing adjacent bodies drawn with a partially common boundary. As an aside, the presence of forces between all contacting objects might be more believable if people understood that all solid objects indeed act like springs, in that they deform, although usually by minute amounts.

A closely related concept, which is relevant to freshman physics, but which is not explicitly articulated at least in connection to the FCI, is that every force is, in fact, a mechanical interaction between two specific bodies, which are usually in contact. Including a force in, say, a free body diagram or in an equation of equilibrium, but failing to attribute that force to two specific bodies, is one of the two most significant sources of errors in Statics. This misconception also gives rise to errors that might appear to have other origins. Specifically, students often fail to observe Newton’s third law regarding action and reaction pairs; as pointed out in [3], this is often the last misconception in freshman physics to be overcome. Newton's third law is extremely important in Statics, where the disassembly of connected bodies naturally reveals such pairs; students often fail to track these pairs and to ensure they are equal and opposite. Failure to ascribe a force to an interacting pair of bodies can make it even more difficult to impose the third law properly. Say body 2 exerts a force on body 1. Say the student draws this force in a free body diagram of body 1, but does not recognize this force as applied by body 2. Then, it is unlikely that the student is going to know to draw the equal and opposite force should the need arise to construct a free body diagram of body 2.

The remaining relevant category of misconceptions cited in the FCI pertains to the concatenation of influences: how one determines the net effect of a combination of forces. Instead of summing forces, students might believe that the largest force or the last force dominates the others [3]. These misconceptions are largely overcome by the time students are in Statics. Rather, the issue for Statics is even more fundamental: which forces are to be concatenated? It is too often unappreciated that the equilibrium principle (or, more generally, Newton's 2^{nd} law) must be applied to *a body*, not to some random collection of forces. The body may be any assembly of material one chooses; but once chosen, the forces to be concatenated are all the forces exerted by bodies interacting directly with the chosen body. Failure to identify a body when applying the equilibrium conditions is the second of the two most significant sources of errors in Statics.

Thus, the concepts C1 and C4 articulated in this paper, namely that each force is between two bodies and that the equilibrium principle is applied to a chosen body, are at least tacitly a part of Newtonian mechanics as taught in freshman physics. If a student had a perfect understanding of physics at this level, these concepts would pose no difficulty in Statics. Yet, freshman physics, as typically taught, rarely addresses problems which severely test student understanding of these concepts, such as situations in which two or more bodies contact one another and more than one body is to be analyzed. Since the concepts articulated in C1 and C4 are slightly different from those in the taxonomies in [3], they are explicitly identified here. By contrast, the other two concepts explicitly identified here, C2 and C3, which address how forces between bodies can be simplified, have virtually no counterpart in freshman physics.

POTENTIAL BENEFITS OF ESTABLISHING A CONCEPTUAL STRUCTURE FOR STATICS

There are benefits for both assessment and instruction in establishing a conceptual basis for Statics. Instructors can look at errors in student solutions to typical problems with greater insight as to their underlying causes. Also, novel ways of assessing student understanding and thinking can be devised. For example, questions can be devised which test sub-components of reasoning, rather than the ability to solve a typical Statics problem. Concept questions, such as those in the FCI, are examples of this approach. Since, as seen above, many errors could originate from conceptual lapses or from inadequate skills, it would be important to formulate questions that permit such a distinction. For example, this might mean that the system of bodies is sufficiently simple, and their connections sufficiently obvious, that the student has no trouble parsing the system. Errors would be due only to conceptual lapses. Concept questions, generally can be particularly valuable if the instructor is able to obtain more than just an overall score, but also insight into how well students handle particular concepts or whether particular misconceptions are consistently exhibited.

The traditional instructional sequence in Statics, as judged by textbooks, addresses the properties and manipulation of vectors, which includes working with forces and moments. This is followed by a presentation of the conditions of equilibrium, and then the analysis of equilibrium problems of increasing complexity, usually with due emphasis on the usefulness of the free body diagram. It is hoped that this paper has convinced the reader of the subtlety and depth of the concepts and skills that form the basis for effective performance in Statics. This analysis may also suggest the benefits of Statics classes addressing these concepts and skills in a way that goes beyond the rather narrow traditional treatment. In addition, at least some of these issues are ones that can benefit from active-engagement techniques, such as have become more widespread in physics. The author's own teaching of Statics is increasingly shaped by reference to the concepts, skills and errors, identified here.

CONCLUSIONS

In sum, we believe that an articulation of the concepts and skills of Statics can be valuable, particularly if it serves to organize both assessment and instruction. This paper offers such an articulation, the particulars of which are justified on their ability to explain commonly observed student errors. Yet, there does not appear to be a unique way to decide whether this articulation of concepts and skills is valid. Indeed, other instructors must ultimately judge the usefulness of this work. We suggest that the following questions be a part of this evaluation: Are the errors listed genuinely typical of students of Statics? Are the explanations of these errors based on the set of concepts and skills plausible or does an alternative set of concepts and skills have more power to explain observations? Finally, does instruction that explicitly addresses a set of fundamental concepts and skills lead to better learning of Statics?

ACKNOWLEDGEMENTS

The author is grateful to Anna Dollár and Marina Pantazidou for their thoughtful comments on this paper. Support by the Department of Mechanical Engineering at Carnegie Mellon University is gratefully acknowledged.

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