

Competitive Poaching in Sponsored Search Advertising and Its Strategic Impact on Traditional Advertising

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Abstract

An important decision for a firm is how to allocate its advertising budget among different types of advertising. Traditional advertising, such as advertising on television and in print, serves the purpose of building consumer awareness and desire about the firm's products. With recent developments in technology, sponsored search (or paid search) advertising at search engines in response to a keyword searched by a user has become an important part of most firms' advertising efforts. An advantage of sponsored search advertising is that, since the firm advertises in response to a consumer-initiated search, it is a highly targeted form of communication. Therefore, the sales-conversion rate is typically higher than in traditional advertising. However, a consumer would search for a specific product or brand only if she is already aware of the same due to previous awareness-generating traditional advertising efforts. Moreover, competing firms can use sponsored search to free-ride on the awareness-building efforts of other firms by directly advertising on their keywords and therefore "poaching" their customers. Anecdotal evidence shows that such poaching is a frequent occurrence. In other words, traditional advertising builds awareness, while sponsored search is a form of technology-enabled communication that helps to target consumers in a later stage of the purchase process, which induces competitors to poach these potential customers.

Using a game theory model, we study the implications of these tradeoffs on the advertising decisions of competing firms, and on the design of the sponsored search auction by the search engine. We find that symmetric firms may follow asymmetric advertising strategies, with one firm focusing on traditional advertising and the other firm focusing on sponsored search with poaching. Interestingly, the search engine benefits from handicapping poaching, i.e., it benefits from *discouraging* competition in its own auctions. This intriguing result explains why search engines such as Google, Yahoo! and Bing use "keyword relevance" scores to under-weight the bids of firms bidding on competitors' keywords. We also obtain various other interesting insights on the interplay between sponsored search advertising and traditional advertising.

1 Introduction

Online advertising is the fastest growing channel of advertising, likely to exceed 30% of the total US advertising expenditure by 2015 (eMarketer 2012a). This rapid growth in online advertising is impressive given that television advertising, which firms have been using for decades, has a market share of about 35%. On aggregate, firms allocate nearly half of their online advertising spend to sponsored search advertising (eMarketer 2012b). There are several unique advantages of sponsored search advertising, including effective targetability, ease of setting up a campaign and ease of measurement of ROI. Given its unique characteristics and spectacular growth, sponsored search advertising is gaining increasing attention from researchers and practitioners. However, while firms are dedicating progressively larger fractions of their advertising budgets to sponsored search advertising at the expense of traditional channels of advertising, the strategic implications of the interactions between these two types of advertising have not been carefully researched.

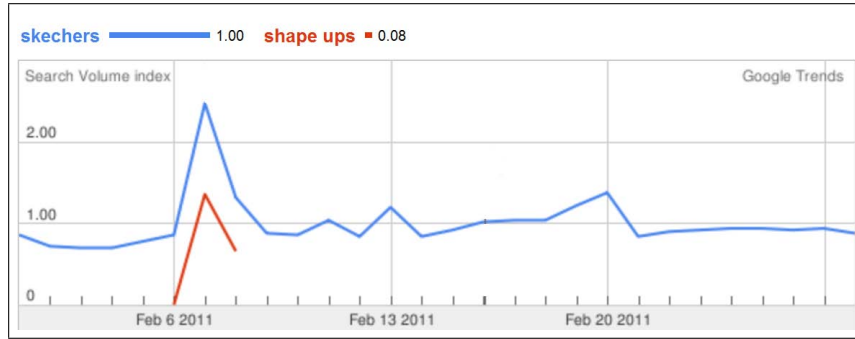
Consumers go through several stages of involvement before purchasing a product, and different types of advertising influence consumers differently in these stages. A widely employed marketing framework that captures the various sequential stages of a typical consumer's decision process before final purchase is the awareness-interest-desire-action (AIDA) model. Traditional channels of advertising, such as television, newspapers, radio and billboards, generate awareness among consumers and are directed more towards the initial stages of the AIDA model. In traditional advertising, communication with potential consumers is initiated by the firm to make them aware of and interested in the firm's brand or product. Sponsored search, however, is directed more towards the last stages of the AIDA model and it influences a consumer close to the purchase decision. Sponsored search effectively targets the consumers who are already aware of the product and have shown some interest or desire in the product by searching for an associated keyword at a search engine. In the context of the AIDA model, traditional advertising can be interpreted as "upstream advertising," while sponsored search can be interpreted as "downstream advertising." Thus, traditional awareness-generating advertising and sponsored search advertising are inter-related and play complementary roles in successfully consummating the sale of a product.

In a strategic market with competing firms, creating awareness has benefits as well as perils, especially when the awareness created through traditional advertising for one brand can be exploited

by a competing firm through sponsored search advertising. Competitors, instead of allocating their advertising budget to create awareness about their own products, can advertise in sponsored search on the keywords of a firm in the same industry that is creating awareness by investing in traditional advertising, trying to steal the latter’s potential customers. We refer to this as “poaching” in sponsored search.

We provide anecdotal examples where such poaching is evident. The shoe company Skechers advertised its Shape Ups model during Super Bowl 2011 (held on February 6, 2011). The upper panel of Figure 1(a) shows the effect of the television ad on the search volumes of the keywords “Skechers” and “Shape Ups” on Google in the days after Super Bowl. It can be easily seen that the advertising created considerable awareness resulting in heavy keyword search on the internet. Interestingly, while Skechers spent millions of dollars for the Super Bowl commercials, its competitor, Reebok, poached on the keyword “Shape Ups” to advertise its competing model called “Easy Tone,” as shown in the screenshot of a Google search for the keyword “Shape Ups” in the lower panel of Figure 1(a). The same phenomenon can be observed for the keyword “Groupon”—the company Groupon advertised on Super Bowl which led to a significant increase in the search volume of its associated keyword (upper panel of Figure 1(b)), while its competitor LivingSocial poached on its keyword (lower panel of Figure 1(b)). These are only a few of many instances of poaching, which is happening with increasing frequency on the internet.

In summary, poaching happens when a firm creates awareness resulting in pertinent keyword search on the internet, and competing firms aggressively bid on these keywords to display their ads. In fact, since the competitors are spending less to create awareness, they can bid more aggressively on sponsored search keywords (typically sold through position auctions run by the search engine) and thus can even enjoy an advantage over the firm that has attracted the customers in the first place. Such poaching behavior has implications not only for the competing firms’ strategies on the sponsored search and traditional advertising channels, it also strategically affects the search engine’s auction strategy. In this paper, we examine these issues using a game theory framework. We address three broad questions. First, under what conditions will poaching arise and be beneficial for a firm? Second, what are the effects of poaching on competing firms’ decisions for budget allocation among traditional and sponsored search advertising? Third, what are the consequences of poaching for the search engine and how should it adapt its auction mechanism anticipating poaching by competing



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Ads

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 Styles Starting at Just \$39.99!
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(a) Shape Ups



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Get 1 Daily Email w/ Huge Deals On The Best Things To Do In Your City!
www.groupon.com

Get Daily Email Refer a Friend
 How Groupon Works See All Deals

Local 1-Day Coupons - Up to 90% Off the Best Stuff To Do 🔍
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www.livingsocial.com

(b) Groupon

Figure 1: Poaching

advertisers.

We first consider the case in which there are two identical competing firms. We find the existence of an asymmetric equilibrium in which one firm mostly advertises on traditional channels and creates awareness, while its competitor poaches on its keyword in sponsored search. This is because poaching increases the competition in sponsored search auctions which increases the advertising prices of the keywords. This increases the per-customer acquisition cost to the firm from sponsored search, which makes sponsored search a less desired option, and incentivizes the firms to move a larger share of their money to traditional advertising. However, poaching remains a profitable strategy for one firm, which leads to the asymmetric budget allocation. Interestingly, although the competition in sponsored search increases with poaching, the search engine’s revenue may decrease because of the incentive of firms to move some of their advertising budget to traditional advertising. As a result, the search engine may benefit from *discouraging competition* in its own keyword auctions by making poaching harder for the firms. This offers an explanation for why major search engines such as Google, Yahoo! and Bing use “keyword relevance scores” to under-weight the bids of firms bidding on competitors’ keywords.

We extend our analysis to the case of asymmetric firms with different advertising budgets. When the difference between budgets is large enough, the firm with the smaller advertising budget has a greater incentive to poach as compared to the firm with the larger budget, because the latter conducts more traditional advertising and drives more traffic towards its keywords. Interestingly, with asymmetric firms, the search engine may in fact benefit from poaching—its revenue is maximized when the poaching is *controlled* but not prohibited. Knowing that the smaller-budget firm has a larger incentive to poach, the search engine can employ appropriate keyword relevance scores to under-weight the poaching bid and make it more expensive for the smaller-budget firm to poach, but only to the extent that it still continues to poach. This serves a dual purpose: first, it helps the search engine to capture a large part of the advertising budget of the smaller-budget firm through poaching; second, it shields the larger-budget firm by controlling bid escalation due to poaching to some extent, which means that this firm does not move much of its budget away from sponsored search. Therefore, keyword relevance measures could be interpreted as a complex price discrimination mechanism: for the smaller-budget firm, poaching is a very desirable option; for this reason, the search engine can charge the firm a higher price than the larger-budget firm which is creating

the search volume. This result explains why search engines are in support of allowing bidding on trademarked keywords by competitors (Parker 2011), yet still employ keyword relevance measures to handicap poaching firms. We also present empirical and anecdotal support for our key results, and several extensions of our basic model to show the robustness of our results.

A growing theoretical and empirical literature on sponsored search advertising has enhanced our understanding of its different aspects; this includes Athey and Ellison (2009), Chan and Park (2010), Desai et al. (2011), Ghose and Yang (2009), Jerath et al. (2011), Katona and Sarvary (2010), Rutz and Bucklin (2011), Yang and Ghose (2010), Yao and Mela (2009) and Zhu and Wilbur (2011). Our work is distinctly different from the above work because they consider sponsored search advertising in isolation while we model it in a multichannel advertising setting. Goldfarb and Tucker (2011a,b) study substitution between online and offline advertising induced by better targeting in online advertising and advertising bans on offline advertising for certain products. Joo et al. (2011) and Zigmond and Stipp (2010) empirically show that television advertising increases Internet search volume; we use their finding as a building block in our model. Kim and Balachander (2010) model sponsored search in a multichannel setting. However, they do not consider poaching behavior of competing firms, and the resulting strategic response of the search engine (in terms of auction design). In our research, the analysis of these two aspects leads to a rich set of results and insights which have anecdotal support.¹

The rest of the paper is structured as follows. In Section 2, we describe the model. In Section 3, we analyze the model with symmetric firms to develop key insights, and in Section 4, we extend to the case of asymmetric firms. In Section 5, we analyze keyword relevance scores as a strategic

¹We note that “downstream advertising,” where the aim is to reach customers expected to have a high likelihood of purchase, is also possible in certain channels other than sponsored search. For instance, firms may advertise in yellow pages to reach customers when they are specifically looking for the provider of a product or service before making a purchase. However, targetability is weak in yellow pages which makes it difficult to poach a competitor’s customers; for instance, among the customers who are consulting yellow pages, firms cannot distinguish between those who are interested in a competitor versus those who are already interested in the firm itself. Similarly, “checkout coupons” used in retail stores, powered by technology from providers such as Catalina Marketing, target customers based on their profiles (purchase history, gender, location, etc.). This allows targeting consumers who purchased a competitor’s product in a category (Pancras and Sudhir 2007). However, in this case, the identification of the customer and subsequent targeting is after the current purchase is made (with the idea of making the customer switch at the next purchase occasion), which makes poaching less effective. Sponsored search, on the other hand, makes for a unique combination of features that make it an extremely effective channel for poaching—based on the keyword searched consumers self-identify whether they are interested in the competitor, the firm itself, or the category; the consumers can be targeted after they are interested in the product but still before the purchase; based on the keyword searched, different consumers can be targeted differently by showing them different ad copies and landing pages upon clicking.

device used by a search engine to control poaching. In Section 6, we present some empirical and anecdotal evidence supporting our main results. In Section 7, we consider several extensions of the basic model and show that the key insights are unchanged under each extension, while we obtain additional interesting results. In Section 8, we conclude with a discussion and lay out some possible directions for future research.

2 Model

Our model consists of three entities: the firms, the users, and the search engine. We start with a simple model with symmetric firms to communicate some key concepts and insights, and later proceed to the case of asymmetric firms. Two firms, Firm 1 and Firm 2, produce identical products. Each firm has an exogenously specified total budget B allocated for advertising,² and has to decide how to allocate its advertising budget to traditional advertising and sponsored search advertising to maximize total sales. For Firm i , we denote the money spent on traditional advertising by T_i and the money spent on sponsored search by S_i , where $T_i + S_i = B$. Note that we bundle all non-sponsored search channels of advertising together into traditional advertising.

As discussed earlier, we focus on the awareness-creating aspect of traditional advertising. When Firm i spends T_i on traditional advertising, it generates awareness for its product among $(1 + \alpha)T_i$ customers, where $\alpha > 0$.³ These customers either buy the product directly or search for the product at a search engine. Each firm is associated with a specific keyword which consumers use to search for it on the search engine. For instance, if Apple sells the iPad and Samsung sells the Galaxy Tab, then the keywords associated with Apple and Samsung are “iPad” and “Galaxy Tab,” respectively.⁴ The transaction of a customer who searches the product is either influenced by the sponsored links or not. In our model, a customer who purchases directly from the firm after being exposed to a traditional ad is equivalent to a customer who searches before purchasing but is not influenced by the sponsored search results (e.g., is influenced only by the organic results). Without loss of generality, we assume that all the customers who search at the search engine are influenced

²We make the budget endogenous in Section 7.4 and confirm that the results of our basic model are robust.

³Note that we are implicitly assuming that advertising response function is linear; however, our results apply to other response functions proposed in the literature such as concave and S-shaped functions as well. Details of this analysis are available by request.

⁴In section 7.1, we consider the extension in which there is a third keyword which is the category keyword, such as “tablet” in the above example.

by sponsored search results. Specifically, we assume that out of the $(1 + \alpha)T_i$ customers made aware by traditional advertising, αT_i customers buy the product independent of what they see in sponsored search, and the remaining T_i customers search for Firm i 's keyword at a search engine and purchase the product that they see advertised in the sponsored section of the search results (which may or may not be Firm i 's product).

For simplicity, we assume that there is only one advertising slot available for each keyword, i.e., only one firm is shown in response to a keyword search. This simplification does not impact the insights from the model. When a customer searches for a keyword, the search engine uses a pay-per-click second-price auction with exogenous reserve price R to sell the advertising slot for that keyword to the firm that bids higher. However, when a consumer clicks on the sponsored link, the winner has to pay the loser's bid or the reserve price, whichever is higher.

Our assumption is that out of the $(1 + \alpha)T_i$ customers who are exposed to traditional advertising, only αT_i make a purchase directly; the remaining T_i customers overflow to the sponsored search channel and all of them purchase from the firm whose ad they see in response to their search. Therefore, the customers who are exposed to traditional advertising and are "upstream" in the AIDA framework have a smaller purchase-conversion rate (conversion rate equal to $\alpha/(1 + \alpha)$) than the customers who are "downstream" in the AIDA framework and are exposed to sponsored search advertising (conversion rate equal to 1). Consequently, there is a trade-off between traditional advertising and sponsored search. On the one hand, the firm can decide to create awareness through traditional advertising and obtain some direct sales. On the other hand, the firm may choose to advertise on the competitor's keyword in sponsored search and rely on the awareness created by the competitor.

Key Intermediate Result

Due to each firm's traditional advertising, T_i customers search keyword i at the search engine. These customers arrive sequentially at the search engine and it runs a separate auction for each customer. In other words, the search engine sequentially runs T_i auctions for each keyword. In each auction, the firms submit their bids simultaneously. Each firm decides its bid in an auction based on the budget it has allocated for the keyword and how much of it is remaining when the customer arrives. Using subgame-perfect equilibrium, we show in Theorem A2 in the appendix

that the unique outcome of this sequential second-price auction coincides with the outcome of a *market-clearing-price mechanism*. We state this result in the lemma below.

Lemma 1 *Assume that Firm 1 spends L_1 and Firm 2 spends L_2 on keyword i searched by T_i consumers. If $L_1 + L_2 \geq T_i R$ then $L_1 / (L_1 + L_2) \cdot T_i$ customers purchase from Firm 1, and $L_2 / (L_1 + L_2) \cdot T_i$ customers purchase from Firm 2. If $L_1 + L_2 < T_i R$ then L_1 / R customers purchase from Firm 1 and L_2 / R customers purchase from Firm 2.*

This result is interesting in itself and, to the best of our knowledge, is new to the auctions literature. This is also a very useful result as, for the analysis in the rest of the paper, it allows us to reduce bidding in a complicated sequential auction to a much simpler form that abstracts away from the auction and, in fact, represents a simple market-clearing allocation. In the analysis to follow, rather than modeling bidding between competitors in every scenario, we will simply use this lemma repeatedly.

Defining the Firms' Strategies

In general, a firm's strategy is any splitting of its advertising budget between the traditional channel, its own keyword in sponsored search, and the competitor's keyword in sponsored search. For simplicity, we restrict the strategy space of the firms to three strategies, each focusing on one of the three channels. This assumption simplifies the analysis without sacrificing any insights obtained from the model. Specifically, we allow a firm to follow one of the following three pure strategies. A firm's strategy can also be a mixed strategy, meaning that each of the strategies below will be played with a certain probability.

1. **Own Keyword Focus (Own):** The firm focuses on its own keyword in sponsored search advertising, trying to target the consumers who are in late stages of their purchase processes.
2. **Traditional Focus (Traditional):** The firm focuses on traditional advertising, trying to create awareness and interest about the product.
3. **Poaching Focus (Poach):** The firm focuses on the competitor's keyword in sponsored search advertising, trying to steal potential customers of the competitor.

Let T^O , T^T and T^P be the amount of money that a firm spends on traditional advertising when using the Own, Traditional and Poach strategies, respectively;⁵ the superscripts O , T and P stand for Own, Traditional and Poach respectively. We now consider these strategies one by one.

In the *Own strategy*, the firm focuses on its own keyword. A natural definition would be to assume that the firm spends all of its budget for advertising on its own keyword in sponsored search. However, this implies that the keyword will have zero search volume because nothing has been spent on awareness-generating traditional advertising, which implies that there will be no revenue. In other words, even when the firm wants to “maximally focus” on its own keyword, it should not spend all of its budget on its keyword in sponsored search, and should use some of its budget for traditional advertising to generate the necessary search volume for its keyword. In fact, we will define the budget allocation in the Own strategy in such a way that *any* strategy that allocates more budget to the firm’s own keyword as compared to the Own strategy will always be *weakly dominated* by the allocation of the Own strategy. In this sense, the “Own Keyword Focus” strategy has “maximal focus” on the firm’s own keyword. We now derive this allocation.

According to the second-price auction of the search engine, the firm has to pay at least R per customer in sponsored search advertising. And according to the model, if it spends T_i on traditional advertising, T_i customers would search the product. Therefore, the firm has to spend at least $T^O = \frac{B}{R+1}$ on traditional advertising even if it wants to focus on sponsored search advertising of its own keyword. Consequently, the firm spends $B - T^O = \frac{RB}{R+1}$ on its own keyword when using Own strategy. It can be proved that spending more than $B - T^O$ for sponsored search advertising of own keyword is *weakly dominated* by spending $B - T^O$.

In the *Traditional strategy*, the firm focuses on traditional advertising. Similar to the Own strategy, we can show that spending all of its budget on traditional advertising is a dominated strategy. As before, we define the Traditional strategy in a way that the firm has “maximal focus” on traditional advertising, i.e., under no conditions should the firm have an incentive to allocate more to traditional advertising than what it allocates in this strategy. Suppose that the firm has budget B and its competitor has budget B' . In the Traditional strategy, the amount of budget that

⁵Note that if $R > 1/\alpha$, then it is a strictly dominant strategy for the firms to spend all of their budget on traditional advertising. In other words, if the reserve price is so high that the cost of attracting a customer in sponsored search is higher than the cost in traditional advertising, the firms should spend all of their budgets on traditional advertising in any strategy. In reality, this situation is unlikely to happen because it means that the search engine has set the reserve price of sponsored search advertising so high that no advertiser wants to advertise through sponsored search.

the firm spends on traditional advertising is defined as $T^T = \min(B, \frac{B+B'}{R+1}, B + B' - \sqrt{\frac{B'(B+B')}{1+\alpha}})$. It can be proved that spending more than T^T on traditional advertising is weakly dominated by spending T^T on traditional advertising. In other words, no matter what strategy the competitor uses, the return to the firm when spending T^T on traditional advertising is always greater than or equal to the return it gets when spending more than T^T on traditional advertising. Consequently, in the Traditional strategy, the firm spends $B - T^T$ for sponsored search advertising on its own keyword.

Finally, in the *Poach strategy*, the firm spends all of its budget for sponsored search advertising on the competitor's keyword. Hence, we have $T^P = 0$, i.e., no money is spent on traditional advertising.

It is critically important to note here that the above allocation of budget is *not* something that the players of the game are doing as part of the game. On the contrary, we as researchers are defining the strategies to set the restricted strategy space such that the pure strategies are undominated, i.e., when a firm is following a strategy of focusing on one of the three forms of advertising, the allocation is such that the firm spends maximum amount of budget on that form of advertising but still uses an undominated strategy. The restricted strategy space simplifies analysis while still allowing us to generate the key insights.

Also note that, for simplicity, we are not considering any pure strategies other than the above three. In Section A5 in the appendix, we consider a more general model in which we allow firms to choose any arbitrary allocation of budget among the three forms of advertising considered (traditional advertising, advertising on its own keyword in sponsored search, and poaching by advertising on the competitor's keyword in sponsored search). We show that the results and insights obtained in our basic model here are not affected. Specifically, we show that, under mild conditions on the parameters, the equilibria of this simpler model are also equilibria of the more general model. This highlights the appropriateness of the strategy space chosen here. We continue to use the simpler model because the key insights are easier to communicate with this model, and it is easier to incorporate extensions into it.

The order of moves in the model is as follows. First, the search engine announces the rules of the auction (that it is a second-price, pay-per-click auction).⁶ Second, the two firms simultaneously decide their budget allocation strategies.⁷ Third, consumers see traditional ads and a fraction $\alpha/(1 + \alpha)$ of them buy directly from the firm whose ad they saw. Fourth, the remaining consumers go to the search engine sequentially and search the keyword of the firm whose traditional ad they saw, and the sequential second-price auction is played out. Fifth, each consumer who searches, purchases from the firm that is shown to her in the sponsored search results.

Finally, note that we have assumed the price of the product to be exogenous. We make this choice to focus solely on competition between firms in the sponsored search auction, and not confound it with price competition. In Section 7.5, we allow for price competition as well and confirm that the insights we obtain from our basic analysis hold.

3 Analysis with Symmetric Firms

In this section, we examine the case of symmetric firms to develop basic insights into the problem.

3.1 Revenue Analysis

We use $\Pi^{i,j}$, where $i, j \in \{O, T, P\}$, to denote the revenue of a firm that is using strategy i while its competitor is using strategy j . For example, $\Pi^{P,O}$ denotes the revenue of a firm playing the Poach strategy whose competitor is playing the Own strategy. Note that the firms are sales maximizers.

According to the definition of the Own strategy, the revenue of a firm that is playing the Own strategy, as long as its competitor does not poach, will be $\Pi^{O,O} = \Pi^{O,T} = T^O(1 + \alpha)$. However, if the competitor poaches, by Lemma 1, its revenue will be $\Pi^{O,P} = T^O(\alpha + \frac{B-T^O}{2B-T^O})$. The search engine's revenue from a firm that is playing the Own strategy will be $B - T^O$.

According to the definition of the Traditional strategy, the revenue of a firm that is playing the Traditional strategy, as long as the other firm does not poach, will be $\Pi^{T,O} = \Pi^{T,T} = \alpha T^T + \min(\frac{B-T^T}{R}, T^T)$. However, if the competitor poaches, by Lemma 1, its revenue will be $\Pi^{T,P} =$

⁶In the "keyword relevance" extension in Section 5, the search engine also decides and announces the relevance score multiplier for a bid by a firm on a competitor's keyword.

⁷Our results do not change qualitatively if firms decide traditional and sponsored search allocations sequentially, and can observe each others' traditional advertising efforts when deciding on sponsored search advertising.

$\alpha T^T + T^T \left(\frac{B-T^T}{2B-T^T} \right)$. The search engine's revenue from a firm that is playing the Traditional strategy will be $B - T^T$.

Now consider a firm that is playing the Poach strategy. If the competitor also poaches simultaneously, no money is spent on traditional advertising and hence no customer is gained. Since firms are sales maximizers, the revenue of both will be zero, $\Pi^{P,P} = 0$. However, if the competitor plays the Own strategy, the revenue of the poaching firm will be $\Pi^{P,O} = T^O \frac{B}{2B-T^O}$. If the competitor plays the Traditional strategy, the revenue of the poaching firm will be $\Pi^{P,T} = T^T \frac{B}{2B-T^T}$. Notice that a firm should not play Poach if the competitor is playing Poach (because $\Pi^{P,P} = 0$, which is less than both $\Pi^{O,P}$ and $\Pi^{T,P}$). Similarly, a firm should not play Poaching if the competitor is playing Own (because $\Pi^{P,O} < \Pi^{O,O}$). We find that the only way that a firm can benefit from playing Poach is if the competitor plays Traditional, i.e., $\Pi^{P,O} < \Pi^{O,O}$. Furthermore, playing Poach can be beneficial only if $\Pi^{P,T} > \Pi^{O,T}$ (note that $\Pi^{O,T} > \Pi^{T,T}$ already holds), which gives the condition

$$R > \frac{1}{1+2\alpha} \left(\sqrt{2(1+\alpha)^3} - \alpha \right). \quad (1)$$

The above condition implies that poaching is profitable only if R is large enough. Intuitively, if R is small, the firm finds it more profitable to conduct its own traditional advertising and capture the customers that overflow into search at the low reservation price, thus avoiding competition with the other firm. However, as R becomes larger, the customers from sponsored search do not come cheap any more. When R is large enough, the firm finds it more profitable to free ride on the awareness generation of the other firm (i.e., not spend anything from its own budget on awareness generation) and, in fact, use all of its advertising budget to compete with the other firm in the auction. Finally, the search engine's revenue from a firm playing Poach will always be B unless the other firm is also playing Poach, in which case the search engine's revenue will be zero.

3.2 Equilibrium Analysis

Given the three pure strategies for each firm, we obtain the two-player normal-form game depicted in Table 1.

Nash Equilibria: The game has both pure- and mixed-strategy Nash equilibria. Since $\Pi^{O,O} \geq \Pi^{P,O}$ and $\Pi^{O,O} \geq \Pi^{T,O}$, both firms using Own strategy is always a pure-strategy Nash equilibrium.

		Firm 2		
		Poach	Own	Traditional
Firm 1	Poach	$(\Pi^{P,P}, \bar{\Pi}^{P,P})$	$(\Pi^{P,O}, \bar{\Pi}^{O,P})$	$(\Pi^{P,T}, \bar{\Pi}^{T,P})$
	Own	$(\Pi^{O,P}, \bar{\Pi}^{P,O})$	$(\Pi^{O,O}, \bar{\Pi}^{O,O})$	$(\Pi^{O,T}, \bar{\Pi}^{T,O})$
	Traditional	$(\Pi^{T,P}, \bar{\Pi}^{P,T})$	$(\Pi^{T,O}, \bar{\Pi}^{O,T})$	$(\Pi^{T,T}, \bar{\Pi}^{T,T})$

Table 1: Payoff matrix of the firms' strategies

If the reserve price R is large enough such that the inequality in (1) holds, since $\Pi^{P,T} \geq \Pi^{O,T} \geq \Pi^{T,T}$ and $\Pi^{T,P} \geq \Pi^{O,P} \geq \Pi^{P,P}$, one firm using Poach and the other firm using Traditional is also a pure-strategy Nash equilibrium; otherwise, it is not. Note that, even though the two firms are symmetric, the above is an asymmetric equilibrium in which one firm spends all of its budget on poaching while the other firm *accommodates* this poaching by spending a larger portion of its budget on traditional advertising (as compared to the case when the competitor is not poaching). In fact, increasing the spend on traditional advertising drives even more traffic to its keyword, which makes poaching more profitable for the other firm.

The equilibria of the game always conform to the following pattern. When R is small, only the (Own, Own) equilibrium is obtained in which no firm is poaching. As R increases and the inequality in (1) holds, two new types equilibria arise. One is the asymmetric (Poach, Traditional) equilibrium discussed above (and its counterpart, the (Traditional, Poach) equilibrium). The third type of equilibrium is a mixed-strategy equilibrium in which one firm mixes between Poach and Own, and the other firm mixes between Traditional and Own. Note that the mixed-strategy equilibrium is also an asymmetric equilibrium. As R increases further and becomes larger than $1/\alpha$, both firms allocate all their budget to traditional advertising, as explained earlier.

Figures 2(a) and 2(b) show the revenues of the firms. For clarity in the graphs for the cases with asymmetric equilibria, we designate one firm as the “poaching firm” (this firm always poaches in the (Poach, Traditional) equilibrium and mixes between Poach and Own in the mixed equilibrium) and the other firm as the “traditional firm” (this firm always uses Traditional in the (Poach, Traditional) equilibrium and mixes between Traditional and Own in the mixed equilibrium). We can observe from Figures 2(a) and 2(b) that when poaching equilibria exist, the poaching firm’s revenue is higher than in the non-poaching (Own, Own) equilibrium, while the non-poaching firm’s revenue is lower. In other words, in this asymmetric equilibrium, the non-poaching firm is accommodating the poaching firm’s free-riding behavior.

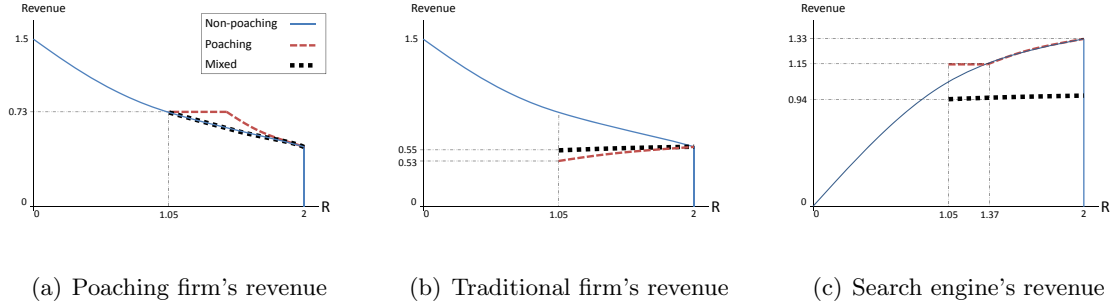


Figure 2: Revenue plots for $B = 1$ and $\alpha = 0.5$.

Search Engine's Revenue: Different equilibria of the game have different revenue expressions for the search engine. Table 2 summarizes the search engine's revenue in each of the outcomes.

		Firm 2		
		Poach	Own	Traditional
Firm 1	Poach	0	$2B - T^O$	$2B - T^T$
	Own	$2B - T^O$	$2B - 2T^O$	$2B - T^O - T^T$
	Traditional	$2B - T^T$	$2B - T^O - T^T$	$2B - 2T^T$

Table 2: Search engine's payoff matrix

The search engine's revenue is depicted in Figure 2(c). First, note that this revenue increases in R until $R = 1/\alpha$ where it drops to zero. Second, there is no poaching for low values of R . For high values of R , multiple equilibria exist and the search engine's revenue is the same from the (Own, Own) and the (Poach, Traditional) equilibria, which is larger than the revenue from the mixed equilibrium. Since there is a positive likelihood of the low-revenue mixed-strategy equilibrium existing, the existence of poaching may lower the search engine's revenue for high values of R (even though poaching is increasing competition in the search engine's auction). This happens because the traditional firm allocates more of its advertising budget to traditional advertising. The following proposition summarizes this important result.

Proposition 1 *Symmetric firms may follow asymmetric budget allocation strategies in which one firm allocates a larger fraction of its advertising budget to poaching on its competitor in sponsored search advertising, while the other firm allocates a larger fraction of its advertising budget to awareness-generating traditional advertising. Furthermore, the search engine's revenue may decrease in the presence of poaching.*

4 Asymmetric Firms with Different Advertising Budgets

In this section, we assume that the two firms have different advertising budgets. Without loss of generality, through scaling, we assume that the budget of the smaller-budget firm is 1, and the budget of the larger-budget firm is $B \geq 1$.⁸ For expositional simplicity, from this point on we call the smaller-budget firm as the “weak” firm, denoted by subscript the W , and the larger-budget firm as the “strong” firm, denoted by the subscript S .

Definitions of Strategies

In this asymmetric firms case, we rederive the budget allocations for the Own, Traditional and Poach strategies for the strong and weak firms. As in Section 2, the budget allocation is based on the idea that when a firm is focusing on one of the three options (namely, Own, Traditional and Poach), the allocation is such that the firm spends the maximum amount of budget on that option but still uses an undominated strategy. Accordingly, for the Own strategy, we have $T_W^O = \frac{1}{R+1}$ and $T_S^O = \frac{B}{R+1}$. For the Traditional strategy, we have $T_W^T = 1+B - \sqrt{\frac{B(1+B)}{1+\alpha}}$, if $\frac{B+1}{R+1} \geq 1+B - \sqrt{\frac{B(1+B)}{1+\alpha}}$, and $T_W^T = \frac{B+1}{R+1}$ otherwise. Similarly, we have $T_S^T = 1+B - \sqrt{\frac{1+B}{1+\alpha}}$, if $\frac{B+1}{R+1} \geq 1+B - \sqrt{\frac{1+B}{1+\alpha}}$, and $T_S^T = \frac{B+1}{R+1}$, otherwise. For the Poach strategy, we have $T_W^A = T_S^A = 0$.

Revenue Analysis

For the Own strategy, if no firm poaches on the keyword of the other, the situation is very similar to the symmetric case. Particularly, we have $\Pi_W^{O,O} = \Pi_W^{O,T} = T_W^O(1+\alpha)$ and $\Pi_S^{O,O} = \Pi_S^{O,T} = T_S^O(1+\alpha)$. If the strong firm poaches, $\Pi_W^{O,P} = T_W^O(\alpha + \frac{1-T_W^O}{B+1-T_W^O})$. Similarly, if the weak firm poaches $\Pi_S^{O,P} = T_S^O(\alpha + \frac{B-T_S^O}{1+B-T_S^O})$. For the Traditional strategy, the revenue of the weak firm, if the other firm poaches, is $\Pi_W^{T,P} = \alpha T_W^T + T_W^T \frac{1-T_W^T}{B+1-T_W^T}$. However, if the strong firm does not poach, the revenue of the weak firm is $\Pi_W^{T,O} = \Pi_W^{T,T} = \alpha T_W^T + \min(T_W^T, \frac{1-T_W^T}{R})$. The revenue of the strong firm is $\Pi_S^{T,P} = T_S^T \alpha + (\frac{B-T_S^T}{1+B-T_S^T})T_S^T$, and $\Pi_S^{T,O} = \Pi_S^{T,T} = \alpha T_S^T + \min(T_S^T, \frac{B-T_S^T}{R})$. For the Poach strategy, for the strong firm, we have $\Pi_S^{P,P} = 0$, $\Pi_S^{P,O} = T_W^O(\frac{B}{B+1-T_W^O})$ and $\Pi_S^{P,T} = T_W^T(\frac{B}{B+1-T_W^T})$.

⁸Note that we define “smaller” and “larger” budget with respect to the campaigns that the firms are running at a given time, not their total advertising budgets. For instance, in the Skechers and Reebok example in the introduction, around the time of Super Bowl 2011, Skechers spent millions of dollars to advertise on TV during Super Bowl and generate internet traffic for its keyword while Reebok did not; therefore, in this case Skechers would be the larger-budget firm even though Reebok may have a larger annual advertising budget.

For the weak firm, we have $\Pi_W^{P,P} = 0$, $\Pi_W^{P,O} = T_S^O(\frac{1}{1+B-T_S^O})$, $\Pi_W^{P,T} = T_S^T(\frac{B}{1+B-T_S^T})$.

Equilibrium Analysis

We solve the game with asymmetric firms fully analytically. We provide the details in Section A1 in the appendix, and highlight the interesting results and insights here. When describing equilibria, for brevity, we assume that the first firm is the weak firm, and the second firm is the strong firm. For example, by (Poach, Traditional) equilibrium we refer to an equilibrium in which the weak firm uses Poach and the strong firm uses Traditional. We will also see two types of mixed equilibria. In the first mixed equilibrium, which we call the Weak-Poach-mixed equilibrium, the weak firm mixes between Poach and Own, and the strong firm mixes between Traditional and Own. In the second mixed equilibrium, which we call the Strong-Poach-mixed equilibrium, the weak firm mixes between Traditional and Own, and the strong firm mixes between Poach and Own.

Different equilibria that arise for different values of the strong firm’s budget B and the reserve price R can be understood by jointly examining Table 3 and Figure 3. Based on the different sets of equilibria that arise in different regions of the parameter space, there are four regions in Figure 3, labeled \mathcal{A} , \mathcal{B} , \mathcal{C} and \mathcal{D} ; these equilibria are shown in the second column of Table 3. Within each region, there are sub-regions based on the impact of poaching on the search engine’s profit. This impact is indicated in the third column of Table 3—the “Y” means that poaching can increase the search engine’s profit, “N” means that poaching decreases the search engine’s profit and “Y/N” means that poaching can increase or decrease the search engine’s profit depending on equilibrium selection.⁹ Analytical expressions for R^W , R^S , R^* , R^{**} , R_m^* and \bar{R} , used to define the regions in Figure 3, and analytical definitions of low, medium and high asymmetry, are provided in Section A1 in the appendix.

We find that poaching equilibria are present in all regions except region \mathcal{A} . Furthermore, any time there is a poaching equilibrium, there is an equilibrium in which the weak firm poaches, while

⁹More precisely, “Y” means that the set of equilibria with poaching equilibria weakly dominates (from the search engine’s perspective) the set of equilibria without poaching equilibria. “N” means that set of equilibria without poaching equilibria weakly dominates the set of equilibria with poaching equilibria. “Y/N” means that the sets of equilibria with and without poaching equilibria cannot be compared, i.e., depending on equilibrium selection, either option may have higher revenue for the search engine. See the appendix for the definition of “weak dominance” to compare sets of equilibria. Also note that for region \mathcal{D} (Poach, Traditional) is the unique equilibrium, so we can trivially say that poaching increases search engine’s revenue. For region \mathcal{A} , we cannot make such a comparison since there is no poaching equilibrium, and we use the symbol “–” to denote this.

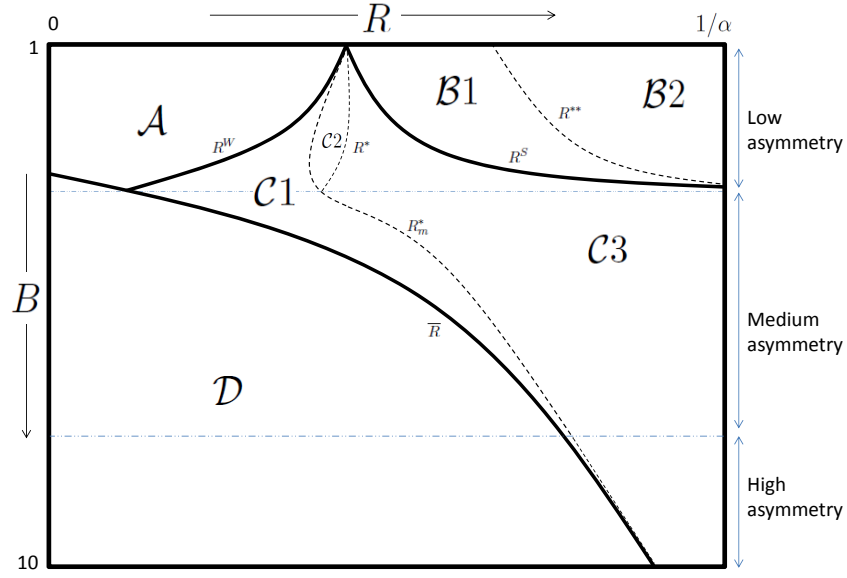


Figure 3: Poaching regions and search engine's revenue for different levels of budget asymmetry and different values of reserve price.

Region	Equilibria	Poaching Beneficial for Search Engine
\mathcal{A}	(O, O)	–
\mathcal{B}	(O, O), (P, T), (T, P), Weak-Poach-mixed, Strong-Poach-mixed	$\mathcal{B}1$: Y/N $\mathcal{B}2$: N
\mathcal{C}	(O, O), (P, T), Weak-Poach-mixed	$\mathcal{C}1$: Y $\mathcal{C}2$: Y/N $\mathcal{C}3$: N
\mathcal{D}	(P, T)	Y

Table 3: Description of the regions in Figure 3

the strong firm poaches only in region \mathcal{B} . We also show that the relative gain from poaching of the weak firm is larger than that of the strong firm (i.e., $\frac{\Pi_W^{P,T}}{\Pi_W^{O,\sigma}} \geq \frac{\Pi_S^{P,T}}{\Pi_S^{O,\sigma}}$). Moreover, the weak firm's incentive to poach increases with increasing budget asymmetry (i.e., $\frac{\Pi_W^{P,T}}{\Pi_W^{O,\sigma}}$ is an increasing function of B). This is intuitive because the strong firm, which uses the Traditional strategy in response to poaching by the weak firm, has a relatively large search volume and spends a large amount of money on its own keyword; therefore, given the market-clearing-price rule from Lemma 1, poaching by the weak firm does not significantly affect the sponsored search price for this keyword, and in turn, allows poaching at a relatively low price. In fact, if the firms are quite asymmetric, the incentive to poach is so high that (Poach, Traditional) is the only equilibrium of the game (region \mathcal{D}). We

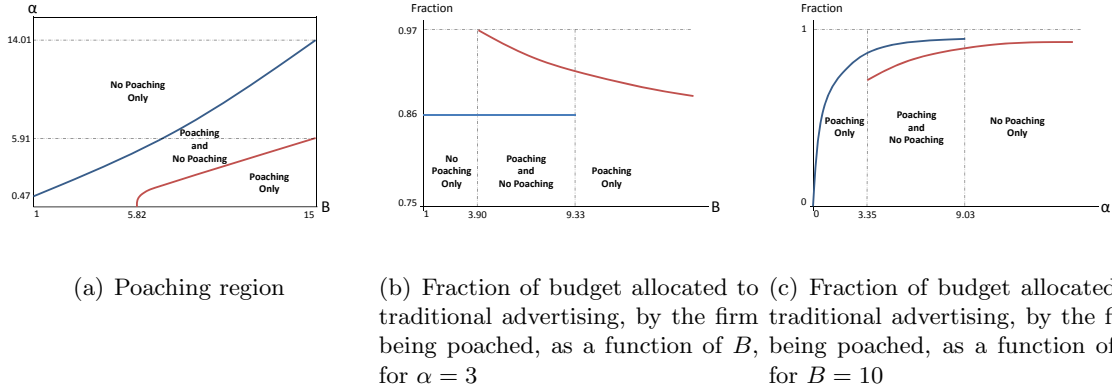


Figure 4: Poaching region and the strategy of the firm being poached.

state this in the proposition below.

Proposition 2 *The lower-budget firm has a larger incentive to poach on the higher-budget firm, and the higher-budget firm’s best response is to accommodate this poaching rather than to retaliate.*

In the case of budget asymmetry, poaching may be beneficial for the search engine. Intuitively, this is because the weak firm can only steal a small fraction of the strong firm’s customers. As a result, the strong firm’s response to the weak firm’s poaching is not as significant as in the case of symmetric firms, i.e., the strong firm does not shift a lot of its budget from sponsored search to traditional advertising in response to poaching. On the other hand, the weak firm spends all of its money on sponsored search. Hence, the search engine’s revenue may increase in the presence of poaching. We state this in the proposition below.

Proposition 3 *If the asymmetry in the advertising budgets of firms is large enough, poaching increases the search engine’s revenue.*

We now examine the firms’ strategies as functions of B (the budget asymmetry between firms) and α (the relative number of consumers who purchase without using sponsored search after viewing a traditional ad) for given R . The plot in Figure 4(a) is representative of the regions in which poaching occurs in the B - α space (for reserve price small enough). Two interesting observations can be made from this figure. First, for a given level of α , poaching happens only if budget asymmetry is large enough. Intuitively, poaching becomes more attractive for a firm as its competitor’s search volume becomes larger. Second, for a fixed level of B , poaching does not happen if α is large enough,

i.e., if the proportion of the consumers who are influenced by sponsored search is small, trying to compete for and poach on those consumers is not a good strategy.¹⁰ These two observations are summarized in the following proposition.

Proposition 4

- (a) For a given proportion α of consumers who are not influenced by sponsored search advertising, poaching happens only if the budget of one firm is enough larger than the budget of the other firm.
- (b) For a given level of budget asymmetry B between the firms, poaching happens only if the proportion of consumers who are influenced by sponsored search advertising is large enough.

Figure 4(b) and Figure 4(c) show the strong firm’s budget allocation to traditional advertising as function of B and α , respectively. Because of the existence of multiple equilibria (poaching and non-poaching), there are two curves depicting the advertising strategy of the firm, each corresponding to one equilibrium. In Figure 4(b), if $B < 3.90$, there is no poaching, if $3.90 \leq B < 9.33$, both poaching and non-poaching equilibria exist, and if $B \geq 9.33$, only the poaching equilibrium exists. Interestingly, the strong firm’s strategy is not monotone in B . The jump in the percentage of budget allocated to traditional advertising in the poaching equilibrium is because the strong firm switches to the Traditional strategy from the Own strategy. After the jump, we see that the percentage gradually decreases as B increases. This is because as the strong firm’s budget increases, it is hurt less by poaching, and its response to poaching is moderated.

In Figure 4(c), if $\alpha < 3.35$, only the poaching equilibrium exists, if $3.35 \leq \alpha < 9.03$, both poaching and non-poaching equilibria exist, and if $\alpha > 9.03$, there is no poaching. Note that within each regime (poaching or non-poaching), the percentage of budget allocated by the stronger firm to traditional advertising increases with α , as expected. However, when switching from the poaching to the non-poaching regime, the percentage of budget that the strong firm allocates to traditional advertising suddenly drops, because the strong firm changes its strategy from Traditional to Own. However, the percentage again increases as α increases further.

¹⁰Mathematically, for poaching to happen we need $\Pi_W^{P,T} \geq \Pi_W^{O,O}$. In other words, poaching must be more profitable than the Own strategy for the weak firm to poach, given that the strong firm uses Traditional. It is easy to verify that $\Pi_W^{P,T} - \Pi_W^{O,O}$ is increasing in B and decreasing in α .

5 Keyword Relevance Measures

All the major search engines, including Google, Yahoo! and Bing, transform an advertiser’s submitted bid into an *effective bid* before determining the outcome of the sponsored search auction. A multiplier is typically used to compute the effective bid, and this multiplier depends on many parameters including the advertiser’s past performance in terms of the click-through rate, the quality and reputation of the advertiser’s product or website, and the *relevance of the keyword* being bid on to the advertiser. Our focus here is on the last parameter and we explain it using the example below.

Consider the keyword “iPad” and the two firms Apple and Samsung. Apple is much more relevant to this keyword than Samsung, since Apple produces the iPad while Samsung only sells a competing product, namely Galaxy Tab, in the same category (electronic tablets). Therefore, if the relevance of Apple to the keyword “iPad” is 1 on a scale from 0 to 1, the relevance of Samsung to this keyword should be less than 1 and is, say, 0.5. For simplicity, assume that both firms have the same scores on other parameters used as multipliers for calculating the effective bid (click-through rate, quality reputation, etc.). Suppose that Apple bids \$1 per click and Samsung bids \$1.5 per click to be displayed in response to the keyword “iPad.” It seems natural that the search engine should prefer to display Samsung instead of Apple in this case (assuming only one is displayed) as Samsung should generate more revenue than Apple for it. However, surprisingly, in a situation like this, the search engine calculates Samsung’s effective bid as $\$1.5 \times 0.5 = \0.75 and Apple’s effective bid as $\$1 \times 1 = \1 ; Apple wins the keyword auction and has to pay only \$0.75 per click. Essentially, the search engine decides Apple as the winner because of higher relevance to the keyword. In fact, Samsung will have to bid and pay at least $\$1/0.5 = \2 to win this keyword.

One explanation for the existence of “relevance measures” is that the search engine wants to improve user experience by showing ads most directly relevant to the keyword searched by the users. Although this is a reasonable explanation, we argue that it is probably not the only explanation. We provide an alternative explanation—a search engine may use keyword relevance measures to handicap poaching selectively and to the extent it wants, and increase its revenue in the process. To simplify and focus on the effect of relevance measures, we assume that both firms have the same click-through rate, and the same quality and website reputation; this assumption does not impact

our results qualitatively.

We assume that if a firm wants to bid on the keyword of the other firm, its bid (i.e., the bid of the poaching firm) will be multiplied by $0 \leq \gamma \leq 1$. If $\gamma = 1$, we are in the framework that we have been in so far, i.e., firms poach on each others' keywords without any handicap. On the other extreme, if $\gamma = 0$, firms cannot bid on each others' keywords, which effectively implies that bidding on trademarked keywords is not possible. We study the effect of intermediate values of γ on the search engine's revenue. To allow for asymmetric firms, we stay with the setting where one firm has budget $B \geq 1$ while the other firm has budget 1.

Definitions of Strategies

As before, we first define the pure strategies—Own, Traditional and Poach—by deriving the budget allocations for strong and weak firms based on the condition that when a firm is following a strategy of focusing on one of the three types of advertising, the allocation is such that the firm spends the maximum from its budget on that form of advertising but still uses an undominated strategy. The Own and Poaching strategies remain the same. However, the Traditional strategy changes slightly because the firm using Traditional strategy now knows that the bid of the Poaching firm is reduced by the handicap multiplier. As a result, we have $T_W^T = 1 + \gamma B - \sqrt{\frac{\gamma B(1+\gamma B)}{1+\alpha}}$, if $\frac{\gamma B+1}{R+1} \geq 1 + \gamma B - \sqrt{\frac{\gamma B(1+\gamma B)}{1+\alpha}}$, and $T_W^T = \frac{\gamma B+1}{R+1}$, otherwise, and $T_S^T = \gamma + B - \sqrt{\frac{\gamma(\gamma+B)}{1+\alpha}}$, if $\frac{B+\gamma}{R+1} \geq \gamma + B - \sqrt{\frac{\gamma(\gamma+B)}{1+\alpha}}$, and $T_S^T = \frac{B+\gamma}{R+1}$, otherwise. The expressions are essentially the same as derived in Section 4, except that when the weak firm uses Traditional against the strong firm's Poach it takes the strong firm's budget as γB instead of B and, similarly, when the strong firm uses Traditional against the weak firm's Poach it takes the weak firm's budget as $\gamma \cdot 1 = \gamma$ instead of 1.

Revenue and Equilibrium Analysis

In this case, there is an additional initial stage in the game in which the search engine decides the value of γ . After this stage, the analysis proceeds in the same way as in the previous cases, for which we construct and analytically solve the 3×3 strategic-form game. We omit the details here and focus on the results, specifically, the value of γ chosen by the search engine and the resulting impact on poaching.

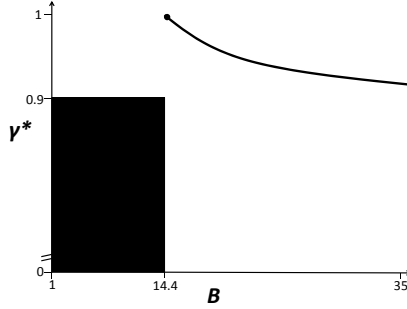


Figure 5: Optimal value of the relevance multiplier, γ , for different levels of budget asymmetry.

Figure 5 shows the optimal values of the relevance multiplier, labeled as γ^* , chosen by the search engine for different values of budget asymmetry, B . This representative plot, with $R = 1.5$ and $\alpha = 0.5$, gives the following insights. When the budget asymmetry is not too large, the search engine wants to prevent poaching by the weak firm. It achieves this by setting γ small enough (i.e., the keyword relevance multiplier reduces the poaching bid significantly). In the figure, this is the dark-shaded region, which indicates that γ can be set to any value below 0.9 for $B < 14.4$. The reason to prevent poaching for low asymmetry is the same as the reason in Section 3—the weak firm has the incentive to poach, and this poaching would make it more expensive for the strong firm to obtain a customer through sponsored search; in response the strong firm would allocate more budget to traditional advertising. This would hurt the search engine’s profit, and therefore it wants to prevent poaching by sufficiently handicapping it.

However, if $B \geq 14.4$, the search engine sets γ at a medium value to maximize its profit, as indicated by the curve in the plot. In this case, poaching is penalized to some degree, but not so much that it does not happen. Intuitively, in spite of being penalized, the weak firm poaches (to free ride on the other firm’s awareness generation), in response to which the strong firm uses Traditional and moves more of its budget to traditional awareness-generating advertising. However, the “keyword relevance penalty” protects the strong firm to some extent by reducing the effective bid of the weak firm on its keyword, thus keeping bids from escalating too high. Shielded in this way, the strong firm does not need to shift as large a portion of its money to the traditional channel (as it would have done if poaching were not penalized). However, at the same time, the weak firm has to pay a higher price per click and the search engine extracts all the budget of the weak firm. This leads to the following interesting result.

Proposition 5 *If the advertising budget of the strong firm is large enough compared to that of the weak firm, the search engine handicaps poaching by competitors but does not prohibit it.*

The above discussion also shows that the practice of using keyword relevance measures could be interpreted as a complex *price discrimination mechanism*—poaching is a very desirable option for the weak firm, and the search engine can use the keyword relevance multiplier to charge the weak firm a higher price per click than the strong firm.

To summarize, our basic model shows that firms in an industry, especially firms with relatively smaller advertising budgets, have the incentive to poach in sponsored search. The best response of competitors is to accommodate this poaching behavior. Surprisingly, even though poaching leads to more competition in the search engine’s auctions, under certain conditions it has the incentive to handicap poaching.

6 Empirical and Anecdotal Support

The examples in the introduction section clearly show the existence of poaching in sponsored search advertising. We now provide empirical and anecdotal evidence to support some of our results. The evidence presented here is not a direct test of hypotheses from our model, and is only meant to offer directional support for certain results. We leave more careful empirical analysis for future work.

Short-Term Poaching Behavior

We show that if a firm runs an effective traditional advertising campaign providing an impetus to search activity for its keywords on search engines, competitors respond with increased poaching on this firm’s keywords in sponsored search advertising. To show this, we consider the time periods before and after Super Bowl 2012. Advertising on TV during the Super Bowl is very salient and is known to reach millions of viewers. Which firms will advertise on Super Bowl in a particular year is known a few days in advance of the event. Based on the buzz in the popular press, we collected keywords related to the names of some companies and their specific products for which ads were expected to be shown during Super Bowl 2012. We also collected keywords related to the names of companies and products that are close competitors of the advertisers but were known to *not* be advertising during Super Bowl 2012. This gave us the following lists:

- Advertisers: Camry, Toyota, CR-V, Honda, Chrysler, GS 350, Lexus, Audi, Acura, Volt, Chevy, BMW (cars); Dannon (yogurt); Taxact (tax software); Go Daddy (WWW domain name registration); Etrade (online trading);
- Non-advertisers: Ford, Infiniti, Nissan, Mercedes Benz (cars); Yoplait (yogurt); TurboTax, H&R Block (tax software); Networksolutions (WWW domain name registration); TD Ameritrade, Scottrade, Fidelity (online trading).

Super Bowl 2012 was held on February 5, 2012. For each keyword we consider, for three days before and after the game was aired (using the local time of the start of the game to determine “before” and “after”) we collected data on its search volume at Google using Google Analytics (note that Google provides an index of search volume for every keyword, not the exact search volume). For these six days, we also repeatedly queried Google with each of the above keywords, roughly once every hour, and crawled all the sponsored search results.¹¹ Using the data collected, we analyzed changes in keyword traffic and the degree to which each keyword is poached by competitors, and found the following clearly discernible patterns.

For the keywords in the advertised category, the average search volume was higher by 44.8% in the three days after Super Bowl as compared to the three days before Super Bowl, while for the keywords in the non-advertised category, the average search volume showed only a modest increase of 3.7% (possibly due to spillovers). In terms of poaching activity, poaching on keywords in the advertised category increased by 55.6% on average in the days after Super Bowl, while poaching on keywords in the non-advertised category decreased by 39.4% on average. We also find that, roughly between three days to a week after Super Bowl, search traffic and poaching activity return to pre-Super Bowl levels. This analysis shows that firms that run effective short-term traditional advertising campaigns leading to an increase in online keyword search activity are poached upon more by their competitors, supporting our results.

Long-Term Poaching Behavior

We now show that, over a long time span, “weaker” firms in an industry poach more and are poached upon less, while “stronger” firms poach less and are poached upon more. We consider the

¹¹Google is known to employ algorithms to modify the ads displayed in the sponsored search list when repeated requests come from the same IP address. To circumvent this issue to the extent possible, we queried using randomized IP identities.

	Insurance	Online Trading	E-Readers	Pornography
traffic vs. being poached	0.58	0.72	0.95	0.94
traffic vs. poaching others	-0.14	-0.22	-0.33	-0.66

Table 4: Correlation table for long-term poaching behavior

following industries and firms:

- Insurance: Geico, Progressive, Allstate, Statefarm
- Online Trading: Etrade, Scottrade, TD Ameritrade, Sharebuilder
- E-Readers: Kindle, Nook, Playbook
- Pornography: Playboy, Penthouse, Hustler, Fling

To find the pattern of interest, we collect the following data. We take the names of each of the above companies as keywords searched on the search engine Google.¹² For the time period from August 2010 to November 2011, we collect two types of data for each firm. First, using Google Insights, we collect monthly search traffic on Google for every company’s keywords. Second, using the website Spyfu.com, we collect monthly poaching data for every company’s keyword. Spyfu periodically queries Google with millions of keywords and crawls the sponsored search results. From these data, for each industry, we can reconstruct who poached on whom and how frequently, and construct monthly indices for the same. (Note that all the keywords of our interest were crawled by Spyfu.)

Using the monthly data for the sixteen months, we find the correlations between: (i) keyword traffic for a firm and the frequency with which this firm’s keyword was poached by competitors in its industry, and (ii) keyword traffic for a firm and the frequency with which it poached the keywords of competitors in its industry. The resulting numbers are provided in Table 4. The first row of the table shows that, for every industry, the correlation between keyword traffic and frequency of being poached is positive. The second row of the table shows that, for every industry, the correlation between keyword traffic and frequency of poaching others is negative. Assuming that higher keyword traffic is correlated with higher levels of awareness-generating advertising (as found by Joo et al. (2011) and Zigmond and Stipp (2010) for television advertising), our analysis indicates that firms that spend more on awareness-generating advertising poach less, and vice versa, which supports our findings from the analytical model.

¹²We also consider alternative ways of spelling these names. For example, for the company Etrade, we use the keywords “Etrade,” “E-Trade” and “E Trade.”

Search Engine Policy on Poaching

The policy that search engines follow regarding poaching is somewhat perplexing—they *allow* poaching by competitors on trademarked keywords (such as brand and company names), *yet still handicap* poaching by employing keyword relevance measures. Interestingly, our result in Proposition 5 suggests that this should exactly be the policy of search engines because it gives them the flexibility to allow and make profit from poaching when beneficial to them (cases in which a smaller-budget firm has a large incentive to poach even if the search engine makes it more expensive to poach through the keyword-relevance penalty), and prevent poaching when it hurts their profit (cases in which firms’ budgets are comparable). Furthermore, we find from the model that the weak firm practices poaching and benefits from it, the strong firm is hurt from poaching, and the search engine benefits from poaching by the weak firm. These results support the observation that some leading firms in their respective industries (e.g., Rosetta Stone and Louis Vuitton) sued search engines in an effort to prevent them from following a policy of allowing bidding on trademarks by competitors (paidContent.org 2010, 2011). The search engines won these lawsuits and have continued to allow poaching on trademarked keywords; at the same time, they continue to use keyword relevance scores to handicap poaching. In the above examples, the predictions from our model are in close agreement with the actual behavior of the strong firms, the weak firms, and the search engines.

Having established empirical and anecdotal support for our results, we now proceed to some extensions of the basic model.

7 Extensions

7.1 Category Keyword

In this extension, we assume that there is a category keyword which attracts customers from the traditional advertising of both firms. For example, in the context of tablets, “iPad” and “Galaxy Tab” are keywords specific to the companies Apple and Samsung, respectively, while “tablet” is a category keyword that describes both products. Some customers who see traditional awareness-generating ads of iPad or Galaxy Tab may search the keyword “tablet” instead of searching the

product name. In accordance with this, we assume that some fraction of the customers who are exposed to traditional advertising of each firm search the category keyword instead of the firm-specific keyword. The insights obtained in Section 3 (asymmetric budget allocation strategies and reduction in search engine revenue due to poaching) also hold under this extension. More details are available in Section A2 in the appendix.

7.2 Reputation Effects

In the basic model, we assumed that a firm needs to advertise on the traditional channel to generate awareness and have non-zero search volume on the search engine. In this section, we drop this simplification and assume that a firm may have search volume even without recently-conducted awareness-generating advertising, say due to previous reputation. In other words, we assume that V customers search a firm’s product even if it does not advertise on the traditional channel. We also let the firms to be asymmetric in this aspect by assuming that the reputation-based search volume of the “strong” firm is $V > 0$ while the “weak” firm has no reputation-based search volume (i.e., the weak firm’s keyword will be searched only if it does awareness-generating advertising).

The detailed analysis of this extension is included in Section A3 in the appendix. We confirm that the insights obtained from the basic model hold. Furthermore, if V is large enough (the strong firm has much larger reputation-based search volume than the weak firm), only the weak firm wants to poach, and its incentive to poach increases with V . The strong firm, which already has high customer awareness, accommodates this poaching by spending more on traditional advertising. This gives us the following counter-intuitive result.

Proposition 6 *The firm that has larger reputation-based customer awareness spends even more on awareness-generating advertising as a best response to poaching by its competitor.*

7.3 Reduced Effectiveness of Poaching Advertisements

In the basic model, we assume that consumers purchase the product from the company whose ad they see in response to their keyword search, even if it is a poaching ad. However, it could be the case that if the name of the company or brand in the displayed ad does not match with the keyword searched, there is a smaller chance that the consumer will click on the link and convert

after clicking. Another factor contributing to the reduced effectiveness of poaching could be that the organic results in response to the keyword search will have content related directly to the original keyword. If the consumer views the organic results as well before clicking on the sponsored link, then a poaching ad may reduce the consumer’s confidence in the poaching firm. For these and other reasons, a poaching ad may be less effective than a non-poaching ad. The reduced effectiveness of poaching ads, however, does not change our results qualitatively—while poaching will become a less desirable strategy, it will still be employed as a viable strategy by firms and all the associated insights will hold (unless the reduction in effectiveness is very severe, which is not always the case). Analytically, this is similar to the case in Section 5 in which the poaching firm is handicapped. We omit the details due to space constraints.

7.4 Endogenous Budget

In this section, we relax the assumption that the advertising budget of each firm is exogenously given, and allow each firm to decide how much to spend on advertising while trying to maximize its profit. We assume that spending more on advertising becomes harder as the firm spends more (Fernandez-Corugedo 2002), reflecting the fact that it is increasingly difficult to raise larger amounts of money. The profit of Firm i is $\Pi_i = \alpha T_i + \min(T_i \frac{S_i}{S_i + P_j}, \frac{S_i}{R}) + \min(T_j \frac{P_i}{S_j + P_i}, \frac{P_i}{R}) - \eta(T_i + S_i + P_i)^\rho$, where $j = 3 - i$ represents the index of the other firm, T_i , S_i and P_i , respectively, represent the level of advertising on traditional channel, sponsored search of own keyword, and sponsored search of competitor’s keyword. The parameter $\rho > 1$ captures the fact that increasing the advertising budget becomes harder as this budget becomes larger. By numerically calculating the equilibria of this revised formulation, we confirm that the results presented in Sections 3, 4 and 5 are robust to budget endogeneity. In particular, there exist asymmetric equilibria in which one firm focuses more on traditional advertising while its competitor poaches on its keyword. The slight difference from the exogenous budget setting is that symmetric firms with endogenous budget may poach on each others’ keywords at the same time. However, there are equilibria in which the degree of poaching is different for symmetric firms, with one firm poaching more than the other one. Results on allowing but handicapping poaching by the search engine also carry over qualitatively.

7.5 Consumers' Purchase Model and Price Competition

In our basic model, we assumed that product prices are determined exogenously, and consumers purchase passively at the price offered to them by the firm whose traditional or sponsored advertisement they see most recently. In this extension, we model price competition between firms using a model in which consumers are horizontally differentiated. We assume that consumers get aware of a firm only if they see an ad of the firm, which can either be a traditional ad or an ad in sponsored search. Therefore, consumers that are poached become aware of both firms and compare prices across firms while making their purchase decisions, which leads to price competition. The consumers who see ads from only one firm do not compare prices as they are not aware of the second firm. More details of the model are available in Section A4 in the appendix.

We solve the model numerically and confirm that our original results are robust under price competition—equilibria exist in which one firm focuses on traditional advertising and the other focuses on poaching, and the search engine's revenue is maximized with a medium level of penalty on poaching. A new interesting result from this model is that the poaching firm sets a lower price than the other firm. The poaching firm can maximize the effect of poaching by pricing below its competitor because it can win more of the comparison shoppers. The firm that is being poached does not decrease the price as much because it is benefiting from the customers who are not aware of the product of the poaching firm (i.e., customers not influenced by sponsored search).

8 Conclusions and Discussion

In this paper, we study the poaching behavior of firms in sponsored search advertising. A firm can spend on traditional channels of advertising such as television, print and radio to create awareness, attract customers, and increase the search volume of its keyword at a search engine. Alternatively, a firm may limit its awareness-creating activities and spend its budget on stealing the potential customers of its competitor by advertising on the competitor's keyword in sponsored search, which we call poaching.

We find that even when firms are identical, they may follow different advertising strategies—one firm focuses on the traditional channel and spends a larger fraction of its budget for creating awareness, while the other firm spends a larger fraction of its budget on poaching. Surprisingly, the

best response of the firm being poached is to accommodate poaching rather than retaliate. Although poaching seems to be beneficial for the search engine by increasing competition for keywords, we find that it actually may decrease the search engine’s revenue. This is because poaching increases prices (bids) in sponsored search, thus increasing per-customer acquisition costs in this channel, which induces firms to spend less on sponsored search. Therefore, the search engine may increase its revenue by making poaching harder for the firms and keeping bids in check.

When the firms are asymmetric, and the advertising budget of one firm is significantly larger than the advertising budget of the other firm, there is an interesting twist in the above results. First, the stronger firm does not want to poach while the weaker firm has much more incentive to poach (as compared to the symmetric case). Furthermore, unlike the case of symmetric firms, poaching may increase the search engine’s revenue. Since the stronger firm has a large search volume, the effect of the weaker firm’s poaching is small. In other words, poaching of the weaker firm does not make sponsored search much less efficient for the stronger firm. Thus, the stronger firm keeps almost the same portion of its budget in sponsored search. On the other hand, the weaker firm does not need to create awareness and can spend its entire budget in sponsored search, which increases the search engine’s revenue.

We find that in the asymmetric firms’ case, the best strategy for the search engine is to handicap poaching but not too much so that the weak firm still prefers to poach. This handicap can be implemented by charging the poaching firm a higher price than the non-poaching firm for the same keyword. Interestingly, we see that well-known search engines, e.g., Google, Yahoo! and Bing, have already implemented such penalties through “keyword relevance” multipliers. A firm has to pay higher price than its competitor for appearing in response to the competitor’s keyword, even if it has the same click-through rate and quality measures as its competitor. By including keyword relevance measures in our model, we find that it may indeed be optimal for the search engine to use a medium level of penalty to maximize its revenue. Our results agree with the industry observations that the search engines, when sued by firms for allowing poaching, defended their practice of allowing bids on trademarked keywords, but are also penalizing poaching through keyword relevance multipliers.

We also consider various extensions of the model which confirm the robustness of our results and provide additional insights. Specifically, we consider an extension in which one firm has some search volume for its keyword even without recent awareness-generating advertising (say, because

of previous reputation). We find that, surprisingly, the firm that has higher exogenous search volume due to reputation-based customer awareness has greater incentive to invest in traditional advertising to drive even more search volume to its keyword. In another extension, where the firms compete on price, we find that the poaching firm has incentive to set a lower price than its competitor.

Our work sheds light on the poaching behavior of firms in a multi-channel advertising setting. There are many other related problems that may be studied in future work. In particular, the firms are not vertically differentiated in our model. Perhaps a joint model of our work and Desai et al. (2011), that allows differentiation in a multi-channel advertising model, would be an interesting direction for future work. Another interesting direction to consider is to understand the consequences of poaching among *partners*. For example, online travel agencies such as Orbitz bid on keywords such as “Sheraton Hotel in San Francisco,” trying to *steal and resell* the potential customers of Sheraton back to Sheraton. This poaching not only decreases Sheraton’s profit from its own customers (because it has to share a part of the revenue with Orbitz for delivering this customer), but also increases the price of sponsored search advertising for Sheraton. It would be interesting to know how partners should react to such poaching behavior.

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Appendix

A1 Derivations for the Asymmetric Firms Case

We use the following terminology for brevity. When describing equilibria, we assume that the first firm is the weak firm, and the second firm is the strong firm. For example, by (Poach, Traditional) equilibrium we mean an equilibrium in which the weak firm poaches and the strong firm uses Traditional. We will also see two types of mixed equilibria. In the first mixed equilibrium, which we call the Weak-Poach-mixed equilibrium, the weak firm mixes between Poach and Own, and the strong firm mixes between Traditional and Own. In the second mixed equilibrium, which we call the Strong-Poach-mixed equilibrium, the weak firm mixes between Traditional and Own, and the strong firm mixes between Poach and Own.

Because of the existence of multiple equilibria in our setting, we define the *weak dominance* concept to compare sets of equilibria. We say that equilibrium set S_1 weakly dominates equilibrium set S_2 , from Player P 's perspective, if for every equilibrium $e_1 \in S_1$ and $e_2 \in S_2$, Player P 's profit in e_1 is greater than or equal to her profit in e_2 , with the inequality being strict for at least one pair (e_1, e_2) . This definition is particularly useful when comparing the revenue of the search engine with and without the presence of poaching.

We start the analysis by assuming a low level of asymmetry between the firms' advertising budgets. Then, we show how the results are generalized for higher levels of asymmetry.

Low Level of Asymmetry

To aid the exposition of the derivation, we use Figure A1, which plots the firms' and the search engine's revenues for $\alpha = 0.5$ and $B = 1.5$ (which is a low asymmetry case). In Figure A1(a) we see the existence of multiple equilibria for $R > 0.60$. For $R \geq 0.6$, since $\Pi_W^{P,T} > \Pi_W^{O,O}$, the weak firm may poach on the strong firm's keyword. Similarly, for $R > 1.58$ since $\Pi_S^{P,T} > \Pi_S^{O,O}$, the strong firm may poach on weak firm's keyword. In general, let R^W be the threshold value of R for which $\Pi_W^{P,T} > \Pi_W^{O,O}$ if $R > R^W$.^{A1} Similarly, let R^S be the threshold value of R for which $\Pi_S^{P,T} > \Pi_S^{O,O}$

^{A1} R^W is the value of R at which $\Pi_W^{P,T} = \Pi_W^{O,O}$, which gives $R^W = \sqrt{\frac{(1+\alpha)^3(1+B)}{(\alpha+B+\alpha B)^2}} + \frac{1-(1+\alpha)B}{\alpha+B+\alpha B}$.

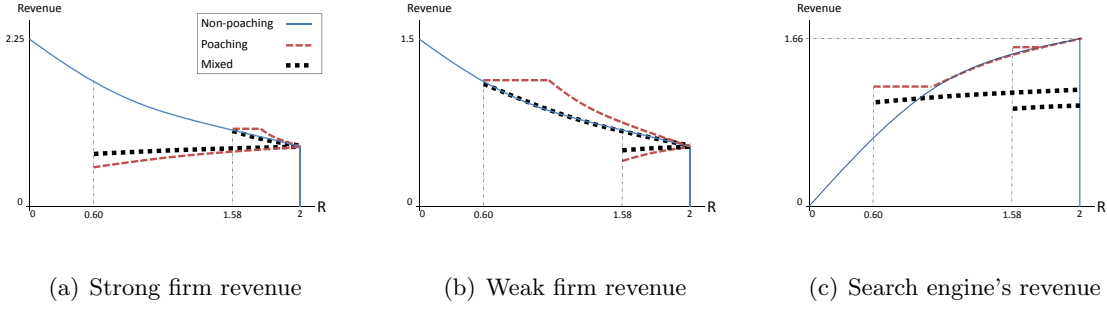


Figure A1: Revenue plots when the Strong firm has budget $B \geq 1$ and the weak firm has budget 1, for parameters $B = 1.5$ and $\alpha = 0.5$.

if $R > R^S$.^{A2} Using elementary calculus, it can be proved that $R^W < R^S$. In other words, the weak firm starts poaching for lower values of R . In the example of Figure A1, $R^W = 0.6$ and $R^S = 1.58$. When $R < R^W$, the unique equilibrium is (Own, Own). When R is between R^W and R^S , there are three equilibria: (Own, Own), (Poach, Traditional) and Weak-Poach-mixed. When R is larger than R^S , there are five equilibria: (Own, Own), (Poach, Traditional), (Traditional, Poach), Weak-Poach-mixed and Strong-Poach-mixed.

Figure A1(c) shows the revenue of the search engine for different equilibria as functions of reserve price R . In (Own, Own) equilibrium the search engine's revenue is $1 + B - T_S^O - T_W^O$. In (Poach, Traditional) equilibrium (for $R > R^W$), the search engine's revenue is $1 + B - T_S^T$. And, in (Traditional, Poach) equilibrium (for $R > R^S$), the search engine's revenue is $1 + B - T_W^T$. In the Weak-Poach-mixed equilibrium, the search engine's revenue is $1 + B - (1 - p_W^*)T_W^O - p_S^*T_S^T - (1 - p_S^*)T_S^O$, where p_W^* and p_S^* represent the probability of poaching of weak firm and Traditional of strong firm, respectively. Similarly, in the Strong-Poach-mixed equilibrium, the search engine's revenue is $1 + B - (1 - p_S^{**})T_S^O - p_W^{**}T_W^T - (1 - p_W^{**})T_W^O$, where p_W^{**} and p_S^{**} represent the probability of Traditional of weak firm and poaching of strong firm, respectively. Note that the probabilities p_W^* , p_S^* , p_W^{**} and p_S^{**} can be calculated analytically, using the equilibrium conditions, as follows.

$$p_S^* \Pi_W^{P,T} + (1 - p_S^*) \Pi_W^{P,O} = p_S^* \Pi_W^{O,T} + (1 - p_S^*) \Pi_W^{O,O} \Rightarrow p_S^* = \frac{\Pi_W^{P,O} - \Pi_W^{O,O}}{\Pi_W^{P,O} + \Pi_W^{O,T} - \Pi_W^{P,T} - \Pi_W^{O,O}}$$

$$p_S^{**} \Pi_W^{T,P} + (1 - p_S^{**}) \Pi_W^{T,O} = p_S^{**} \Pi_W^{O,P} + (1 - p_S^{**}) \Pi_W^{O,O} \Rightarrow p_S^{**} = \frac{\Pi_W^{T,O} - \Pi_W^{O,O}}{\Pi_W^{T,O} + \Pi_W^{O,P} - \Pi_W^{T,P} - \Pi_W^{O,O}}$$

^{A2} R^S is the value of R at which $\Pi_S^{P,T} = \Pi_S^{O,O}$, which gives $R^S = \sqrt{\frac{(1+\alpha)^3 B(1+B)}{(1+\alpha+\alpha B)^2}} + \frac{B-\alpha-1}{1+\alpha+\alpha B}$.

$$p_W^* \Pi_S^{T,P} + (1 - p_W^*) \Pi_S^{T,O} = p_W^* \Pi_S^{O,P} + (1 - p_W^*) \Pi_S^{O,O} \Rightarrow p_W^* = \frac{\Pi_S^{T,O} - \Pi_S^{O,O}}{\Pi_S^{T,O} + \Pi_S^{O,P} - \Pi_S^{T,P} - \Pi_S^{O,O}}$$

$$p_W^{**} \Pi_S^{P,T} + (1 - p_W^{**}) \Pi_S^{P,O} = p_W^{**} \Pi_S^{O,T} + (1 - p_W^{**}) \Pi_S^{O,O} \Rightarrow p_W^{**} = \frac{\Pi_S^{P,O} - \Pi_S^{O,O}}{\Pi_S^{P,O} + \Pi_S^{O,T} - \Pi_S^{P,T} - \Pi_S^{O,O}}$$

Let $R^* = \frac{\sqrt{\frac{1+B}{1+\alpha}}}{1+B-\sqrt{\frac{1+B}{1+\alpha}}}$. Using the expressions derived for search engine's revenue, we have that the revenue of (Poach, Traditional) equilibrium is the same as the revenue of (Own, Own) equilibrium for any value of R larger than R^* . In other words, for $R \geq R^*$, we have $1 + B - T_S^T = 1 + B - T_W^O - T_S^O$. In Figure A1(c), we have $R^* = 1.07$, showing the point where the curve representing the poaching equilibrium joins the curve representing the non-poaching equilibrium. We can similarly define $R^{**} = \frac{\sqrt{\frac{B(1+B)}{1+\alpha}}}{1+B-\sqrt{\frac{B(1+B)}{1+\alpha}}}$ to be the threshold value of R beyond which the search engine's revenue of (Traditional, Poach) equilibrium is equal to the revenue of (Own, Own) equilibrium. In other words, for $R \geq R^{**}$, we have $1 + B - T_W^T = 1 + B - T_W^O - T_S^O$. In Figure A1(c), we have $R^{**} = 1.72$, indicating the point where the curve representing the (Traditional, Poach) equilibrium joins the curve representing (Own, Own) equilibrium. When $R > R^*$ and $R < R^S$, not allowing poaching weakly dominates allowing poaching from search engine's perspective. In this region, the revenues of (Own, Own) equilibrium and (Poach, Traditional) equilibrium are the same, and larger than the revenue of the mixed equilibrium. Similarly, when $R > R^{**}$, not allowing poaching weakly dominates allowing poaching. In this region, (Poach, Traditional), (Traditional, Poach) and (Own, Own) equilibria have the same revenue for the search engine; but they are higher than the revenues of the two mixed equilibria.

Let R_m^* be the value of R at which $1 + B - T_S^O - T_W^O = 1 + B - (1 - p_W^*) T_W^O - p_S^* T_S^T - (1 - p_S^*) T_S^O$. In other words, R_m^* is the value of R at which the revenue of the search engine in the Weak-Poach-mixed equilibrium is equal to the revenue of the search engine in (Own, Own) equilibrium. In Figure A1(c), we have $R_m^* = 0.98$. For $R > R^W$ and $R < R_m^*$, revenues of the search engine from the Weak-Poach-mixed equilibrium and from (Poach, Traditional) equilibrium are larger than the revenue from (Own, Own) equilibrium. In other words, for $R^W \leq R < R_m^*$, the set of equilibria in presence of poaching (when poaching is allowed) weakly dominates the set of equilibria without poaching (when poaching is not allowed), from search engine's perspective.

To summarize, R can be in one of the following intervals:

1. $[0, R^W)$: The unique equilibrium is (Own, Own).
2. $[R^W, R_m^*)$: There are three equilibria: (Own, Own), (Poach, Traditional) and the Weak-Poach-mixed. Allowing poaching weakly dominates not allowing poaching from search engine's perspective.
3. $[R_m^*, R^*)$: There are three equilibria: (Own, Own), (Poach, Traditional) and the Weak-Poach-mixed. Search engine's revenue from the mixed equilibrium is lower than (Own, Own), and revenue of (Poach, Traditional) equilibrium is higher than (Own, Own) equilibrium.
4. $[R^*, R^S)$: There are three equilibria: (Own, Own), (Poach, Traditional) and the Weak-Poach-mixed. Not allowing poaching weakly dominates allowing poaching from search engine's perspective.
5. $[R^S, R^{**})$: There are five equilibria. Search engine's revenue may be lower or higher in presence of poaching, depending on equilibrium selection.
6. $[R^{**}, \frac{1}{\alpha}]$: There are five equilibria. Not allowing poaching weakly dominates allowing poaching from search engine's perspective.
7. $(\frac{1}{\alpha}, \infty)$: Both firms spend all of their budget on traditional channel. Search engine's revenue is 0.

In case of symmetric firms, $R^W = R^S$. In other words, intervals 2, 3 and 4 do not exist.

Medium and High Levels of Asymmetry

On increasing the degree of budget asymmetry, the results remain qualitatively similar. However, there are interesting effects on the size and location of the intervals of R . The first interesting observation is that R^S is increasing in B . In other words, as budget asymmetry increases, the intervals in which the strong firm poaches on weak firm's keyword (intervals 5 and 6) shrink. If B is large enough, R^S becomes larger than $1/\alpha$. In other words, if one firm is enough larger than the other firm, the strong firm does not poach on the weak firm's keyword under any condition. The reverse of this effect exists for R^W . As B increases, R^W decreases. If B is large enough, R^W becomes zero. In other words, if one firm is enough larger than the other firm, weak firm

poaching on strong firm's keyword is always an equilibrium. These changes in interval thresholds are consistent with Proposition 2 that says weak firm's incentive to poach increases and strong firm's incentive to poach decreases as budget asymmetry increases.

Mathematically speaking, if $B \geq \frac{1}{\alpha}$ then $R^S > \frac{1}{\alpha}$. Under this condition, strong firm does not poach on weak firm's keyword. Furthermore, $B \geq \frac{3+3\alpha+\alpha^2}{1+\alpha}$ implies $R^W = 0$. Under this condition, weak firm poaching on strong firm's keyword is always an equilibrium. We define the values of B where $B < \max(\frac{1}{\alpha}, \frac{3+3\alpha+\alpha^2}{1+\alpha})$ as *low* level of budget asymmetry. For such values of B , the results are what we discussed in the previous section. However, when $B \geq \max(\frac{1}{\alpha}, \frac{3+3\alpha+\alpha^2}{1+\alpha})$ we have *medium* or *high* level of asymmetry.

For medium level of asymmetry, R can be in one of the following intervals.

1. $[0, \bar{R})$: There is one equilibrium: (Poach, Traditional). Allowing poaching has the same revenue as not allowing poaching for the search engine.
2. $[\bar{R}, R_m^*)$: There are three equilibria: (Own, Own), (Poach, Traditional) and Weak-Poach-mixed. Allowing poaching weakly dominates not allowing poaching from search engine's perspective.
3. $[R_m^*, \frac{1}{\alpha}]$: There are three equilibria: (Own, Own), (Poach, Traditional) and Weak-Poach-mixed. Not allowing poaching weakly dominates allowing poaching from search engine's perspective.
4. $(\frac{1}{\alpha}, \infty)$: Both firms spend all of their budget on traditional channel. Search engine's revenue is zero.

A condition that could not exist for low level of asymmetry and could occur for high level of asymmetry is $\Pi_W^{P,O} > \Pi_W^{O,O}$. In other words, if the firms are asymmetric enough, even if the strong firm uses Own strategy, the weak firm prefers to poach. In this situation, (Own, Own) cannot be an equilibrium anymore. Using simple calculus, we see that this condition is satisfied if $B \geq 1 + \alpha$ and $R < \frac{-1-\alpha+B}{1+\alpha+\alpha B}$. Define $\bar{R} = \frac{-1-\alpha+B}{1+\alpha+\alpha B}$. Note that \bar{R} converges to $\frac{1}{\alpha}$ as B increases. This means that for large enough values of B , the only equilibrium is when weak firm poaches on strong firm's keyword for *almost* all values of R (except, of course for $R > \frac{1}{\alpha}$ where no firm uses sponsored search advertising at all). In summary, for high level of asymmetry, R can be in one of

the following intervals.

1. $[0, \bar{R})$: There is one equilibrium: (Poach, Traditional). Allowing poaching has the same revenue as not allowing poaching for the search engine.
2. $[\bar{R}, \frac{1}{\alpha}]$: There are three equilibria: (Own, Own), (Poach, Traditional) and Weak-Poach-mixed. Not allowing poaching weakly dominates allowing poaching from search engine's perspective.
3. $(\frac{1}{\alpha}, \infty)$: Both firms spend all of their budget on traditional channel. Search engine's revenue is 0.

A2 Category Keyword

We extend our model and assume that there exists a category keyword which some customers search after viewing a traditional ad by either firm. Therefore, there are three categories of customers: (1) customers who purchase the product directly after seeing the traditional ad without being influenced by sponsored search, (2) customers who search the product keyword online after viewing a traditional ad and purchase from the firm listed at the top in the sponsored search results, and (3) customers who search the category keyword online after viewing a traditional ad and purchase from the firm listed at the top in the sponsored search results. We assume that the “scaled probability” that a customer is in Category 1 is α , in Category 2 is 1, and in Category 3 is β . Therefore, if a firm spends x on traditional advertising, there will be αx , x and βx customers in Categories 1, 2 and 3, respectively.

Before proceeding further, we state and prove the following lemma, which we use subsequently.

Lemma A1 *Suppose that there are two investment options. The revenue of the first one has the functional form $\alpha Q \frac{x}{\alpha C + x}$ if x is invested, while the revenue of the second one is $\beta Q \frac{x}{\beta C + x}$. Then, the optimal way to split x between the two options is to invest $\frac{\alpha x}{\alpha + \beta}$ in the first one and $\frac{\beta x}{\alpha + \beta}$ in the second one.*

Proof: The proof directly follows from first-order conditions, and the fact that, for any $x \geq 0$ and fixed $C \geq 0$, the function $\frac{x}{C+x}$ is monotonically increasing and concave in x . \square

Definitions of Strategies

We rederive the budget allocations for strong and weak firms based on the core idea that when a firm is following a strategy of focusing on one of the three types of advertising, the allocation is such that the firm spends maximum amount of budget on that form of advertising but still uses an undominated strategy.

Let C^J be the amount spent on the category keyword in sponsored search in strategy J , where $J \in \{N, P, D\}$; all other notation is carried over from the basic model.

Own Strategy: First, assume that there is only one firm in the market. If the firm spends x in traditional advertising, to win the customers of the product keyword she has to spend at least xR on sponsored search of the product keyword, and βxR on sponsored search of the category keyword, where R is the reserve price. The optimal amount of money to be spent on traditional advertising in this case is $T^O = \frac{B}{1+R(1+\beta)}$. Consequently, since in Own strategy, the firm does not advertise on the keyword of the other firm (i.e., $S_2^O = 0$), the amount of money that it spends on sponsored search is $S_1^O + C^O = B - T^O = \frac{BR(1+\beta)}{1+R(1+\beta)}$. Since the the number of queries to the product keyword and the category keyword are proportional to 1 and β , by Lemma A1, we have $S_1^O \beta = C^O$, which gives $S_1^O = \frac{BR}{1+R(1+\beta)}$ and $C^O = \frac{BR\beta}{1+R(1+\beta)}$.

Poaching Strategy: The poaching firm's spending on its own keyword and on traditional advertising is zero, i.e., $T^P = S_1^P = 0$. Using Lemma A1, the poaching firm's spending on the competitor's keyword is $S_2^P = B/(1 + \beta)$, and on the category keyword is $C^P = \beta B/(1 + \beta)$.

Traditional Strategy: In the Traditional strategy, the firm assumes that the other firm poaches; given this assumption, the firm's revenue is $\alpha T + (B - T)/R$ if $2B - T < T(1 + \beta)R$, and is $\alpha T + T(1 + \beta)\frac{B-T}{2B-T}$ otherwise. Assuming $\alpha < (1 + \beta)/R$, the optimal solution to this problem is $T^T = B(2 - \sqrt{2(1 + \beta)/(1 + \alpha + \beta)})$ if $\frac{2}{R(1+\beta)+1} \geq 2 - \sqrt{2(1 + \beta)/(1 + \alpha + \beta)}$, and $T^T = \frac{2B}{(1+\beta)R+1}$ otherwise. Since $S_1^T + C^T = B - T^T$, by Lemma A1, $S_1^T = \frac{1}{1+\beta}(B - T^T)$ and $C^T = \frac{\beta}{1+\beta}(B - T^T)$.

Revenue Analysis

Both Firms Own: If both firms choose Own strategy, the revenue of each firm is $\Pi^{O,O} = T^O(1 + \alpha + \beta)$.

Both Firms Traditional: If both firms choose Traditional strategy, the revenue of each firm is

$$\Pi^{T,T} = \alpha T^T + \min(T^T, \frac{S_1^T}{R}) + \min(\beta T^T, \frac{C^T}{R}).$$

Both Firms Poaching: The revenue of both firms in this case is of course zero, i.e., $\Pi^{P,P} = 0$.

One Firm Poaching, One Firm Own: In this case, the number of queries on the product keyword is T^O and on the category keyword is βT^O . Therefore, the Own firm's revenue is $\Pi^{O,P} = \alpha T^O + T^O \frac{S_1^O}{S_1^O + S_2^P} + \beta T^O \frac{C^O}{C^P + C^O}$, and the Poaching firm's revenue is $\Pi^{P,O} = T^O \frac{S_2^P}{S_1^O + S_2^P} + \beta T^O \frac{C^P}{C^P + C^O}$.

One Firm Traditional, One Firm Own: In this case, the number of queries on the Traditional firm's product is T^T and on the Own firm's product is T^O . Also, the number of queries on the category keyword is $\beta(T^O + T^T)$. Hence, the Own firm's revenue is $\Pi^{O,T} = \alpha T^O + T^O + \min(\frac{C^O}{R}, (\frac{C^O}{C^O + C^T})\beta(T^O + T^T))$, and the Traditional firm's revenue is $\Pi^{T,O} = \alpha T^T + \min(T^T, \frac{S_1^T}{R}) + \min(\frac{C^T}{R}, (\frac{C^T}{C^O + C^T})\beta(T^O + T^T))$.

One Firm Poaching, One Firm Traditional: In this case, the price will be greater than or equal to R for category keyword and the product keyword; therefore, $\Pi^{T,P} = \alpha T^T + T^T \frac{S_1^T}{S_1^T + S_2^P} + \beta T^T \frac{C^T}{C^T + C^P}$ and $\Pi^{P,T} = T^T \frac{S_2^P}{S_1^T + S_2^P} + \beta T^T \frac{C^P}{C^T + C^P}$.

We use the above expressions to analyze the equilibrium of the two-player normal-form game as before. We find that the results and insights from the basic model in Section 3 continue to hold.

A3 Reputation Effects

Suppose that Firm i has some exogenous search volume V_i for its keyword, which is independent of how much it has recently spent on creating awareness for its product. This may be, for instance, because of the previous reputation that the firm holds. For simplicity, we assume that $V_1 = V$ and $V_2 = 0$, i.e., V customers search the keyword of the “strong” firm (denoted by subscript S) without traditional advertising, while no customers search the keyword of the “weak” firm (denoted by subscript W) without traditional advertising. As before, we assume that spending x on awareness-generating traditional advertising creates search volume x ; therefore, if the strong firm spends x on awareness advertising, the search volume for its keyword will be $V + x$.

Definitions of Strategies

We rederive the budget allocations for strong and weak firms as before.

Own Strategy: For the weak firm, the Own strategy has not changed and is as before: $T_W^O = \frac{B}{R+1}$.

However, for the strong firm, if $B \leq VR$, we have $T_S^O = 0$; otherwise, $T_S^O = \frac{B-VR}{R+1}$.

Traditional Strategy: For the weak firm, the Traditional strategy does not change. However, notice that when the strong firm poaches, Traditional is not necessarily the best response from the weak firm as it may want to poach too. If $\frac{2}{R+1} \geq 2 - \sqrt{2/(1+\alpha)}$, $T_W^T = B(2 - \sqrt{2/(1+\alpha)})$; otherwise, $T_W^T = 2B/(R+1)$. For the strong firm, recall that in Traditional strategy the firm assumes that the other firm poaches. Given this assumption, the firm's revenue is $\alpha T + (B - T)/R$ if $2B - T < (T + V)R$, and is $\alpha T + (T + V)\frac{B-T}{2B-T}$ if $2B - T \geq (T + V)R$. Therefore, if $\frac{2B-VR}{R+1} \geq B(2 - \sqrt{\frac{2B+V}{B(1+\alpha)}})$, then $T_S^T = B(2 - \sqrt{\frac{2B+V}{B(1+\alpha)}})$; otherwise, if $2B \geq VR$, $T_S^T = \frac{2B-VR}{R+1}$; otherwise, $T_S^T = 0$.

Poaching Strategy: By definition, $T_W^P = T_S^P = 0$.

Revenue Analysis

Own Strategy: For the weak firm, as long as it is not poaching, V does not have an impact. Therefore, $\Pi_W^{O,O} = \Pi_W^{O,T} = T_W^O(1 + \alpha)$ and $\Pi_W^{O,P} = T_W^O(\alpha + \frac{B-T_W^O}{2B-T_W^O})$. For the strong firm, if $B \geq VR$, $\Pi_S^{O,O} = \Pi_S^{O,T} = T_S^O(1 + \alpha) + V$; otherwise, $\Pi_S^{O,O} = \Pi_S^{O,T} = B/R$. Similarly, if $B \geq VR$, $\Pi_S^{O,P} = T_S^O\alpha + \frac{B-T_S^O}{2B-T_S^O}(V + T_S^O)$; otherwise, $T_S^O = 0$ and hence, if $2B \geq VR$, $\Pi_S^{O,P} = \frac{B}{2B}V = V/2$; otherwise $\Pi_S^{O,P} = B/R$.

Traditional Strategy: Nothing changes for the weak firm compared to the basic model without V , which implies $\Pi_W^{T,P} = \alpha T_W^T + T_W^T \frac{B-T_W^T}{2B-T_W^T}$ and $\Pi_W^{T,O} = \Pi_W^{T,T} = \alpha T_W^T + \min(T_W^T, \frac{B-T_W^T}{R})$. For the strong firm, if $2B - T_S^T \geq (V + T_S^T)R$, $\Pi_S^{T,P} = T_S^T\alpha + \frac{B-T_S^T}{2B-T_S^T}(T_S^T + V)$; otherwise, as in the previous case, $\Pi_S^{T,P} = T_S^T\alpha + \frac{B-T_S^T}{R}$. Similarly, if $B - T_S^T \geq (V + T_S^T)R$, $\Pi_S^{T,O} = \Pi_S^{T,T} = T_S^T(1 + \alpha) + V$; otherwise, $\Pi_S^{T,O} = \Pi_S^{T,T} = T_S^T\alpha + \frac{B-T_S^T}{R}$.

Poaching Strategy: If the strong firm poaches, value of V does not affect its utility. Therefore, $\Pi_S^{P,P} = 0$, $\Pi_S^{P,O} = T_W^O(\frac{B}{2B-T_W^O})$ and $\Pi_S^{P,T} = T_W^T(\frac{B}{2B-T_W^T})$. If the weak firm poaches, if $B \geq VR$, $\Pi_W^{P,P} = V$; otherwise, $\Pi_W^{P,P} = B/R$. Similarly, if $2B - T_S^O \geq (V + T_S^O)R$, $\Pi_W^{P,O} = \frac{B}{2B-T_S^O}(V + T_S^O)$; otherwise, $\Pi_W^{P,O} = B/R$. Finally, if $2B - T_S^T \geq (V + T_S^T)R$, $\Pi_W^{P,T} = \frac{B}{2B-T_S^T}(V + T_S^T)$; otherwise, $\Pi_W^{P,T} = B/R$.

We use the above expressions to analyze the equilibrium of the two-player normal-form game as before, and do not present the details here.

A4 Consumers' Purchase Model and Price Competition

We consider a Hotelling line of length 1, with consumers distributed uniformly on it and each firm located at one end of the line. We assume that the valuation of each consumer for either firm's product is $V = 1$, and travel cost (misfit cost) along the line is $t > 0$ per unit distance traveled by a consumer. Initially, the consumers do not know about the existence of the firms/products. A firm can make consumers aware of its product through traditional advertising. More specifically, if Firm i spends T_i on traditional advertising, $(1 + \alpha)T_i$ consumers become aware of Firm i 's product, and we assume that these consumers are uniformly distributed on the Hotelling line.^{A3} After being exposed to traditional advertising, some consumers search the firm's keyword on a search engine, in response to which they may see this firm's ad or the competing firm's ad. Some of the consumers who become aware of both firms (through one firm's traditional ad and the other firm's sponsored ad) compare prices before purchasing, which leads to price competition.

The consumers who eventually purchase the product from Firm i could be in one of the following categories ($j = 3 - i$ is the index of Firm i 's competitor):

1. Exposed to traditional advertising of Firm i , not influenced by sponsored search advertising and purchase from Firm i ;
2. Exposed to traditional advertising of Firm i , influenced by sponsored search advertising, see sponsored search ad of Firm i and purchase from Firm i ;
3. Exposed to traditional advertising of Firm j , influenced by sponsored search advertising, see sponsored search ad of Firm i , do not compare prices and purchase from Firm i ;
4. Exposed to traditional advertising of Firm i , influenced by sponsored search advertising, see sponsored search ad of Firm j , compare prices and purchase from Firm i ;
5. Exposed to traditional advertising of Firm j , influenced by sponsored search advertising, see sponsored search ad of Firm i , compare prices and purchase from Firm i .

Consumers in Category 1 are not influenced by sponsored search. Consumers in Categories 2, 3, 4 and 5 are influenced by sponsored search. Price competition between firms is only due to

^{A3}For simplicity, we assume that the total consumer population is large enough that it is unlikely that a consumer is exposed to traditional advertising of both firms. Therefore, after traditional advertising, each consumer knows about at most one product.

Categories 4 and 5. This feature of the model implies that consumers who are poached, and therefore become aware of both firms, also compare prices across firms, which leads to price competition.

Let $C_{a,b}$ (where $a, b \in \{1, 2\}$) be the number of customers who are exposed to traditional advertising of Firm i and sponsored search advertising of Firm j , where each $C_{a,b}$ is a function of the firms' advertising budget allocations. There is a total of $C_{1,2} + C_{2,1}$ customers who are exposed to advertising (traditional or sponsored search) of both firms. We assume that χ fraction of them compare the prices of the two firms while $1 - \chi$ fraction purchase from the firm that is shown in sponsored search.^{A4} From Firm i 's point of view, Categories 1, 2, 3, 4 and 5 have $\alpha T_i, C_{i,i}, (1 - \chi)C_{j,i}, \chi C_{i,j}$ and $\chi C_{j,i}$ consumers, respectively. The number of consumers in each category depend on the firms' advertising budget allocations. Using the formulation of Section 2, we have $C_{i,j} = \min(T_i \frac{P_j}{S_i + P_i}, \frac{P_j}{R})$, where T_i, S_i and P_i represent how much Firm i spends on traditional advertising, sponsored search advertising of its own keyword, and poaching on competitor's keyword, respectively.

For Categories 1, 2 and 3, in which consumers do not compare prices across firms, $(1 - p_i)/t$ of the consumers purchase. For Categories 4 and 5, in which consumers compare prices, $1/2 + (p_j - p_i)/(2t)$ of the consumers purchase from Firm i (and the rest from Firm j).^{A5} Therefore, assuming that the marginal cost of production is zero, the profit of Firm i is:

$$\Pi_i = p_i \left((\alpha T_i + C_{i,i} + (1 - \chi)C_{j,i}) \frac{1 - p_i}{t} + \chi(C_{i,j} + C_{j,i}) \left(\frac{1}{2} + \frac{p_j - p_i}{2t} \right) \right).$$

The parameter χ captures the price competition between the firms due to poaching. Note that if $\chi = 0$, the model collapses to the model in Section 2 (and the optimal price is $1/2$).

We solve the above model numerically and confirm that the results presented in Sections 3 and 4 are robust under price competition. (While solving the model, we consider boundary effects as needed.) We see that symmetric firms may use different strategies in equilibrium with one firm focusing on traditional advertising and the other firm focusing on poaching. Moreover, as in Section 5, the search engine's revenue is maximized with a medium level of penalty on poaching.

^{A4}Note that those customers who are exposed to advertising of both firms but without comparing prices purchase from the firm that did traditional advertising are already counted in αT_i and are categorized in the Category 1.

^{A5}Let x be a consumer's distance from Firm i , and p_i and p_j be the prices of Firms i and j , respectively. If this consumer considers only Firm i , she purchases if $1 - tx - p_i \geq 0$. If she considers both firms, she purchases from Firm i if $1 - tx - p_i \geq 1 - t(1 - x) - p_j$.

A new interesting result from this model is that the poaching firm sets a lower price than the other firm. In this way, the poaching firm can maximize the effect of poaching on its competitor’s keyword and win more of the comparison shoppers. The firm that is being poached does not decrease the price as much because it is benefiting from the customers who are not aware of the product of the poaching firm (i.e., customers not influenced by sponsored search).

A5 Strategy Space Discretization

In our model, we discretize the game by restricting the strategy space of each firm to three strategies, namely, Own, Poaching and Traditional. Although we allow the firms to use mixed strategies, we do not allow them to split their budgets among the channels as a pure strategy. The purpose of the discretization is to make the model easier to solve and understand. In contrast, we could use a continuous strategy space, allowing each firm to allocate arbitrary portions of its budget among the channels. In this section, we show that the results and insights we present in the paper are robust if we use a continuous strategy space. Specifically, we show that, under mild conditions on the parameters, the equilibria we discuss in the discrete game are also equilibria of the continuous game. Therefore, the insights obtained from the analysis of the discrete game also hold in the continuous game.

Let game G be the discrete game, where we also allow the search engine to use a keyword relevance multiplier γ . Let game H be the continuous version of game G , i.e., in game H , each firm decides how to arbitrarily allocate its budget to traditional advertising, sponsored search advertising on its own keyword, and sponsored search advertising on its competitor’s keyword. The parameters of the two games are $B > 0, \alpha > 0, R > 0$ and $0 \leq \gamma \leq 1$. In the theorem below, we divide our analysis into two parts: $\alpha \geq 1$ and $\alpha < 1$. Since α is the ratio of consumers not influenced by sponsored search to those influenced by sponsored search, $\alpha \geq 1$ refers to the case in which, after traditional ads are shown, a larger fraction of consumers directly purchase the product than go through sponsored search, while $\alpha < 1$ refers to the reverse case. Note that, given the current market structure, we can expect the case with $\alpha \geq 1$ to be the dominant case in reality. For this case, i.e., $\alpha \geq 1$, we show that for all allowed values of the other parameters, namely, B, r and γ , both poaching and non-poaching equilibria of game G are always also equilibria of game H . For the

other case, i.e., $\alpha < 1$, we show that for $R \geq \frac{2B-T^T}{T^T}$ the poaching equilibria of game G , which are the equilibria of interest, are also equilibria of game H with no conditions on the other parameters. Note that $\frac{2B-T^T}{T^T} < \frac{1}{\alpha}$ holds, implying that there is always a range of values of R in which G and H have the same poaching equilibria. Furthermore, in the case of handicapping poaching, we show that for $\gamma \leq \frac{(1+\alpha)R}{1+R}$ the non-poaching equilibria of game G are also equilibria of game H with no conditions on the other parameters. We state the above results in the following theorem.^{A6}

Theorem A1 *Consider discrete game G and continuous game H . If $\alpha \geq 1$, all equilibria of game G are also equilibria of game H . If $\alpha < 1$, then all poaching equilibria of G are also equilibria of H for $R \geq \frac{2B-T^T}{T^T}$ and, in the presence of a keyword-relevance multiplier γ , non-poaching equilibrium of G is also an equilibrium of H for $\gamma \leq \frac{(1+\alpha)R}{1+R}$.*

Proof: First consider the case where $\alpha \geq 1$. We start by proving that the poaching equilibrium of G is always an equilibrium of H . Consider a poaching equilibrium of game G where Firm 1 poaches on Firm 2's keyword. Firm 2's response, Traditional strategy, is calculated over the continuous strategy space and hence is a best response to poaching of Firm 1 in the continuous game H as well, by definition. Game G having a poaching equilibrium means $\Pi^{P,T} \geq \Pi^{O,T}$. This, together with $\alpha \geq 1$ gives us the condition $R \geq \frac{2B-T^T}{T^T}$. Therefore, firm 1's utility from poaching all of its budget is $\frac{B}{R}$. If Firm 1 deviates and spends x on poaching and $B - x$ on traditional channel and its own keyword, assuming that it splits $B - x$ optimally between traditional channel and its own keyword, its utility will be $\frac{x}{R} + (1 + \alpha)\frac{B-x}{R+1}$. From Section 3 we know that $\alpha < \frac{1}{R}$. This proves that the deviation is dominated for any $x < B$. Therefore, poaching with all of the budget is best response to Traditional. Consequently, poaching equilibrium of discrete game G is also an equilibrium of continuous game H when $\alpha \geq 1$.

Next, we prove that when $\alpha \geq 1$, non-poaching equilibrium of game G is also an equilibrium of game H . Consider the non-poaching equilibrium of G in which each firm uses Own strategy. We know, by definition, that as long as the firms do not want to poach on each others' keywords, Own strategy is the optimum way of splitting budget between traditional channel and own keyword on sponsored search. Consider a deviation where Firm 1 spends $x > 0$ poaching on Firm 2's

^{A6}The theorem only addresses the case with symmetric firms. The result and proof for the case with asymmetric firms are on the same lines.

keyword while Firm 2 is playing Own strategy. Also, assume that Firm 1 splits the remaining $B - x$ optimally between traditional channel and its own keyword. Firm 1's utility after deviation is $\gamma T^O \frac{x}{B-T^O+x} + (1 + \alpha) \frac{B-x}{R+1}$ while before deviation it is $(1 + \alpha) \frac{B}{R+1}$. Using elementary calculus we see that the deviation is beneficial if and only if $\gamma \geq \frac{(1+\alpha)(BR+x+Rx)}{B(1+R)}$. Therefore, if $\gamma \leq \frac{(1+\alpha)R}{1+R}$ then non-poaching equilibrium of game G is also an equilibrium of game H . If $\alpha \geq 1$, it suffices for γ to be not more than 1, which is satisfied by definition of γ . Therefore, when $\alpha \geq 1$, non-poaching equilibrium of game G is also an equilibrium of game H .

Now consider the case where $\alpha < 1$. We first prove that for sufficiently large R , poaching equilibrium of G is also an equilibrium of H . Consider a poaching equilibrium of game G where Firm 1 poaches on Firm 2's keyword. Firm 2's response, Traditional strategy, is calculated over the continuous strategy space and hence is a best response to poaching of Firm 1 in the continuous game H as well, by definition. If $R \geq \frac{2B-T^T}{T^T}$, firm 1's utility from poaching all of its budget is $\frac{B}{R}$. If Firm 1 deviates and spends x on poaching and $B - x$ on traditional channel and its own keyword, assuming that it splits $B - x$ optimally between traditional channel and its own keyword, its utility will be $\frac{x}{R} + (1 + \alpha) \frac{B-x}{R+1}$. From Section 3 we know that $\alpha < \frac{1}{R}$. This proves that the deviation is dominated for any $x < B$. Therefore, poaching with all of the budget is best response to Traditional. Consequently, poaching equilibrium of discrete game G is also an equilibrium of continuous game H when $\frac{2B-T^T}{T^T}$.

Next, we prove that when $\gamma \leq \frac{(1+\alpha)R}{1+R}$, non-poaching equilibrium of game G is also an equilibrium of game H . Consider the non-poaching equilibrium of G in which each firm uses Own strategy. We know, by definition, that as long as the firms do not want to poach on each others' keywords, Own strategy is the optimum way of splitting budget between traditional channel and own keyword on sponsored search. Consider a deviation where Firm 1 spends $x > 0$ poaching on Firm 2's keyword while Firm 2 is playing Own strategy. Also, assume that Firm 1 splits the remaining $B - x$ optimally between traditional channel and its own keyword. Firm 1's utility after deviation is $\gamma T^O \frac{x}{B-T^O+x} + (1 + \alpha) \frac{B-x}{R+1}$ while before deviation it is $(1 + \alpha) \frac{B}{R+1}$. Using elementary calculus we see that the deviation is beneficial if and only if $\gamma \geq \frac{(1+\alpha)(BR+x+Rx)}{B(1+R)}$. Therefore, if $\gamma \leq \frac{(1+\alpha)R}{1+R}$ then non-poaching equilibrium of game G is also an equilibrium of game H .

□

A6 On the Equivalence of Sequential Second-Price Auction and Market-Clearing-Price Mechanism

We consider a slightly more general version of Lemma 1 here. Suppose that a seller want to sell n units of an item. The seller can sell the units one by one, each in a second-price auction. We call this mechanism a *sequential second-price auction*. This mechanism captures the essence of the mechanism that search engines use to sell their advertising slots. Whenever a consumer searches a keyword, the search engine runs a (generalized) second-price auction to sell the advertising slot. The seller can instead sell the n units using a *market-clearing-price mechanism*. In the market-clearing-price mechanism, the seller sets the highest price p at which demand meets supply. The following theorem proves that the two mechanisms lead to the same outcome.

Theorem A2 *Suppose that n identical items are sold in a sequential second-price auction with reserve price R . Two bidders 1 and 2 with budgets B_1 and B_2 are participating in the auctions, and each bidder wants to maximize the number of items that she wins. The outcome of any subgame perfect equilibrium of the game is equivalent to the outcome of market clearing price mechanism with reserve price R .*

Proof: First suppose $\lfloor B_1/R \rfloor + \lfloor B_2/R \rfloor \geq n$, i.e., the market clearing price is at least R . Let p be the market clearing price; i.e., $\lfloor B_1/p \rfloor + \lfloor B_2/p \rfloor = n$. Note that if the first player bids p in all rounds, he can make sure that he wins at least $n - \lfloor B_2/p \rfloor = \lfloor B_1/p \rfloor$ items because his opponent has to pay p for every item that he wins. Similarly, if the second player bids p in all rounds, he can make sure that he wins at least $n - \lfloor B_1/p \rfloor = \lfloor B_2/p \rfloor$ items. Since, $\lfloor B_1/p \rfloor + \lfloor B_2/p \rfloor = n$, we see that player i cannot win more than $\lfloor B_i/p \rfloor$ items, which means that he wins exactly $\lfloor B_i/p \rfloor$ items.

Now, consider the case where $\lfloor B_1/R \rfloor + \lfloor B_2/R \rfloor < n$. In this case, we know that if the largest bid in the auction is smaller than R , the item in that round will be left unallocated. Also, if the larger bid is at least R , but the smaller bid is less than R , the item will be allocated, but at price R (instead of the second-highest bid). Given this information, bidding anything below R , in any round, is weakly dominated. Also, by bidding R , bidder i can make sure that he wins at least $\lfloor B_i/R \rfloor$ items. Since bidder i can never win more than $\lfloor B_i/R \rfloor$ items, in any subgame perfect equilibrium, he wins exactly $\lfloor B_i/R \rfloor$ items. \square

Note that the subgame perfect equilibrium of a sequential second-price auction is not unique, and there are many different optimal actions that the players may take in each period. However, they all eventually lead to the same outcome described in Theorem A2. The result above is also robust to different variations to the model. For instance, if all of the customers arrive at once, or if the firms cannot change the bids for each customer, or if the search engine uses a first-price auction instead of a second-price auction, we get the same outcome. The result can also be extended to two slots per keyword instead of one slot under the condition $B_1/(B_1 + B_2) \leq c_1/(c_1 + c_2)$, where c_1 and c_2 are the click-through rates of slots 1 and 2, respectively, with $c_1 \geq c_2 > 0$, and $B_1 \geq B_2 > 0$. This condition basically implies that asymmetry in budgets should not be too large. Moreover, if the search engine allows one firm’s ads to be placed in more than one slot (e.g., a search for “Toyota” may return ads from “toyota.com” and “buyatoyota.com,” both of which actually belong to Toyota) then the above condition is not needed for the theorem to hold in the case of multiple slots. The result is also robust to unequal valuations of the advertisers for clicks, as long as the valuations are high enough (given their budgets and the number of units being sold) to guarantee a unique market-clearing price. In the case of equal valuations, we assume that the search engine uses a rule that if bids are equal, it will choose the advertiser with the larger remaining budget at the time as the winner. This assumption parallels the practice of “bid throttling” by search engines. While search engines do not reveal the details of their mechanisms, they claim that their mechanisms try to keep the budgets of the advertisers non-zero until the end of the campaign, which is the essence of our assumption above. While the advertisers benefit from this because their campaigns keep running until the end of the campaign, their non-winning bids are being used against their competitors by the search engine, i.e., ensuring that no advertiser runs out of money too soon keeps the auction competitive for a longer time. More details about the different aspects of the proof are available from the authors by request.