Today's marketplace needs skilled graduates capable of solving real problems of innovation in a changing environment. Puzzle-Based Learning (PBL) is a new and emerging model of teaching critical thinking and problem-solving. We have run courses and workshops on PBL in the United States, Australia, and the Middle-East (Qatar and Abu Dhabi) in a range of formats. Preliminary assessment indicates that the PBL approach is assisting students by providing a framework to explore critical thinking, as well as being fun and engaging. In addition to our home institutions, several universities world-wide have introduced courses on PBL based on our curriculum or have introduced PBL themes in existing courses. Companies and organizations are also finding PBL as an effective way of exposing their executives (managers and engineers) to an innovative approach towards honing their problem-solving skills. In this paper we discuss our experience of the past three years and outline PBL's effectiveness with our preliminary evaluations.

Puzzle-Based Learning: An Introduction to Critical Thinking and Problem Solving

by Zbigniew Michalewicz and Nickolas Falkner, University of Adelaide; and Raja Sooriamurthi, Carnegie Mellon University

Zbigniew Michalewicz *is a professor at the University of Adelaide, Australia, and serves as chairman of the Board for SolveIT Software. He has published over 200 articles and 15 books including* How to Solve It: Modern Heuristics, Adaptive Business Intelligence; *as well as* Puzzle-Based Learning: An Introduction to Critical Thinking, Mathematics, and Problem Solving; *and* Winning Credibility: A Guide for Building a Business from Rags to Riches. *Currently he is involved in designing and teaching puzzle-based learning courses at the University of Adelaide.*

zbyszek@cs.adelaide.edu.au

Nickolas Falkner *is a lecturer at the University of Adelaide, Australia. His research interests include knowledge representation, network taxonomy, data models, wireless sensor networks, computer science education, data stream management and semantic annotation for data management. He lectures in a range of courses, including Puzzle Based Learning, advanced C/C++ programming, computer networking and distributed systems, and has presented a number of public seminars for business and academia in puzzle-based learning.*

nickolas.falkner@adelaide.edu.au

Raja Sooriamurthi *is an associate teaching professor with Carnegie Mellon's Information Systems Program and also has a faculty appointment at Carnegie Mellon's Heinz College. His research interests are in artificial intelligence, case-based reasoning, system development, decision support, and CS/IS pedagogy. His pedagogical efforts have been recognized with several awards for distinguished teaching. He has taught courses and made a number of presentations on puzzle-based learning to audiences ranging from middle-school to industry practitioners.*

raja@cmu.edu

While students are trained to recognize familiar problems with known solutions, they may not be sufficiently prepared to address novel real-world problems. What is missing in many of the curricula that we have examined is coursework focused on the development of *general* problem-solving skills. Further, courses that introduce elements of problem-solving skills often do so at the third or fourth level of the programs after students have already faced the majority of their in-academy intellectual challenges. While some courses with a design content emphasis may meet this requirement, many students do not learn how to think about solving problems in *general*. Throughout their education they are often constrained to concentrate on textbook questions at the end of each chapter, solved using material discussed earlier in the chapter. This constrained form of "problem solving" is not sufficient preparation for addressing real-world problems—on entering the real world, students find that problems do not come with instructions or guidebooks.

As a step towards addressing this situation, we have created and experimented with a new approach, *Puzzle-Based Learning*, that is aimed at getting students to think about how to frame and solve unstructured problems. The pedagogical goal is to increase students' analytical awareness and general problem-solving skills by employing puzzles, which are educational, engaging, and thought provoking.

What is Puzzle-Based Learning?

Consider the following puzzles:

- Given two eggs, for a 100-story building, what would be an optimal way to determine the highest floor above which an egg would break if dropped?
- Suppose you buy a shirt at a discount. Which is more beneficial to us: apply the discount first and then apply sales tax to the discounted amount or apply the sale tax first and then discount the taxed amount? What do stores do?
- If you have a biased coin (say, it comes up heads 70 percent of the time and

tails 30 percent), is there a way to work out a fair 50/50 toss?
- A farmer sells 100kg of mushrooms for $1 per kg. The mushrooms contain 99 percent moisture. A buyer makes an offer to buy these mushrooms a week later for the same price. However, a week later the mushrooms would have dried out to 98 percent of moisture content. How much will the farmer lose if he accepts the offer?

What is common to all of the above? Apart from being fun to ponder, solutions to these puzzles exemplify several problem-solving heuristics. What general problem-solving strategies can we learn from the way we solve these puzzles? As entertaining and engaging puzzles inherently are, they are just a means to our pedagogical end of fostering general domain-independent reasoning and critical thinking skills that can lay a foundation for problem solving in future course work.

Pedagogical Innovations

The puzzle-based learning approach aims to encourage students to *think* about how to frame and solve problems that are not encountered at the end of some textbook chapter. Our goal is to motivate students, and also to increase their mathematical awareness and problem-solving skills by discussing a variety of puzzles and their solution strategies. In this course we concentrate on *educational puzzles* that support problem-solving skills and creative thinking. These educational puzzles satisfy most of the following criteria:

1. *Independence:* The puzzles are not specifically tied to a single problem-solving domain.
2. *Generality:* Educational puzzles should explain some universal mathematical problem-solving principles.
3. *Simplicity:* Educational puzzles should be easy to state and easy to remember.
4. *Eureka factor:* Educational puzzles should initially frustrate the problem solver, but with the promise of resolution. A puzzle should be interesting because its result is not immediately intuitive. Eventually a *Eureka!* moment is reached (Martin Gardner's *Aha!*), when the correct path to solving the puzzle is recognized.
5. *Entertainment factor:* Educational puzzles should be entertaining and engaging. Entertainment is often a side-effect of simplicity, frustration, the Eureka factor, and an interesting setting.

Many real-world problems can be perceived as large-scale puzzles. As William Poundstone discusses in his exposition of the famous Microsoft / Silicon Valley interview puzzles (Poundstone, 2000), companies perceive a strong connection between the ability to solve puzzles and the ability to solve industry/business problems.

Puzzle-based Learning vs. Problem-based Learning vs. Project-based Learning

The ultimate goal of puzzle-based learning is to lay a foundation for students to be effective problem solvers in the real world. At the highest level, problem solving in the real world calls into play three categories of skills: dealing with the vagaries of uncertain and changing conditions; harnessing domain specific knowledge and methods; and critical thinking and applying general problem-solving strategies. These three skill categories are captured in the three forms of learning depicted in Figure 1.

In this continuum, each layer of skills builds upon the layers below it (Blumenfeld et al., 1991; Bransford et al., 1986). The focus of puzzle-based learning is on domain independent, transferable skills. In addition, we aim to foster introspection and reflection on the personal problem-solving process. What was I thinking? What is the solution? Why did I not see it?

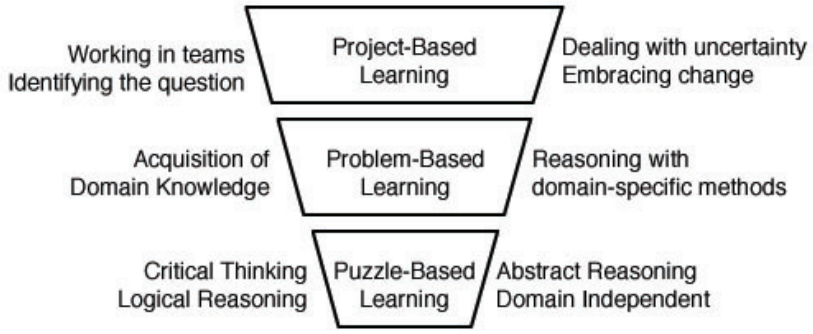
Course Structure: Content and Style

There are a few different versions of the puzzle-based learning course being taught currently. The course can be offered as a full-semester (three units) elective course (typically three contact hours per week, split into lectures and tutorials), a full-semester (three units) freshman seminar (three contact hours per week), one unit freshman seminar, and one unit core module as part of some other course.

One of the important points about puzzle-based learning courses is that the course is not about presenting and discussing a variety of puzzles but rather about presenting, discussing, and understanding problem-solving principles and some mathematical principles in the context of puzzles that serve as entertaining illustrations of the presented concepts. Also, the process of understanding problem-solving principles leads students through a variety of topics, exposing them to many important concepts at early stages of their college education.

Despite a variety of possible offerings of puzzle-based learning, the structure of

Figure 1. A continuum of learning and skills needed for problem solving in the real world.



the course is very much the same. The topics listed below correspond to a 12-week semester regardless whether each topic is allocated one hour or three hours.

1. **Introduction:** What it is all about?
2. **The problem:** What are you after?
3. **Intuition:** How good is it?
4. **Modeling:** Let's think about the problem
5. **Some mathematical principles:** Do you see it?
6. **Constraints:** How old are my children?
7. **Optimization:** What is the best arrangement?
8. **Probability:** Coins, dice, boxes, and bears
9. **Statistically speaking:** What does it mean?
10. **Let's simulate:** Can we generate the answer?
11. **Pattern Recognition:** What is next?
12. **Strategy:** Shall we play?

Each topic is illustrated by a variety of puzzles presented in an interactive manner. The course introduces a few simple problem-solving rules that we refer to in every class. Every week students are presented with homework assignments: one or more puzzles on the topic covered in the class. The following week, at the beginning of a class, the solutions are presented and discussed.

The University of Adelaide Experience. The initial implementation of puzzle-based learning was a one-unit course set up as a component of a three-unit first-year course. A three-unit first-year course for students planning to major in computer science was launched simultaneously in 2009 and made available to all non-engineering students in the University. We refer to the one-unit offering as PBL-E (PBL for engineers) and the three-unit offering as PBL Main. The courses cover the same material, at different levels of depth.

Lectures in PBL follow a set pattern. The first lecture of the week presents the solution to the previous homework, identifies the key points for this week's lectures, and then builds on the topic area. The lecture concludes with the next assignment. PBL Main has a second lecture that develops the themes of the week's topic. Lecture materials are developed in parallel, with the single PBL-E lecture derived from a revision and abridgement of the two PBL Main lectures for that topic to maintain currency between the two courses.

Tutorials are offered for PBL Main and allow students to take part in collaborative problem-solving exercises, with a tutor to provide assistance and guidance. Tutorial groups are up to 25 students, with sub-group formation of five to eight students for problem solving. During these sessions, we introduce fundamental mathematical concepts that are useful in the later course, including counting and the bases of probability, including factorials, combinations, and permutations.

The Carnegie Mellon University Experience. Puzzle-based learning was offered as a three-credit freshman seminar in Spring 2009 and 2010. Given the seminar nature of the spring course, enrollment was capped at 15 but it was encouraging to see that the wait list was longer than the class enrollment. The class had an interdisciplinary mix of students majoring in Information Systems, Computer Science, Psychology, Statistics, Cognitive Science, Economics, and Physics. The class met twice a week for 80 minutes. Given the smaller size of the class we were able to experiment with several alternative themes. For example, after the introductory classes, each session started with a puzzle-of-the-day. One student would present a puzzle of their choice. The class as a whole would try to solve the puzzle with hints and guidance provided by the puzzle poser. Student chosen puzzles ranged across the gamut of logic puzzles to diagrammatic reasoning to physical puzzles. Students had to submit a one-page write-up of their puzzle, solution, and most importantly, their reflection on the puzzle: what did they find interesting in the puzzle, variations, how does the solution tie into the general class discussions, etc.

During our discussion of scientific induction and mathematical induction, given the smaller size of the class, we played Robert Abbott's inductive game of Eleusis that models the process of scientific method. To introduce students to some of the problem-solving thoughts of leaders in the field we watched a few videos. These included Polya's "Let us teach guessing" wherein Polya beautifully illustrates several problem-solving heuristics (Polya, 1945) (that are embraced by puzzle-based learning) in the process of deriving a solution to the five-plane problem; an interview with Nobel-prize-winning economist Herb Simon on being a researcher, with advice to undergraduates; Nobel-prize-winning physicist Richard Feynman on problem solving and induction. We also visited a local Super Computing Center open house to get a glimpse of problem solving in the real world. To emphasize the link between the thought processes involved in solving puzzles and addressing open real-world problems, we examined a few case studies including the recently cracked Netflix Prize (**www.netflixprize.com**) and the classic work of Mosteller in resolving the authorship of some of the disputed Federalist papers.

Evaluation

Early student response shows that students enjoy the course material and that it does develop their thinking skills. The first implementation of PBL as a three-unit course has shown a consistent development of student puzzle-solving skills, culminating in excellent examination performance that outstripped our initial expectations. Figures 2a and 2b show the overall improvement of the students during the semester as we provided personalized feedback as well as overall assignment solutions.

Following are some sample quotes from our students in their end-of-term evaluations (reproduced verbatim): "I think the topic is very interesting. I enjoy coming to class because it is very hands-on and allows me to use critical thinking." "This course seems to be expanding my mind by giving me new ways to interpret and solve problems

Figure 2a. Student results for assignments over the semester.

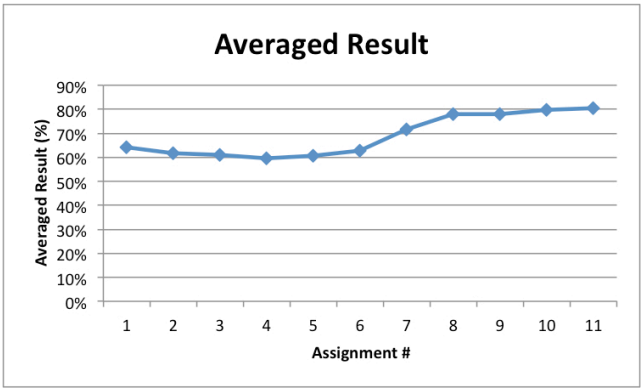
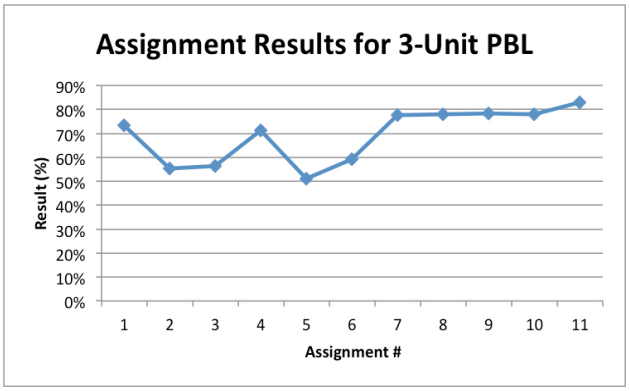


Figure 2b. Averaged student results for assignments over the semester.



that I would be completely lost on." "I like that the course opens my eyes to independent critical thinking and learning new ways to approach problems." "The way you are taught a new way about attempting problems, helps in all courses, not just this one."

Effectiveness and Transferability

Puzzle-based learning originated with the goal of enhancing the general problem-solving and critical-thinking skills of freshmen college students. This student demographic group continues to be our primary focus, and we have personally offered such semester-long courses in the United States, Australia, and Qatar. Our curriculum is supported by a 328-page textbook (Michalewicz, 2008) that details our approach. Sample syllabi, slides, assignments, exams, simulation software are all available off of our website (www.puzzlebasedlearning.edu.au) dedicated to puzzle-based learning. This educational material has been (or is in the process of being) translated to French, Polish, Japanese, and Hebrew to be offered to local communities. Currently, approximately 20 universities worldwide are in the process of offering PBL-themed courses.

In addition, since our original conception, PBL has transferred both downwards, upwards, and outwards from the college freshman curriculum. Two high schools are currently experimenting with PBL courses for their students. A graduate version of PBL targeting Information Systems majors was designed and delivered in the summer of 2010. We have offered several workshops on PBL at conferences to other faculty to introduce them to our approach. We have also delivered training workshops to government and industry employees.

Apart from a full puzzle-based-learning-themed course, portions of these ideas have also been blended into courses on intelligent systems, decision support, system development, and high-school outreach efforts to highlight various problem-solving strategies.

Conclusions

Puzzle-based learning is a pedagogical experiment in progress. The goal is to foster general domain independent reasoning and critical thinking skills that can lay a foundation for problem-solving in future course work (as depicted in Figure 1). As fun as puzzles inherently are, they are just a means to this pedagogical end. Our preliminary experience in different instantiations of the course and educational contexts has been encouraging and well received as we continue to explore this approach. We are in the process of collecting relevant data to demonstrate the benefit of our approach. Early results (Falkner, 2009) indicate that students perceive an improvement in their thinking and general problem-solving skills.

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