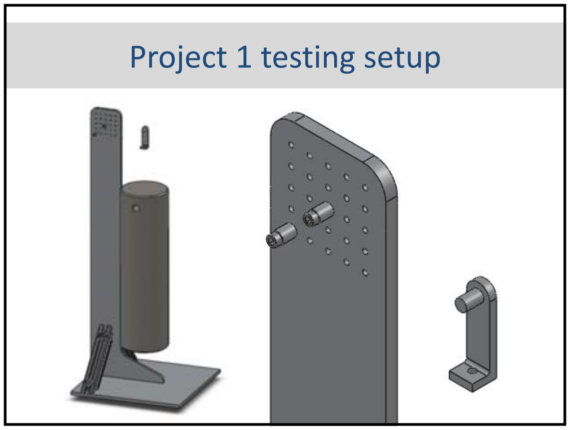
Component Failure

24-370 - Spring 2011
Professor Steve Collins



Project 1 testing setup

Announcements

- HW2 assigned, due next Wednesday
  - Homeworks guide projects
- Project 1 Questions?
- Where we are in the course…
  - Tools: sketching, SolidWorks
  - Analyses: stress, failure (today)
  - Next up: Design!
  - First loop: geometry, mass, and factor of safety
- But first… Failure!

Failure in Mechanical Engineering

- What is failure?
  - Compromisation or degradation of any kind
  - Commonly: breaking or permanently bending
  - Also: bending too much, wear, dynamic failure
- Today: custom component failure
  - Common & most-easily estimated modes
  - Later: catalog component failure

Failed Mechanical Parts

Avoiding Failure

- Reality can be a tricky place to operate…
  - Failure is very difficult to estimate accurately
  - Models: simple, generalizable, but miss details
  - Empirical data: accurate, but case specific
  - Prototyping: best data, but expensive
- What does a Mechanical Design Engineer do?
  - Use models or empirical data first
  - Use appropriate Factors of Safety
  - Prototype and test

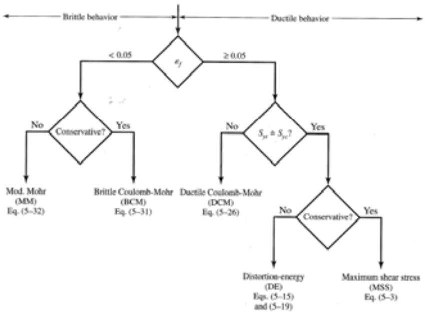
Common Failure Modes

- Static loading
  - – Primarily related to stress
  - – Ultimate stress exceeded → breakage
  - – Yield stress exceeded → plastic deformation
  - – Impermissible deflection
    - Buckling → breakage
- Cyclic loading: fatigue
  - – Similar stress analysis, very different behaviour
  - – Fatigue life (# cycles) exceeded → breakage

Failure due to static loading

- Related to development of excessive stress
  - – Exceeding yield stress is often unacceptable
  - – Which maximum stress to use?
    - Many models, depending on material & risks
    - Typically, von Mises equivalent (distortion-energy)
      - – $\sigma' = (\frac{1}{2} ((\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2))^{\frac{1}{2}}$
      - – $S_{sy} = 0.58\ S_y$
      - – Pure normal stress equivalents

Models of Failure due to Static Loading

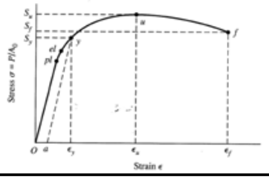


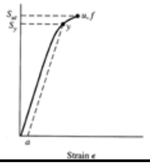
Failure due to static loading

- Related to development of excessive stress
  - – Exceeding yield stress is often unacceptable
  - – Which maximum stress to use?
    - Many models, depending on material & risks
    - Typically, von Mises equivalent (distortion-energy)
  - – Stress concentrations?
    - In Ductile materials, usually not
      - – Stress distributing, strain hardening
    - In Brittle materials, definitely yes
    - (later: also fatigue)
    - And… stress concentrators are everywhere!

Ductile vs Brittle Failure

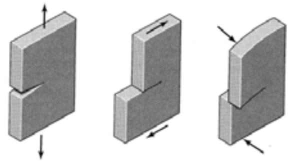
- Ductile materials strain a lot, then fail
- Brittle materials just fail
  - – Typically, $\epsilon_f \leq 0.05$
  - – Often, only σ_u reported





Fracture Mechanics

- Failure usually occurs along cracks
- Cracks are everywhere!
- Brittle materials particularly susceptible
- Research on crack propagation ongoing

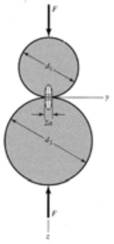


Static Failure Analysis Summary

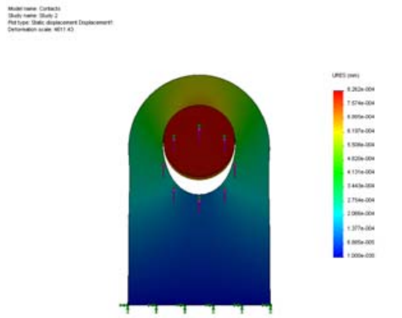
- Use empirical data to determine yield strength
  - Or ultimate strength for brittle materials
- Determine maximum stress in design
  - Simple models (last week)
    - Stress concentrations for conservative or brittle cases
  - FEA analysis (last week)
- Compare to obtain factor of safety
- Design: set $\sigma_m \leq S_y \text{ FOS}^{-1}$
  - Obtaining this relationship… that's the fun part!

Special Case: Contact Stress

- Strong analytical models for some types, e.g. cylindrical:
- I've typically used FEA
- SolidWorks example
  - Pin in hole



SolidWorks contact example

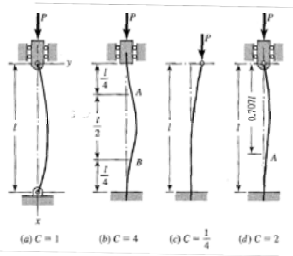


Deflection

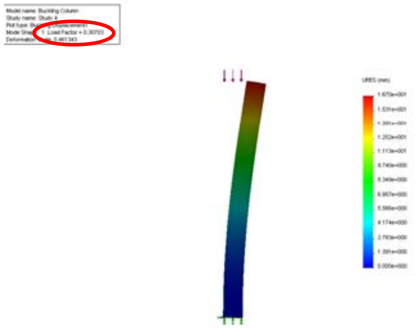
- Possible failure mode: excessive displacement
- Compression or tension: $\delta = F\, L\, A^{-1}\, E^{-1}$
  - Load, length, area, and material stiffness
- Torsion: $\theta = T\, L\, J^{-1}\, G^{-1}$
  - Rotational equivalents
- Bending: depends upon loading
  - Many approaches: see Shigley, others
  - Cantilever: $\delta_e = ⅓\, F\, L^3\, E^{-1}\, I^{-1}$
- Stiffness: rearrange terms

Buckling

- Special deflection
- Long, thin elements in compression
  - Unstable
  - Small deformations increase leverage
- Simple model
  - $F_{cr} = C\, \pi^2\, E\, I\, L^{-2}$
- FEA: SolidWorks



SolidWorks Buckling Example

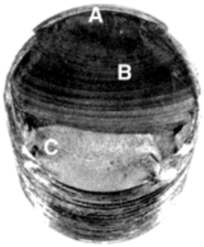


Failure due to Fatigue

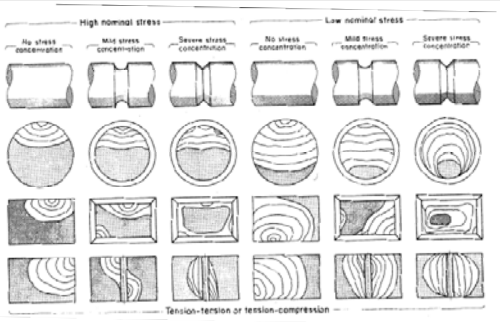
- Everyone knows: push hard and things break
- But lots of little pushes breaking something?
  - Crazy, but true. This is fatigue
- Fatigue occurs below (apparent) yield stresses
  - Same stresses, different effects
- After a number of cycles, the part fails
- Predicting fatigue failure:
  - Difficult to model, but empirical data useful
  - Insight into mechanisms helpful too…

Stages of Fatigue Failure

- Stages:
  - A - Crack initiation
    - At concentrator
    - Or… at crack
  - B - Propogation
    - Often hidden
  - C - Final failure
    - Often brittle



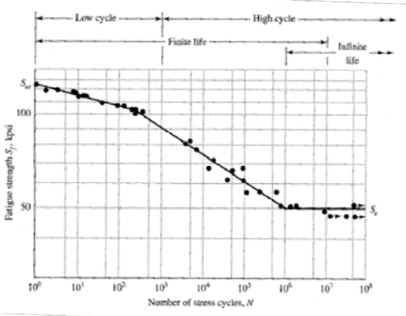
Patterns tell a story



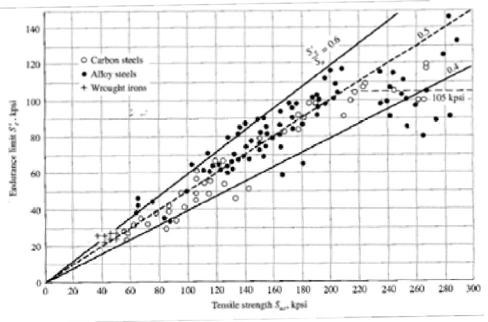
Fatigue Life and Strength

- Need to account for # cycles as well as load
  - Peak sustainable stress, S
  - Number of cycles, N
- Diagrams of S vs. N
  - Material-specific
  - Detailed fatigue data
  - Idealized conditions
- Scalar fatigue strength
  - Peak stress S that may be fully reversed N times

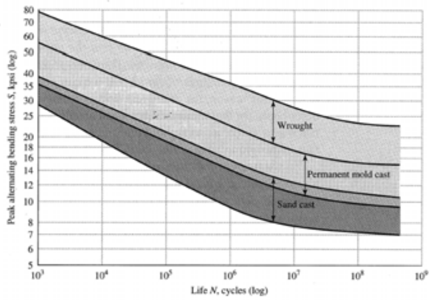
Fatigue Life: Steel S-N Diagram



Endurance Limit for Steels



Aluminum S-N Diagram



Interpreting S-N Diagrams

- Steel has infinite fatigue life region
  - Fatigue strength corresponds to infinite life
- Aluminum (and most other materials) do not
  - Fatigue strength typically means $N = 10^8$
- S-N diagrams useful for sub-limit performance
- Often, we just want $\sigma(N=10^8)$
- Careful: this data is for specific conditions

Fatigue Life Analysis

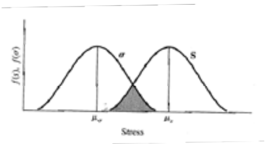
- Stress-life method
  - Simplest, most data, $N \geq 10^3$, but least accurate
  - Other methods: strain life (low N), fracture (crack)
- Obtain material-specific fatigue strength
  - Generally, use empirical data
  - Steel endurance limit: $S_e' \approx 0.5\, S_{ut}$
- Apply strength-reducing factors
  - Due to surface, size, load, temp, freq, etc.
- Compare to peak stress in component
  - Use fatigue stress concentration factors

Fatigue Analysis Example

- Material: Alum 7075-T6, $S_f' = 23$ ksi, $S_u = 83$ ksi
- Fatigue strength modification factors:
  - Loading: bending → $k_c = 1$
  - Surface: ground
    - $k_a = a\, S_u^{\,b}$, $a = 1.34$, $b = -0.085$ → $k_a = 0.92$
  - Size: diameter D = 1”
    - $K_b = 0.879\, D^{-0.107}$ → $k_b = 0.88$
- Part stress
  - Peak bending stress $\sigma_m = 9$ ksi (no concentrators)
- Synthesis: $S_f = k_a\, k_b\, k_c\, S_f' = 18$ ksi → $FOS_f = 2$

Final Caveats

- Failure is stochastic
  - In many ways
  - e.g. actual yield stress
- In design:
  - Modeling useful
  - Emprical data useful
  - Prototyping crucial
  - Iteration is reality



Suggested Reading

- Shigley Chapters 4, 5 & 6
  - Basics for the uninitiated
  - Details for the expert
  - Includes formulae from lecture
  - Accompaniment to HW3 (next week)