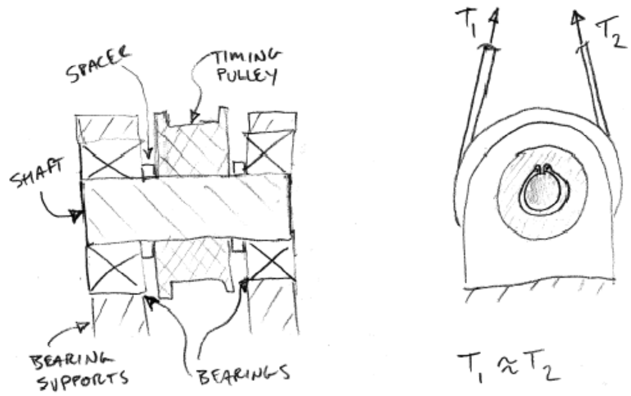
Assignment 6: Catalog Part Selection

24-370 Engineering Design I
Due @ 12:30, Wednesday March 16th 2011

Part 1: Components for a Timing Pulley Assembly

The figure below shows a timing pulley mounted to a shaft, supported by two ball bearings, which are in turn set in a support component. The timing pulley is rigidly attached to the shaft, and held in place axially by round shim washers (shaft spacers) that contact the inner race of the ball bearings.



1.a - Timing pulley. Using Stock Drive Products' online catalog, select an appropriate timing belt pulley. The belt to be used has a pitch of 0.200" (XL). The desired pitch diameter of the pulley is about 1.25". The pulley should have flanges, to keep the belt aligned, but no hub, to keep the width low. The pulley should be made from steel, to avoid fatigue and wear. Finally, the pulley should have as small of a bore (inner diameter) as possible. What part number would you order?

1.b - Ball bearings. Using McMaster Carr, select an appropriate ball bearing set for the assembly. We will choose from open, general purpose ball bearings made from stainless steel. First, match the inner diameter to that of the timing pulley, so that a single shaft diameter may be used. Next, select an appropriate radial load capacity (you may need to select a range before selecting the final value). The belt tension will be approximately 50 lbf on each side of the pulley, and the desired factor of safety is about 3. Finally, select the smallest bearing that meets your requirements. What part number would you order?

1.c - Shaft spacers. Using McMaster Carr, select appropriate shaft spacers for this application. These may be found under shaft spacers --> shims and shim stock --> round shims. First, select the correct Shim Type for our application, which is to contact the inner race of the bearing on one side and the timing pulley on the other. Next, select the appropriate inner diameter. Finally, choose the greatest available thickness, so there will be plenty of room between our pulley and bearing. What part number would you order?

1.d - Shaft. Using McMaster Carr, select an appropriate shaft for the assembly. First, select the needed shaft outer diameter. Next, select stainless steel as the material, so that the shaft will be strong but won't corrode. Next, select the appropriate shaft type. In our application, the timing pulley will be driving another element through torque on the shaft. Next, pick the more corrosion-resistant form of stainless steel. Finally, pick the smallest length that will allow us to cut the shaft to the desired length.

Part 2: Design of a Gear Set using Catalog Gears

In this problem, you will use simple gear stress analysis to select a set of gears from a catalog. You will consider the fatigue life of the gear set. You will attempt to minimize the gear size while meeting a desired output torque.

Before getting started, please open a web browser and navigate to www.sdp-si.com, then click products --> gears --> spur gears (inch). You should now see a spreadsheet with catalog part numbers and a property selection dialog. We will use this interface in the design process.

2.a - Gear material. We want the gears to be small and strong, and to last many cycles. Which material should we choose, of those available? Make this selection in the active spreadsheet.

2.b - Material strength. In order to set the allowable stress in our gear, we must first know the strength of the material. Since the listed material is not specific enough to find using, e.g., matweb.com, we must obtain the specifications from the gear manufacturer. Clicking around on the website, you find the relevant catalog pages: www.sdp-si.com/D790/PDF/D790C01015.pdf. However, the surface strength is only given as a Rockwell C hardness (R_C or H_{RC}). To a decent approximation, Rockwell hardness can be related to ultimate strength using the formulae

$$\boldsymbol{S_u} \approx 66 + 2.8\cdot \boldsymbol{H_{RC}} - 0.061\cdot \boldsymbol{H_{RC}}^2 + 0.0016\cdot \boldsymbol{H_{RC}}^3 \quad (\text{or } \boldsymbol{S_u} \approx -190 + 9.4\cdot \boldsymbol{H_{RB}} - 0.13\cdot \boldsymbol{H_{RB}}^2 + 0.00063\cdot \boldsymbol{H_{RB}}^3)$$

where $\boldsymbol{S_u}$ is the ultimate strength and $\boldsymbol{H_{RC}}$ (or $\boldsymbol{H_{RB}}$) is the Rockwell C (or B) hardness. What is the ultimate stress of the gear material, to two significant digits?

2.c - Endurance strength. Of course, the gear will be cyclically loaded and fatigue is a concern, so we must find the endurance strength of the material. What is the idealized endurance strength, $\boldsymbol{S_e'}$?

Using the formulae and tables found in Ch. 6 and appendix A-15 of Shigley, you determine the following values for endurance strength modification factors:

$K_a \approx 0.6$ (the gear surface is likely machined)
$K_b \approx 1.1$ (using the appropriate size modification factors)
$K_c = 1$ (the teeth are loaded in bending)
$K_d = 1$ (room temperature)
$K_e = 0.9$ (for 90% reliability)
$K_m \approx 1.33$ (using corrections for one-way bending)

What is the corrected endurance strength of the gear material, $\boldsymbol{S_e}$, to two significant digits?

2.d - Stress concentration factor. Because we are considering the fatigue life of the gear, and because hardened materials may become brittle, we must also include a stress concentration factor. The smallest radius on the tooth occurs at the fillet at its root. This fillet typically has a value $r_f \approx 0.3\cdot P^{-1}$, where P is the diametral pitch, or $r_f \approx 0.15\cdot t$, where t is the tooth thickness. Therefore, from figure A-15-6, we have:

$K_t \approx 1.68$ (typical gear stress concentration factor)

Next, we will correct this factor for fatigue. Using figure 6-20, S_u, and an initial estimate of r_f, we find that q ≈ 0.85. Therefore:

$$K_f = 1 + q \cdot (K_t - 1) \approx 1.6$$

Finally, we will add a design factor of safety, ***fos*** = 2. What is the maximum allowable pre-concentration stress, $\boldsymbol{\sigma_{allow}}$, in the gear as a function of $\boldsymbol{S_e}$, $\boldsymbol{K_f}$, and ***fos***? Please give your answer both symbolically and numerically to two significant digits.

2.e - Governing stress equation. What formula should you use to find the peak stress in the gear tooth? Do not account for safety factor or stress concentration factor.

2.f - Symbolic manipulation. You now have a governing equation for maximum stress and an allowable maximum value. However, there appear to be four free design parameters, one of which requires a look-up table! Let's see if we can reduce the degrees of freedom. Counter-intuitively, we will first add two new design variables: ***N*** = the number of teeth; and ***D*** = the pitch diameter. Notice that the number of teeth on the gear may be related to the diametral pitch (number of teeth per inch) and the pitch diameter:

$$\text{Circumference} = \pi \cdot \boldsymbol{D}$$
$$\boldsymbol{N} = \text{Circumference} \cdot \boldsymbol{P} = \pi \cdot \boldsymbol{D} \cdot \boldsymbol{P}$$
$$\boldsymbol{D} = \boldsymbol{N} \cdot (\pi \cdot \boldsymbol{P})^{-1}$$

Next, we can relate the gear tooth load to the applied torque, since torque is force times lever arm:

$$\boldsymbol{T} = \boldsymbol{W_t} \cdot \tfrac{1}{2}\, \boldsymbol{D}$$
$$\boldsymbol{W_t} = 2 \cdot \boldsymbol{T}/\boldsymbol{D} = 2 \cdot \pi \cdot \boldsymbol{P} \cdot \boldsymbol{T} \cdot \boldsymbol{N}^{-1}$$

Finally, let us consider the Lewis Form Factor, ***Y***. We can see from table 14-2 that ***Y*** is only a function of the number of teeth, ***N***. It turns out that, with clever regression:

$$\boldsymbol{Y} \approx \tfrac{1}{4} \cdot (\boldsymbol{N} - 11)^{0.125} \quad (\text{for } \boldsymbol{N} \leq 150)$$

Substitute these relationships into your equation from part 2.e and simplify. You should now have an equation with three free design parameters: ***P***, $\boldsymbol{F_w}$, and ***N***.

2.g - First catalog parameter selection. Consider your equation from 2.f. Which of your design parameters appears to most dramatically influence maximum stress? In the catalog spreadsheet, select the optimal value for this parameter and note your choice.

2.h - Second catalog parameter selection. In our application, space is an important consideration. Select the smallest candidate gear size in terms of diameter (or equivalently teeth) and note your selection.

2.i - Final parameter calculation. In order to meet our design strength requirements, we must now solve for the remaining free parameter. First, solve your equation from 2.f for the remaining parameter. Next, substitute numerical values in for all of the variables. Use a torque, ***T***, of 12 in·lbf. What is the minimum allowable value? Which available value would you select from the catalog?

2.j - Catalog part selection. What catalog part number would you choose? Use the minimum shaft diameter.

2.k - Driven gear. You desire a gear ratio of 2:1 (torque on driven gear : torque on driving gear). If the part you selected in 2.j were the driving gear, what catalog part would you select for the driven gear? Use the same shaft diameter.

2.l - Interpretation. Is it theoretically possible to reduce the diameters of your gears while still maintaining the required fatigue strength? What are the practical limitataions?