Assignment 3: The Geometry-Strength-Mass Design Loop

24-370 Engineering Design I
Due @ 12:30, Wednesday February 9th 2011

Part A: Intuitive part sketching

Using the loading and support sketches on the following pages as a guide, sketch conceptual designs for mass-optimal component structures that could meet pre-defined strength requirements, such as factor of safety. Start with stick figures, then flesh out the structures to enable them to bear compressive, tensile, or bending loads, or to prevent excessive deflection or buckling. For this sketching exercise, imagine that the parts are relatively small, and therefore difficult to design as trusses. In other words, they should be single continuous components, with no super-small features. If you have a strong intuition in trusses, think of solid sections of material as representing planes of triangular linkages in the truss. These exercises are meant to help you to develop your intuition, so spend as much or as little time as you think will benefit you. Grades are for completion, with feedback proportional to answer content.

Part 1: Scratch-Paper Design of a Rod in Tension

Design the diameter of a cylindrical member in tension to achieve minimum mass while meeting a factor of safety requirement.

The Miller sign at Billy Bob's Place got busted again, and he has come to you to try to design a more permanent solution: raising the sign up and out of harm's way. It seems he wants the sign to be supported by a single cylindrical rod. One end of the cylinder will be rigidly fixed to the ceiling, while the other will be rigidly attached to the neon sign weighing 50 pounds. Billy Bob is light on cash and wants to use the least amount of steel possible. What rod diameter, $\boldsymbol{D}$, do you prescribe?

1.a - Load analysis. Draw a Free Body Diagram of the rod, including the load, $\boldsymbol{F}$, and any reaction forces and/or moments. It should be possible to balance forces and/or moments for this static FBD.

1.b - Symbolic derivation. What is the peak stress, $\boldsymbol{\sigma_m}$, in the cylinder, as a function of $\boldsymbol{D}$?

1.c - Inverse problem. The local machine shop has bundles of plain carbon steel rods lying around, with a yield stress of $\boldsymbol{\sigma_y}$ = 30,000 psi. Using a factor of safety, ***fos***, of 10 (after all, we don't want anyone hit with a falling Miller sign) determine the minimum rod diameter $\boldsymbol{D_{min}}$, to three significant digits.

1.d - Reality strikes. The only rod diameters available at the shop are in integer multiples of 1/32 of an inch. What rod diameter should you choose? Please report your answer in fractional form, i.e. x/32.

1.e - Numerical validation. Model the part in SolidWorks and perform a Simulation to determine the factor of safety. Use SolidWorks' built-in material "Plain Carbon Steel". You may find it helpful to create a factor of safety plot: after you have run the simulation, right click on Results and select Define Factor Of Safety Plot. Please report the minimum ***fos*** to two significant digits.

1.f - Interpretation. Billy Bob thinks the sign will be out of reach, but some of his customers look like they might get creative in their attempts to get his goat. How might this part fail, if the loading conditions were not as expected? Please give a qualitative type of loading, in five words or less (e.g. "normal stress").

Part 2: Analytical Optimization of a Height-Varying Beam in Bending

Design the length-varying cross-section of a cantilevered beam to achieve minimum mass within constraints on factor of safety, material and manufacturing.

The consulting firm you work for is helping a civilian contractor to design a search and rescue robot for disaster sites. You are currently designing a gripper to remove debris from the robot's path. You have decided that the main structural component of the gripper will be a set of calipers constructed from 7075-T73 Aluminum. You decide to model the front half of the calipers cantilevered beams, with a force at the tip due to the reaction force from the gripped object. The mass of the gripper is critical, since it must be carried by the machine and lifted by the proximal robotic arm joints. To withstand bending loads with low mass, the component will have an I-beam cross section, with thickness $\boldsymbol{t}$, base width $\boldsymbol{b}$, and overall height $\boldsymbol{h}$. The beam must have length $\boldsymbol{L}$, due to the size of objects to be carried, and will be loaded with force $\boldsymbol{F}$, to produce sufficient friction to lift the desired objects. The machining process will require a minimum thickness, t_{min} = 0.05 inches, in all places. Because moments are low near the loading point and high near the cantilever, we may want to vary the beam height along its length. But what is the best way to do this?

1.a - Load analysis. Determine the reaction moment, $\boldsymbol{M}$, as a function of the axial position, $\boldsymbol{x}$, along the beam. Define $x = 0$ as the loaded end of the beam, and $x = L$ as the cantilever point.

1.b - Symbolic derivation: stress vs position. First, determine the peak stress along the beam, σ_m, as a function of the axial position, $\boldsymbol{x}$, assuming constant cross-section parameters $\boldsymbol{t}$, $\boldsymbol{b}$, and $\boldsymbol{h}$. Use the simplified I-beam area moment of inertia equation derived in class: $I \approx \frac{1}{2} b t h^2$. Next, try linearly increasing the beam height as it approaches the cantilever. Substitute a length-varying height, $h(x) = k \cdot x$, in for $\boldsymbol{h}$, and simplify the resulting equation.

1.c - Inverse problem: base as function of k. Since we aim to minimize mass, the beam should not be any wider than necessary for a given height parameter. To find out what beam width is needed, first substitute a combination of the material's yield stress, σ_y, and the desired factor of safety, $\boldsymbol{fos}$, for σ_m. We now have an equation with only $\boldsymbol{b}$, $\boldsymbol{t}$, and $\boldsymbol{k}$ as free parameters. Solve for $\boldsymbol{b}$. (Later, we will set $\boldsymbol{t}$ and vary $\boldsymbol{k}$ to see the effects on mass.)

1.d - Symbolic derivation: beam mass. Now determine the mass, $\boldsymbol{m}$, of the resulting beam geometry. First, derive the mass of a small slice of the beam, d$\boldsymbol{m}$, for a slice thickness of d$\boldsymbol{x}$ along the length of the beam. Use the simplified I-beam cross-sectional area equation derived in class: $A \approx t\,(2\,b + h)$, and material density, ρ. Next, substitute $h(x) = k \cdot x$ for $\boldsymbol{h}$, since this will affect our integration. You should now have an equation with two terms, in ρ, t, b, k, x, and d$\boldsymbol{x}$. Perform definite integration over the range $x = [0, L]$ to find the total beam mass. Finally, substitute your answer from part 1.c for $\boldsymbol{b}$ to obtain an equation for the beam mass with only two free parameters, $\boldsymbol{t}$ and $\boldsymbol{k}$.

1.e - Numerical solution: mass vs height factor k. Now we will determine the optimal values for thickness and height constant for our beam. Let us first assume that the minimum allowable thickness is optimal, and test this assumption later. This leaves only the beam height factor, ***k***, as a free parameter in our optimization problem. In **Matlab**, enter the following values for constants: rho = 0.3; F = 100; fos = 2; L = 2; sigma_y = 50000; t = 0.05;. Next create a vector of possible values for k, ranging from 0.1 to 1.0, in increments of 0.001: k = [0.1:0.001:1];. Finally, solve for mass m = [your equation here];. You will probably need to use the "./" command (the "dot slash" operator, which divides vectors element by element rather than as whole matrices) or similar. Plot your result using the commands: figure(1); plot(k,m); xlabel('k'); ylabel('m');. Please print your plot and report your answers to three significant digits:

1.e.i - What is the minimum value of ***m***? Use the command [v,i] = min(m), where v is the minimum value and i is the index of that value.

1.e.ii - What is the minimizing value of ***k***? Use k(i) to find out.

1.e.iii - What is the corresponding value for ***b***? Use your equation from part 1.c to find out.

1.e.iii - Finally, test the assumption that mass is minimized with minimum allowable thickness t_{min}. Set t = 0.1; and re-calculate m. What is the new minimum mass ***m***?

1.f - Interpretation. Will the optimal height factor k_{opt} change if the load ***F*** is changed? Please answer "yes" or "no". Are there any obvious problems with the beam cross-section we have defined, at the extremes for instance? Please answer in 10 words or less.

Part 3: Fatigue Design of a Cyclically-Loaded Shaft in Torsion

Design a steel shaft to have the desired (infinite) fatigue life, using feature geometry.

The medical device company you work for is designing a novel shaker for agitating biological and chemical emulsions, and you are currently working on the design of the central drive shaft. This component will have a motor on one end and a circular steel test-tube rack on the other end, which the motor will rotate back and forth cyclically, producing a dynamic, cyclical torsional load with peak torque ***T*** = 195 in lbf on the shaft. The shaft is to be constructed from 440C Stainless Steel, which has a yield strength S_{yt} = 180 ksi, and an ultimate strength S_{ut} = 240 ksi. The outer diameter of the shaft is ***D*** = 0.500 inches, but you also need a groove for a retaining ring just outside the main bearing. The diameter of the shaft inside the groove is ***d*** = 0.476 inches, and the machining process to be employed will result in internal radii at the corners ***r*** = 0.036 inches. Most of the shaft will be polished, but in the critical retaining ring groove the surface will be machined. Customers will use the device at room temperature, and it is expected that neither heat from the motor nor internal energy dissipation from the shaft will change the shaft temperature significantly. The medical staff using the product expects very high quality, high-reliability components, so you aim for a reliability of 99.9% and an infinite fatigue life.

Use appropriate portions of Shigley's "Road Map" for fatigue analysis, copied in the appendix below for your convenience, to answer the following questions:

3.a - Endurance strength. Calculate the endurance strength, S_e, of the shaft under these conditions. Please calculate all modification factors separately, and record them to two significant digits, before performing your final calculations (with the rounded numbers). Please neglect the miscellaneous-effects factor.

3.b - Peak stress and factor of safety. Calculate the peak stress, τ_m, including the fatigue concentration factor. You might find chart A-15-15 useful. For simplicity, please approximate S_u as 200 ksi and r as 0.04 in when calculating q (use the correct values for all other calculations.) What is the factor of safety, ***fos***? Note that you have already converted to shear when calculating S_e, due to k_c.

3.c - Design for fatigue. Describe the design process you would use to achieve a given ***fos*** for minimum D,d.

Part 4: Guess-and-Check method for a Column in Compression

Determine the minimum diameter of a column in compression that will prevent buckling.

You are the in-house mechanical engineer at an electronics start-up whose goal is to put a Squig™ in the home of every American parent. What is the Squig™? Something to do with early development of mathematics skills through the pushing of multicolored buttons, but don't worry about that right now. The main problem with the Alpha prototypes is failure due to toddlers using the Squig™ as a seat or stepping stool. It flattens like a pancake. To strengthen the design for the Beta version, you decide to add a set of internal posts, which rigidly connect opposite faces to one another. The opposing faces are not very stiff relative to one another. In other words, there are no external constraints on the motion of the top of the column with respect to a reference frame fixed at the bottom. The posts will be approximately circular in cross-section, with diameter D and length L = 2 in, molded from ABS (Acrylonitrile Butadiene Styrene) with E = 290 ksi. The maximum toddler load, F, is about 50 pounds force.

4.a - CAD guess and check. Being the SolidWorks maestro that you are, you decide to skip the pencil and paper stage and simply model this thing and tweak it to get the parameters right. Hey, guess and check can help build your intuition too, right? Fire up SolidWorks, throw in an initial guess for D, use the buckling analysis tool in Simulation, and tweak D until you obtain a Load Factor of 2 ± 0.1. What diameter did you arrive at? About how many iterations did you go through?

4.b - Analytical validation. Now use the formula presented in class to solve for the diameter analytically. Please give your answer for D to three significant digits (this should be slightly different from the SolidWorks solution). Which process used more time? Which process would be faster if you found out that the elastic modulus E of the material had to be changed to 340 ksi?

Part 5: Analysis of a Post and Ring in Contact

Demonstrate use of CAD tools to perform contact stress analysis in an assembly.

You are a Mechanical Engineering student that has been assigned a design project that involves contact between a custom part and a set of pegs. Being the awesome, can-do, go-getter that you are, you decide to go the extra mile in your project and consider contact stresses during your design process. But first, you need to play around with the FEA tool that will allow you to analyze them. Using the method we discussed in class, please model a post in contact with a hole in a block. You may use any dimensions and loads that obtain qualitatively interesting results, but please give your block a unique and distinguishing feature of some kind. Please print a screen shot (use Save As --> File Type: JPEG) of the stress distribution result plot, using a deformation constant that makes important features visible.

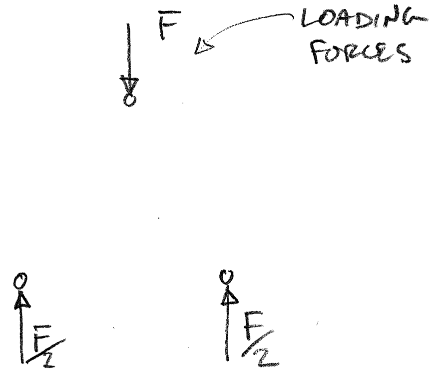
Bonus Question A: SolidWorks Parallel-End Buckling Analysis [Completely optional]

Demonstrate a buckling simulation for a cylindrical shaft in compression, where the base and ceiling portions of the beam are able to move vertically but are constrained in both rotation and horizontal translation. This corresponds to C = 4, or part (b) of the figure on buckling presented in lecture. Please provide a screen shot of the deformed geometry, along with a brief description of the fixtures and loads you applied. [In addition to showing up your sad old Prof, earn up to 10 extra points!]

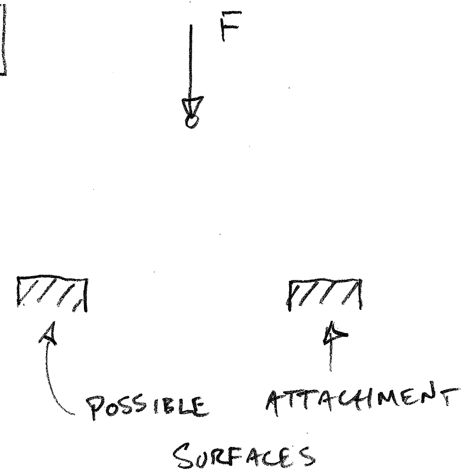
Bonus Question B: Leaf Spring Design [Completely optional]

You are designing a biomimetic robot joint that utilizes a series-elastic actuator with a leaf spring. The leaf spring is a beam rigidly connected to one limb and loaded by a cable at a distance of about 5 inches from the joint. For simplicity (in modeling and manufacturing) try a rectangular cross-section. Derive the relevant design equations, involving geometry, material properties, factor of safety, linear stiffness at the endpoint, and mass. Using material properties for Gordon Composites' GC-67-UB bar stock and a desired stiffness of 600 lbs in^{-1}, determine the spring mass and cross-section parameters. How does mass change with cross-section and length? In addition to establishing your supreme awesomeness, earn up to 15 extra points.]

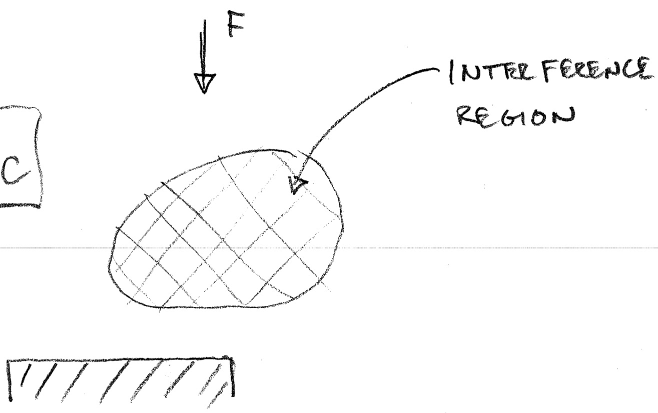
A.a


F
LOADING FORCES
F/2
F/2

A.b


F
POSSIBLE ATTACHMENT SURFACES

A.c

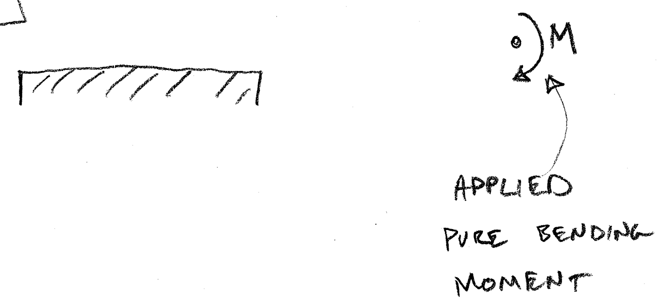

F
INTERFERENCE REGION

A.d

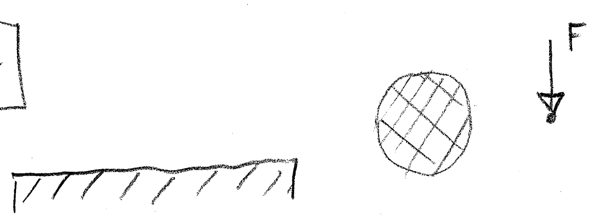

F

A.e

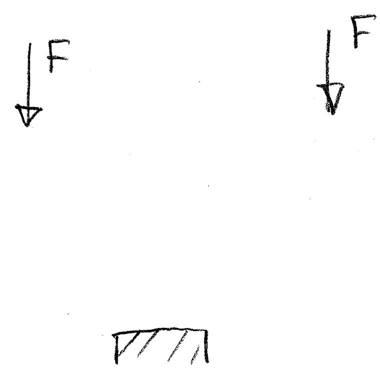
A.f



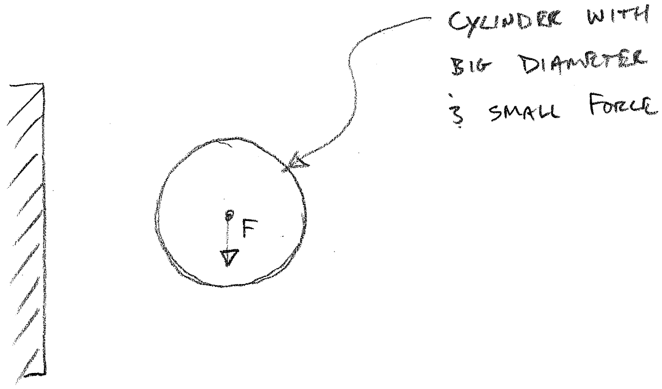
A.g



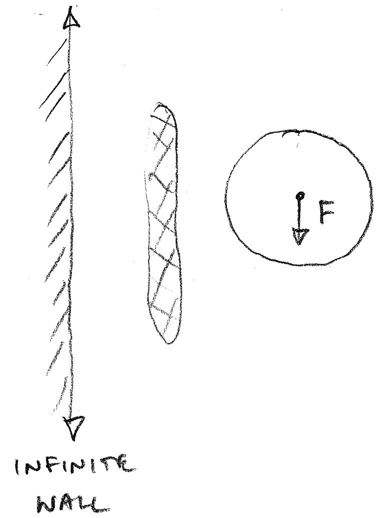
A.h



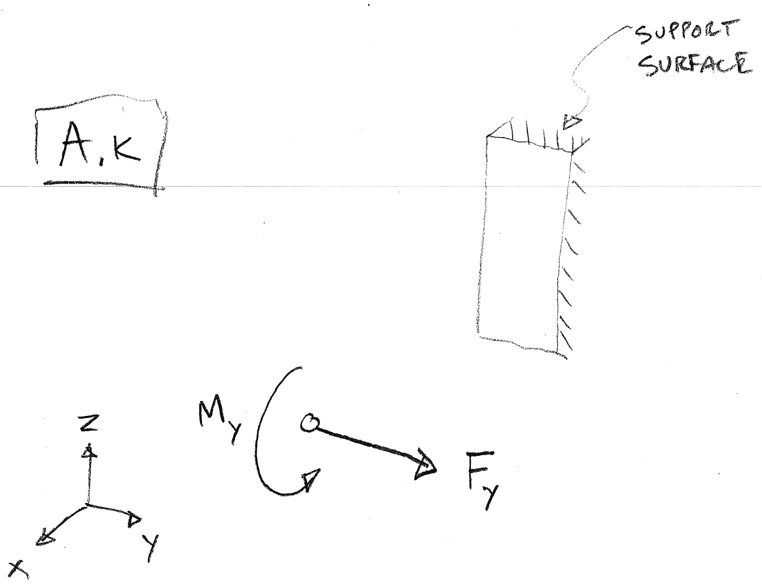
A.i


CYLINDER WITH
BIG DIAMETER
& SMALL FORCE
F

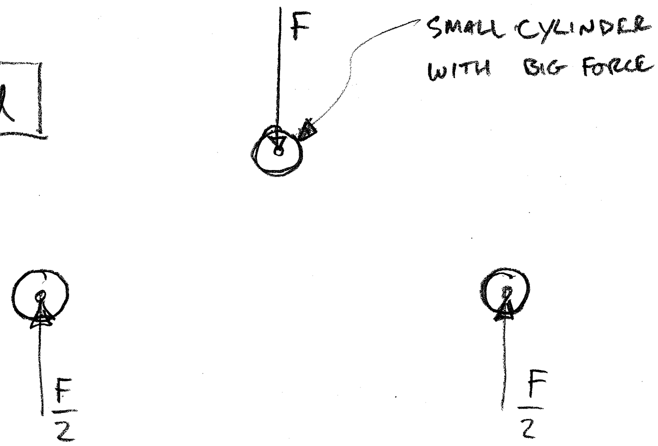
A.j


F
INFINITE
WALL

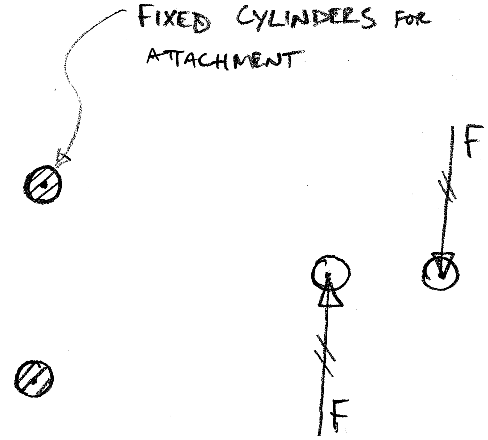
A.k


SUPPORT
SURFACE
Z
X
Y
M_y
F_y

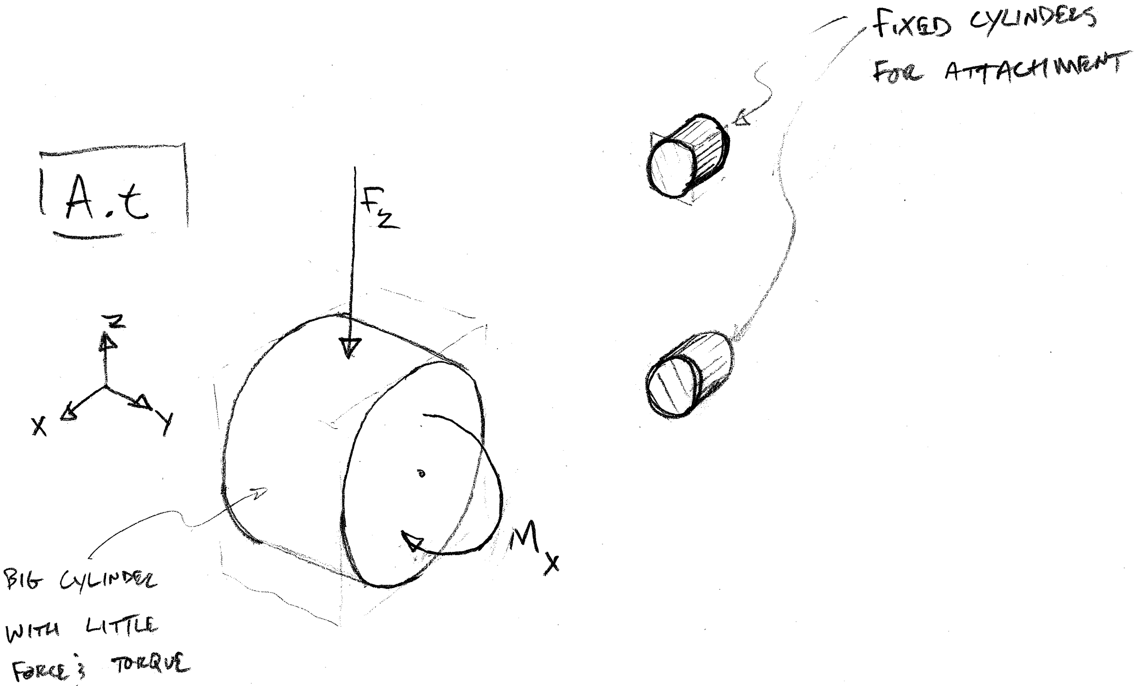
A.l


F
SMALL CYLINDER WITH BIG FORCE
F/2
F/2

A.m


FIXED CYLINDERS FOR ATTACHMENT
F
F

A.t


FIXED CYLINDERS FOR ATTACHMENT
F_z
Z
X
Y
M_x
BIG CYLINDER WITH LITTLE FORCE & TORQUE

Appendix: Fatigue Analysis Equations & Tables

Reproduced from Shigley's Mechanical Engineering Design, Budynas & Nisbett

6–18 Road Maps and Important Design Equations for the Stress-Life Method

As stated in Sec. 6–15, there are three categories of fatigue problems. The important procedures and equations for deterministic stress-life problems are presented here.

Completely Reversing Simple Loading

1 Determine S'_e either from test data or

p. 282
$$S'_e = \begin{cases} 0.5S_{ut} & S_{ut} \le 200 \text{ kpsi } (1400 \text{ MPa}) \\ 100 \text{ kpsi} & S_{ut} > 200 \text{ kpsi} \\ 700 \text{ MPa} & S_{ut} > 1400 \text{ MPa} \end{cases} \tag{6–8}$$

2 Modify S'_e to determine S_e.

p. 287
$$S_e = k_a k_b k_c k_d k_e k_f S'_e \tag{6–18}$$
$$k_a = aS_{ut}^b \tag{6–19}$$

Table 6–2

Parameters for Marin Surface Modification Factor, Eq. (6–19)

Surface Finish	Factor a S_{ut}, kpsi	Factor a S_{ut}, MPa	Exponent b
Ground	1.34	1.58	−0.085
Machined or cold-drawn	2.70	4.51	−0.265
Hot-rolled	14.4	57.7	−0.718
As-forged	39.9	272.	−0.995

Rotating shaft. For bending or torsion,

p. 288
$$k_b = \begin{cases} (d/0.3)^{-0.107} = 0.879d^{-0.107} & 0.11 \le d \le 2 \text{ in} \\ 0.91d^{-0.157} & 2 < d \le 10 \text{ in} \\ (d/7.62)^{-0.107} = 1.24d^{-0.107} & 2.79 \le d \le 51 \text{ mm} \\ 1.51d^{-0.157} & 51 < 254 \text{ mm} \end{cases} \tag{6–20}$$

For axial,

$$k_b = 1 \tag{6–21}$$

Nonrotating member. Use Table 6–3, p. 290, for d_e and substitute into Eq. (6–20) for d.

p. 290
$$k_c = \begin{cases} 1 & \text{bending} \\ 0.85 & \text{axial} \\ 0.59 & \text{torsion} \end{cases} \tag{6–26}$$

p. 291 Use Table 6–4 for k_d, or

$$k_d = 0.975 + 0.432(10^{-3})T_F - 0.115(10^{-5})T_F^2 + 0.104(10^{-8})T_F^3 - 0.595(10^{-12})T_F^4 \tag{6–27}$$

pp. 292–293, k_e

Table 6–5

Reliability Factor k_e Corresponding to 8 Percent Standard Deviation of the Endurance Limit

Reliability, %	Transformation Variate z_a	Reliability Factor k_e
50	0	1.000
90	1.288	0.897
95	1.645	0.868
99	2.326	0.814
99.9	3.091	0.753
99.99	3.719	0.702
99.999	4.265	0.659
99.9999	4.753	0.620

pp. 293–294, k_f

3 Determine fatigue stress-concentration factor, K_f or K_{fs}. First, find K_t or K_{ts} from Table A–15.

p. 295

$$K_f = 1 + q(K_t - 1) \quad \text{or} \quad K_{fs} = 1 + q(K_{ts} - 1) \tag{6–32}$$

Obtain q from either Fig. 6–20 or 6–21, pp. 295–296.

Alternatively,

p. 296

$$K_f = 1 + \frac{K_t - 1}{1 + \sqrt{a/r}} \tag{6–33}$$

For $\sqrt{a}$ in units of $\sqrt{\text{in}}$, and S_{ut} in kpsi

Bending or axial: $\sqrt{a} = 0.246 - 3.08(10^{-3})S_{ut} + 1.51(10^{-5})S_{ut}^2 - 2.67(10^{-8})S_{ut}^3$ (6–35a)

Torsion: $\sqrt{a} = 0.190 - 2.51(10^{-3})S_{ut} + 1.35(10^{-5})S_{ut}^2 - 2.67(10^{-8})S_{ut}^3$ (6–35b)

4 Apply K_f or K_{fs} by *either* dividing S_e by it *or* multiplying it with the purely reversing stress, *not* both.

5 Determine fatigue life constants a and b. If $S_{ut} \geq 70$ kpsi, determine f from Fig. 6–18, p. 285. If $S_{ut} < 70$ kpsi, let $f = 0.9$.

p. 285

$$a = (f\ S_{ut})^2/S_e \tag{6–14}$$

$$b = -[\log(f\ S_{ut}/S_e)]/3 \tag{6–15}$$

6 Determine fatigue strength S_f at N cycles, or, N cycles to failure at a reversing stress σ_{rev}

(*Note*: this only applies to purely reversing stresses where $\sigma_m = 0$).

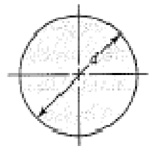
p. 285

$$S_f = aN^b \tag{6–13}$$

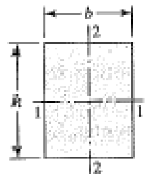
$$N = (\sigma_{rev}/a)^{1/b} \tag{6–16}$$

Table 6–3

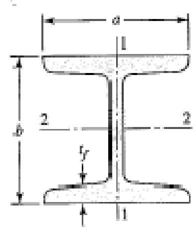
$A_{0.95\sigma}$ Areas of Common Nonrotating Structural Shapes



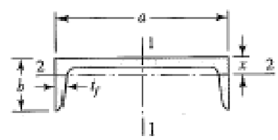
$$A_{0.95\sigma} = 0.01046d^2$$
$$d_e = 0.370d$$



$$A_{0.95\sigma} = 0.05hb$$
$$d_e = 0.808\sqrt{hb}$$



$$A_{0.95\sigma} = \begin{cases} 0.10at_f & \text{axis 1-1} \\ 0.05ba \quad t_f > 0.025a & \text{axis 2-2} \end{cases}$$



$$A_{0.95\sigma} = \begin{cases} 0.05ab & \text{axis 1-1} \\ 0.052xa + 0.1t_f(b - x) & \text{axis 2-2} \end{cases}$$

Table 6–4

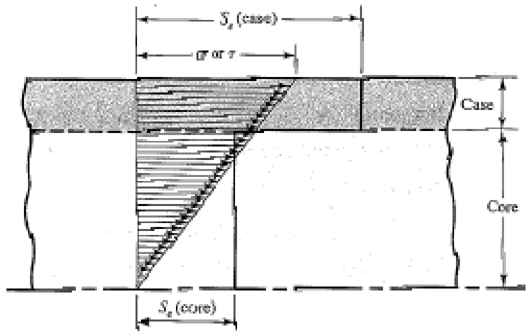
Effect of Operating Temperature on the Tensile Strength of Steel.* (S_T = tensile strength at operating temperature; S_{RT} = tensile strength at room temperature; $0.099 \le \hat{\sigma} \le 0.110$)

Temperature, °C	S_T/S_{RT}	Temperature, °F	S_T/S_{RT}
20	1.000	70	1.000
50	1.010	100	1.008
100	1.020	200	1.020
150	1.025	300	1.024
200	1.020	400	1.018
250	1.000	500	0.995
300	0.975	600	0.963
350	0.943	700	0.927
400	0.900	800	0.872
450	0.843	900	0.797
500	0.768	1000	0.698
550	0.672	1100	0.567
600	0.549		

*Data source: Fig. 2–9.

Figure 6–19

The failure of a case-hardened part in bending or torsion. In this example, failure occurs in the core.



Miscellaneous-Effects Factor k_f

Though the factor k_f is intended to account for the reduction in endurance limit due to all other effects, it is really intended as a reminder that these must be accounted for, because actual values of k_f are not always available.

Residual stresses may either improve the endurance limit or affect it adversely. Generally, if the residual stress in the surface of the part is compression, the endurance limit is improved. Fatigue failures appear to be tensile failures, or at least to be caused by tensile stress, and so anything that reduces tensile stress will also reduce the possibility of a fatigue failure. Operations such as shot peening, hammering, and cold rolling build compressive stresses into the surface of the part and improve the endurance limit significantly. Of course, the material must not be worked to exhaustion.

The endurance limits of parts that are made from rolled or drawn sheets or bars, as well as parts that are forged, may be affected by the so-called *directional characteristics* of the operation. Rolled or drawn parts, for example, have an endurance limit in the transverse direction that may be 10 to 20 percent less than the endurance limit in the longitudinal direction.

Parts that are case-hardened may fail at the surface or at the maximum core radius, depending upon the stress gradient. Figure 6–19 shows the typical triangular stress distribution of a bar under bending or torsion. Also plotted as a heavy line in this figure are the endurance limits S_e for the case and core. For this example the endurance limit of the core rules the design because the figure shows that the stress σ or τ, whichever applies, at the outer core radius, is appreciably larger than the core endurance limit.

Corrosion

It is to be expected that parts that operate in a corrosive atmosphere will have a lowered fatigue resistance. This is, of course, true, and it is due to the roughening or pitting of the surface by the corrosive material. But the problem is not so simple as the one of finding the endurance limit of a specimen that has been corroded. The reason for this is that the corrosion and the stressing occur at the same time. Basically, this means that in time any part will fail when subjected to repeated stressing in a corrosive atmosphere. There is no fatigue limit. Thus the designer's problem is to attempt to minimize the factors that affect the fatigue life; these are:

- Mean or static stress
- Alternating stress
- Electrolyte concentration
- Dissolved oxygen in electrolyte
- Material properties and composition
- Temperature
- Cyclic frequency
- Fluid flow rate around specimen
- Local crevices

Electrolytic Plating

Metallic coatings, such as chromium plating, nickel plating, or cadmium plating, reduce the endurance limit by as much as 50 percent. In some cases the reduction by coatings has been so severe that it has been necessary to eliminate the plating process. Zinc plating does not affect the fatigue strength. Anodic oxidation of light alloys reduces bending endurance limits by as much as 39 percent but has no effect on the torsional endurance limit.

Metal Spraying

Metal spraying results in surface imperfections that can initiate cracks. Limited tests show reductions of 14 percent in the fatigue strength.

Cyclic Frequency

If, for any reason, the fatigue process becomes time-dependent, then it also becomes frequency-dependent. Under normal conditions, fatigue failure is independent of frequency. But when corrosion or high temperatures, or both, are encountered, the cyclic rate becomes important. The slower the frequency and the higher the temperature, the higher the crack propagation rate and the shorter the life at a given stress level.

Frettage Corrosion

The phenomenon of frettage corrosion is the result of microscopic motions of tightly fitting parts or structures. Bolted joints, bearing-race fits, wheel hubs, and any set of tightly fitted parts are examples. The process involves surface discoloration, pitting, and eventual fatigue. The frettage factor k_f depends upon the material of the mating pairs and ranges from 0.24 to 0.90.

Table A–15

Charts of Theoretical Stress-Concentration Factors K_t^*

Figure A–15–1

Bar in tension or simple compression with a transverse hole. $\sigma_0 = F/A$, where $A = (w - d)t$ and t is the thickness.

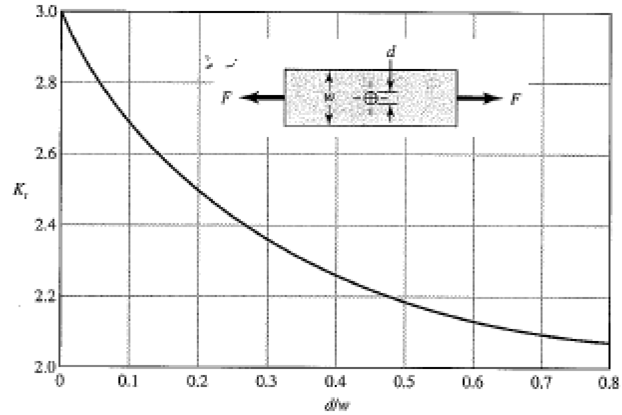


Figure A–15–2

Rectangular bar with a transverse hole in bending. $\sigma_0 = Mc/I$, where $I = (w - d)h^3/12$.

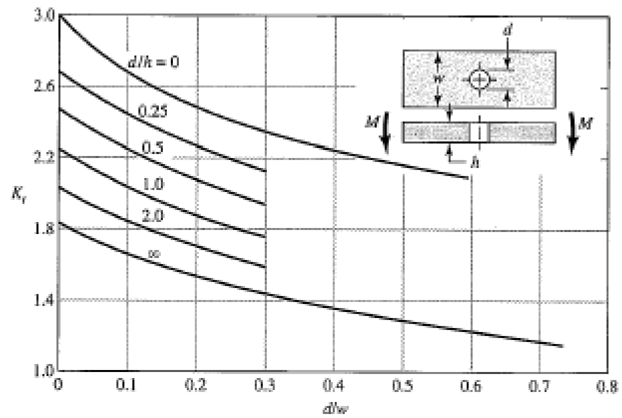


Figure A–15–3

Notched rectangular bar in tension or simple compression. $\sigma_0 = F/A$, where $A = dt$ and t is the thickness.

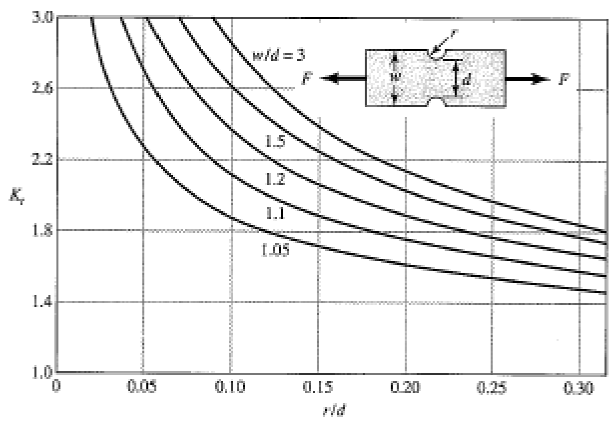


Table A-15

Charts of Theoretical Stress-Concentration Factors K_t^* *(Continued)*

Figure A-15-4

Notched rectangular bar in bending. $\sigma_0 = Mc/I$, where $c = d/2$, $I = td^3/12$, and t is the thickness.

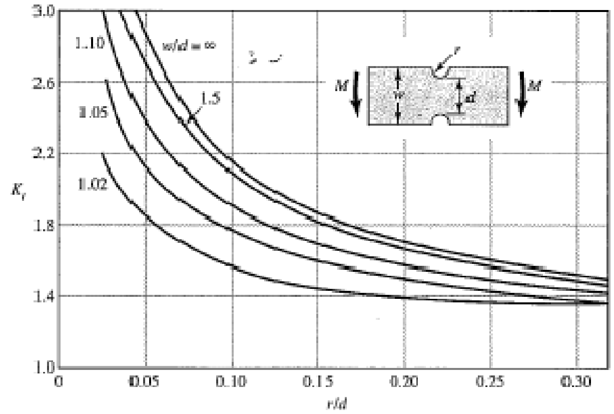


Figure A-15-5

Rectangular filleted bar in tension or simple compression. $\sigma_0 = F/A$, where $A = dt$ and t is the thickness.

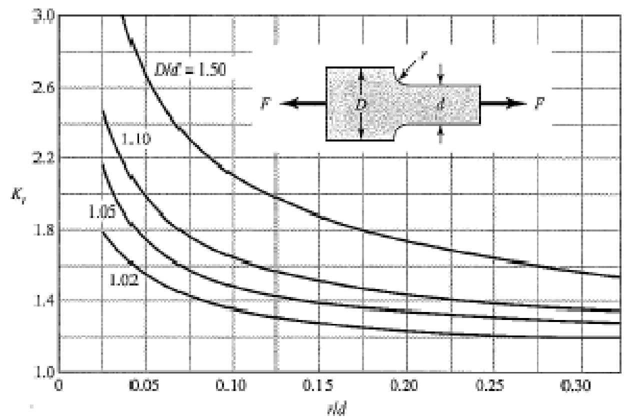
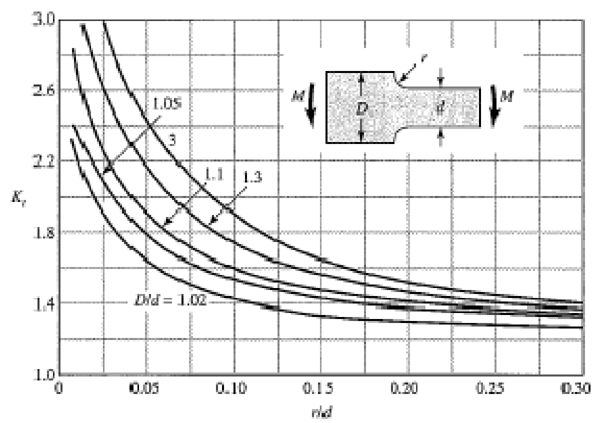


Figure A-15-6

Rectangular filleted bar in bending. $\sigma_0 = Mc/I$, where $c = d/2$, $I = td^3/12$, t is the thickness.



(continued)

*Factors from R. E. Peterson, "Design Factors for Stress Concentration," Machine Design, vol. 23, no. 2, February 1951, p. 169; no. 3, March 1951, p. 161, no. 5, May 1951, p. 159; no. 6, June 1951, p. 173; no. 7, July 1951, p. 155. Reprinted with permission from Machine Design, a Penton Media Inc. publication.

Table A-15

Charts of Theoretical Stress-Concentration Factors K_t^* *(Continued)*

Figure A-15-7

Round shaft with shoulder fillet in tension. $\sigma_0 = F/A$, where $A = \pi d^2/4$.

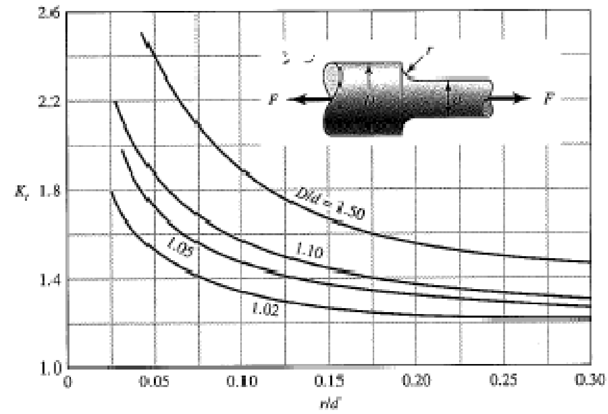


Figure A-15-8

Round shaft with shoulder fillet in torsion. $\tau_0 = Tc/J$, where $c = d/2$ and $J = \pi d^4/32$.

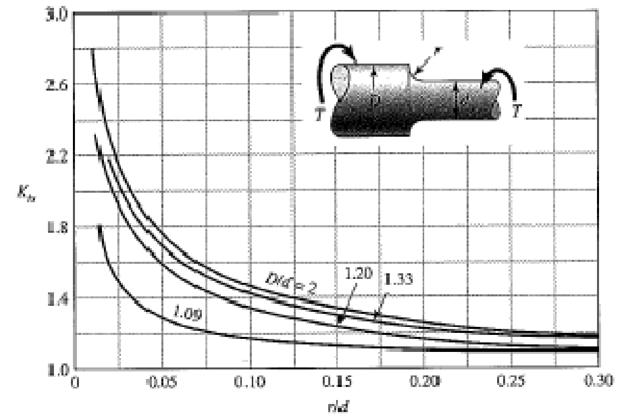


Figure A-15-9

Round shaft with shoulder fillet in bending. $\sigma_0 = Mc/I$, where $c = d/2$ and $I = \pi d^4/64$.

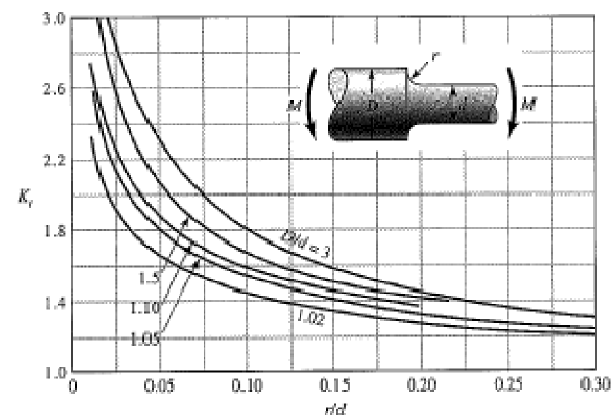


Table A–15

Charts of Theoretical Stress-Concentration Factors K_t^* *(Continued)*

Figure A–15–10

Round shaft in torsion with transverse hole.

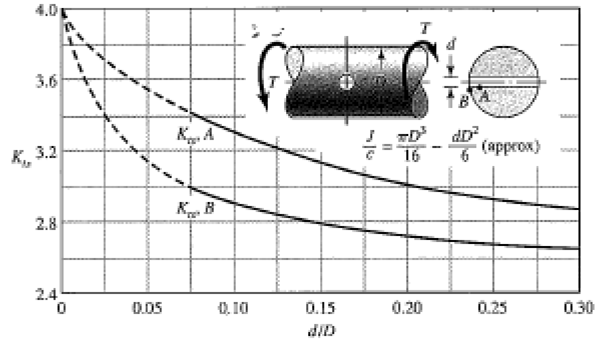


Figure A–15–11

Round shaft in bending with a transverse hole. $\sigma_0 = M/[(\pi D^3/32) - (dD^2/6)]$, approximately.

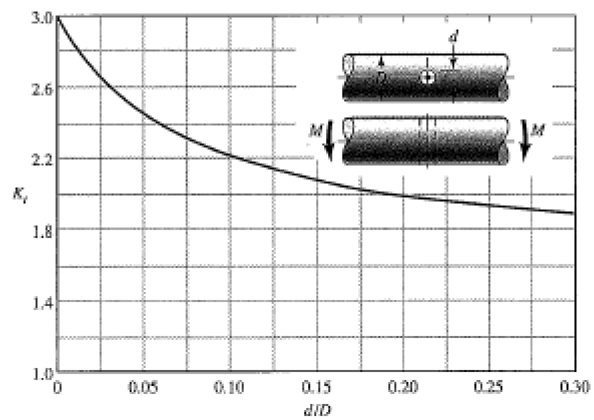
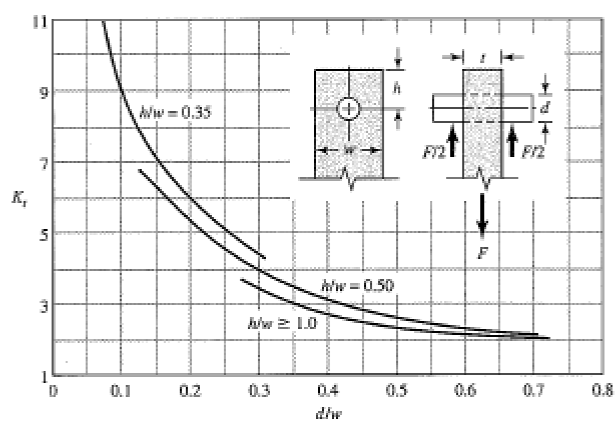


Figure A–15–12

Plate loaded in tension by a pin through a hole. $\sigma_0 = F/A$, where $A = (w - d)t$. When clearance exists, increase K_t 35 to 50 percent. *(M. M. Frocht and H. N. Hill, "Stress-Concentration Factors around a Central Circular Hole in a Plate Loaded through a Pin in Hole," J. Appl. Mechanics, vol. 7, no. 1, March 1940, p. A-5.)*



(continued)

*Factors from R. E. Peterson, "Design Factors for Stress Concentration," Machine Design, vol. 23, no. 2, February 1951, p. 169; no. 3, March 1951, p. 161, no. 5, May 1951, p. 159; no. 6, June 1951, p. 173; no. 7, July 1951, p. 155. Reprinted with permission from Machine Design, a Penton Media Inc. publication.

Table A-15

Charts of Theoretical Stress-Concentration Factors K_t^* *(Continued)*

Figure A-15-13

Grooved round bar in tension. $\sigma_0 = F/A$, where $A = \pi d^2/4$.

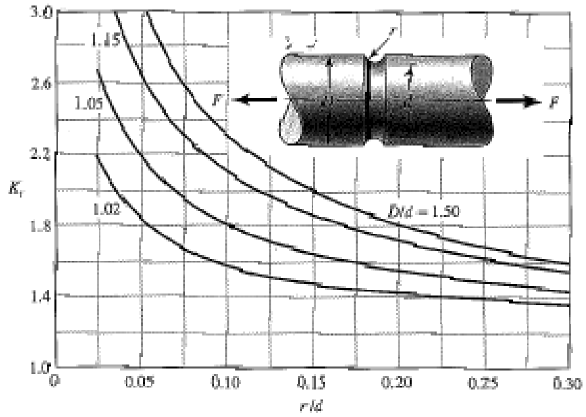


Figure A-15-14

Grooved round bar in bending. $\sigma_0 = Mc/I$, where $c = d/2$ and $I = \pi d^4/64$.

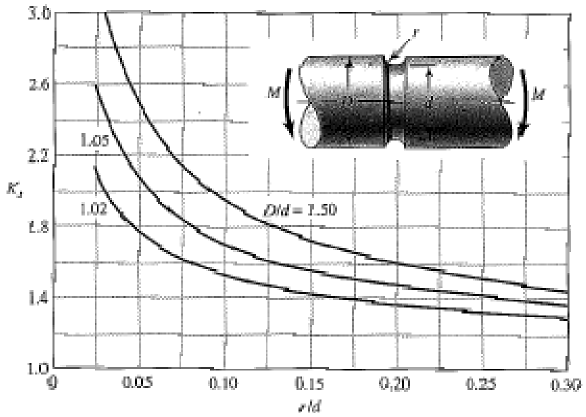
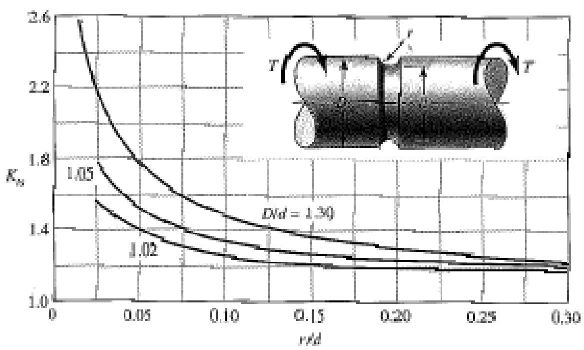


Figure A-15-15

Grooved round bar in torsion. $\tau_0 = Tc/J$, where $c = d/2$ and $J = \pi d^4/32$.



*Factors from R. E. Peterson, "Design Factors for Stress Concentration," Machine Design, vol. 23, no. 2, February 1951, p. 169; no. 3, March 1951, p. 161, no. 5, May 1951, p. 159; no. 6, June 1951, p. 173; no. 7, July 1951, p. 155. Reprinted with permission from Machine Design, a Penton Media Inc. publication.

Table A–15

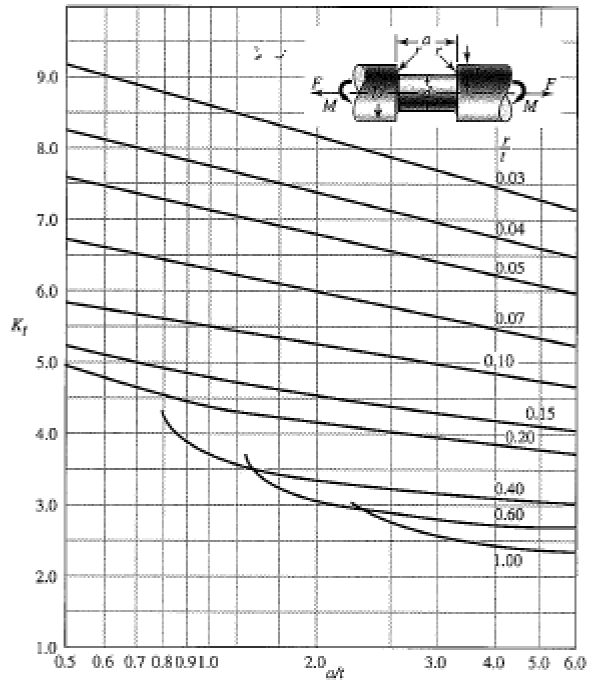
Charts of Theoretical Stress-Concentration Factors K_t^* *(Continued)*

Figure A–15–16

Round shaft with flat-bottom groove in bending and/or tension.

$$\sigma_0 = \frac{4F}{\pi d^2} + \frac{32M}{\pi d^3}$$

Source: W. D. Pilkey, *Peterson's Stress-Concentration Factors*, 2nd ed. John Wiley & Sons, New York, 1997, p. 115.



(continued)

Table A–15

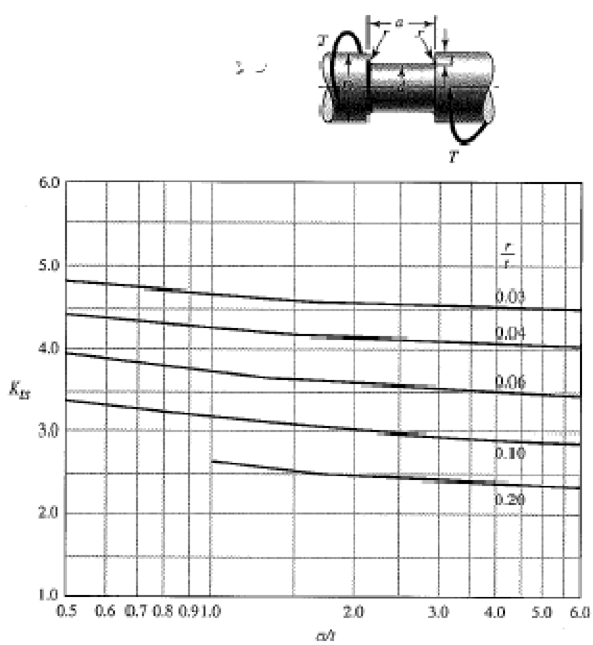
Charts of Theoretical Stress-Concentration Factors K_t^* *(Continued)*

Figure A–15–17

Round shaft with flat-bottom groove in torsion.

$$\tau_0 = \frac{16T}{\pi d^3}$$

Source: W. D. Pilkey, *Peterson's Stress-Concentration Factors*, 2nd ed. John Wiley & Sons, New York, 1997, p. 133



6–10 Stress Concentration and Notch Sensitivity

In Sec. 3–13 it was pointed out that the existence of irregularities or discontinuities, such as holes, grooves, or notches, in a part increases the theoretical stresses significantly in the immediate vicinity of the discontinuity. Equation (3–48) defined a stress-concentration factor K_t (or K_{ts}), which is used with the nominal stress to obtain the maximum resulting stress due to the irregularity or defect. It turns out that some materials are not fully sensitive to the presence of notches and hence, for these, a reduced value of K_t can be used. For these materials, the effective maximum stress in fatigue is,

$$\sigma_{\max} = K_f \sigma_0 \qquad \text{or} \qquad \tau_{\max} = K_{fs} \tau_0 \tag{6–30}$$

where K_f is a reduced value of K_t and σ_0 is the nominal stress. The factor K_f is commonly called a *fatigue stress-concentration factor,* and hence the subscript f. So it is convenient to think of K_f as a stress-concentration factor reduced from K_t because of lessened sensitivity to notches. The resulting factor is defined by the equation

$$K_f = \frac{\text{maximum stress in notched specimen}}{\text{stress in notch-free specimen}} \tag{a}$$

Notch sensitivity q is defined by the equation

$$q = \frac{K_f - 1}{K_t - 1} \qquad \text{or} \qquad q_{\text{shear}} = \frac{K_{fs} - 1}{K_{ts} - 1} \tag{6–31}$$

where q is usually between zero and unity. Equation (6–31) shows that if $q = 0$, then $K_f = 1$, and the material has no sensitivity to notches at all. On the other hand, if $q = 1$, then $K_f = K_t$, and the material has full notch sensitivity. In analysis or design work, find K_t first, from the geometry of the part. Then specify the material, find q, and solve for K_f from the equation

$$K_f = 1 + q(K_t - 1) \qquad \text{or} \qquad K_{fs} = 1 + q_{\text{shear}}(K_{ts} - 1) \tag{6–32}$$

Notch sensitivities for specific materials are obtained experimentally. Published experimental values are limited, but some values are available for steels and aluminum. Trends for notch sensitivity as a function of notch radius and ultimate strength are shown in Fig. 6–20 for reversed bending or axial loading, and Fig. 6–21 for reversed

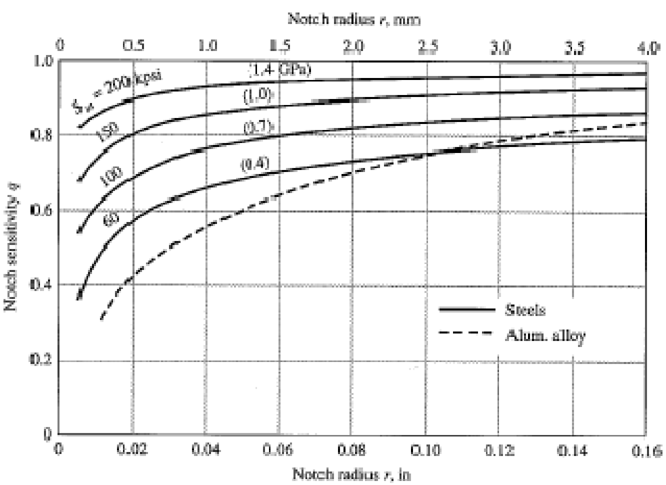
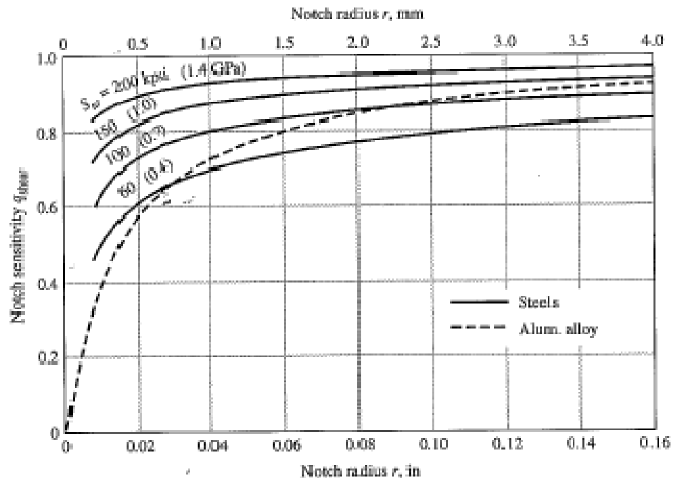


Figure 6–20

Notch-sensitivity charts for steels and UNS A92024-T wrought aluminum alloys subjected to reversed bending or reversed axial loads. For larger notch radii, use the values of q corresponding to the $r = 0.16$-in (4-mm) ordinate. *(From George Sines and J. L. Waisman (eds.),* Metal Fatigue, *McGraw-Hill, New York. Copyright © 1969 by The McGraw-Hill Companies, Inc. Reprinted by permission.)*

Figure 6–21

Notch-sensitivity curves for materials in reversed torsion. For larger notch radii, use the values of q_{shear} corresponding to $r = 0.16$ in (4 mm).



torsion. In using these charts it is well to know that the actual test results from which the curves were derived exhibit a large amount of scatter. Because of this scatter it is always safe to use $K_f = K_t$ if there is any doubt about the true value of q. Also, note that q is not far from unity for large notch radii.

Figure 6–20 has as its basis the *Neuber equation,* which is given by

$$K_f = 1 + \frac{K_t - 1}{1 + \sqrt{a/r}} \qquad (6\text{–}33)$$

where $\sqrt{a}$ is defined as the *Neuber constant* and is a material constant. Equating Eqs. (6–31) and (6–33) yields the notch sensitivity equation

$$q = \frac{1}{1 + \dfrac{\sqrt{a}}{\sqrt{r}}} \qquad (6\text{–}34)$$

correlating with Figs. 6–20 and 6–21 as

Bending or axial: $\sqrt{a} = 0.246 - 3.08(10^{-3})S_{ut} + 1.51(10^{-5})S_{ut}^2 - 2.67(10^{-8})S_{ut}^3$ (6–35a)

Torsion: $\sqrt{a} = 0.190 - 2.51(10^{-3})S_{ut} + 1.35(10^{-5})S_{ut}^2 - 2.67(10^{-8})S_{ut}^3$ (6–35b)

where the equations apply to steel and S_{ut} is in kpsi. Equation (6–34) used in conjunction with Eq. pair (6–35) is equivalent to Figs. (6–20) and (6–21). As with the graphs, the results from the curve fit equations provide only approximations to the experimental data.

The notch sensitivity of cast irons is very low, varying from 0 to about 0.20, depending upon the tensile strength. To be on the conservative side, it is recommended that the value $q = 0.20$ be used for all grades of cast iron.

Figure 6–18

Fatigue strength fraction, f, of S_{ut} at 10^3 cycles for $S_e = S'_e = 0.5S_{ut}$ at 10^6 cycles.