

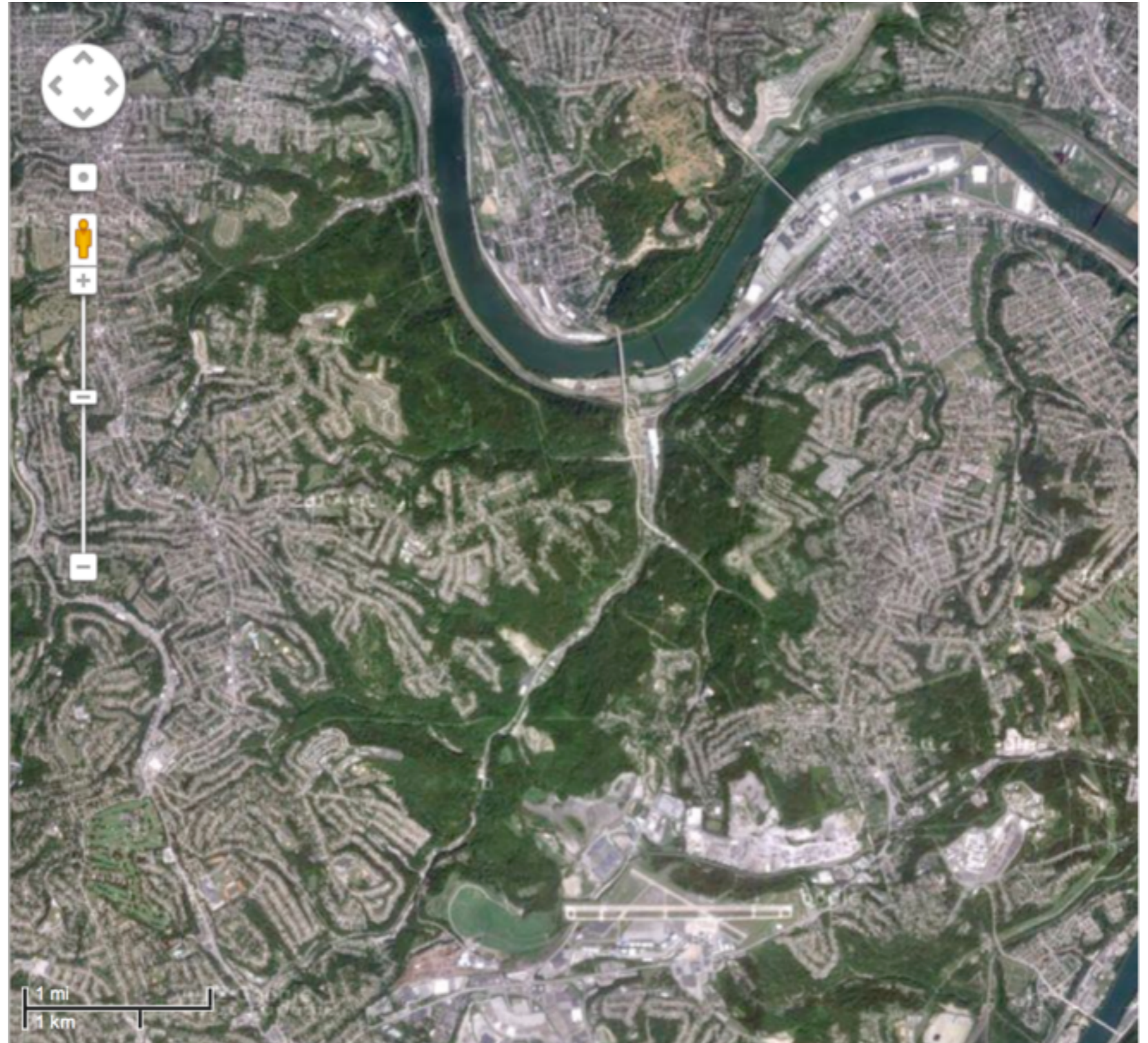


Descriptive Geometry

A typical problem

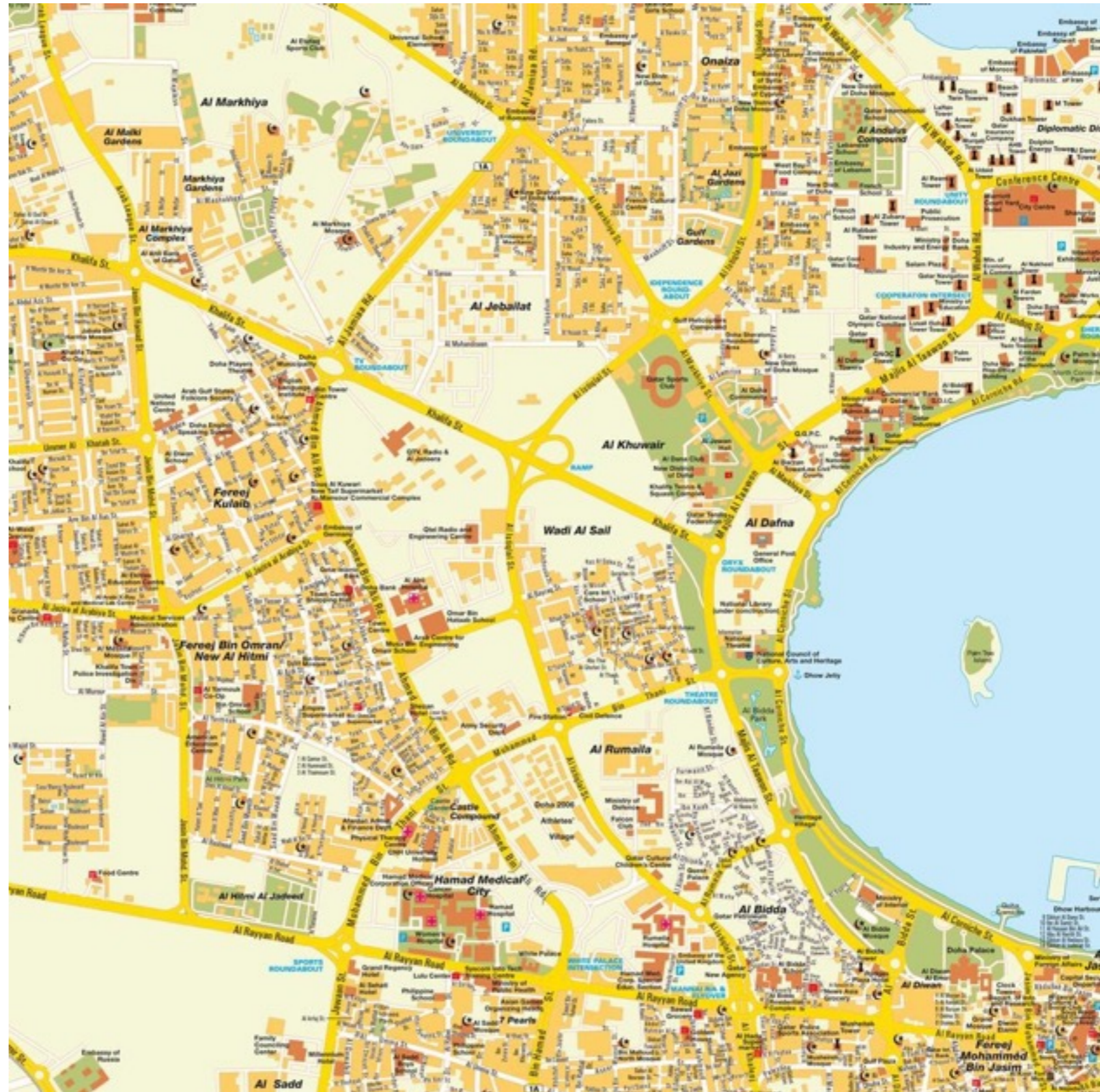


*can you work out
the area of the
green area just
using geometrical
construction?*



► a typical problem

Or the green areas
here?



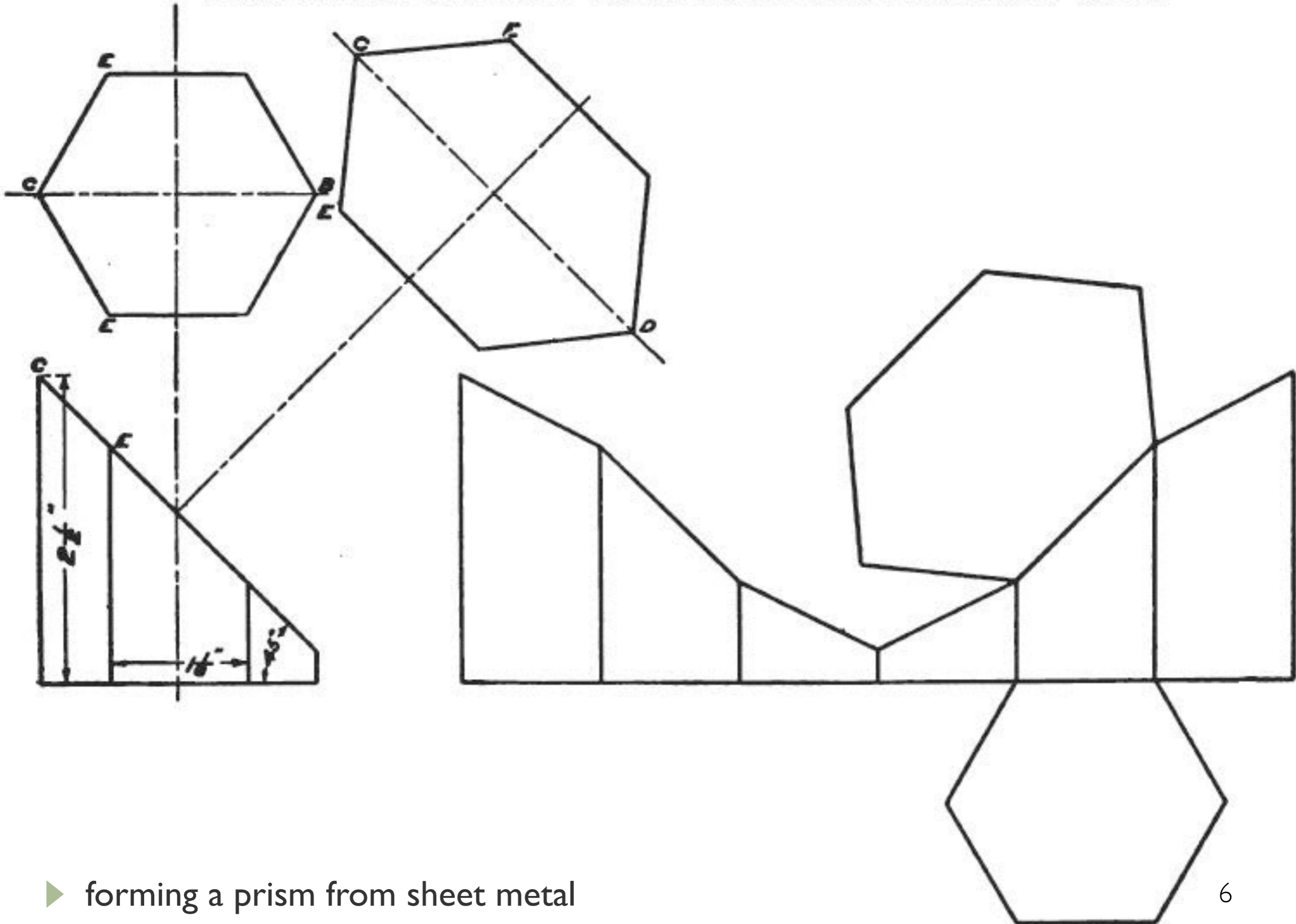
► a typical problem



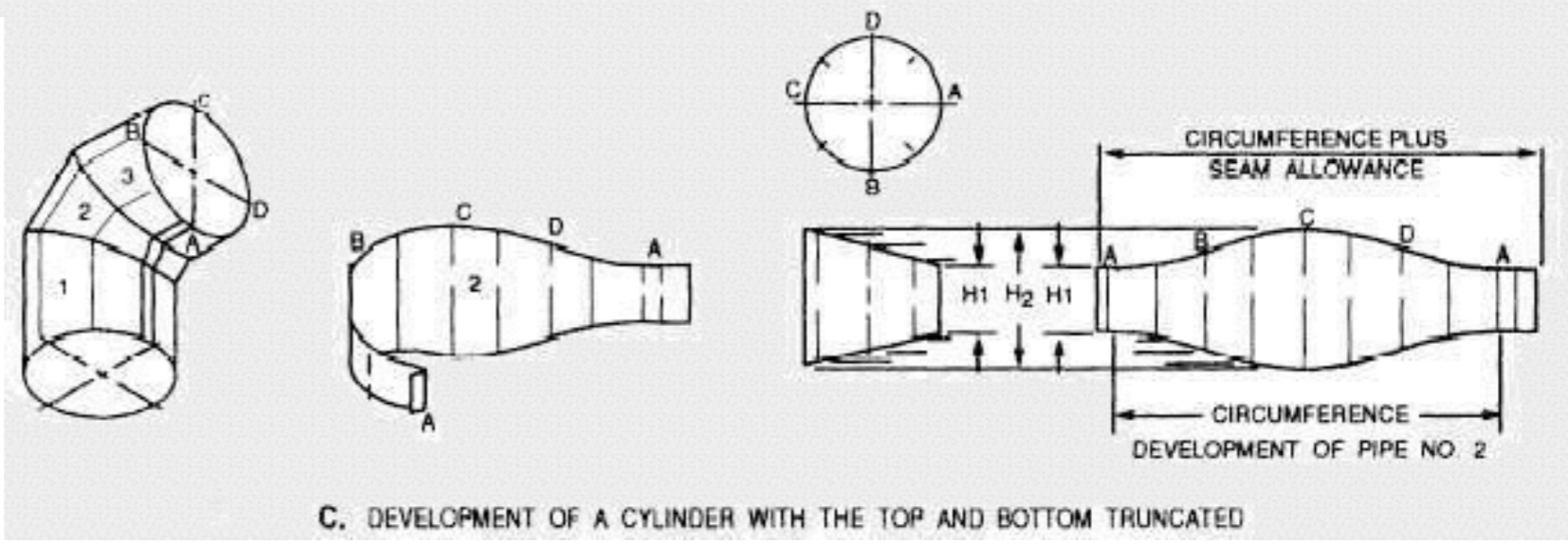
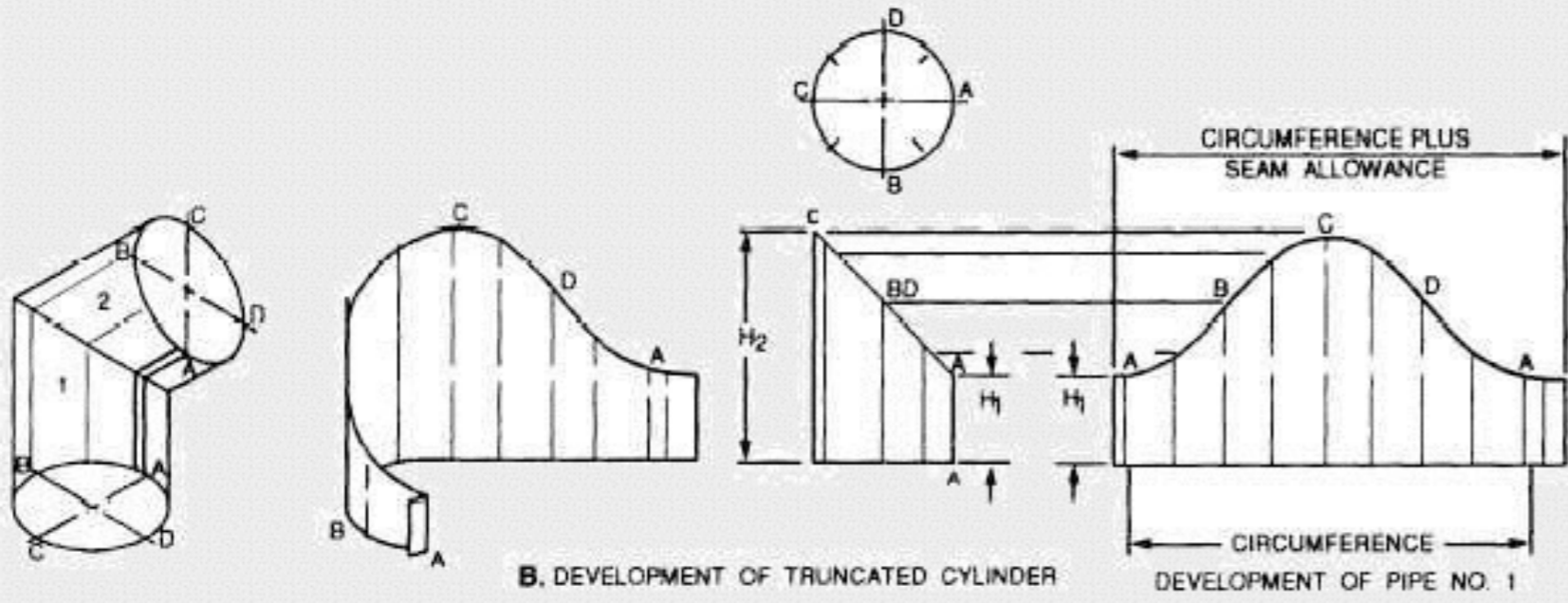
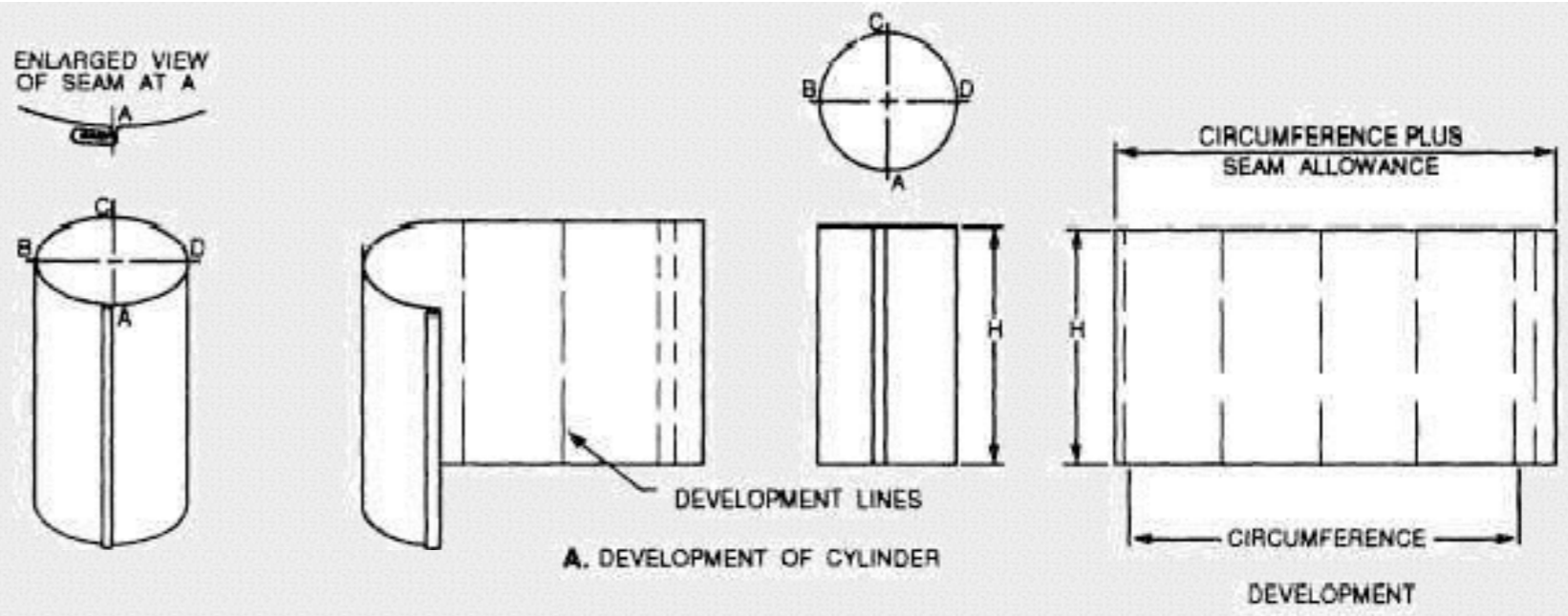
Development of an object



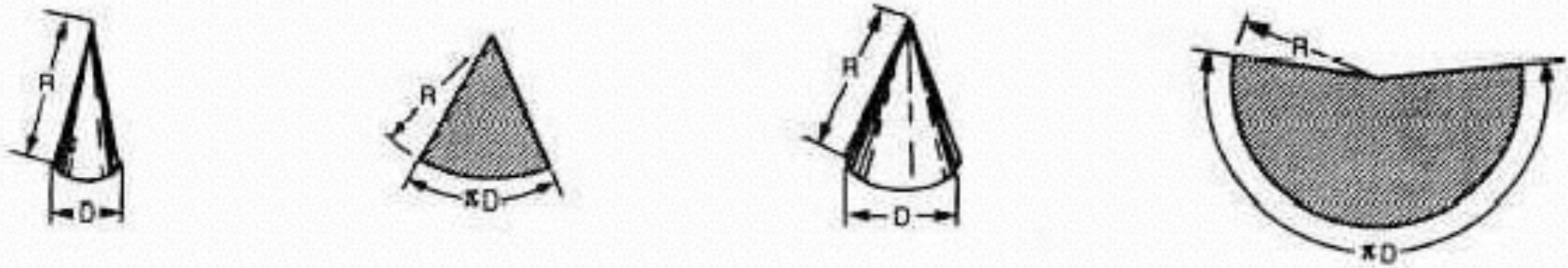
DEVELOPMENT OF A TRUNCATED HEXAGONAL PRISM



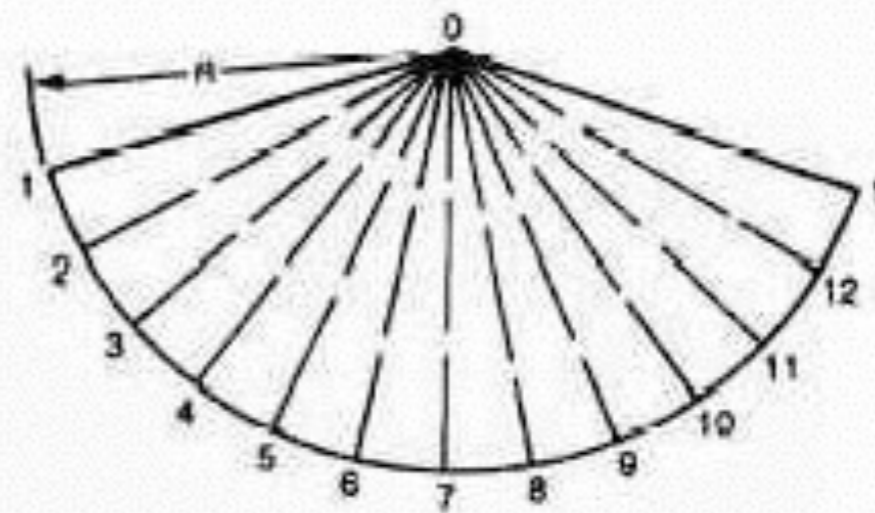
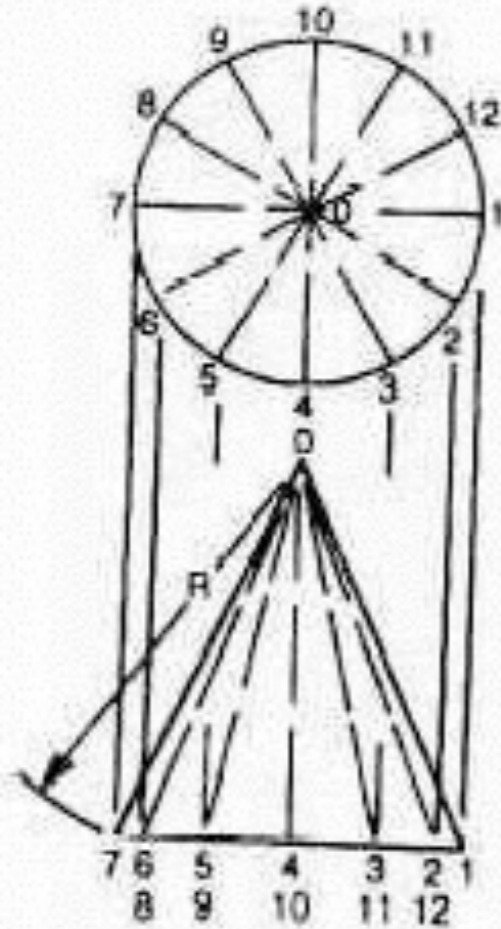
► forming a prism from sheet metal



► development of a cylinder

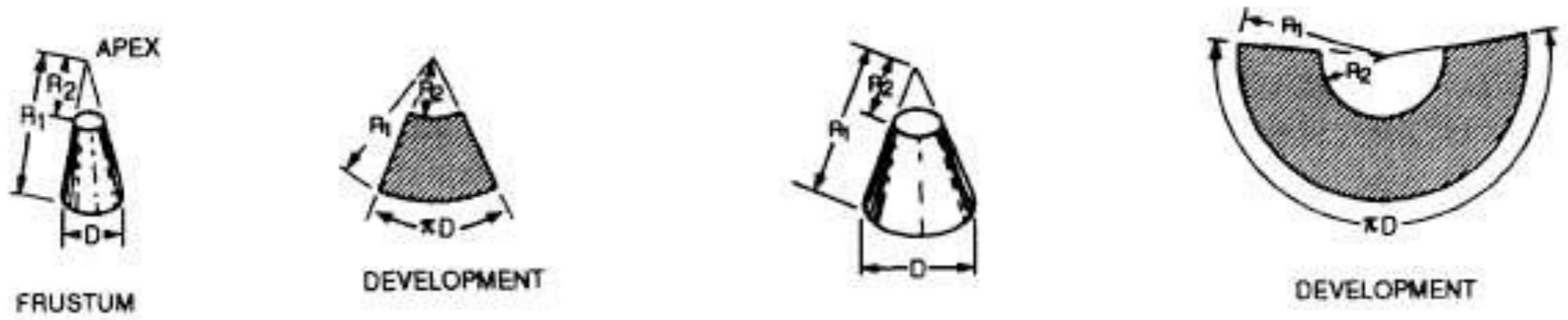


A. PROPORTION OF HEIGHT TO BASE

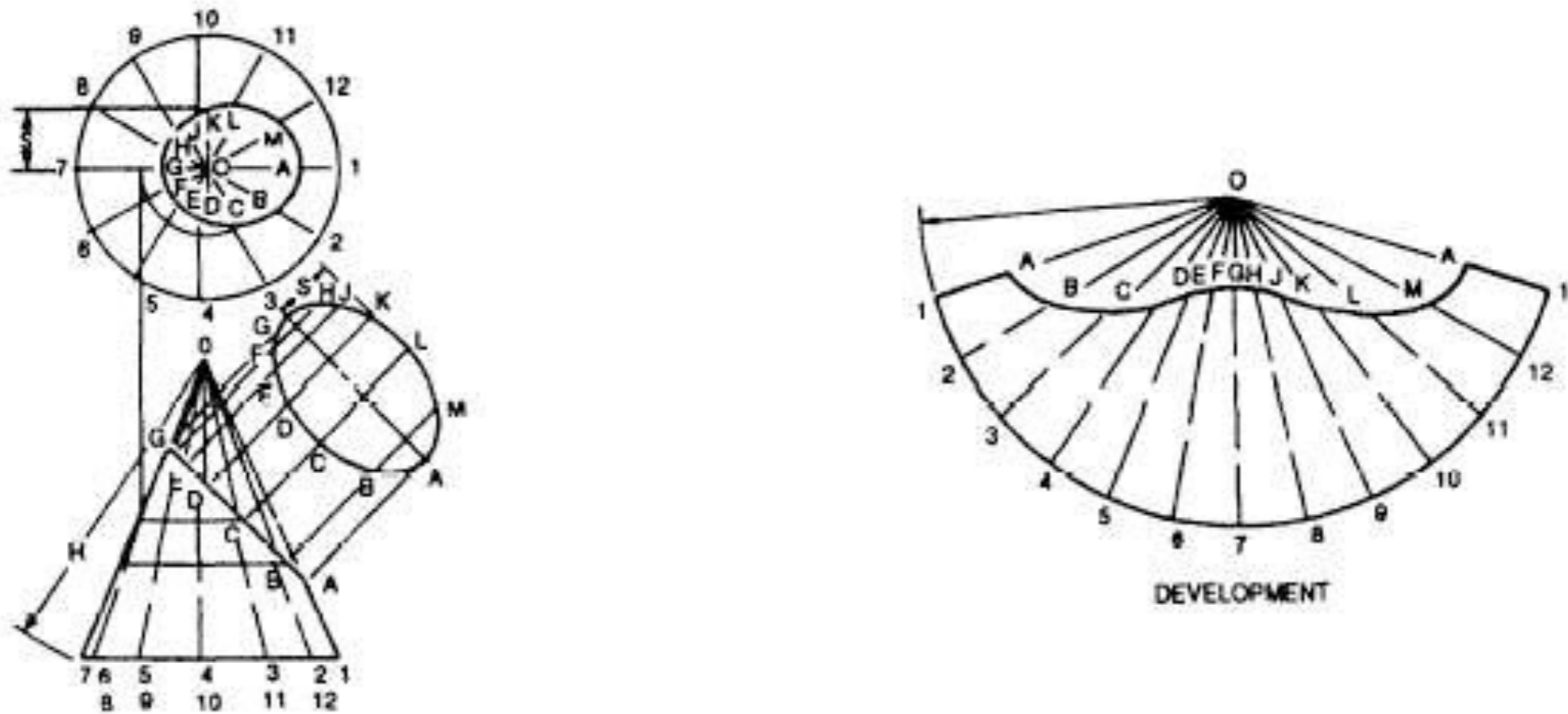


DEVELOPMENT

B. DEVELOPMENT PROCEDURE



A. PROPORTION OF HEIGHT TO BASE



B. DEVELOPMENT PROCEDURE

► development of a truncated cone



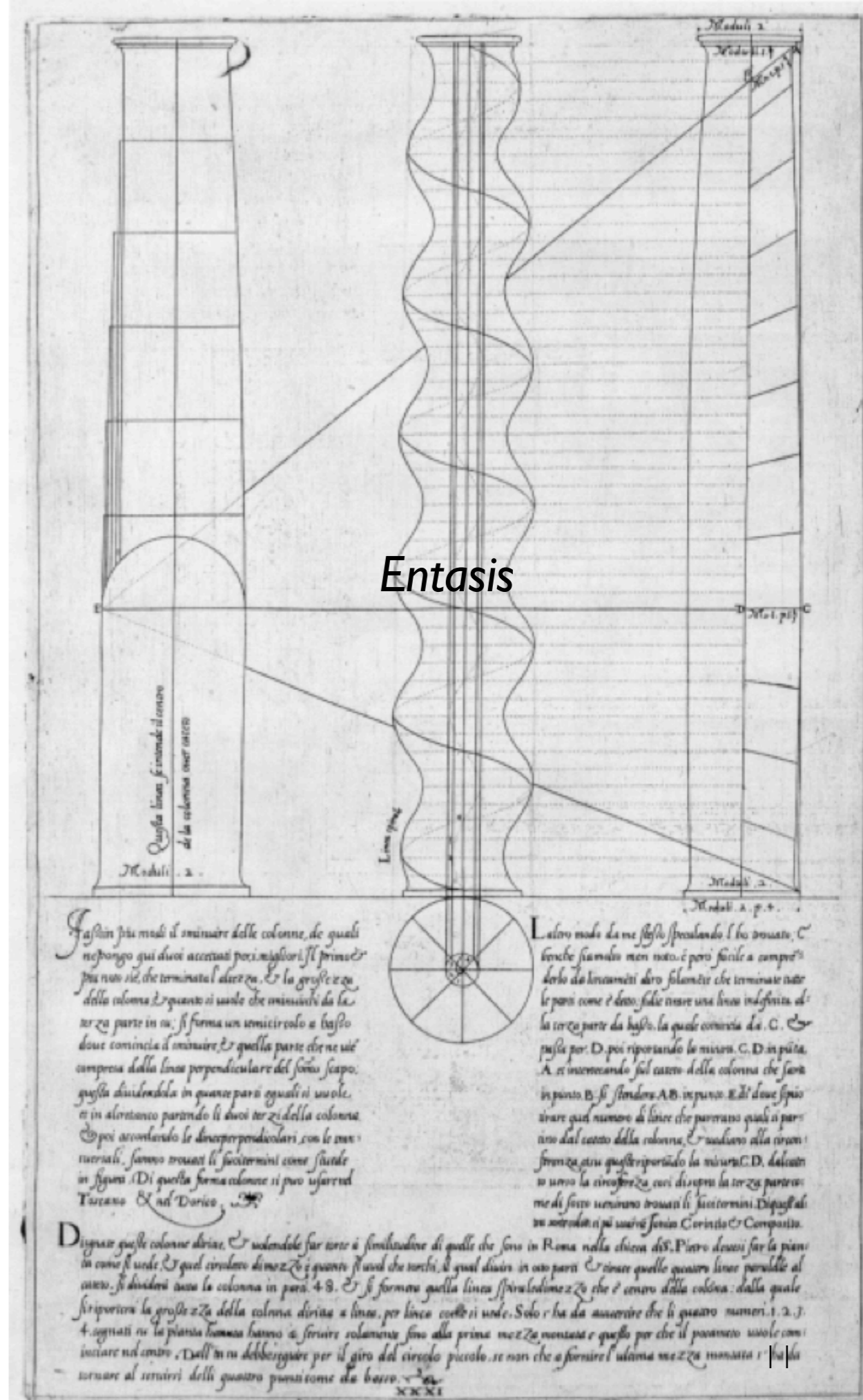
Canons of the Five Orders of Architecture





Giacomo Barozzi da Vignola Canon of the Five Orders of Architecture

- ▶ the use of geometric tools

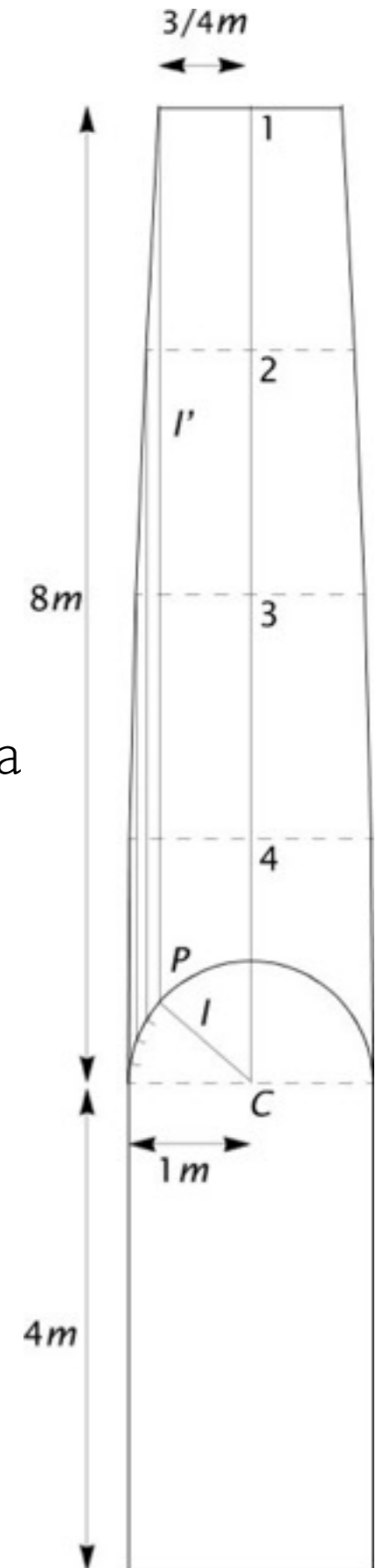


Facem più modi di ornare delle colonne, de quali ne pongo qui dieci accetti per i migliori. Il primo è più nudo, che terminata al di sopra. Et la grossezza della colonna è quanto si vuole che minuisca da la terza parte in su; si forma un semicircolo a basso doue comincia il ornare. Et quella parte che ne uie compresa dalla linea perpendicolare del sotto scapo, quella diuidendola in quattro parti eguali si vuole, et in altrettanto partendo li due terzi della colonna. Et poi accostando le linee perpendicolari con le due verticali, fanno trouar li fuortermi come si uide in figura. Di quella forma colonne si può usar nel Tuscano Et nel Dorico.

Un altro modo da me stesso speculando l'ho trouato. Et benché sia molto men noto, è però facile a comprerlo da li termini altri solamente che terminati alle parti come è detto. Solo resta una linea indifferita di la terza parte da basso, la quale comincia da C. Et passa per D. poi riportando la misura C. D. in parte A. et incrementando sul casso della colonna che sarà in punto B. si stende AB. in parte. Ed è doue si può auere quel numero di linee che potranno quati a partire dal casso della colonna. Et ualendo alla circonferenza di questo rapporto la misura C. D. delimito un arco la cui circonferenza cavi disopra la terza parte: me di sotto ueruno trouati li fuortermi. Di quella forma colonne si può usar in Corinto Et Composito.

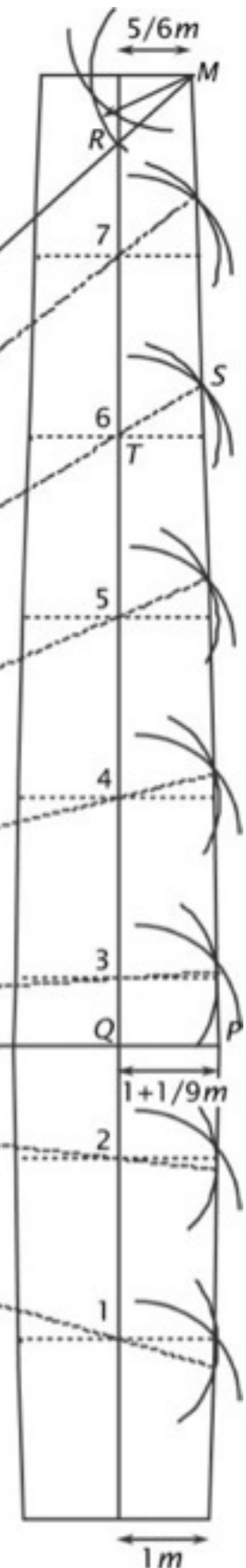
Di queste due colonne diria. Et uolendole far tutte à similitudine di quelle che sono in Roma nella chiesa di S. Pietro douo far la pianta in caso si uede. Et quel circolo di mezzo è quanto si uole che tocchi al qual diuiso in otto parti. Et tirare quelle quattro linee parallele al casso. Si diuiderà douo la colonna in parti 4. 8. Et si formano quella linea spirale douo è che è centro della colonna: dalla quale si riporterà la grossezza della colonna diria a linea, per linea colle si uole. Solo resta da auerire che li quattro numeri 1. 2. 3. 4. segnati in la pianta hanno a seruire solamente fino alla prima mezza montata e quella per che il portamento uole cominciare nel centro. Dall' in su debber girare per il giro del circolo piccolo, se non che a finire l'ultima mezza montata. Et si uolere al seruire delli quattro punti come da basso.

1. Determine height and largest diameter, d . These measures are normally integral multiples of a common module, m .
2. At $\frac{1}{3}$ the height, **draw a line**, l , across the shaft and draw a semi-circle, c , about the center point of l , C , with radius d ($1m$). The shaft has uniform diameter d below line l .
3. Determine smallest diameter at the top of the shaft ($1.5m$ in our case). **Draw a perpendicular**, l' , through an end-point of the diameter. l' intersects c at a point P . The line through P and C defines together with l a segment of c .
4. **Divide the segment into segments of equal size** and divide the shaft above l into the same number of sections of equal height.
5. Each of these segments intersects c at a point. **Draw a perpendicular** line through each of these points and find the intersection point with the corresponding shaft division as shown. *Each intersection point is a point of the profile.*



- Determine height and diameter (or radius) at its widest and top. The base is assumed to be $2m$ wide, the height $16m$. The widest radius occurs at rd of the total height and is $1 + m$. The radius at the top is m .
- Draw a line, l ,** through the column at its widest. Q is the center point of the column on l and P is at distance $1 + m$ from Q on l .
- M is at distance m from the center at the top and on the same side as P . **Draw a circle** centered at M with radius $1 + m$. This circle intersects the centerline of the column at point R .
- Draw a line** through M and R and find its intersection, O , with l .
- Draw a series of horizontal lines** that divide the shaft into equal sections. Any such line intersects the centerline at a point T . **Draw a circle** about each T with radius m .

The point of intersection, S , between this circle and the line through O and T is a point on the profile.

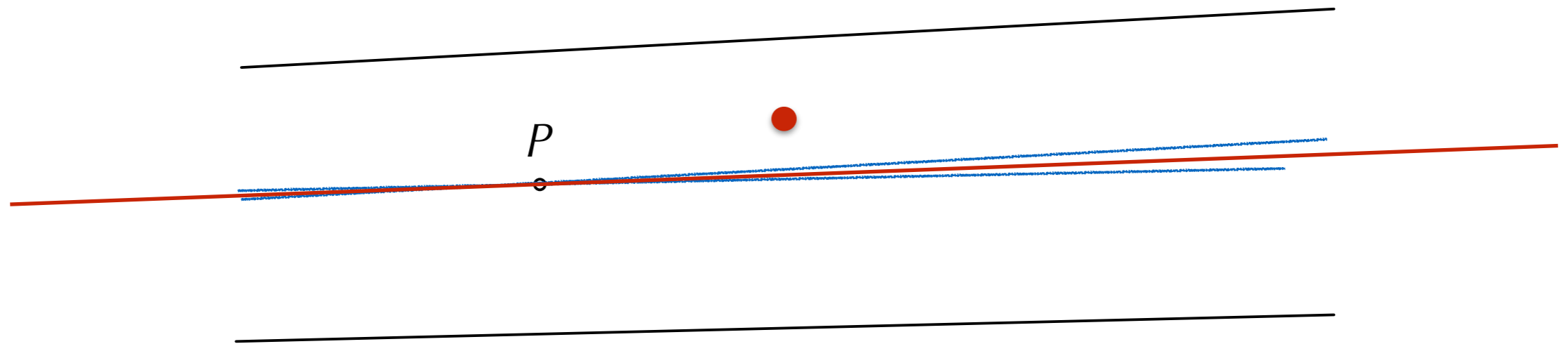




Another typical problem



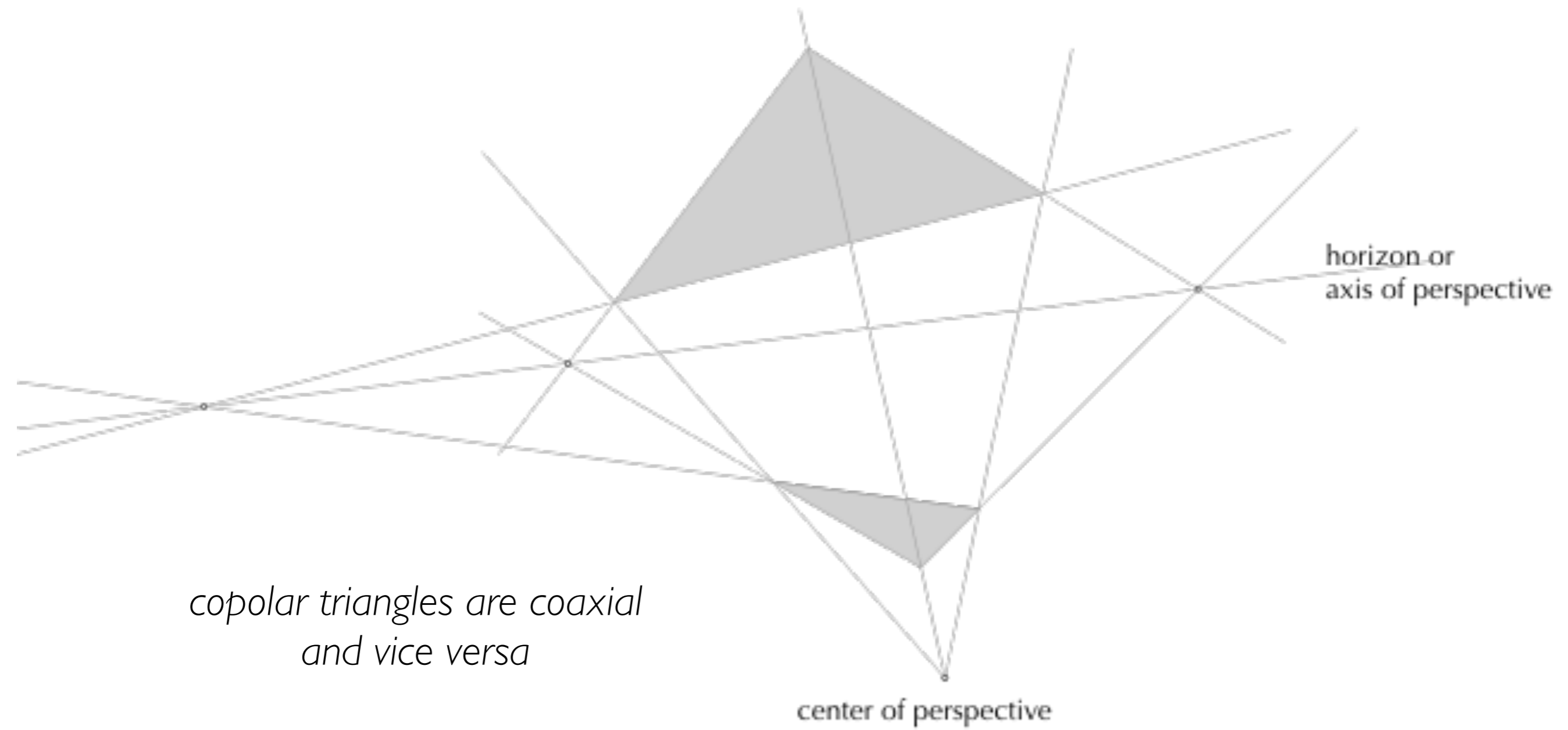
draw a line through P that meets the intersection of the two lines?



a typical problem

hint





Desargues configuration



Computation and Representations

Architecture

but also

Digital Fabrication

Engineering

Product design

CAM

Prototyping

Robot programming

Motion/sensor/... planning

Architecture

Mathematics

Engineering

Computer Science

areas where computation & representation is important

models

general

are representations of physical artifacts, ideas, designs ... of things in

- *User interface*
- *Geometrical and algorithmic level*
- *Arithmetic substratum*

models and **representations**

models are representations of physical artifacts, ideas, designs ...
of things in general

an **artificially constructed object** that makes the observation
of another object easier

**Levels
of modeling**

- World of physical objects
- **Geometrical** modeling space
- Representation space

a geometrical model is an abstraction
— an idealization of real 3D physical objects

models and **representations**

models are **representations** of physical artifacts, ideas, designs ...
of things in general

configurations of elements bearing relationships to one another to give an overall sense of structure

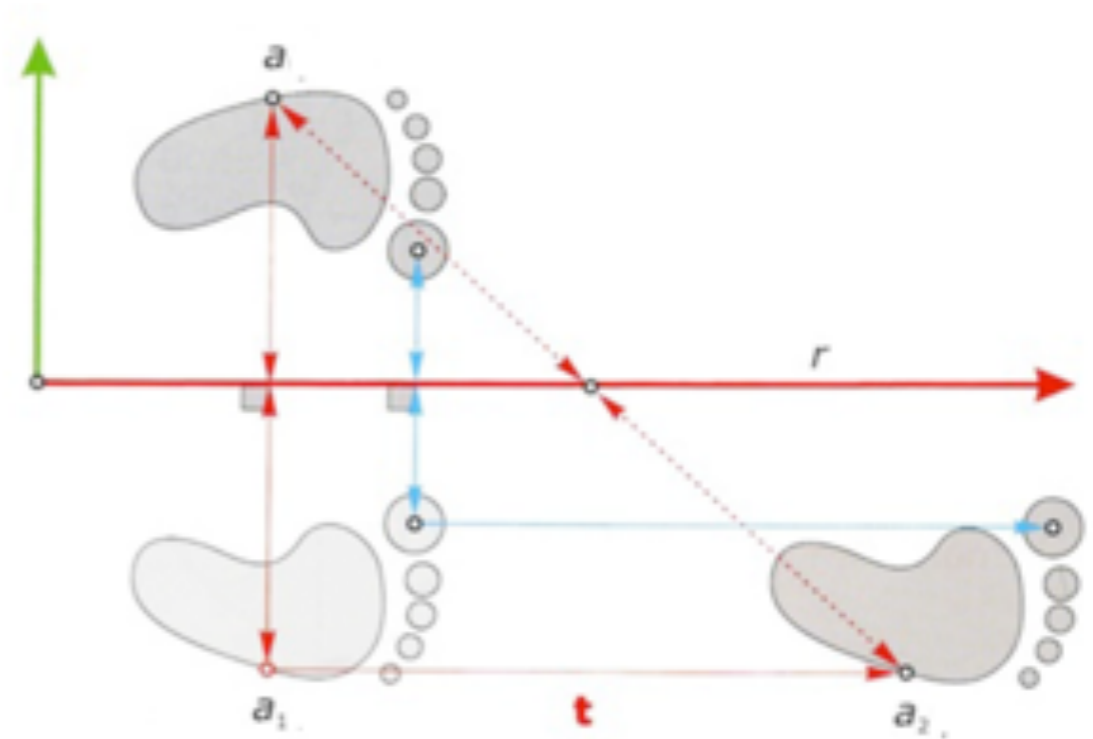
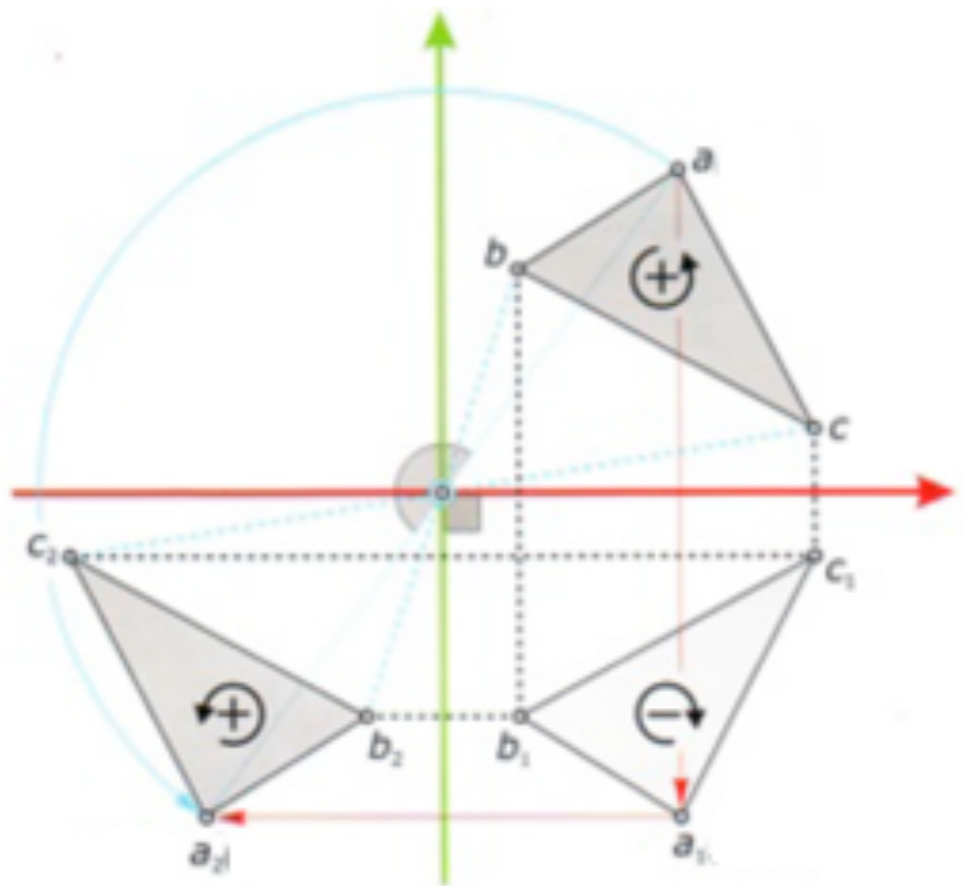
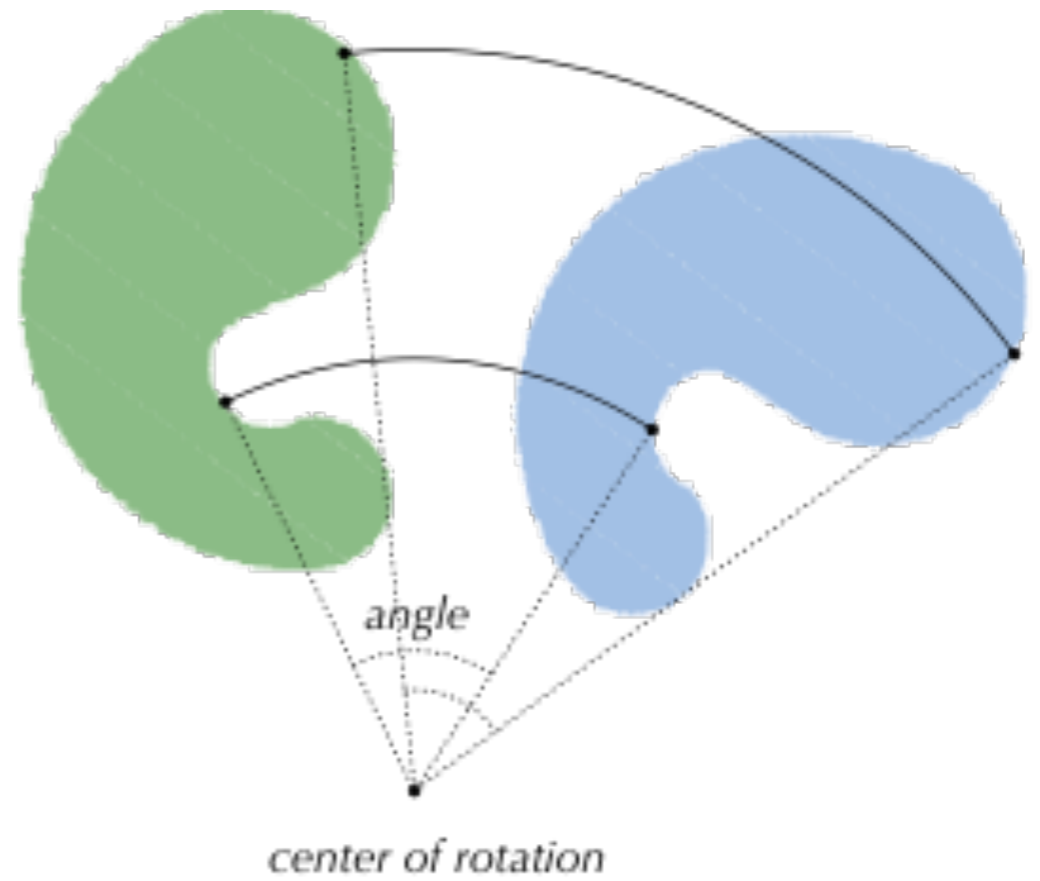
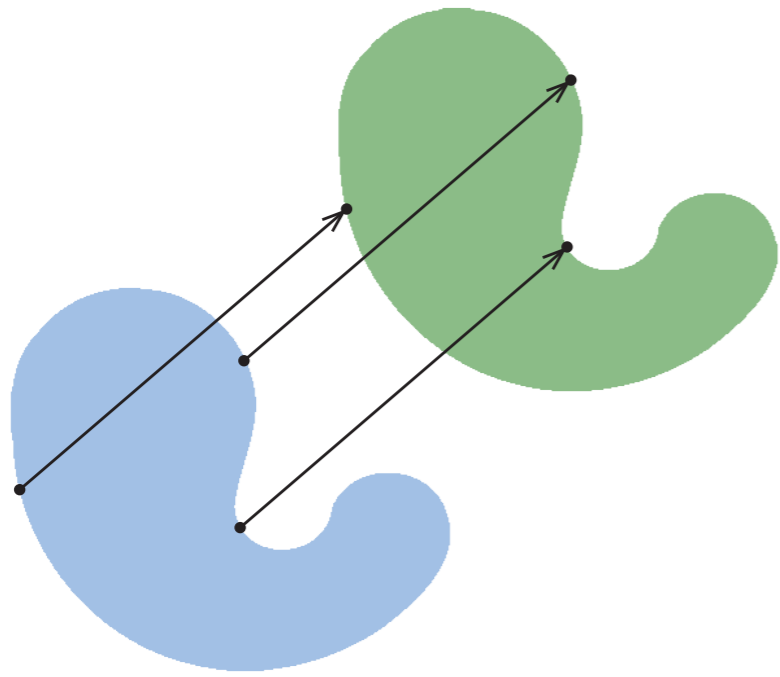
Elements

Relationships between elements

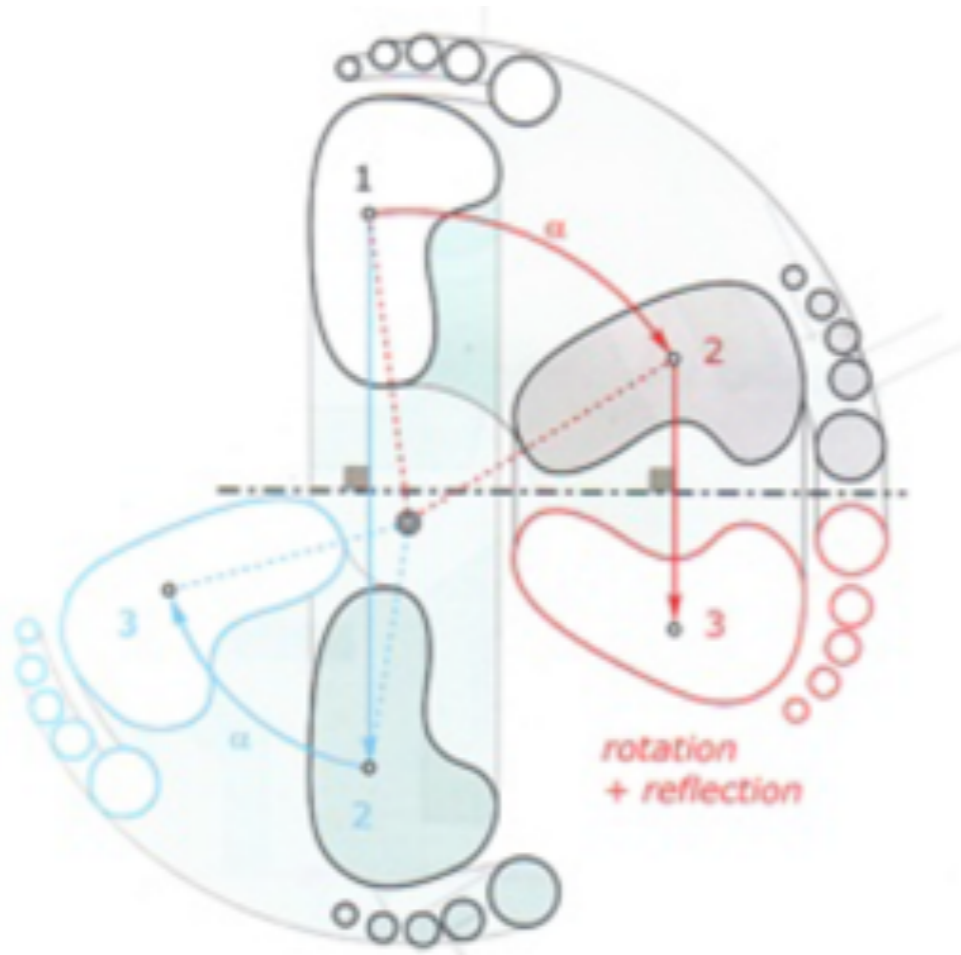
Structure

models and **representations**

<http://www.designboom.com/architecture/ik-studio-conics-canopy/>



geometric transformations



Hint: all you need are mirrors!

rotating an object without using a compass

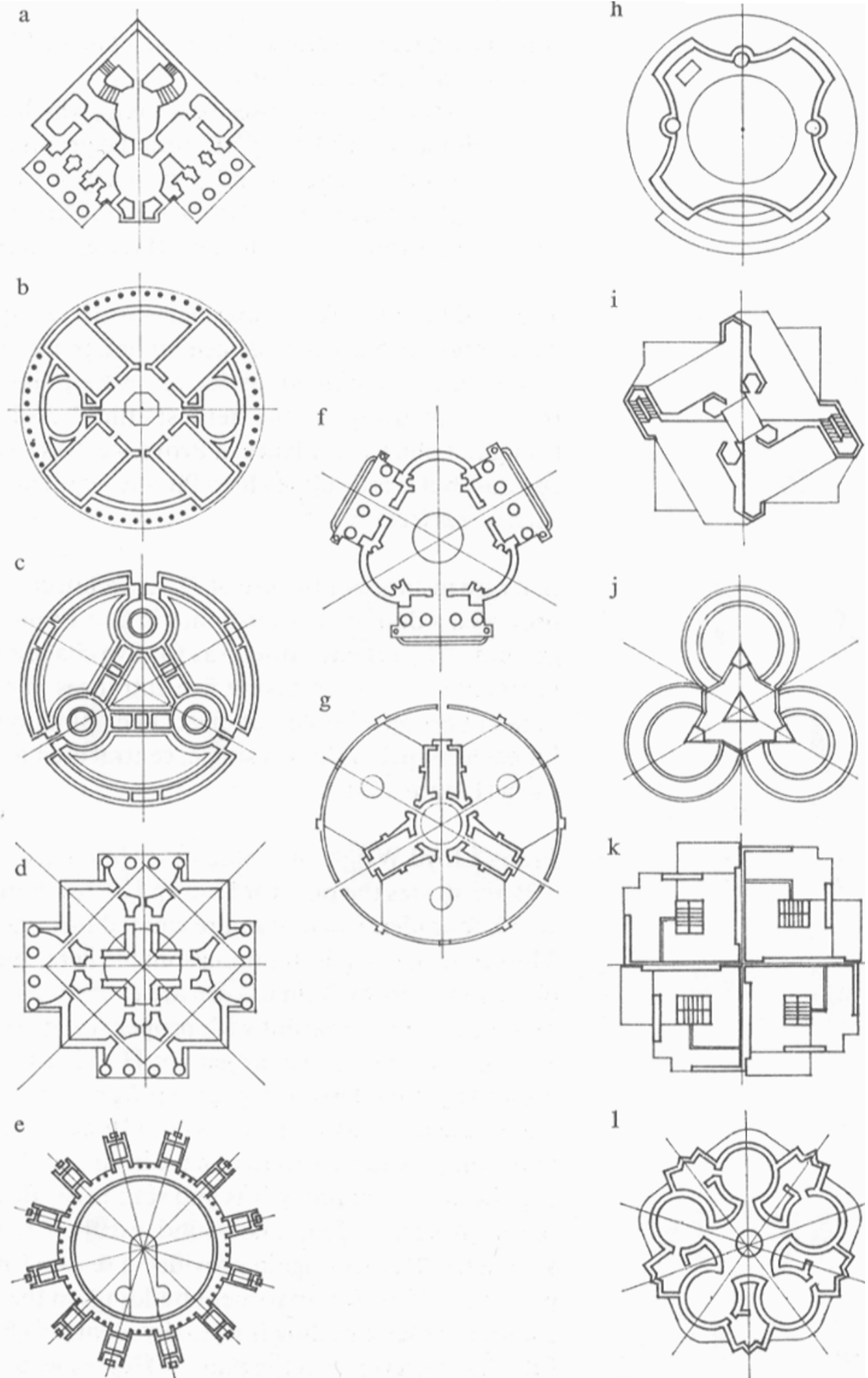
symmetry

Figure 3.2
 Examples of point groups
 in architectural plans:
 a, Montmorency
 Palace, D_1
 b, De Witt House, D_2
 c, Inn St. Marceau, D_3
 d, Barrière de Picpus, D_4
 e, House of Entertain-
 ment (circular colonnade
 and pavilions), D_{12}
 all projects and buildings
 by Claude-Nicolas
 Ledoux:

f, Sepulchral Church, D_3
 g, Kennels, D_3
 both designs by Sir John
 Soane:

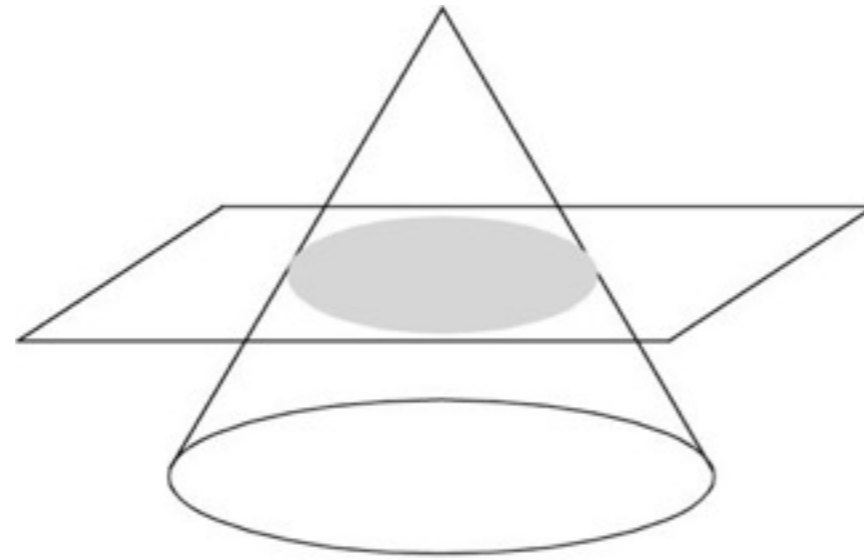
and projects and build-
 ings by Frank Lloyd
 Wright

h, Greek Orthodox
 Temple, C_1
 i, St Mark's Apartment
 Tower, C_2
 j, Huntingdon Hartford
 Clubhouse, D_3
 k, Suntop Homes, C_4
 l, Daphne Mortuary, D_5

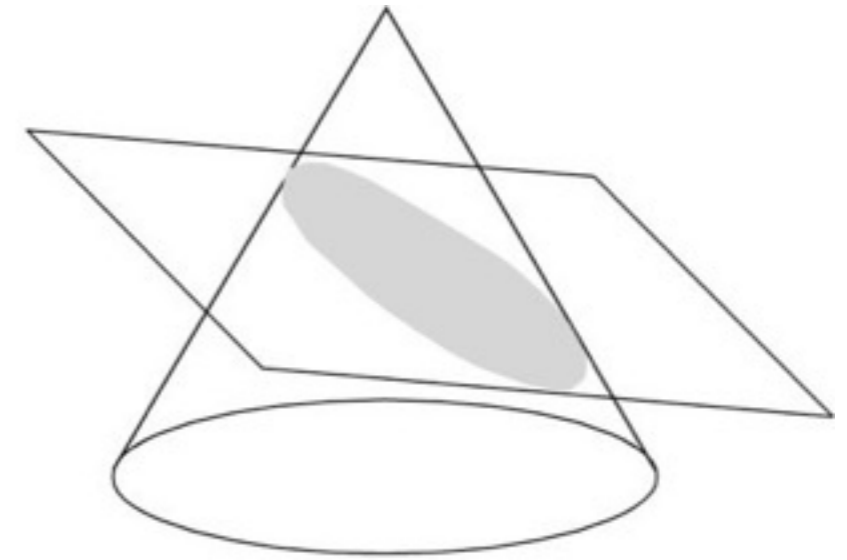




Conic Sections

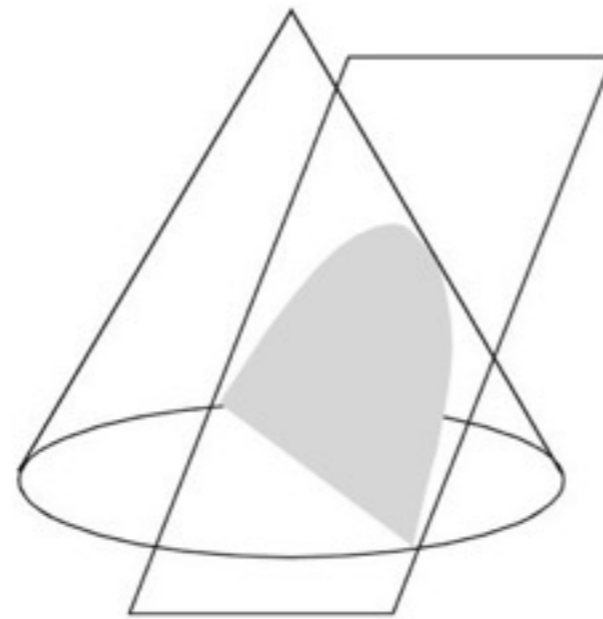


Circle

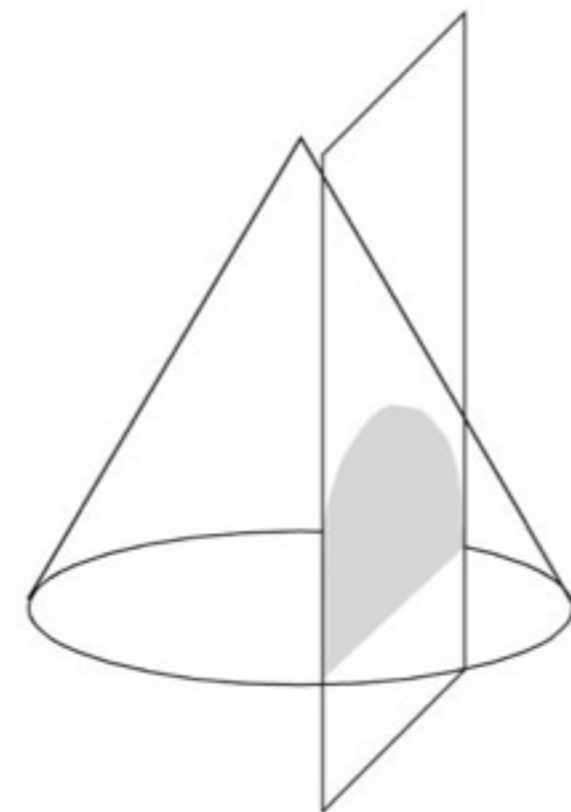


Ellipse

produced by slicing a cone by a cutting plane

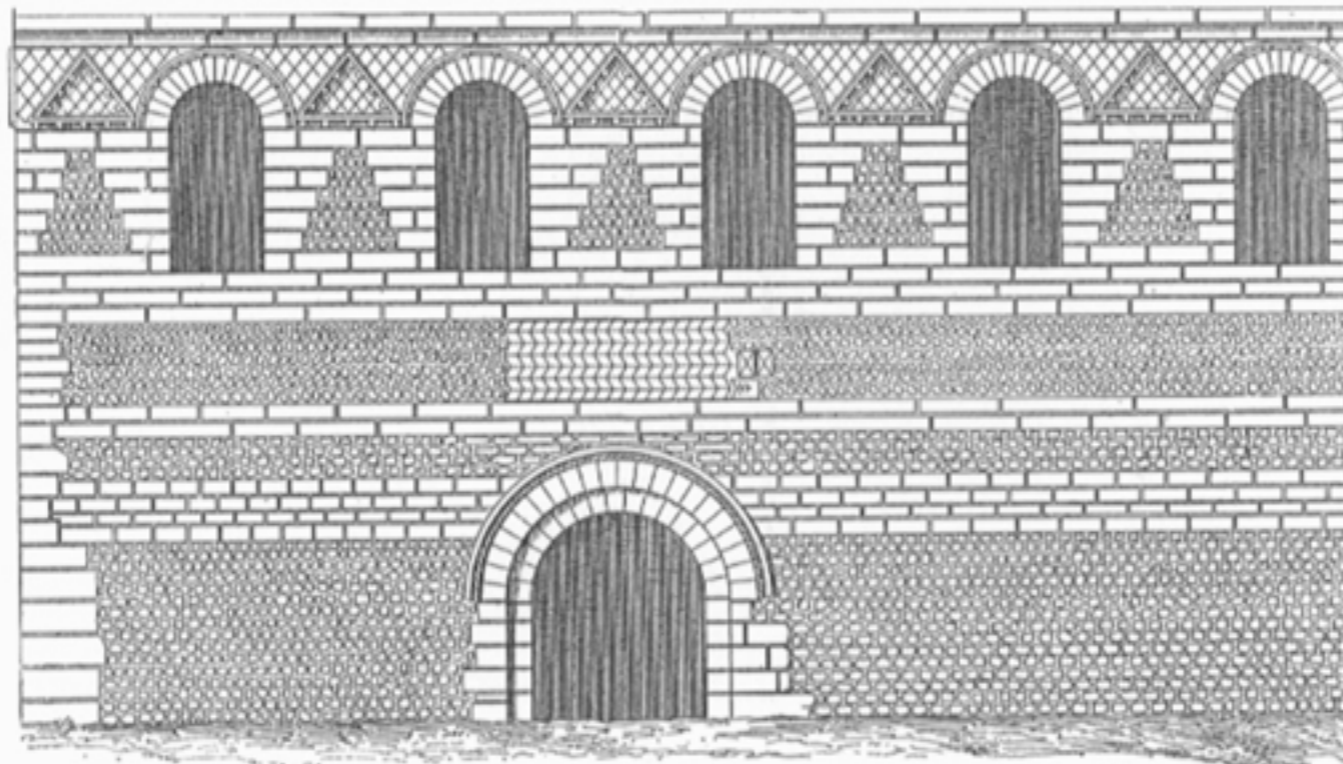
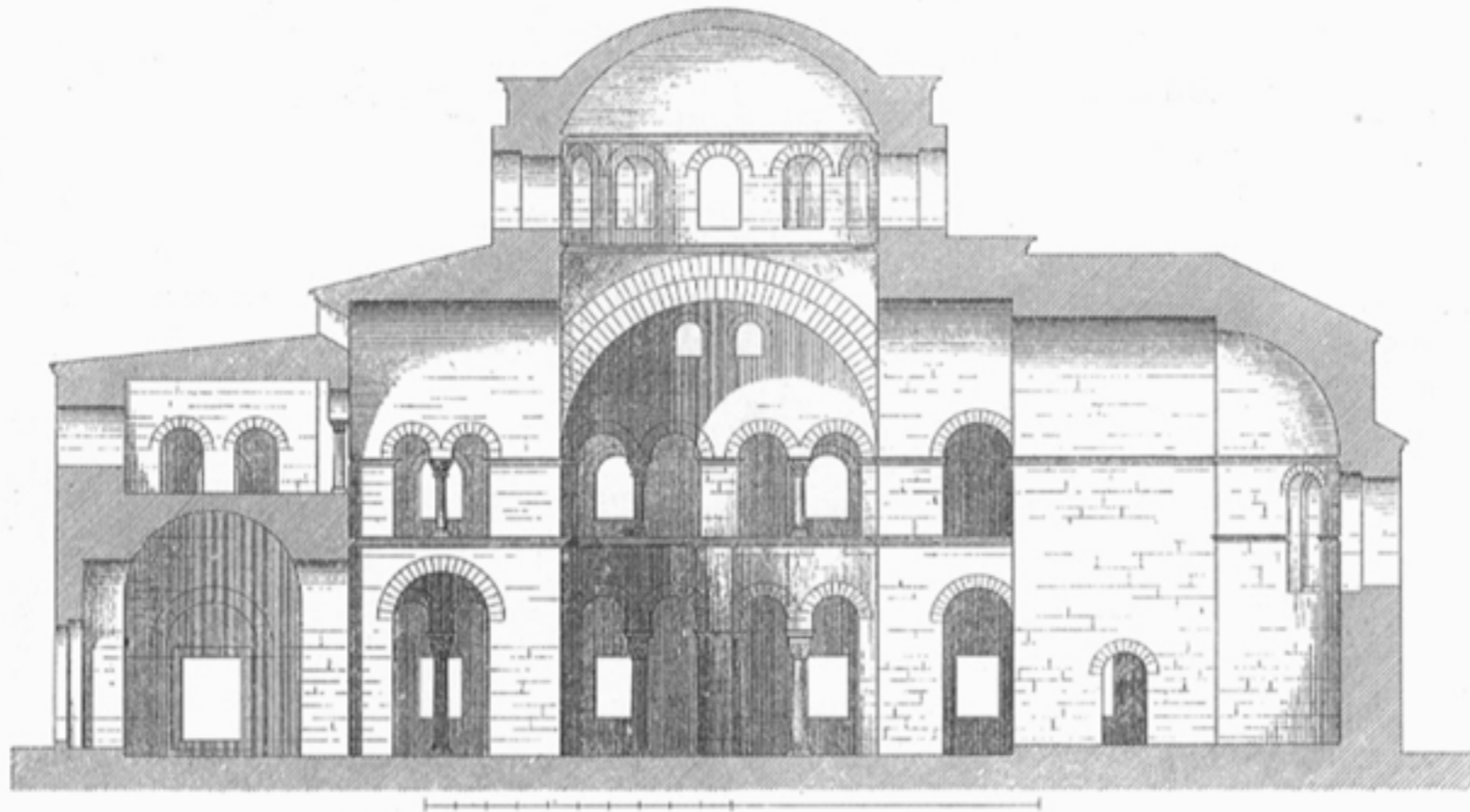


Parabola

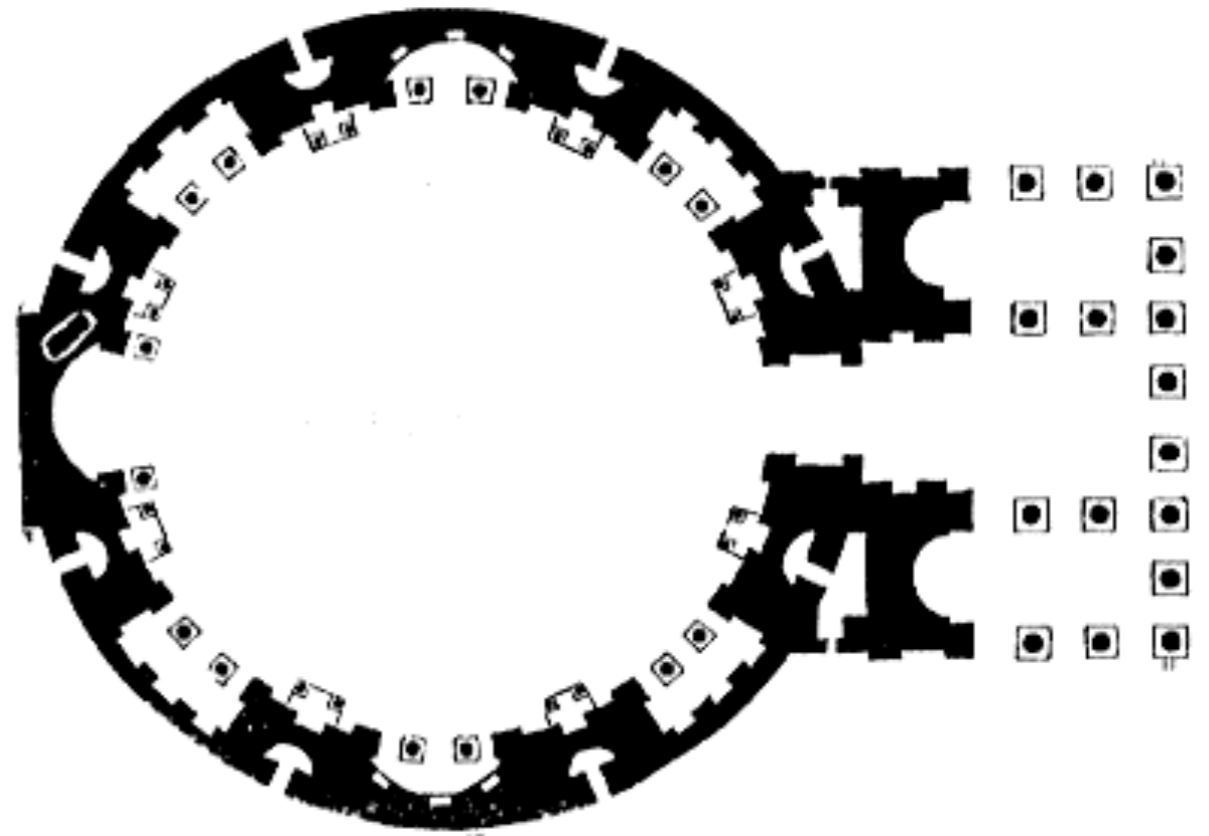


Hyperbola

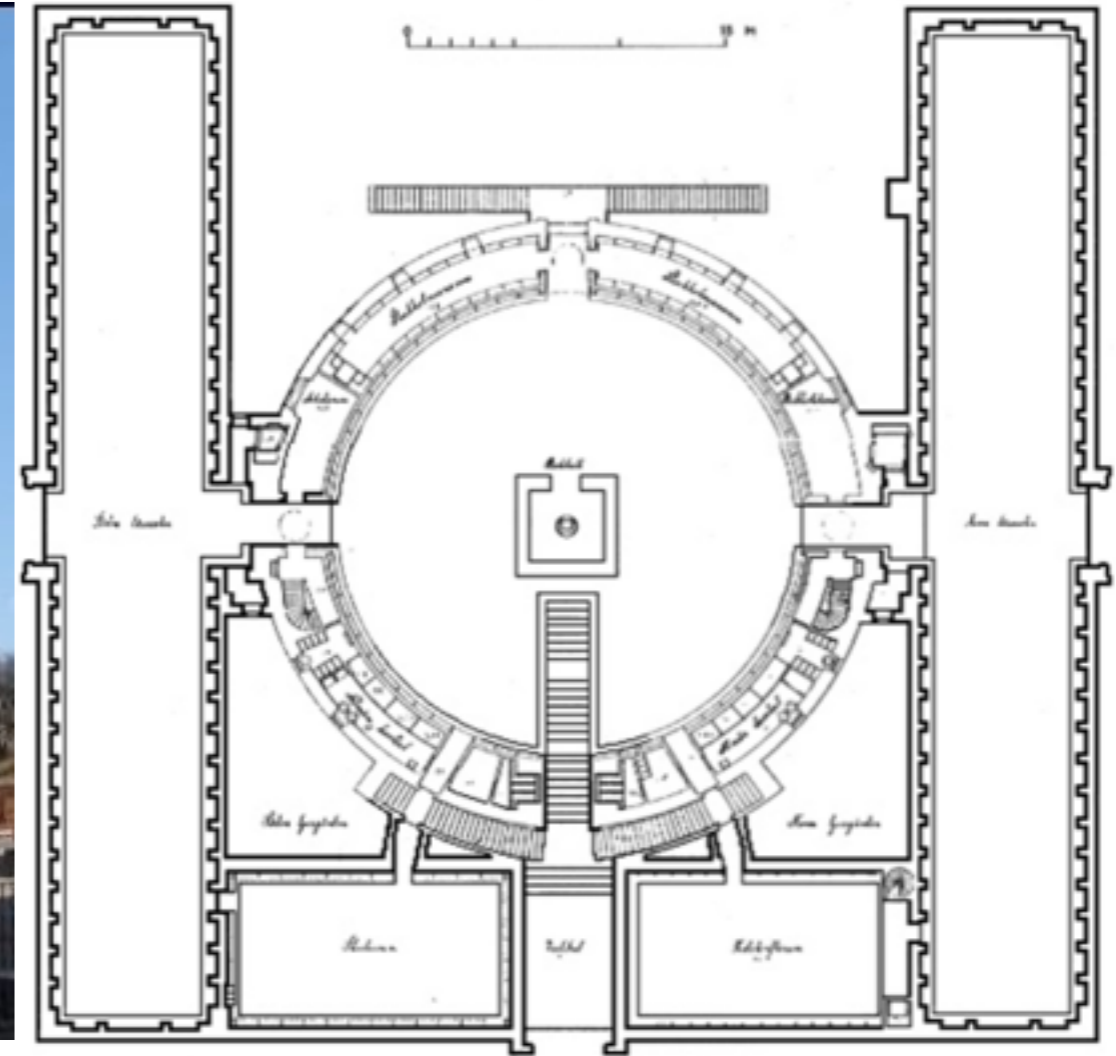
conic sections



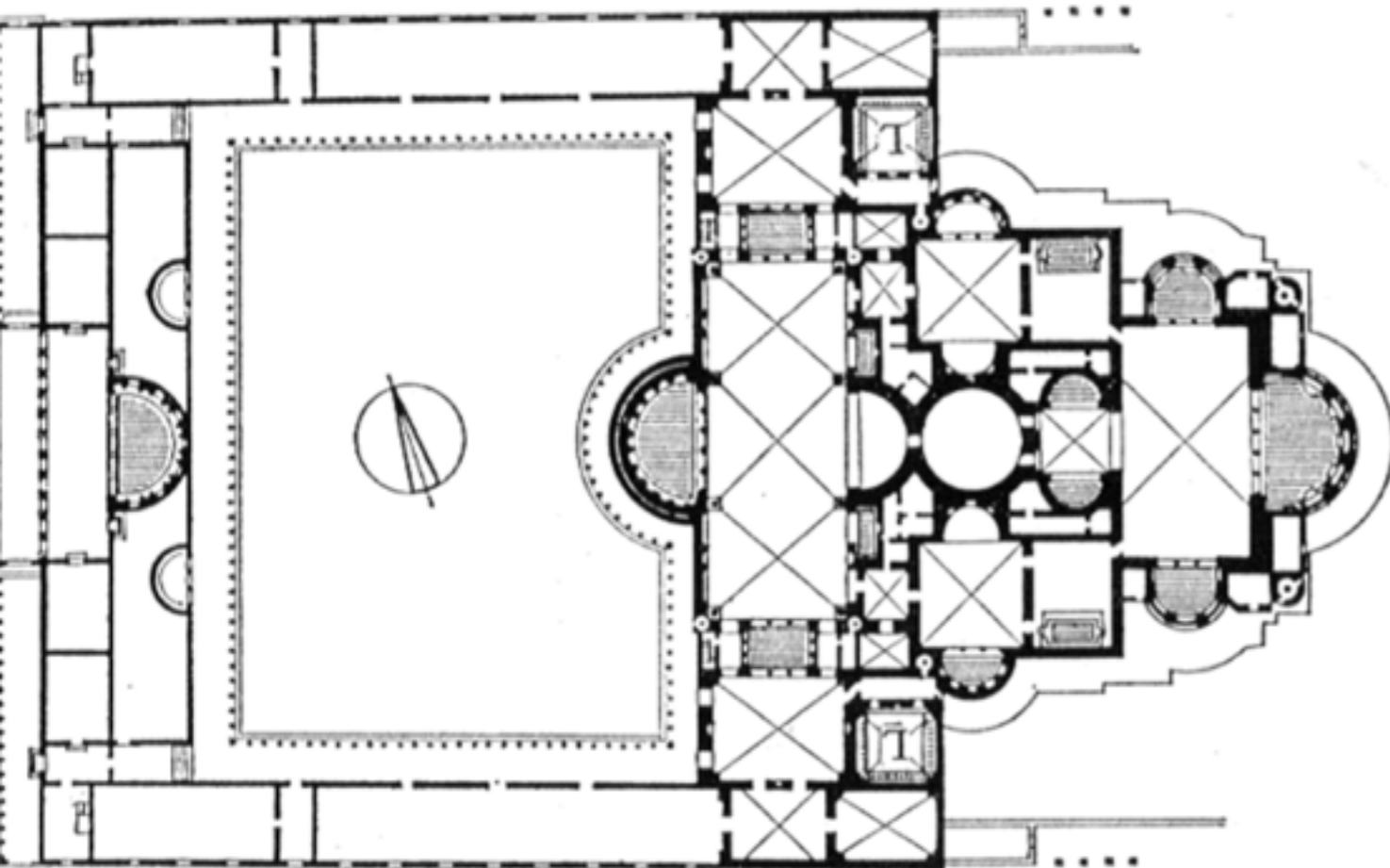
circle



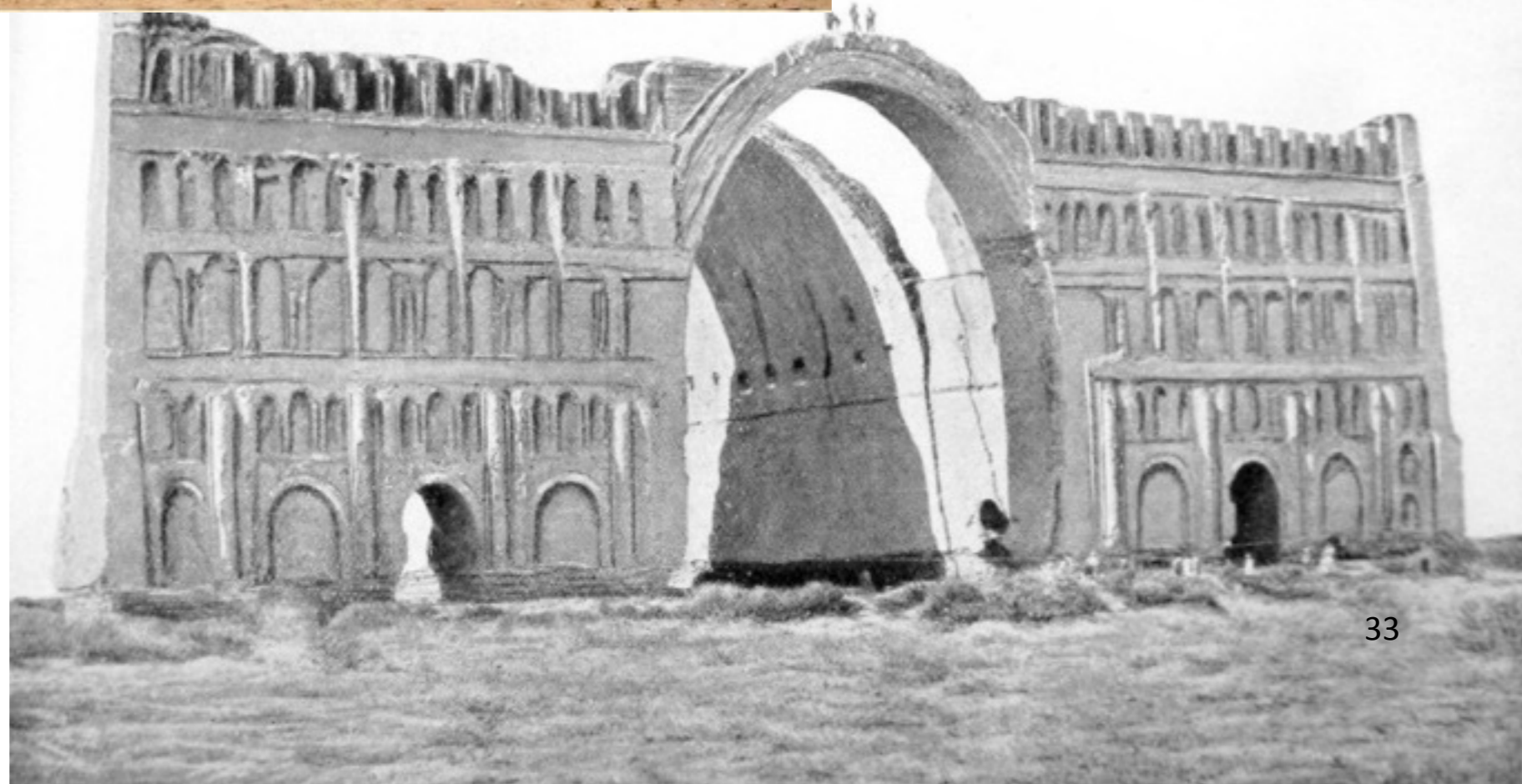
Pantheon



Stockholm Public Library

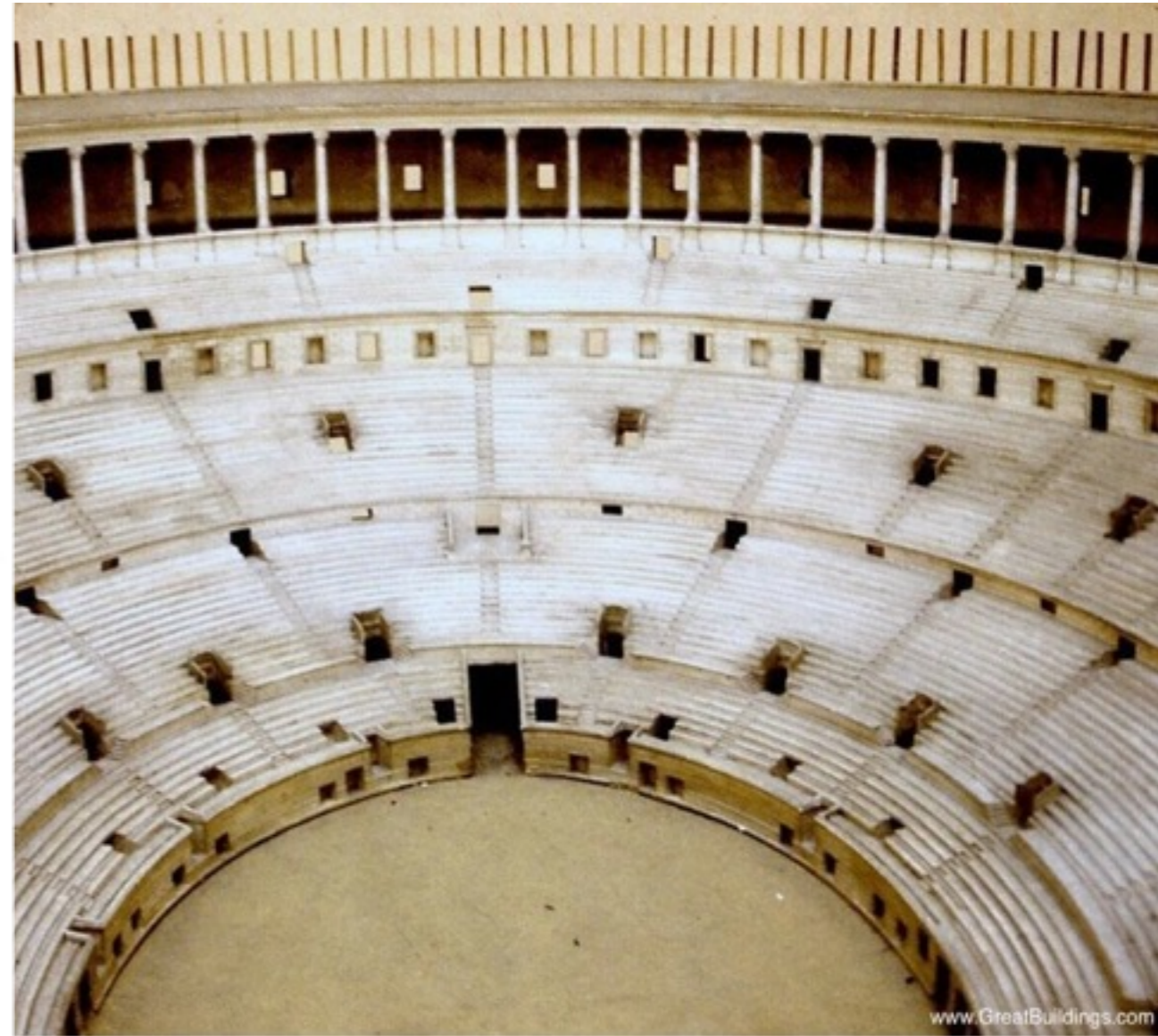
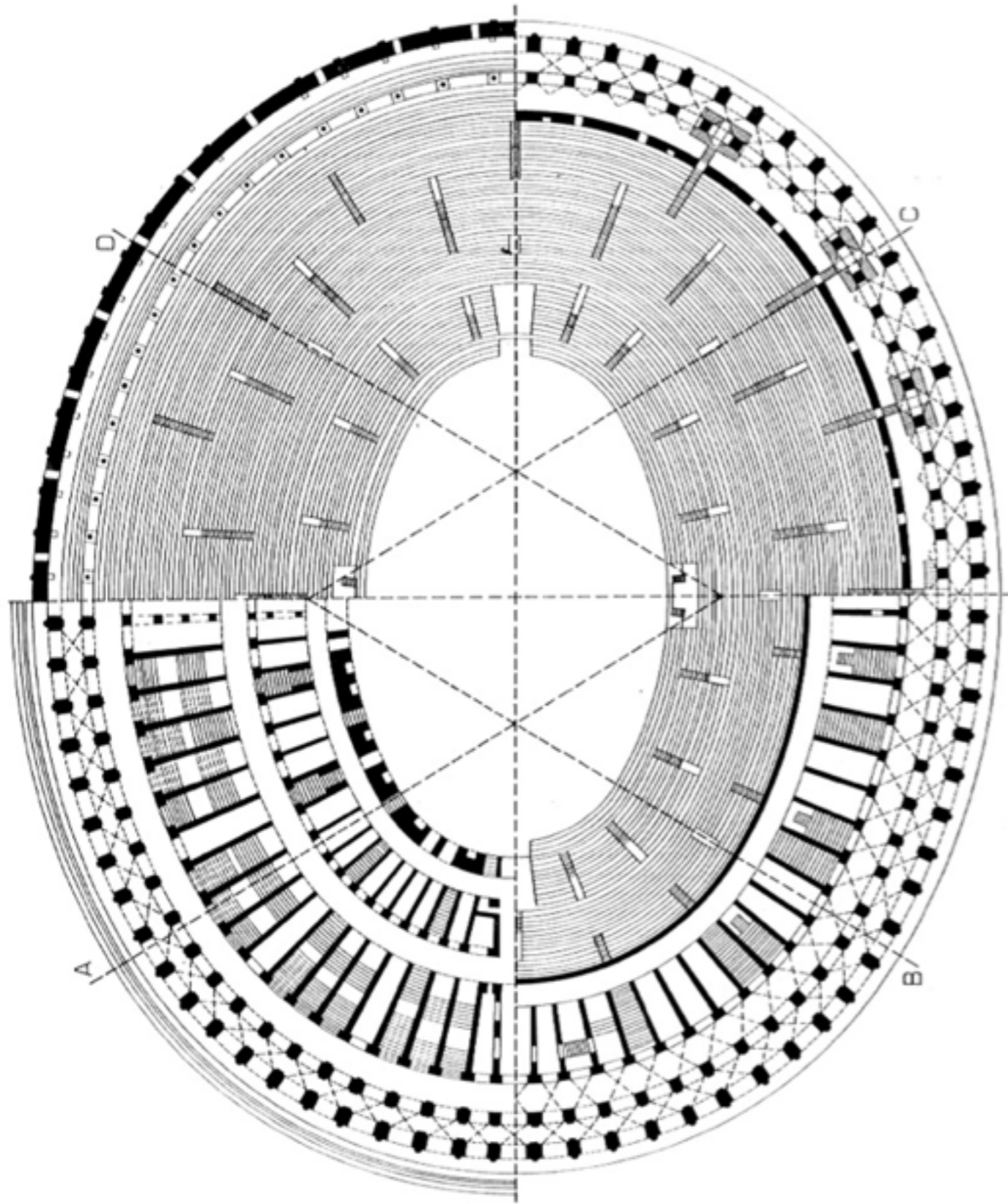


Imperial baths, Trier



Ctesiphon





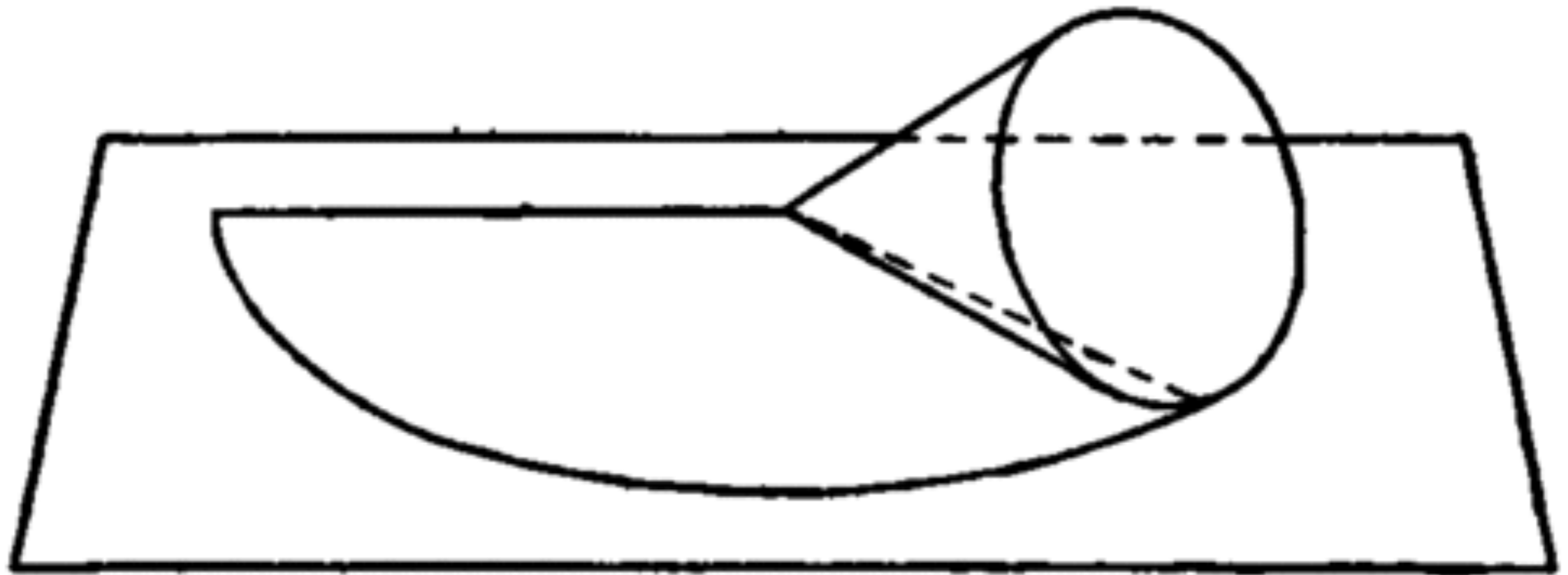
Colosseum



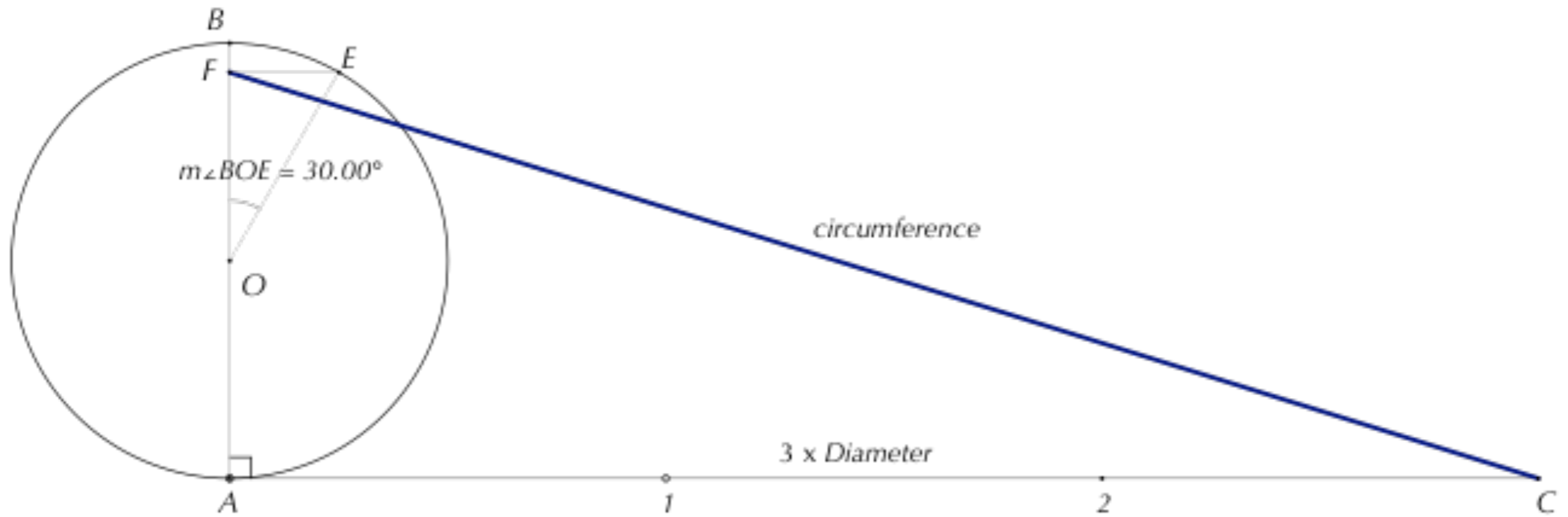
S.Vicente de Paul at Coyoacan



circle



developing a cone

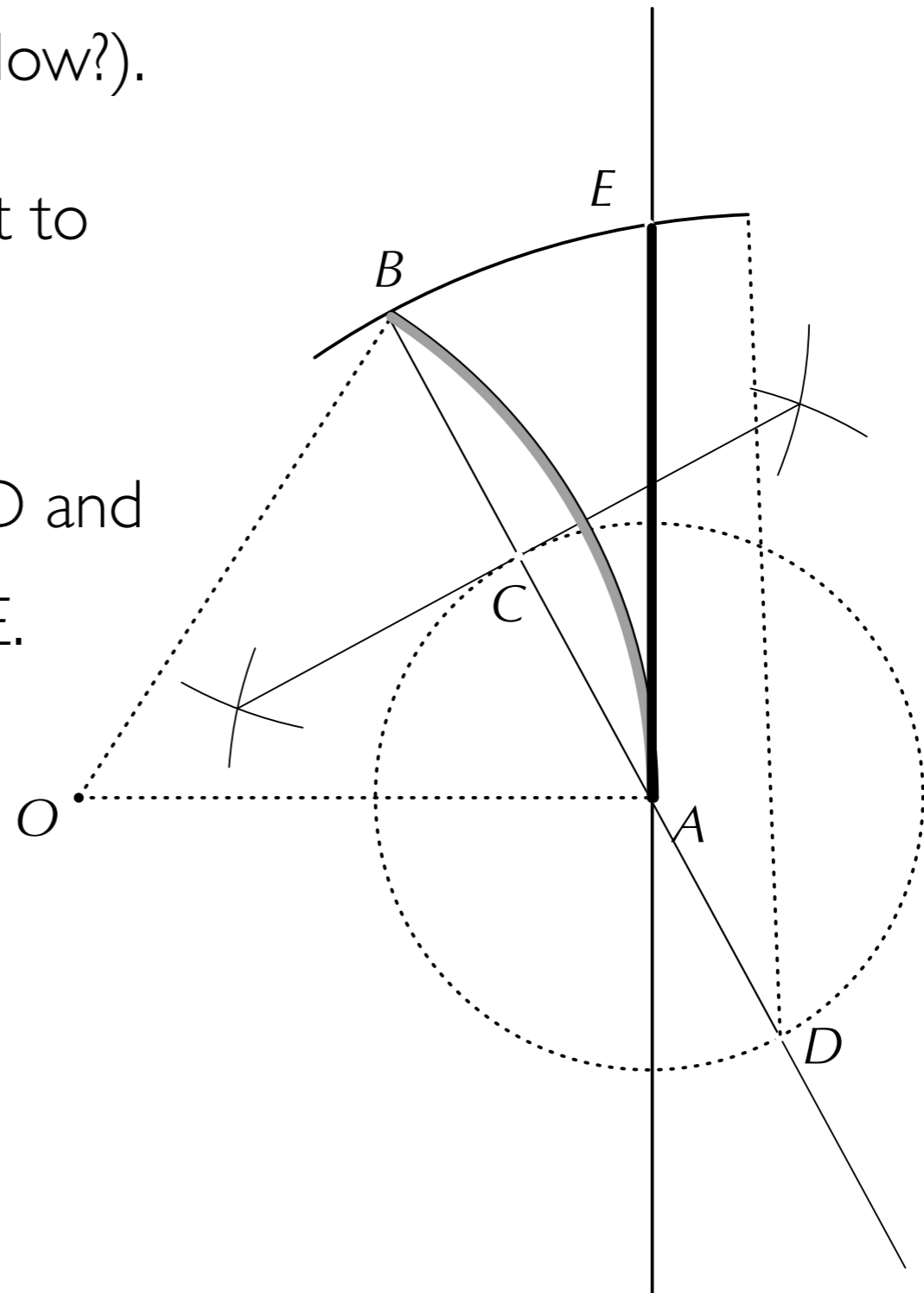


rectifying the circumference of a circle

rectification: approximate length of a circular arc

1. Draw a tangent to the arc at A (How?).
2. Join A and B by a line and extend it to produce D with $AD = \frac{1}{2}AB$.
3. Draw the circular arc with center D and radius DB to meet the tangent at E .

AE is the *required length*



constructions involving circles

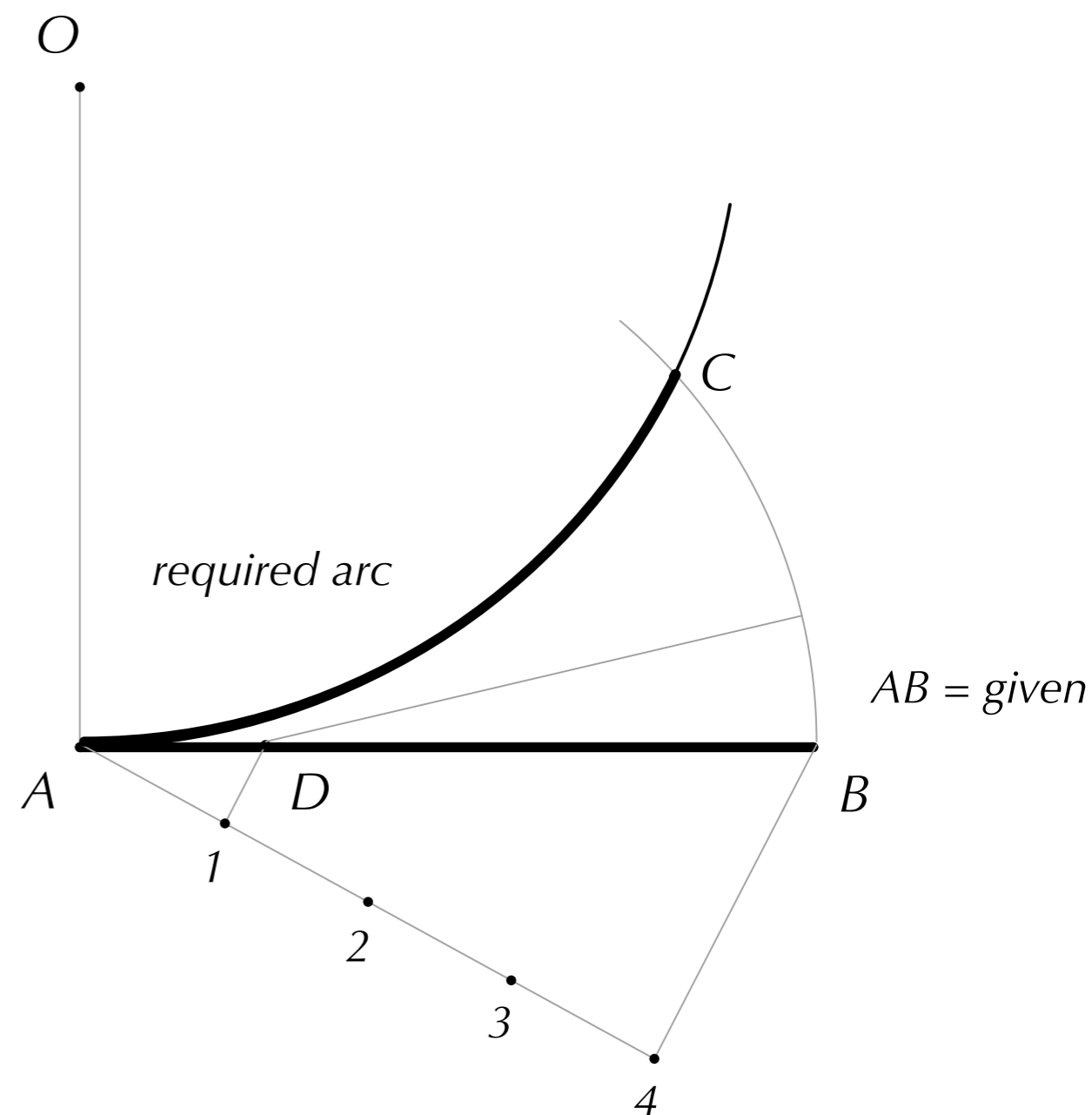
approximate circular arc of a given length

A be a point on the arc.

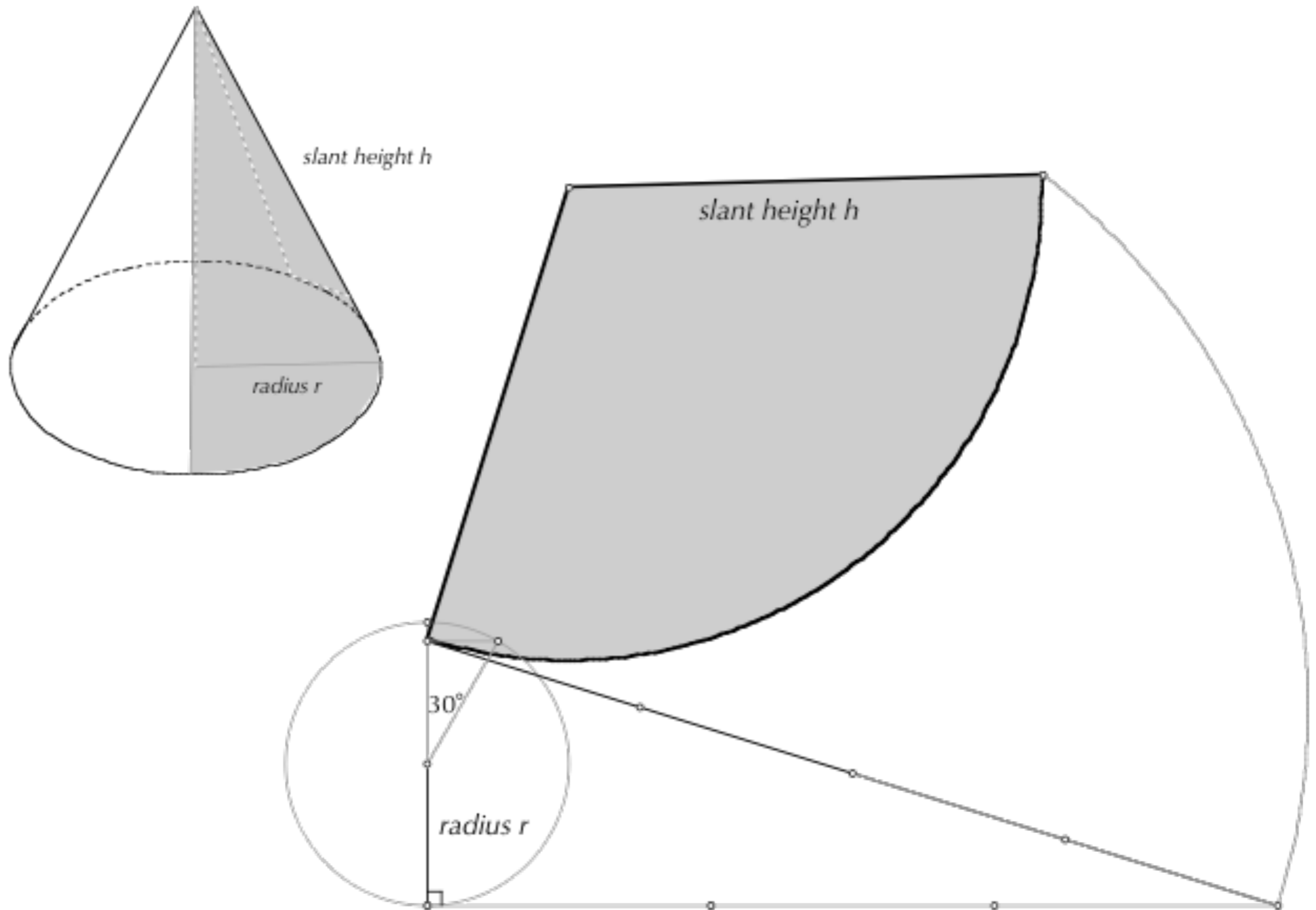
AB is the given length on the tangent at A .

1. Mark a point D on the tangent such that $AD = \frac{1}{4}AB$.
2. Draw the circular arc with center D and radius DB to meet the original at C .

Arc AC is the *required arc*

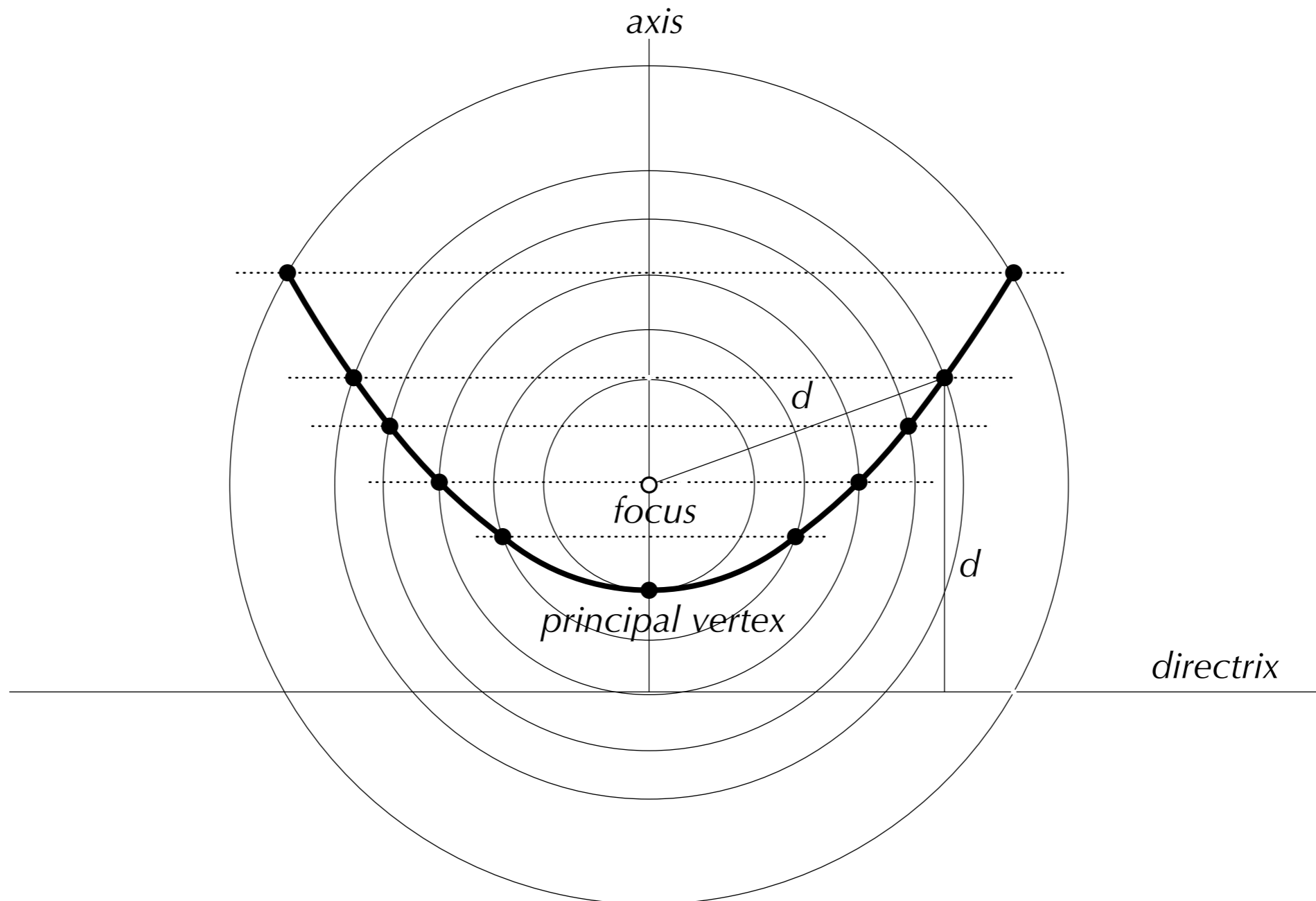


constructions involving circles

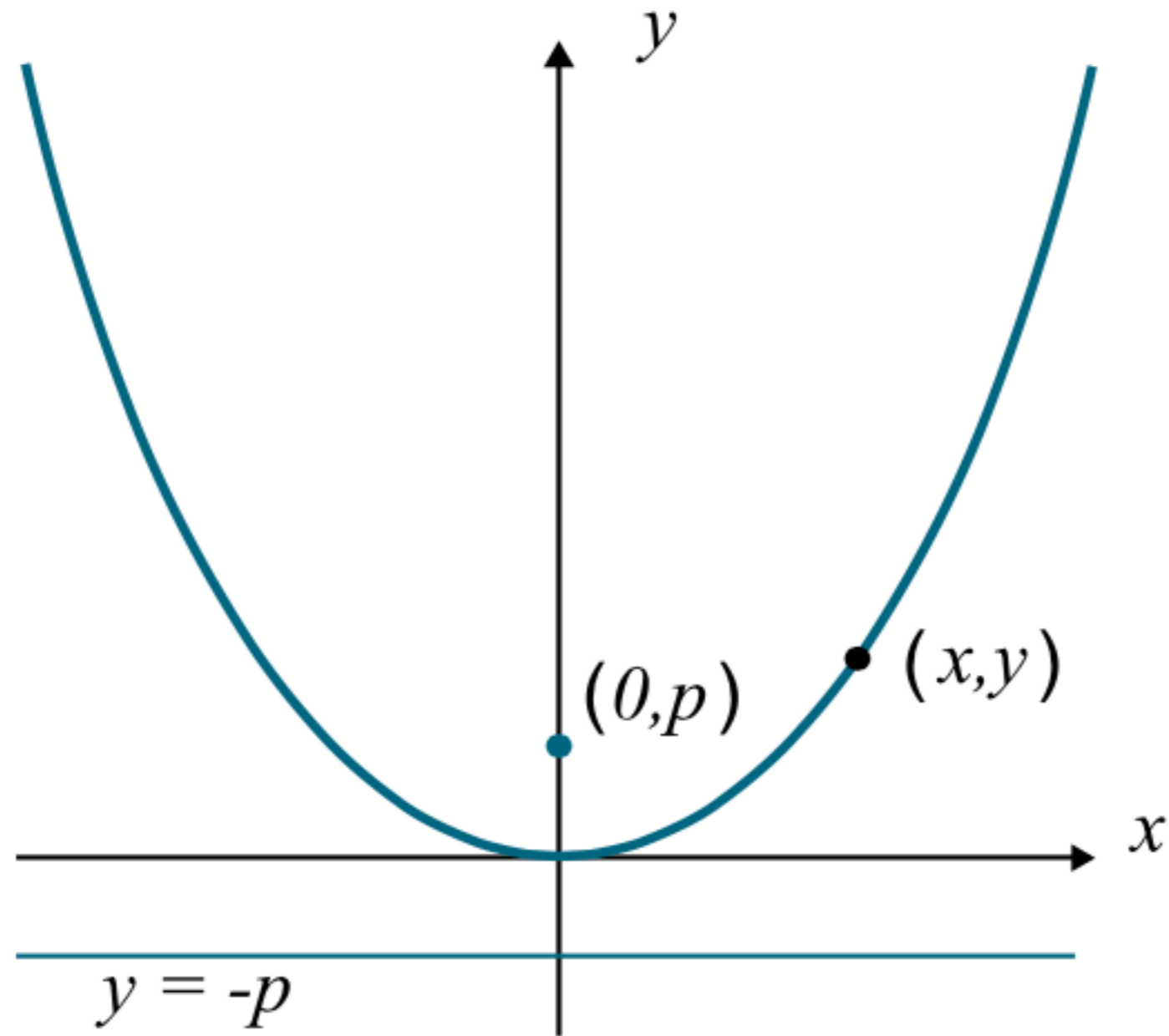


a practical application

parabola



parabola



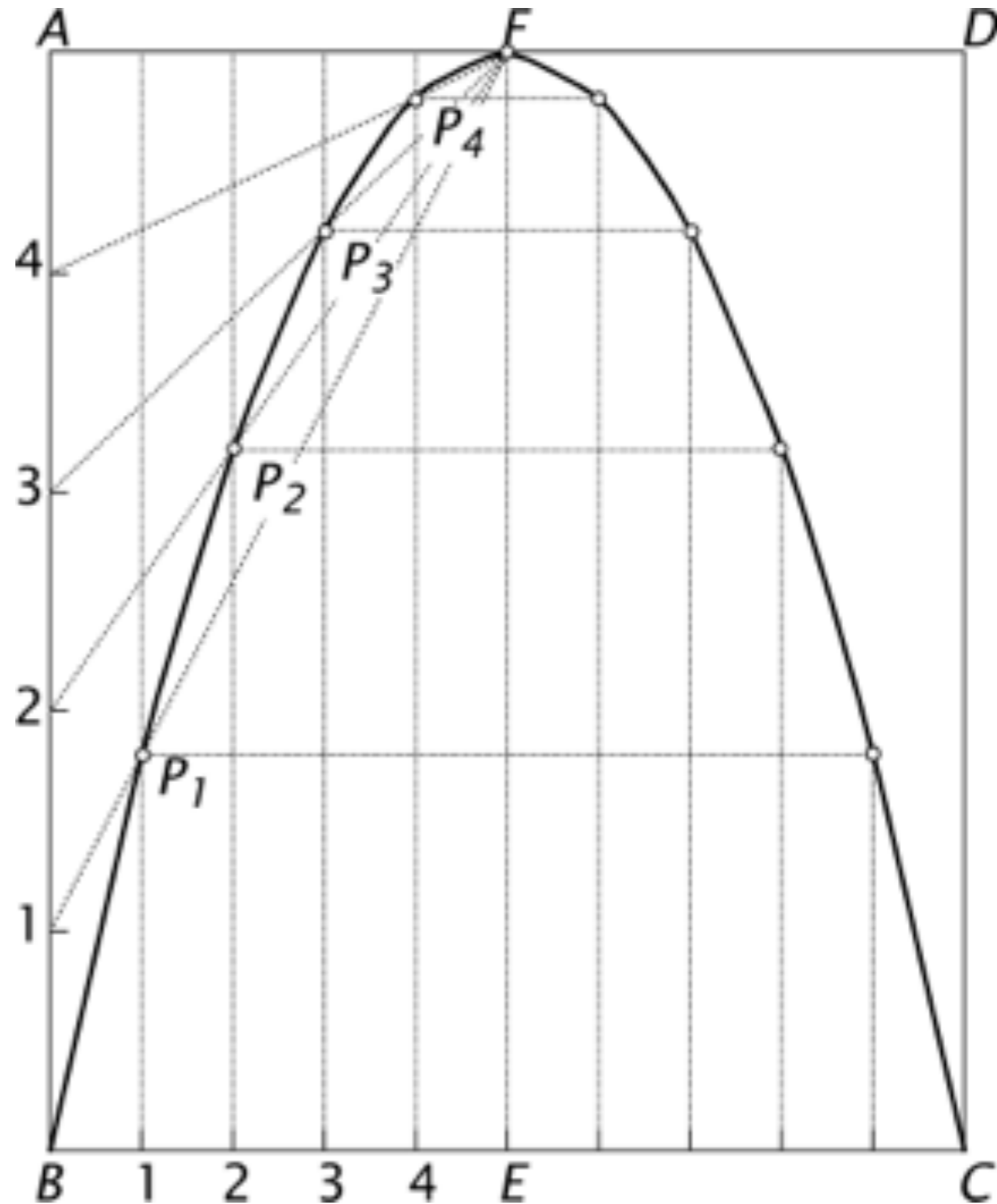
$$\sqrt{x^2 + (y - p)^2} = y + p$$

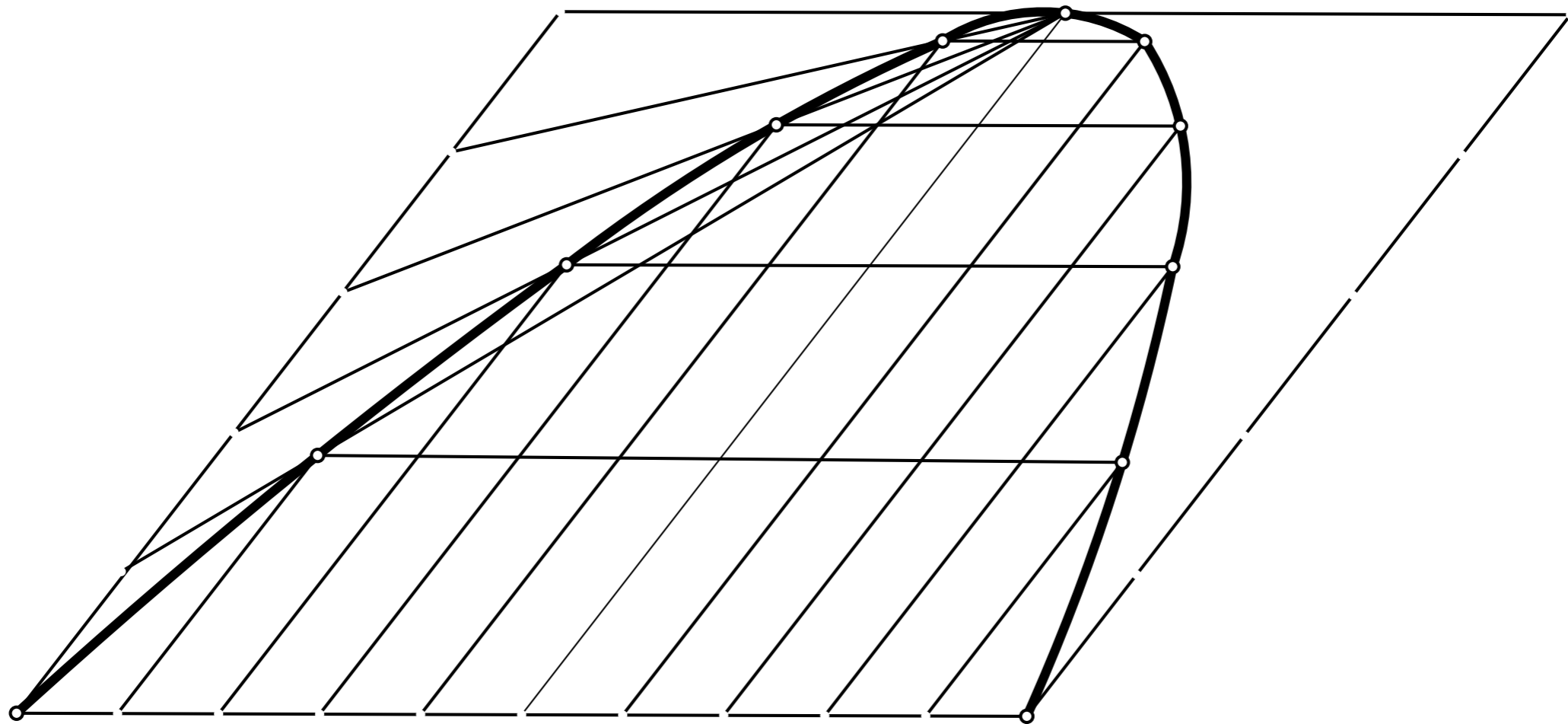
$$x^2 = 4py$$

analytic form

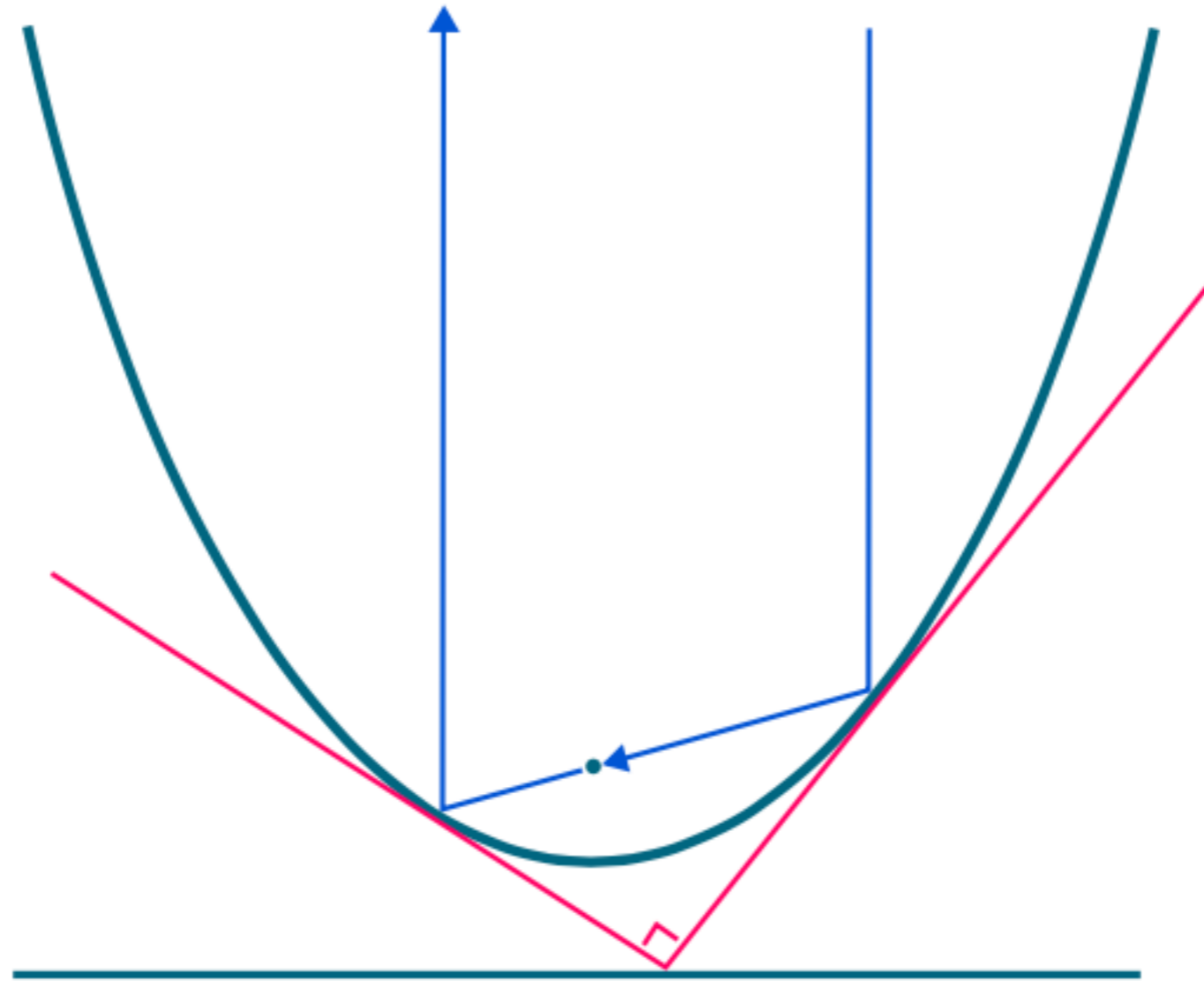
a parabola within a rectangle

- ↔ Bisect the sides AB and CD of the rectangle $ABCD$ and join their midpoints, E and F , by a line segment.
- ↕ Divide segments BE and EC into the same number of equal parts, say $n = 5$, numbering them as shown.
- ↔ Join F to each of the numbered points on BE and EC to intersect the lines parallel to EF through the numbered points on AB and CD at points P_1, P_2, \dots, P_{n-1} as shown.
- These points lie on the required parabola.





constructing an oblique parabola



reflective property of a parabola



kraal in Namibia

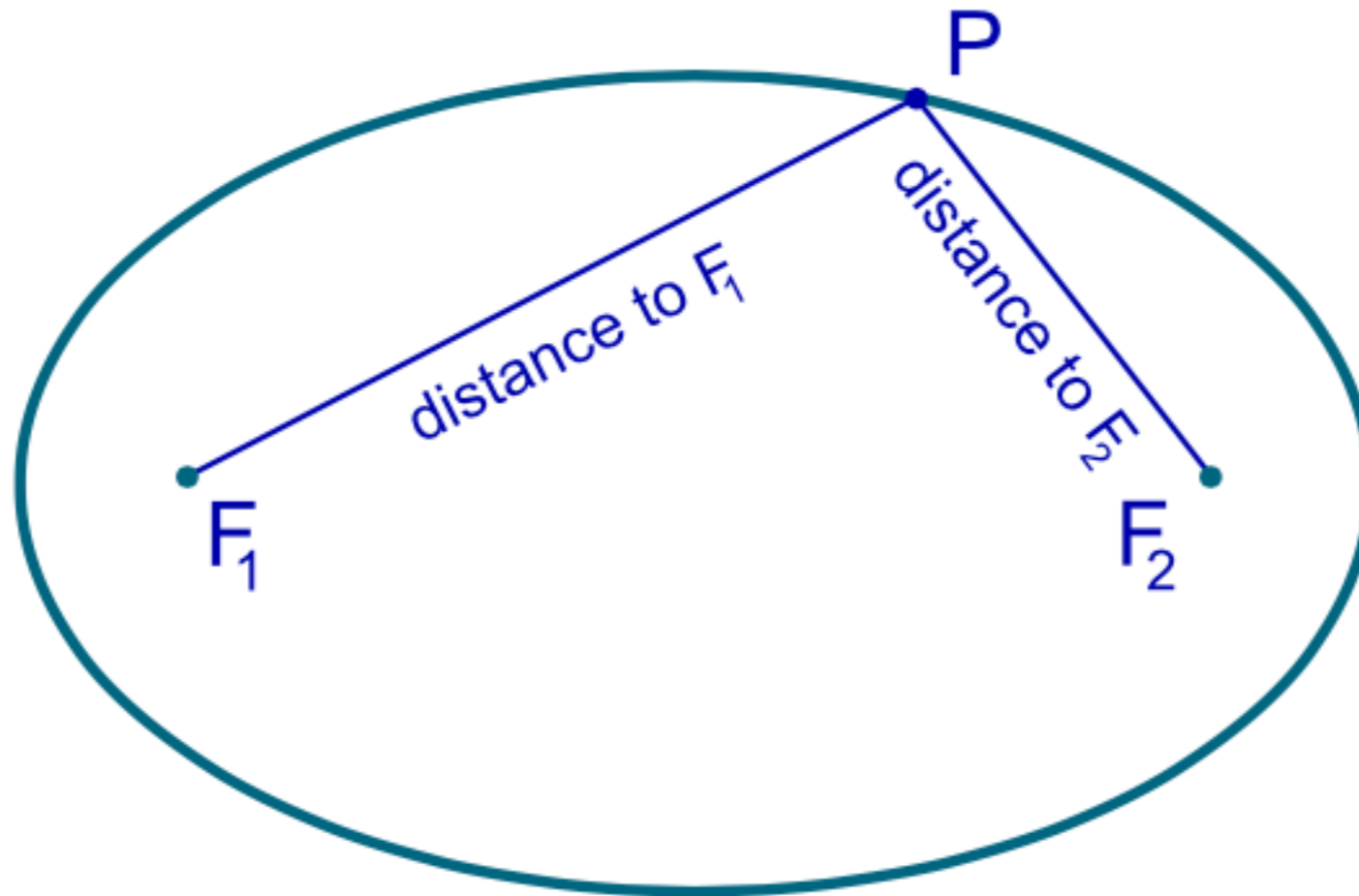


Inuit igloo



ellipse

distance to F_1 + distance to F_2 = a constant

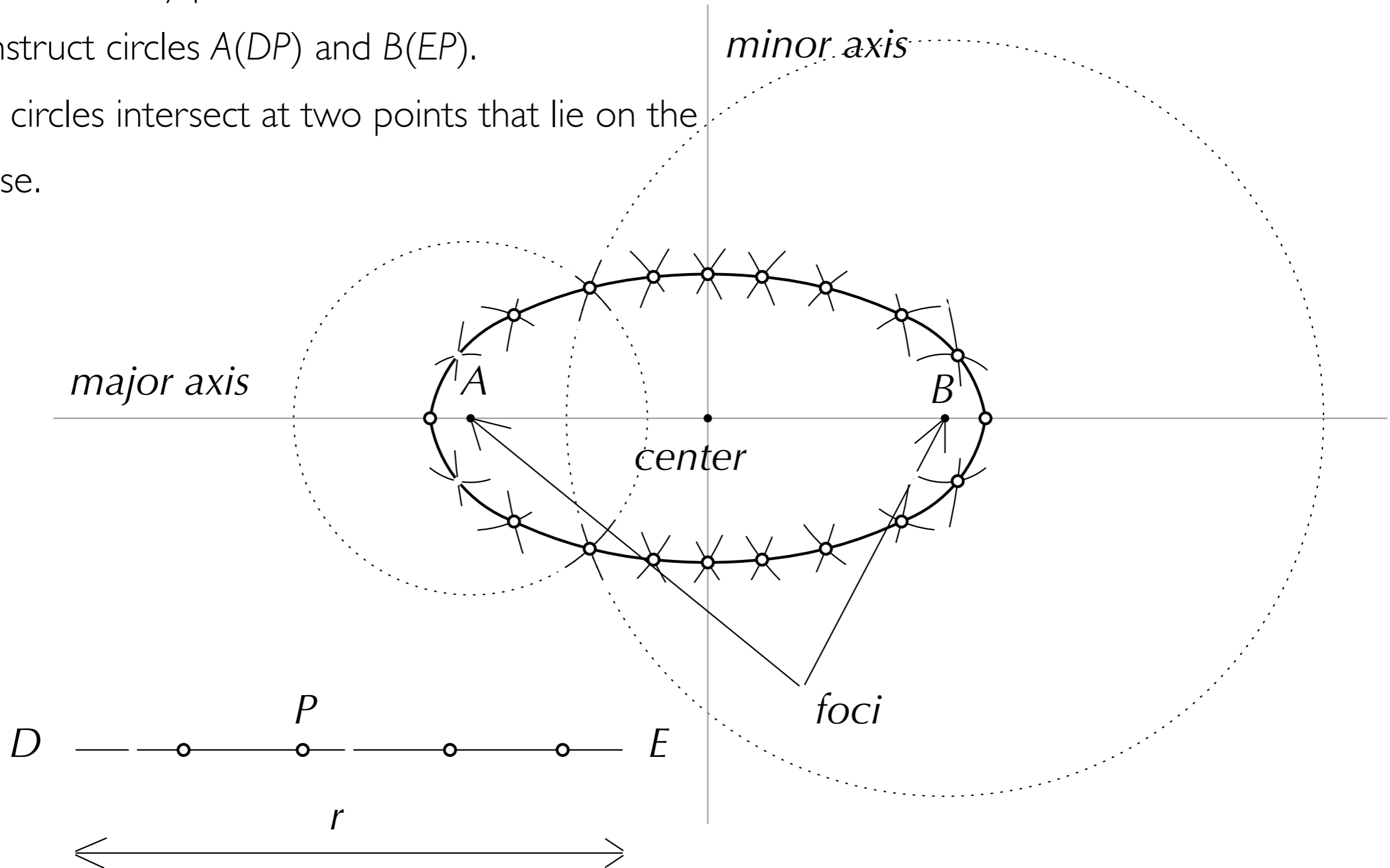


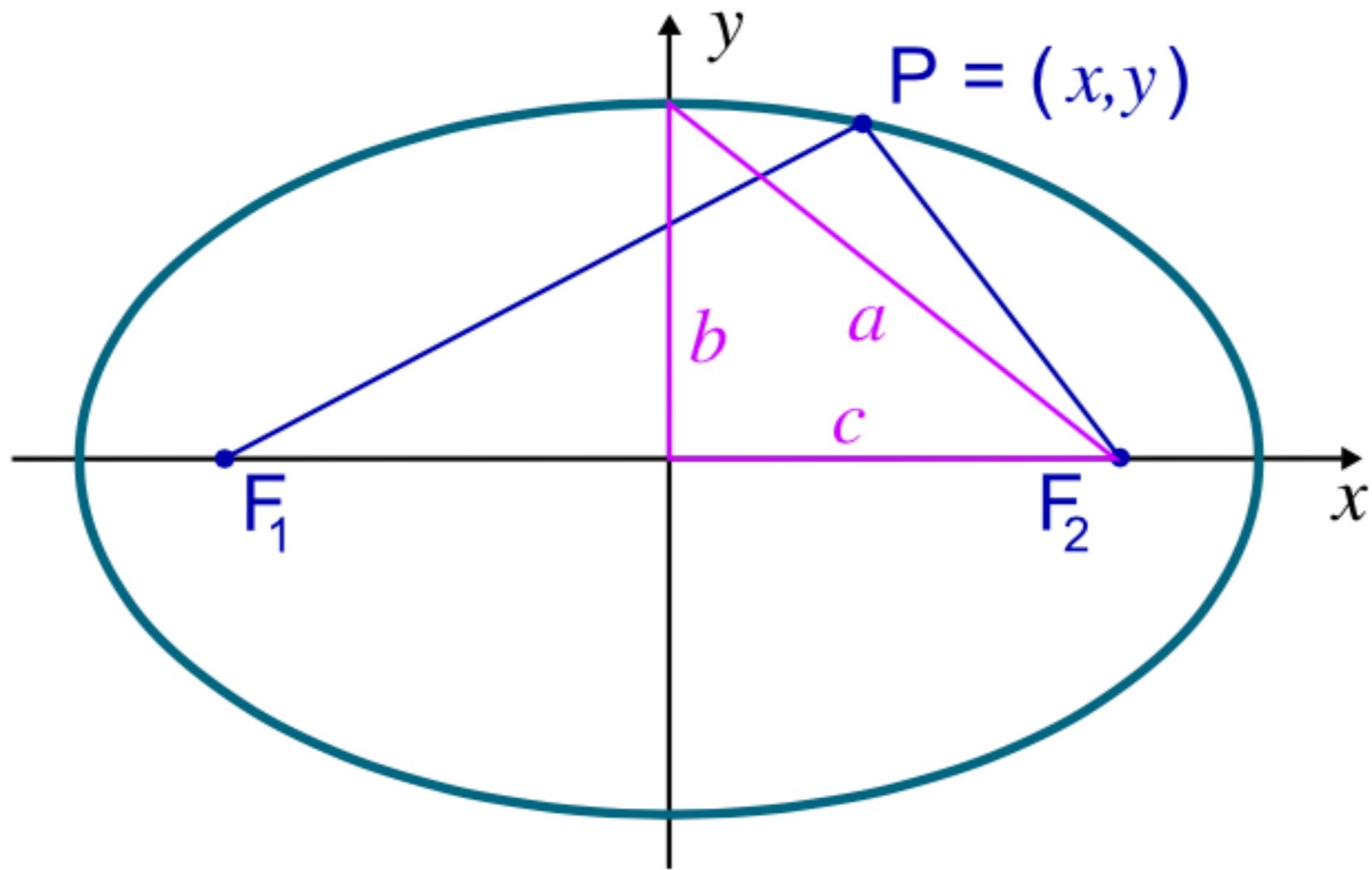
basic property of an ellipse

P is an arbitrary point between D and E .

Construct circles $A(DP)$ and $B(EP)$.

The circles intersect at two points that lie on the ellipse.



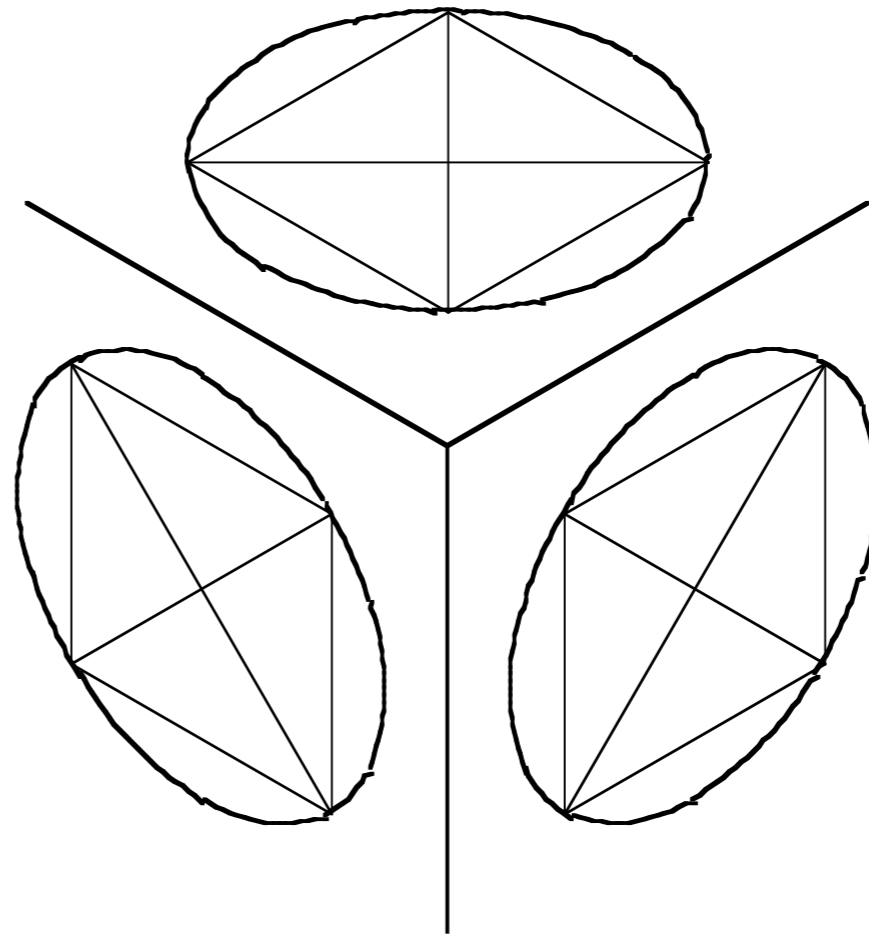
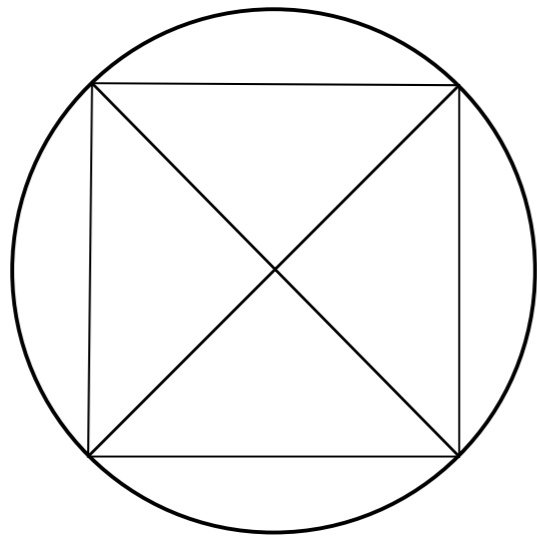


$$2a = \overline{PF_1} + \overline{PF_2}$$

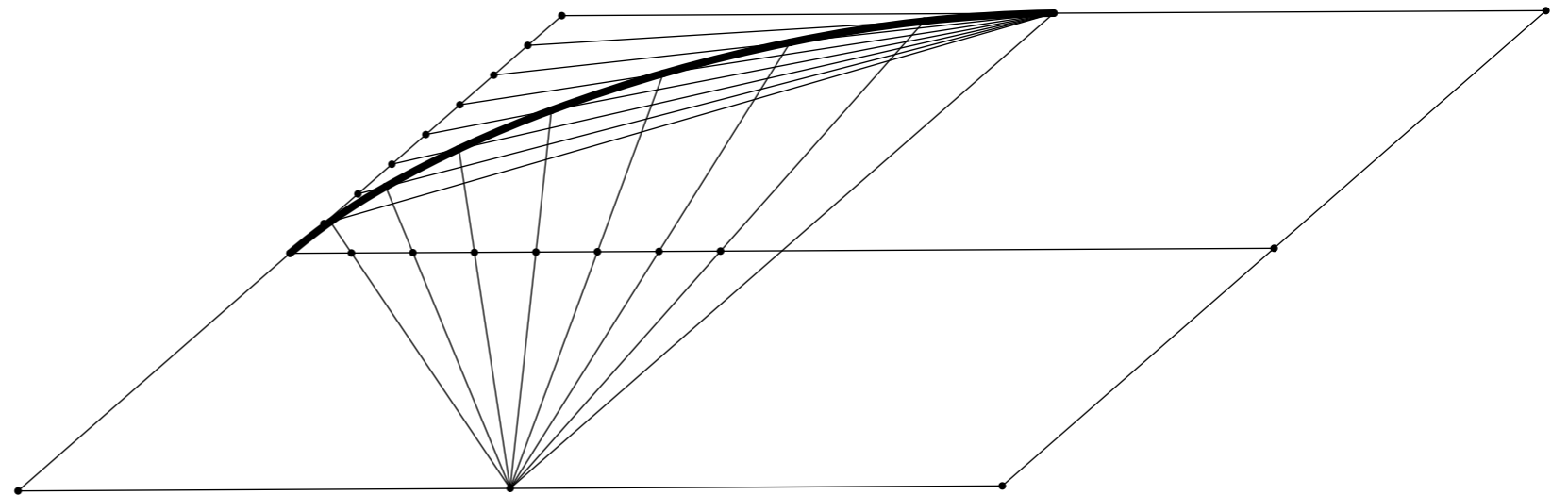
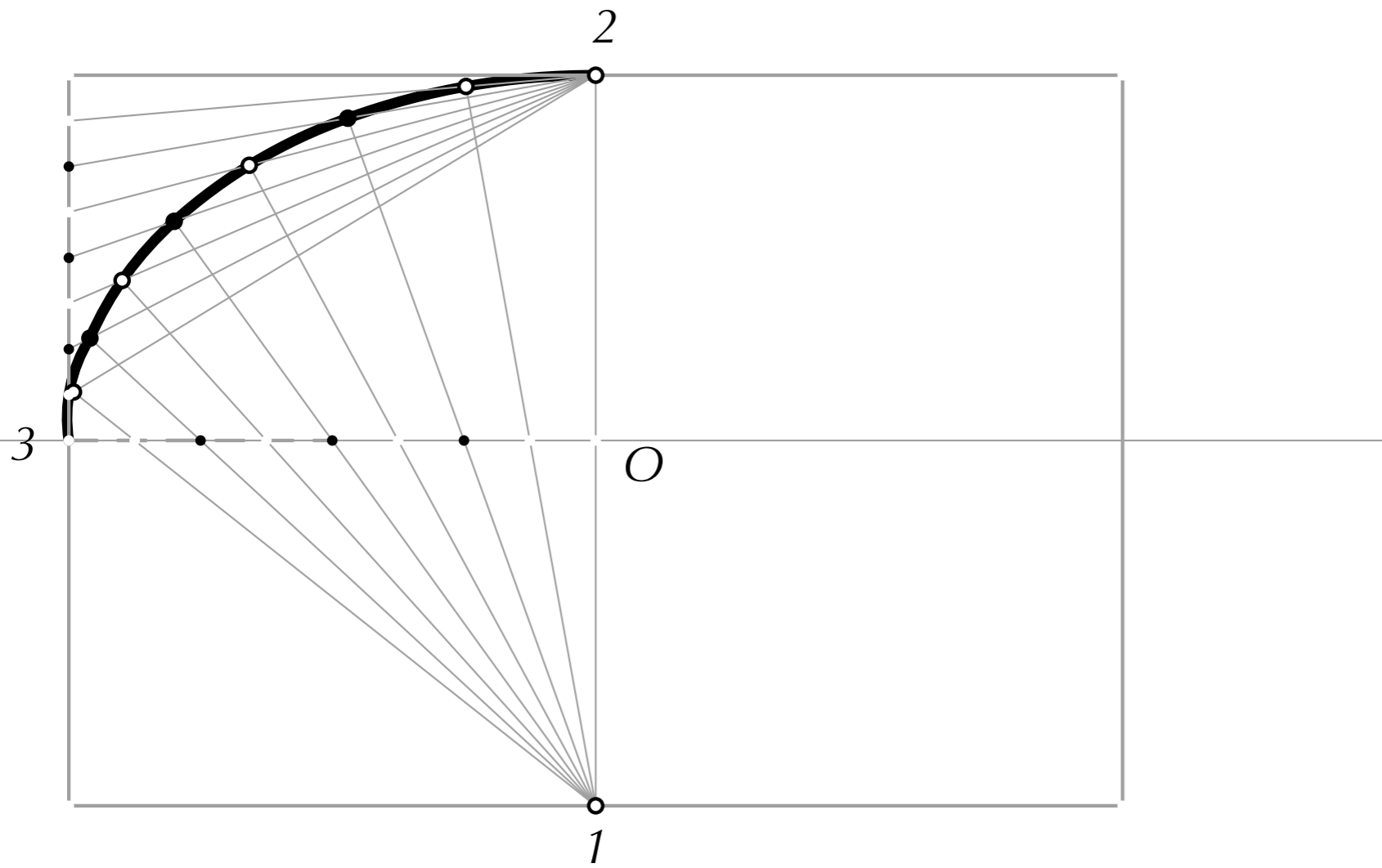
$$= \sqrt{(x+c)^2 + y^2} + \sqrt{(x-c)^2 + y^2}$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

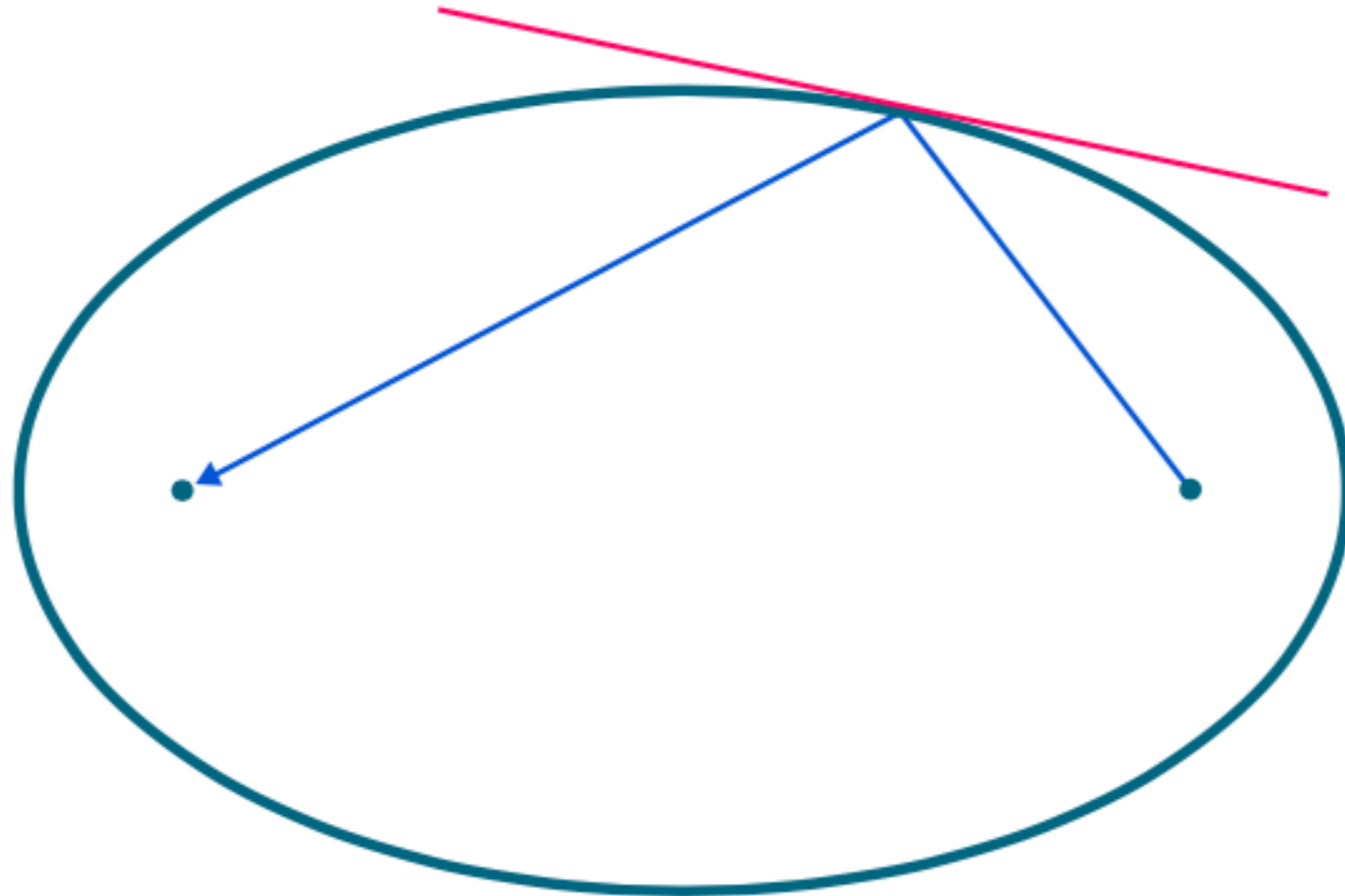
analytic form



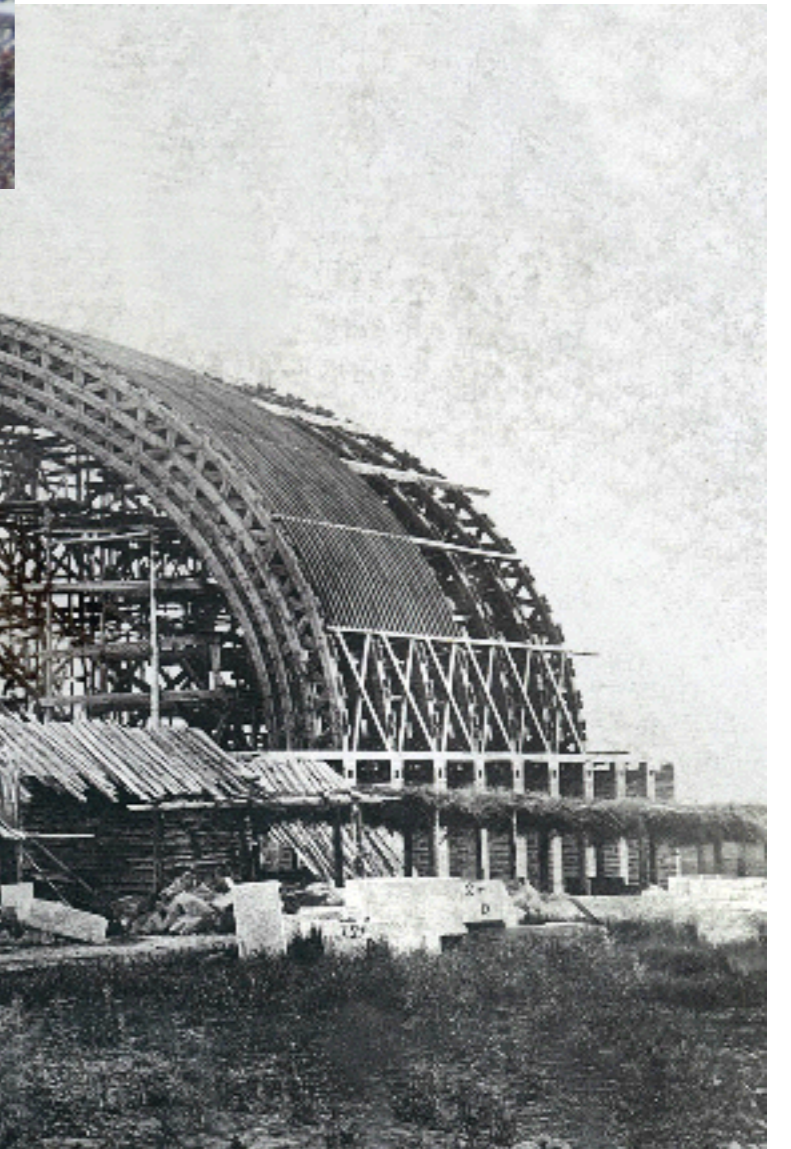
axonometric view of a circle is an ellipse



constructing an ellipse within a rectangle



reflective property of an ellipse



mormon tabernacle



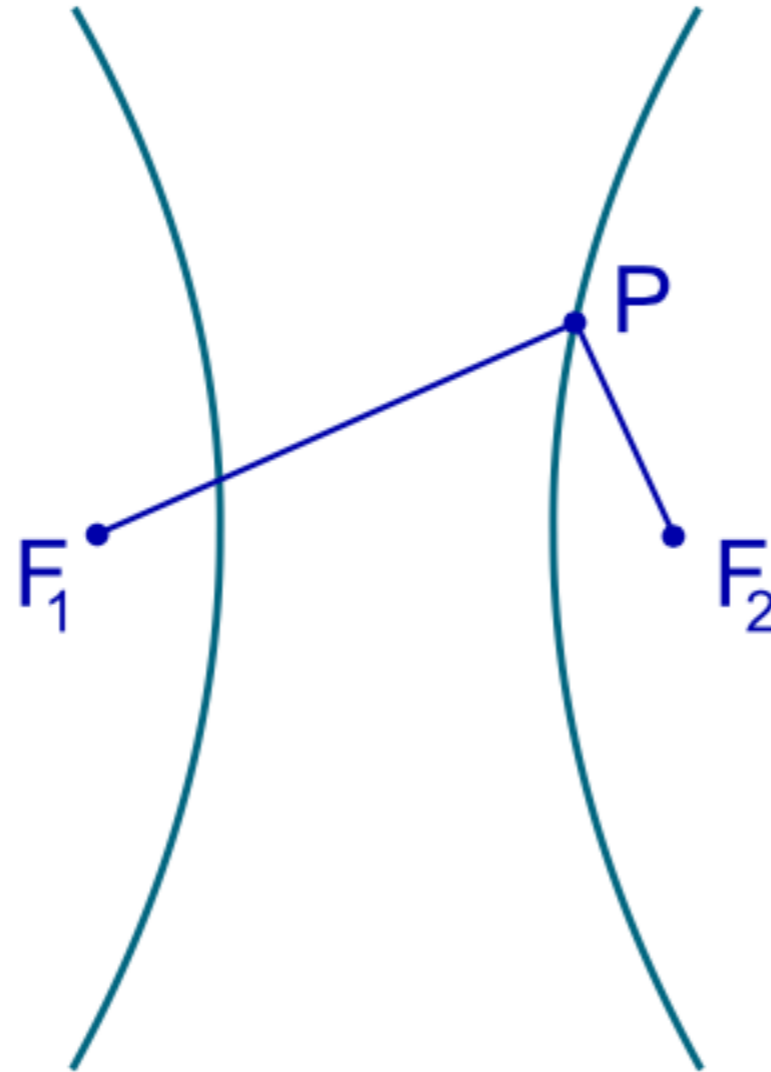
us capitol building

<http://www.loop-the-game.com>

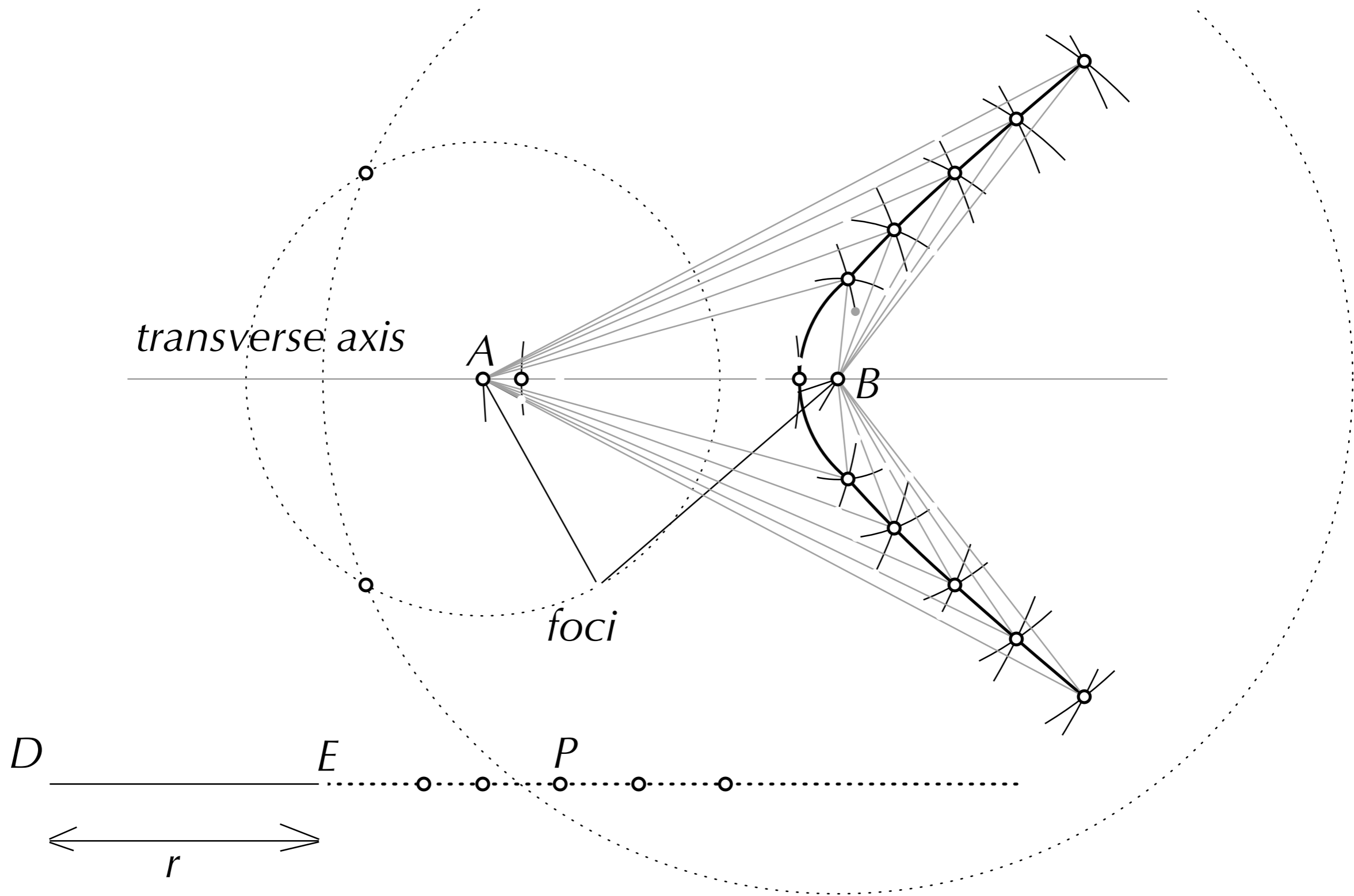


hyperbola

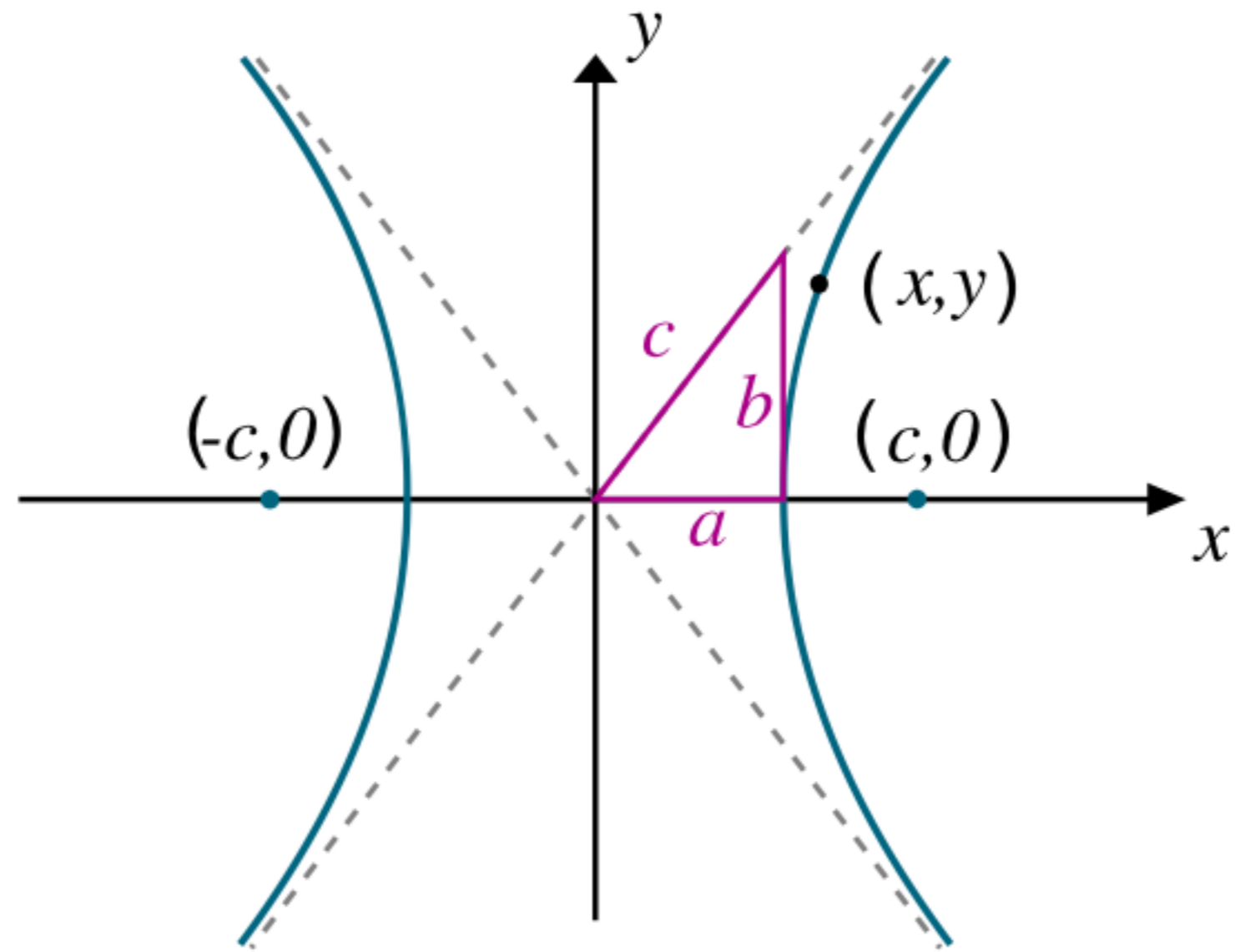
distance to F_1 - distance to F_2 = a constant



hyperbola



hyperbola



$$y = \pm \frac{b}{a}x$$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

analytic form

C is the center and V , one of the vertices. $-C-V-$ is the semi-transverse axis.

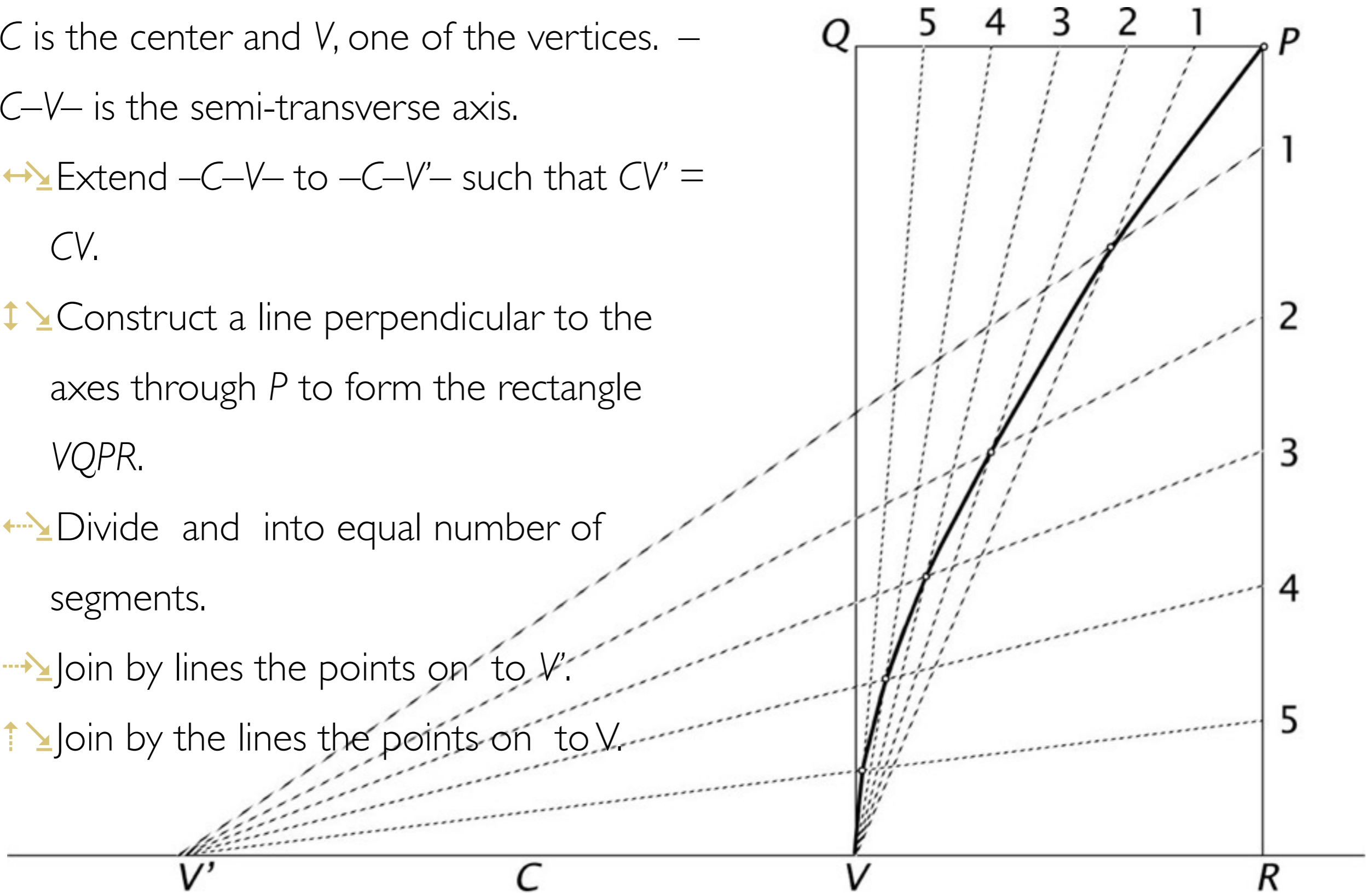
↔ ↘ Extend $-C-V-$ to $-C-V'-$ such that $CV' = CV$.

↕ ↘ Construct a line perpendicular to the axes through P to form the rectangle $VQPR$.

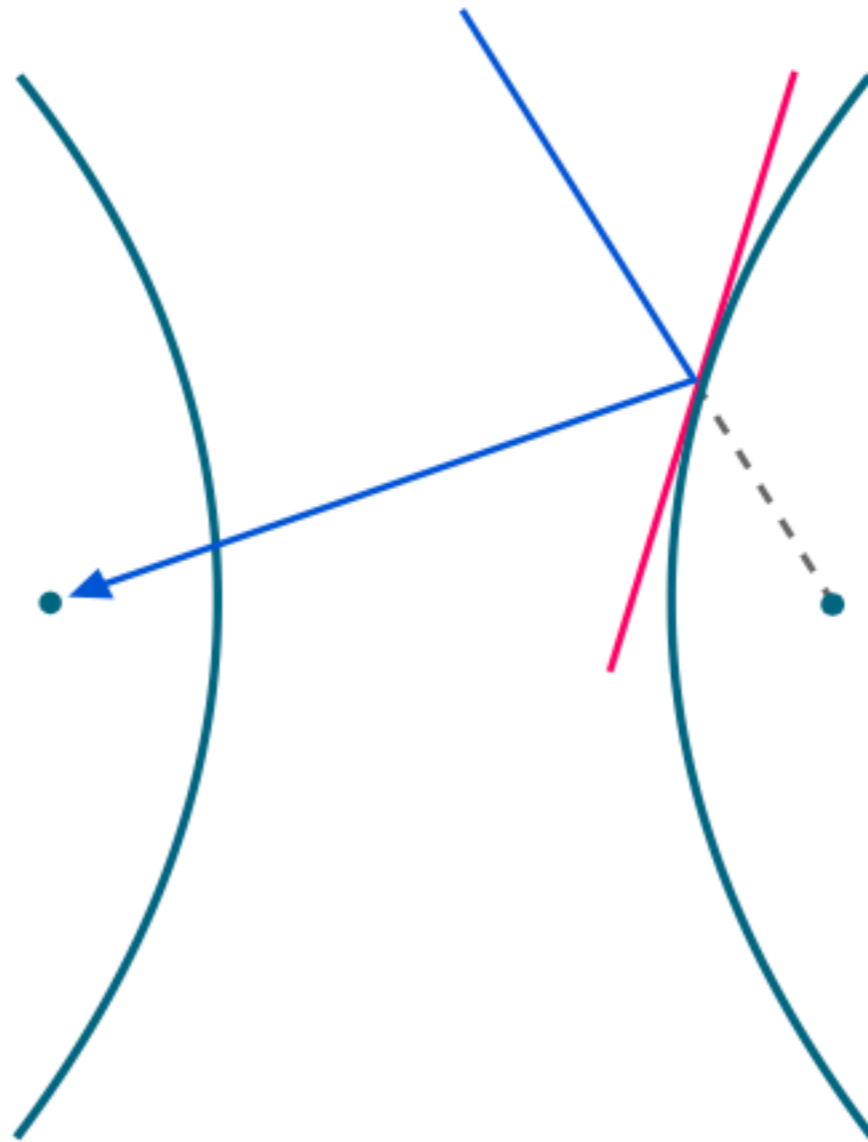
↔ ↘ Divide QP into equal number of segments.

↔ ↘ Join by lines the points on QP to V' .

↕ ↘ Join by the lines the points on QP to V .



hyperbola given semi-transverse axis and a point

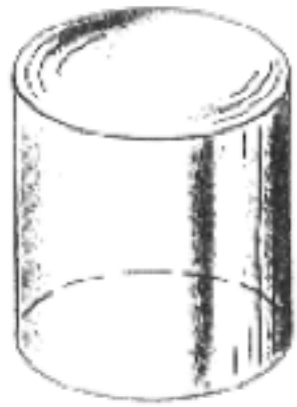


reflective property of a hyperbola



oscar neimeyer

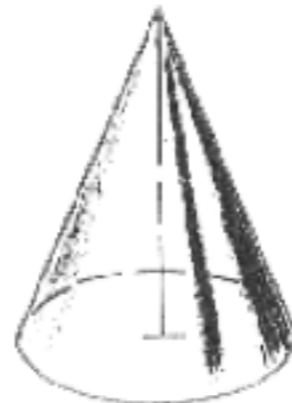
creating surfaces from conic curves



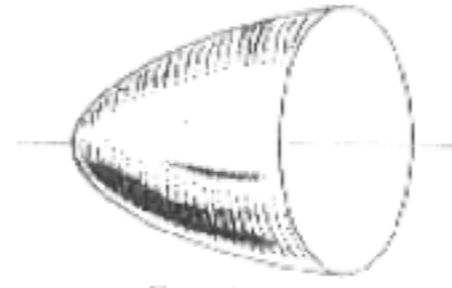
Right Cyl



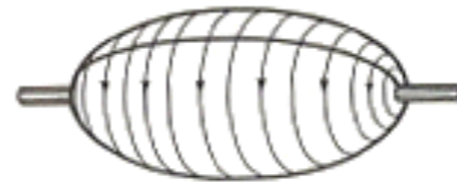
Sphere



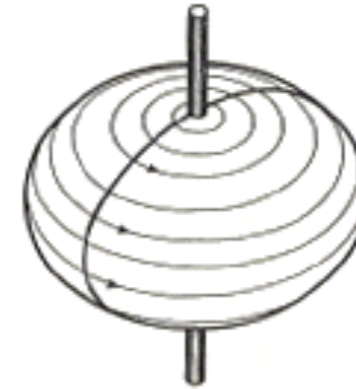
Right Cone



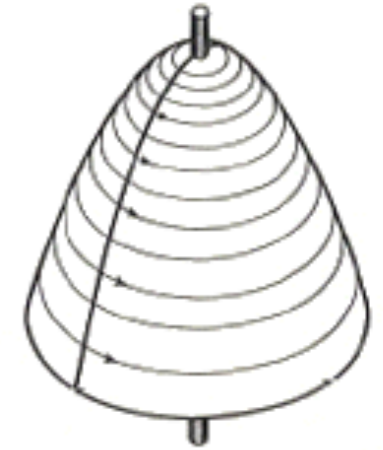
Paraboloid



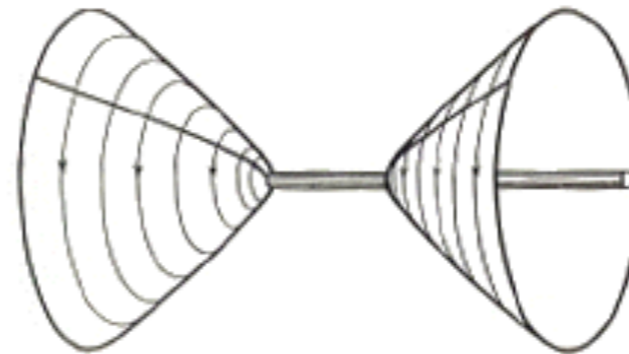
PROLATE SPHEROID



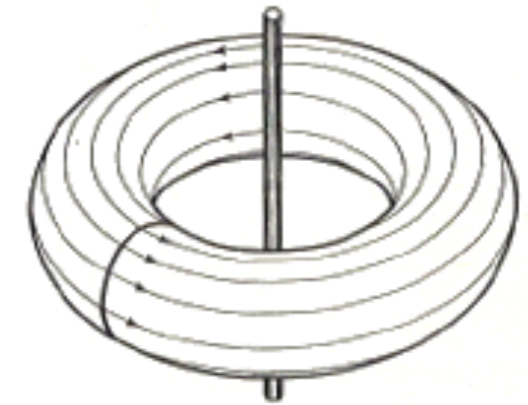
OBLATE SPHEROID



PARABOLOID



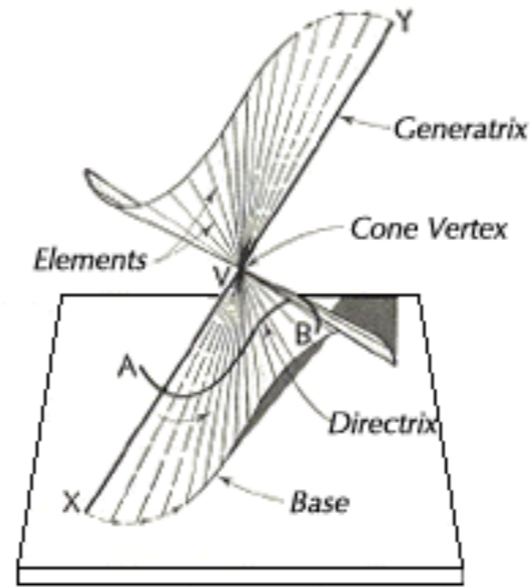
HYPERBOLOID OF TWO NAPPES



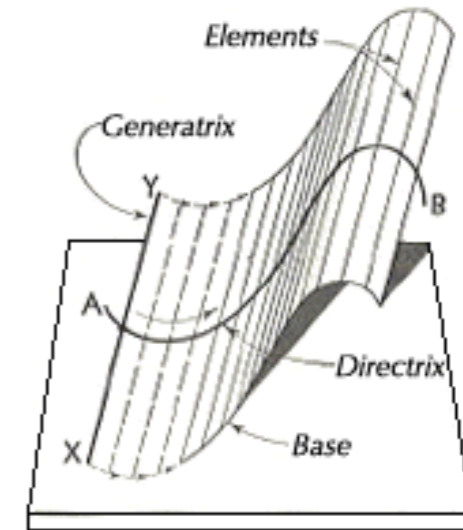
ANNULAR TORUS

by revolving

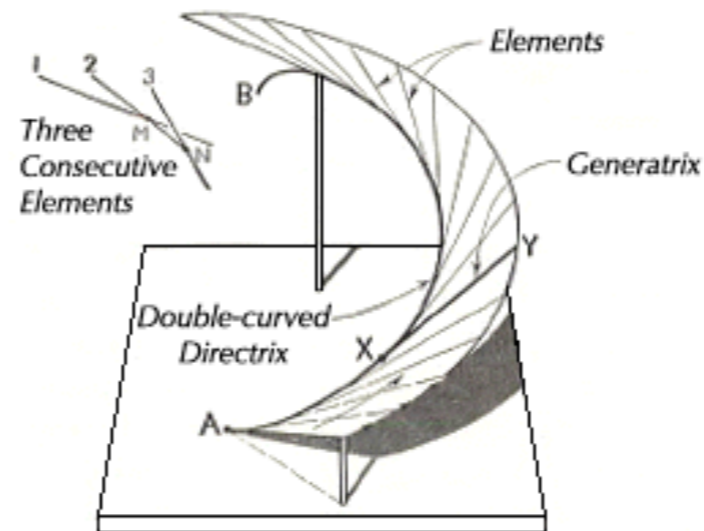
- Is produced when a line is moved in contact with a curve (directrix) in the plane to produce a surface



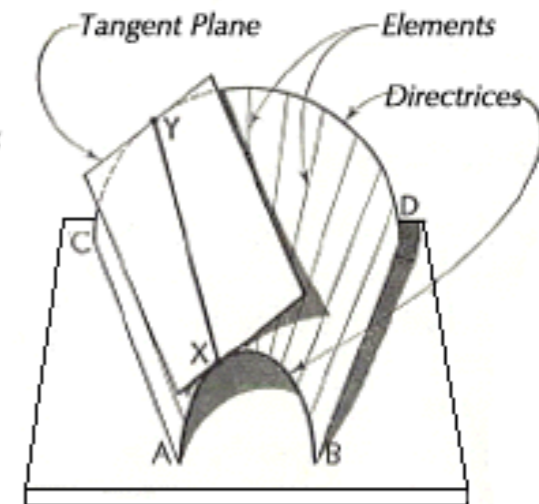
Generating a conical surface



Generating a cylindrical surface



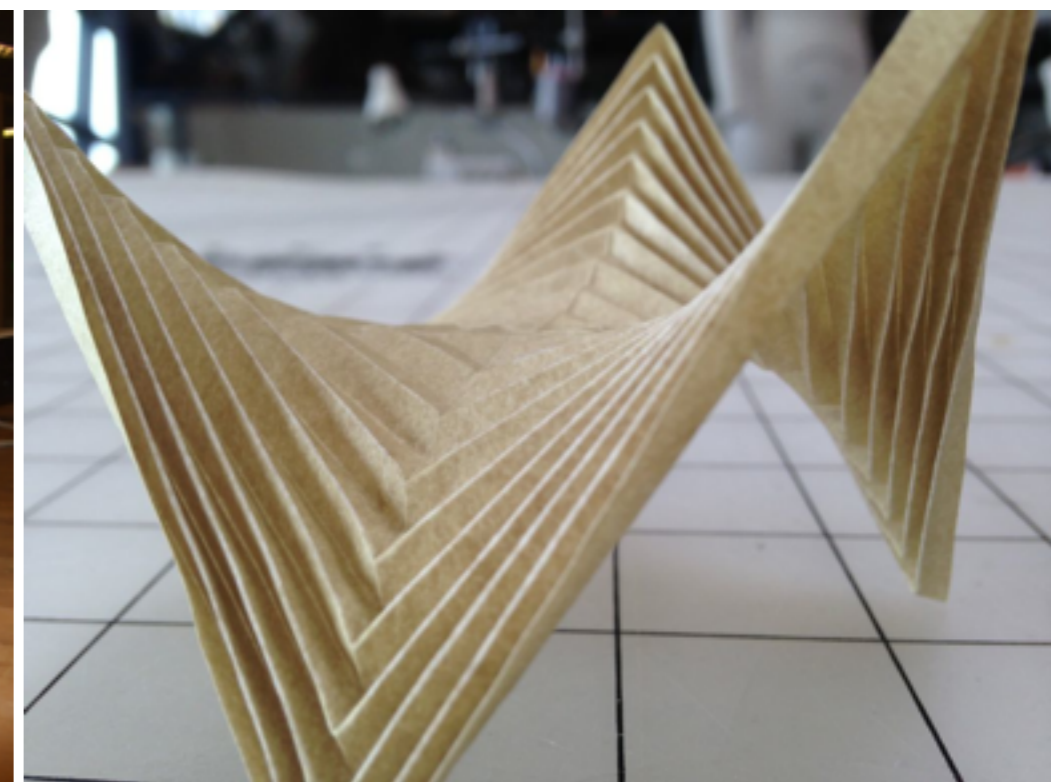
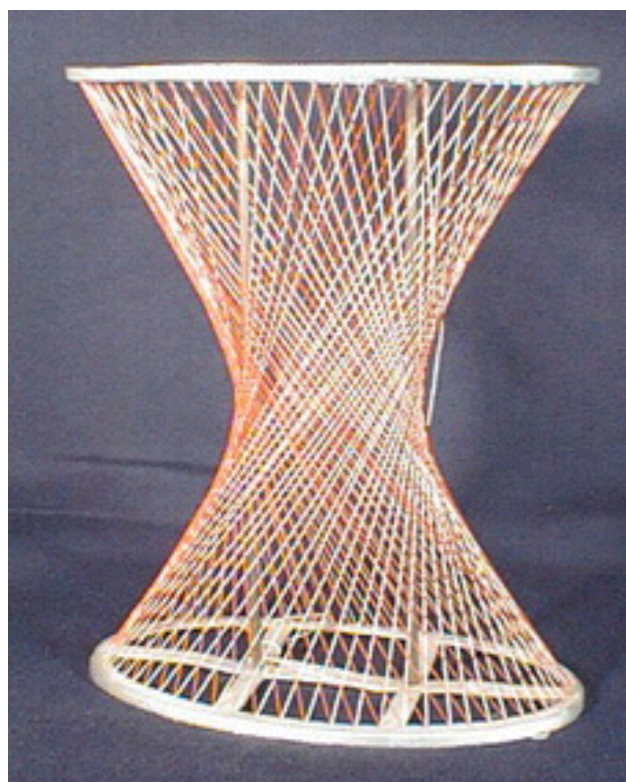
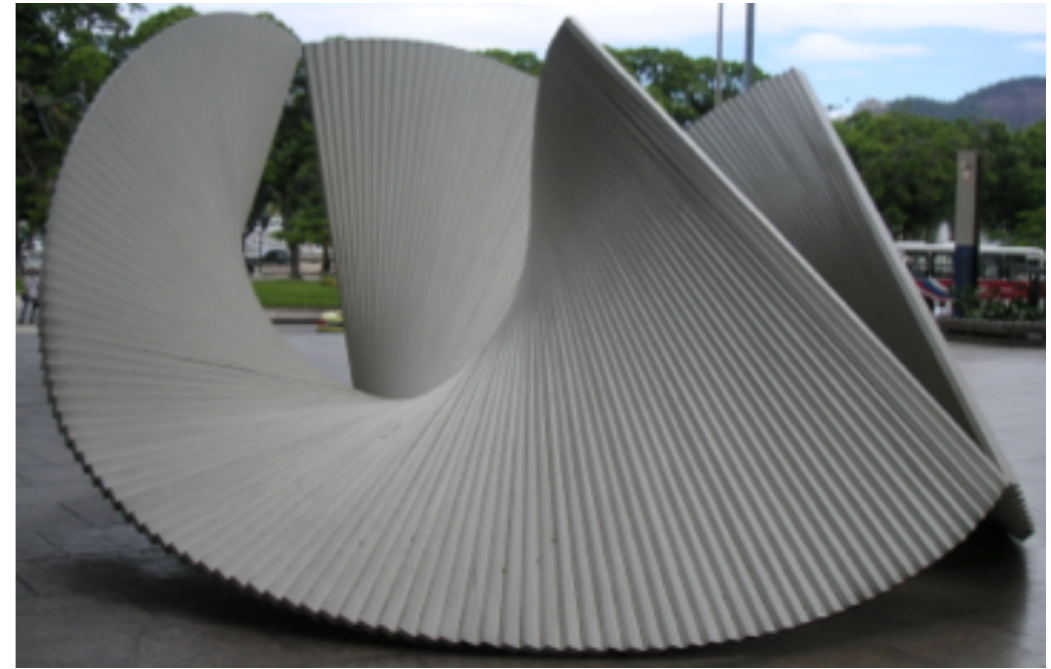
Generating a convolute (Tangent line)



Generating a convolute (Tangent plane)

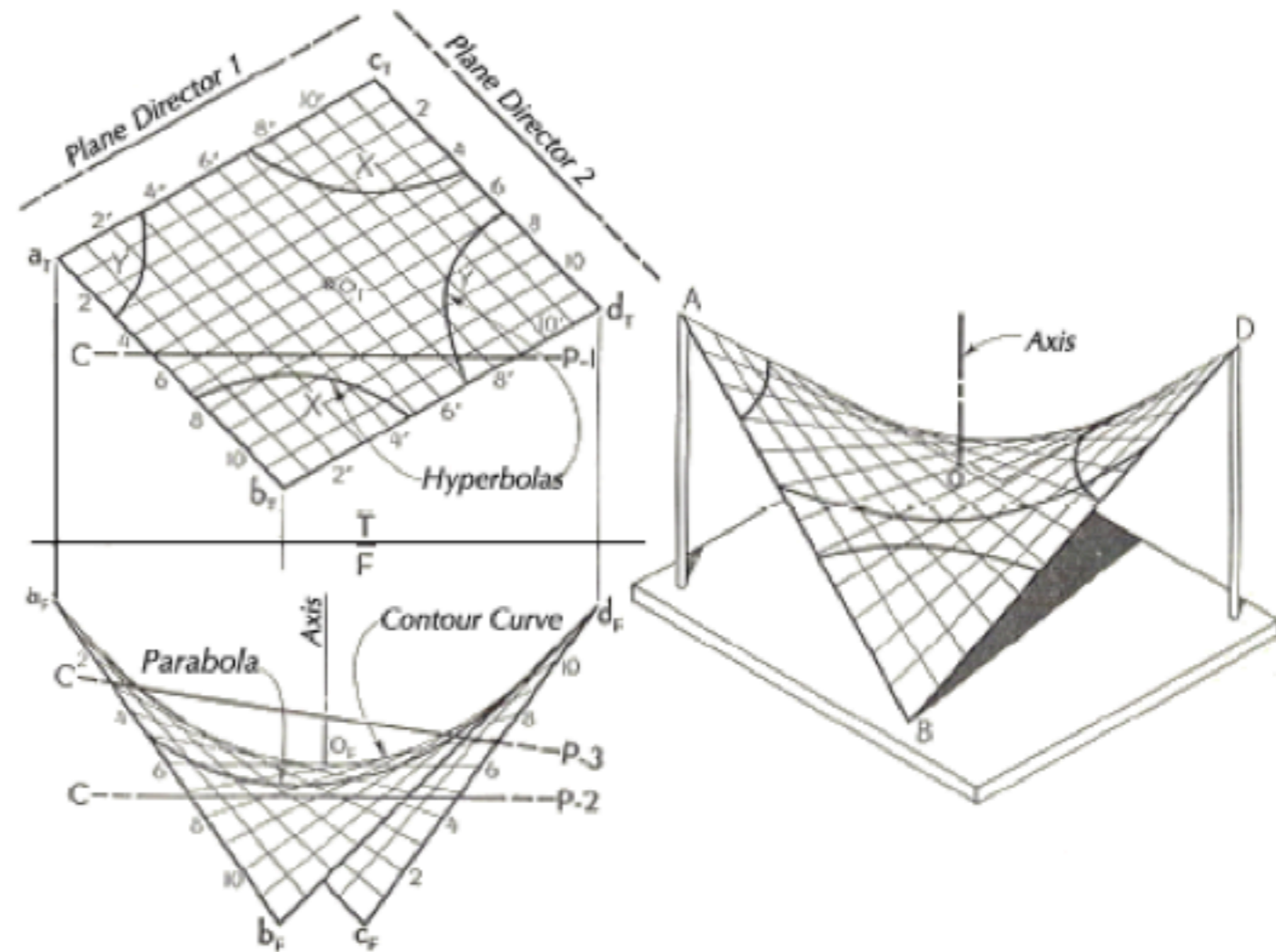
ruled surface

A ruled surface has the property that a straight line on the surface can be drawn through any point on the surface.

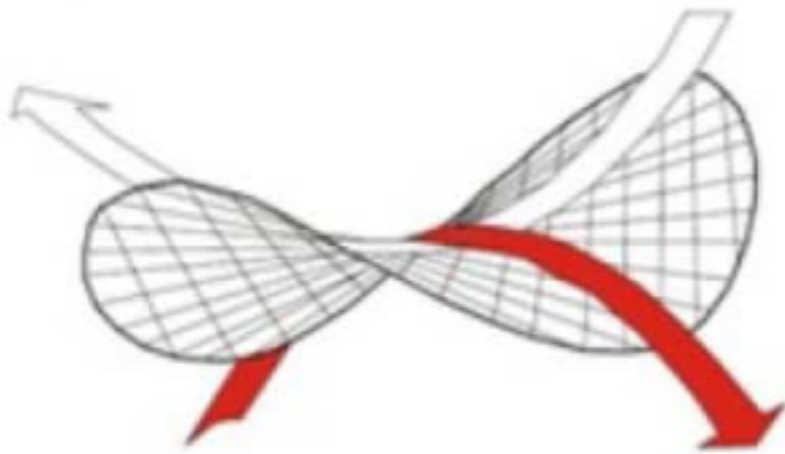


ruled surfaces

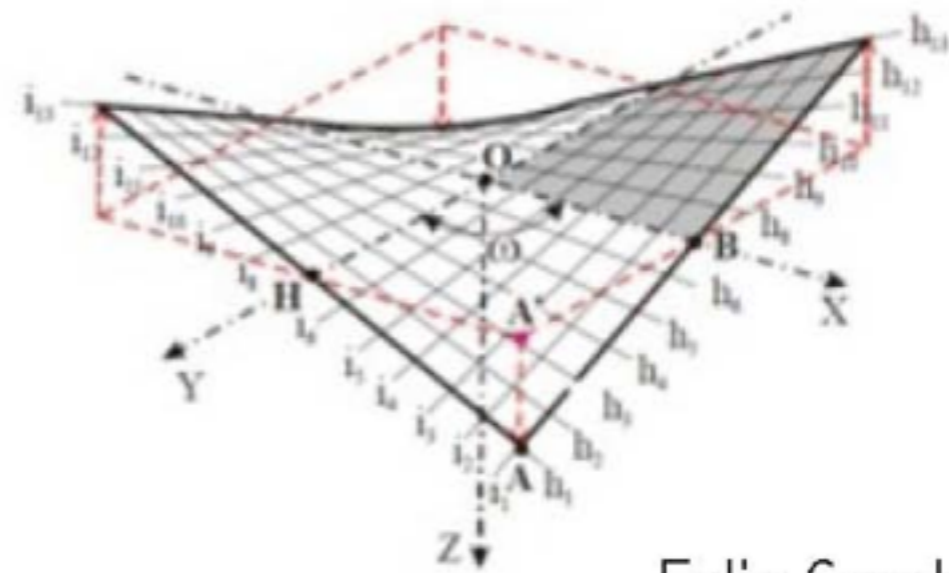
- Is a ruled surface for which two successive elements are neither parallel nor pass through a common point



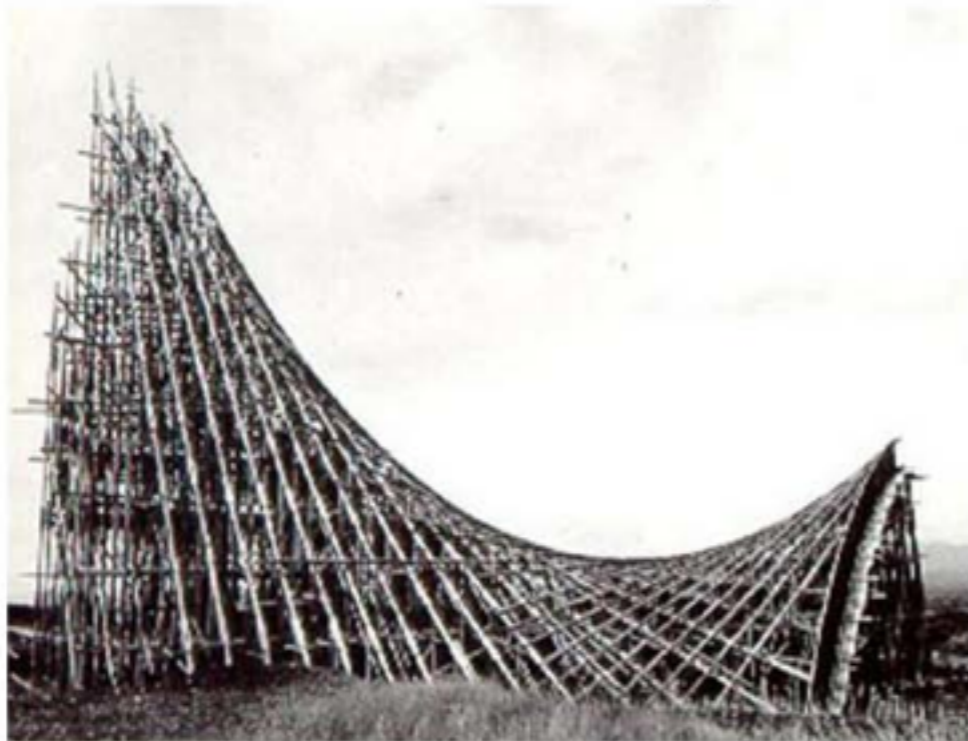
warped surface



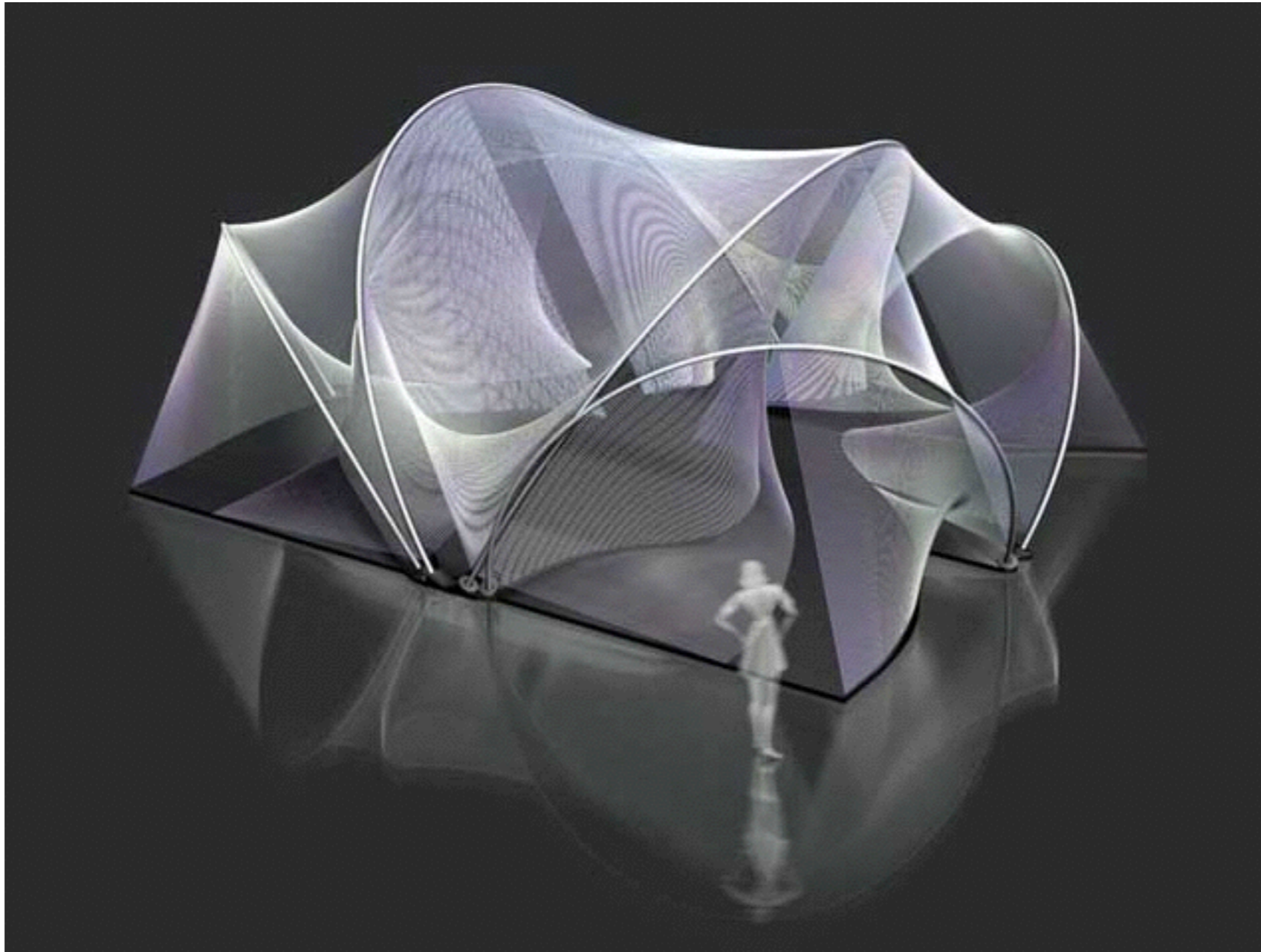
Hyperbolic Paraboloid
Curves constructed from a doubly ruled surface



Felix Candela
Capilla de Palmira, Cuernavaca, Mexico

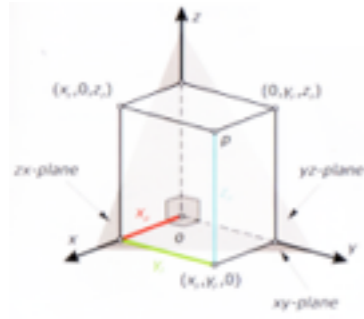


doubly-curved surface

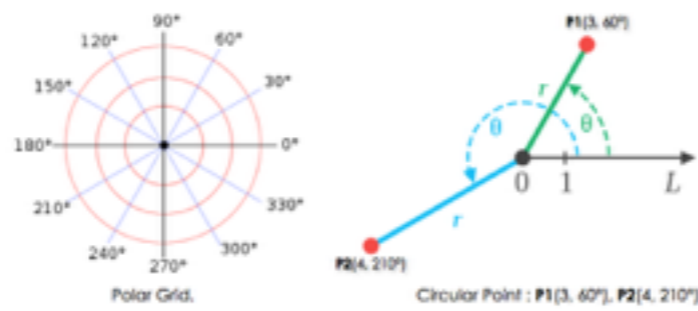


<http://www.achimmenges.net>

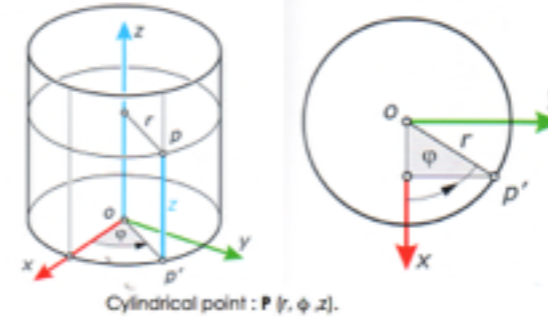
coordinates and transformations



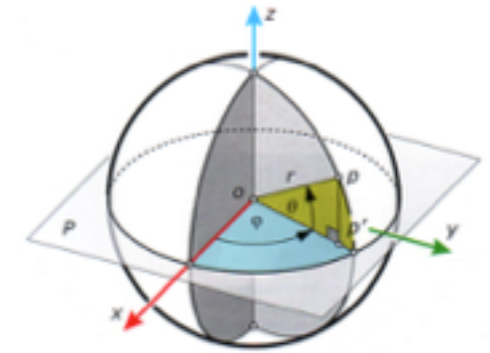
cartesian coordinate system



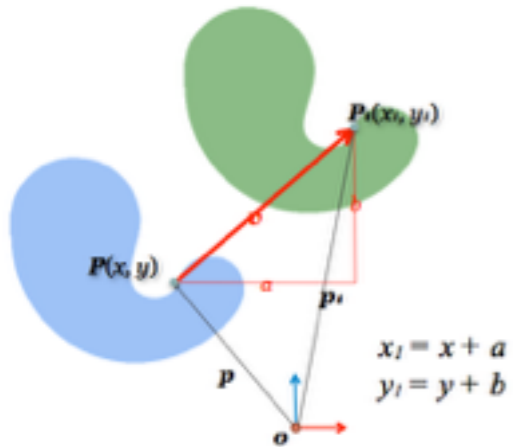
polar coordinate system



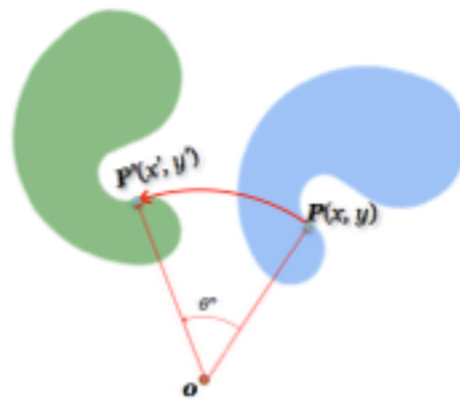
cylindrical coordinate system



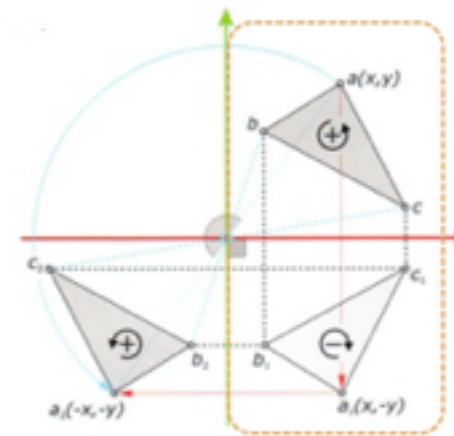
spherical coordinate system



translation



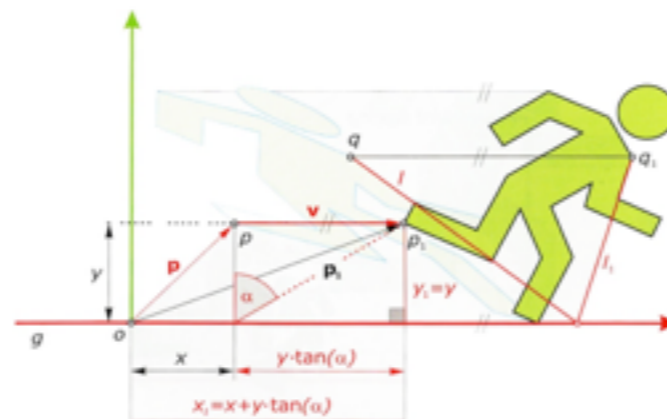
rotation



reflection

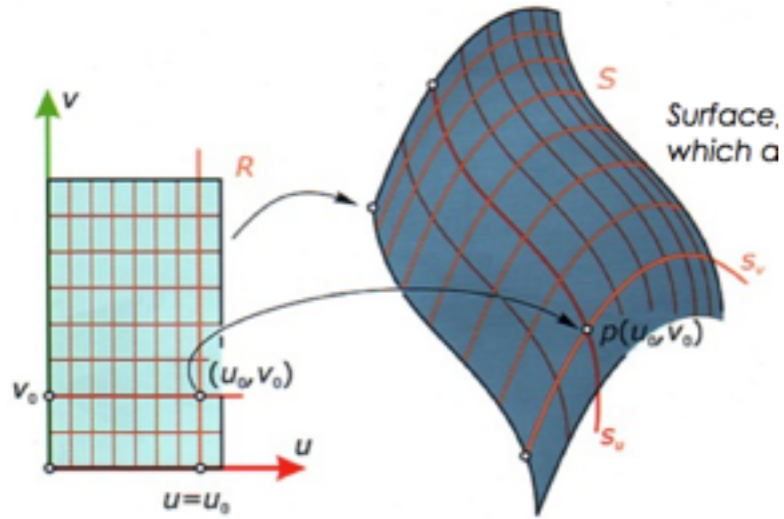


scale transformation

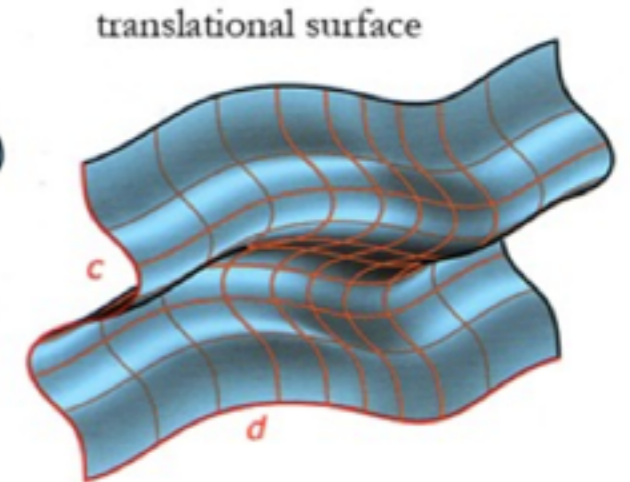
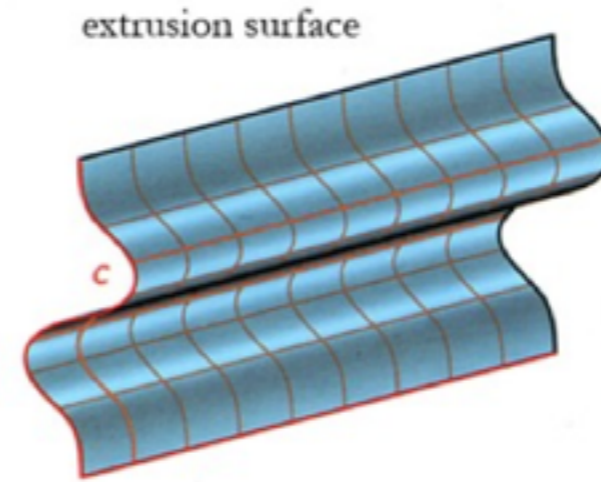


shear transformation

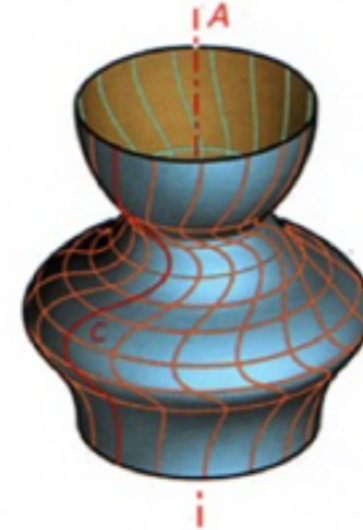
freeform curves to surfaces



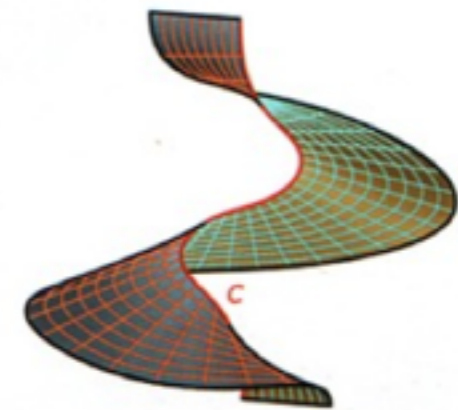
surface parameterization



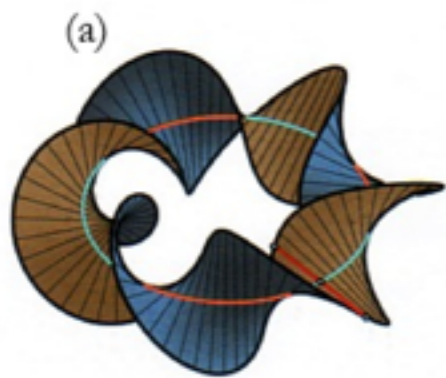
rotational surface



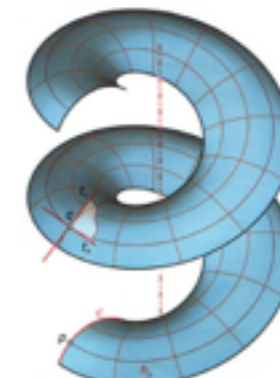
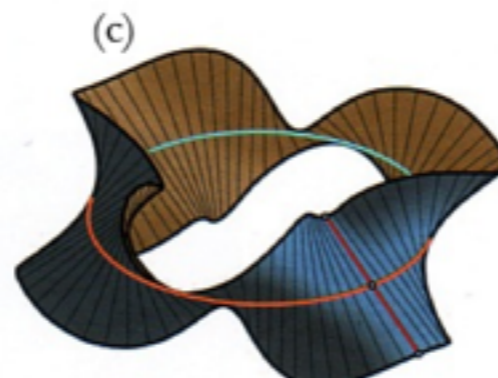
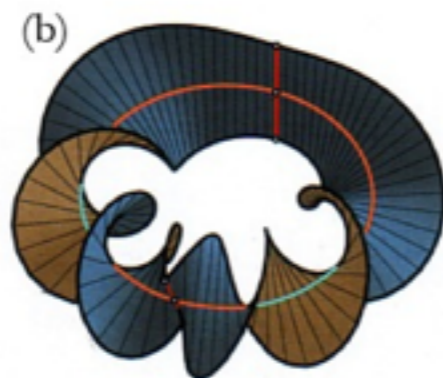
rule surface



surface classes



mobius strip

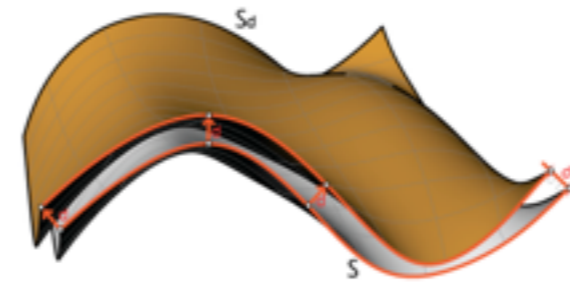


helical surface

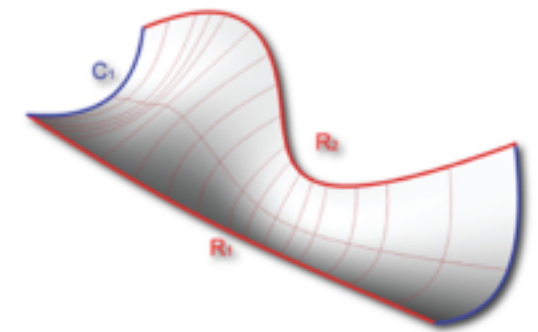


pipe surface

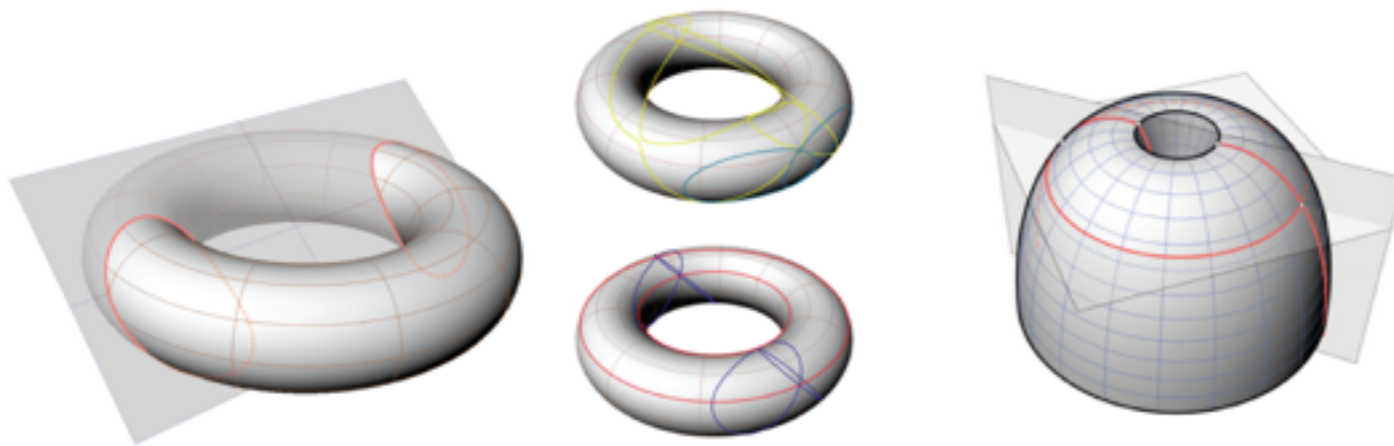
surface constructions



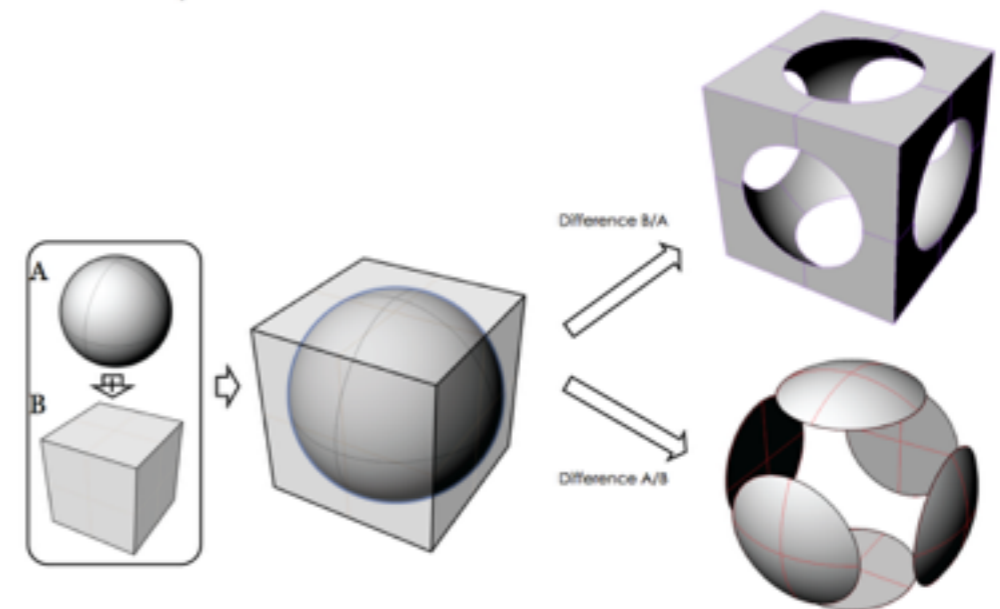
offset surface



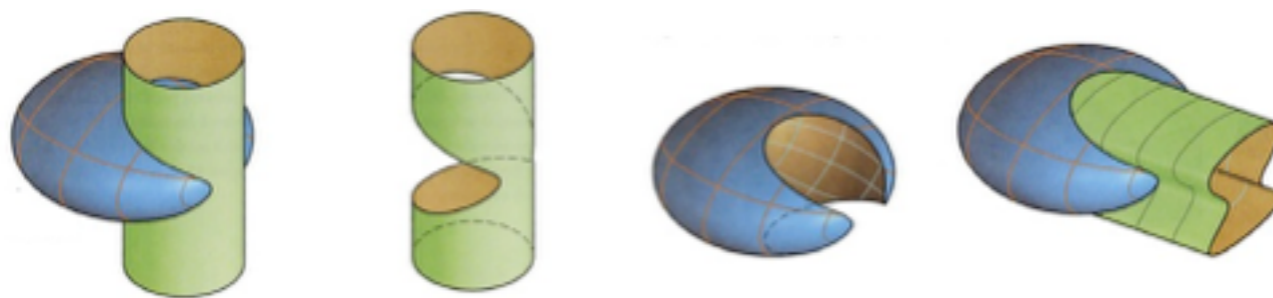
swept surface



intersection curves of surfaces

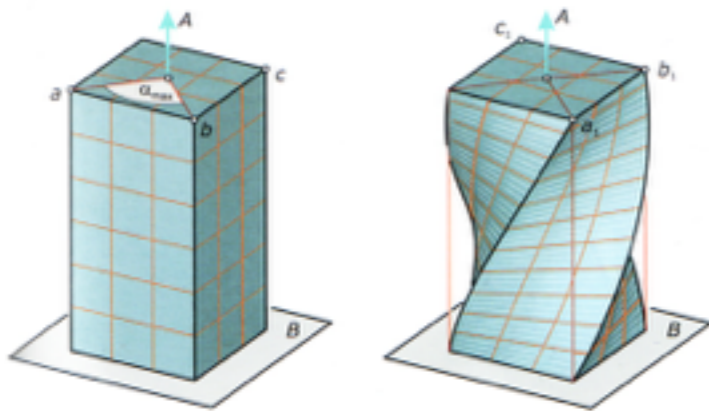


boolean operations

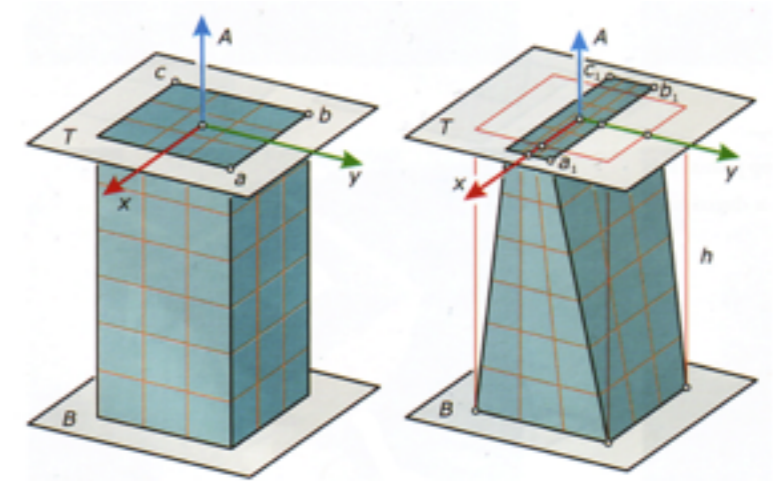
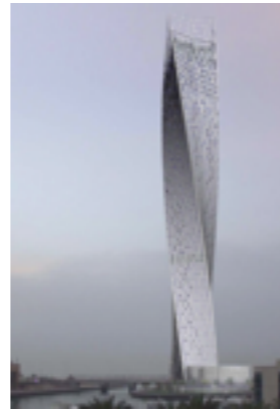


trim and split

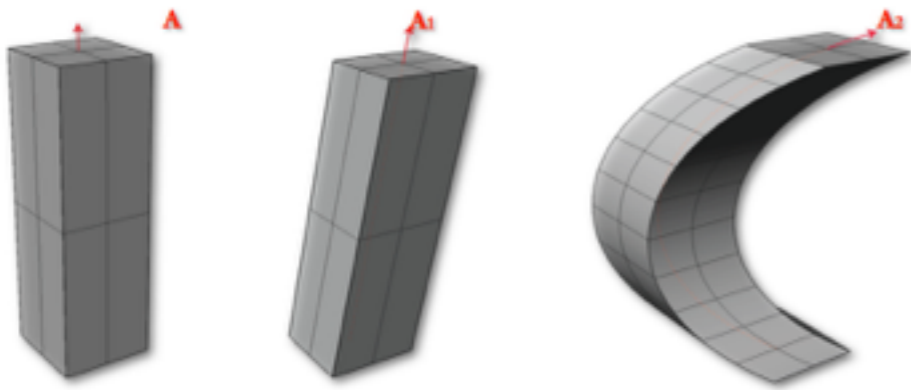
deformations



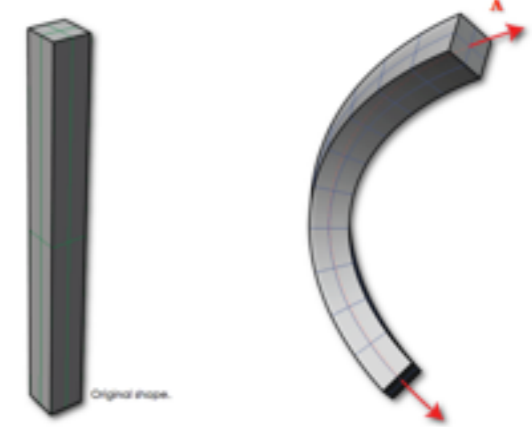
twisting



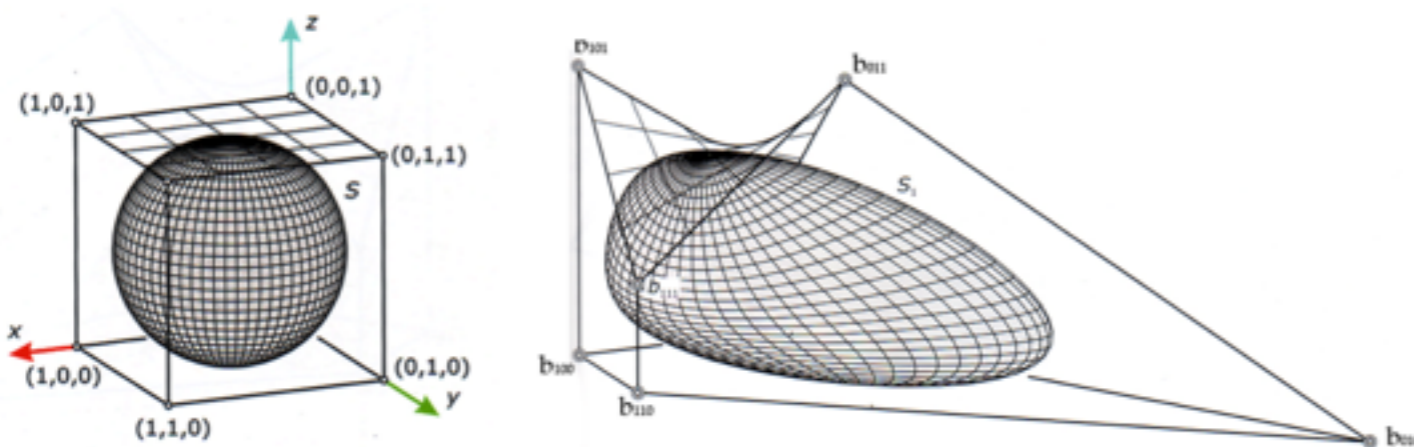
tapering



shear deformations



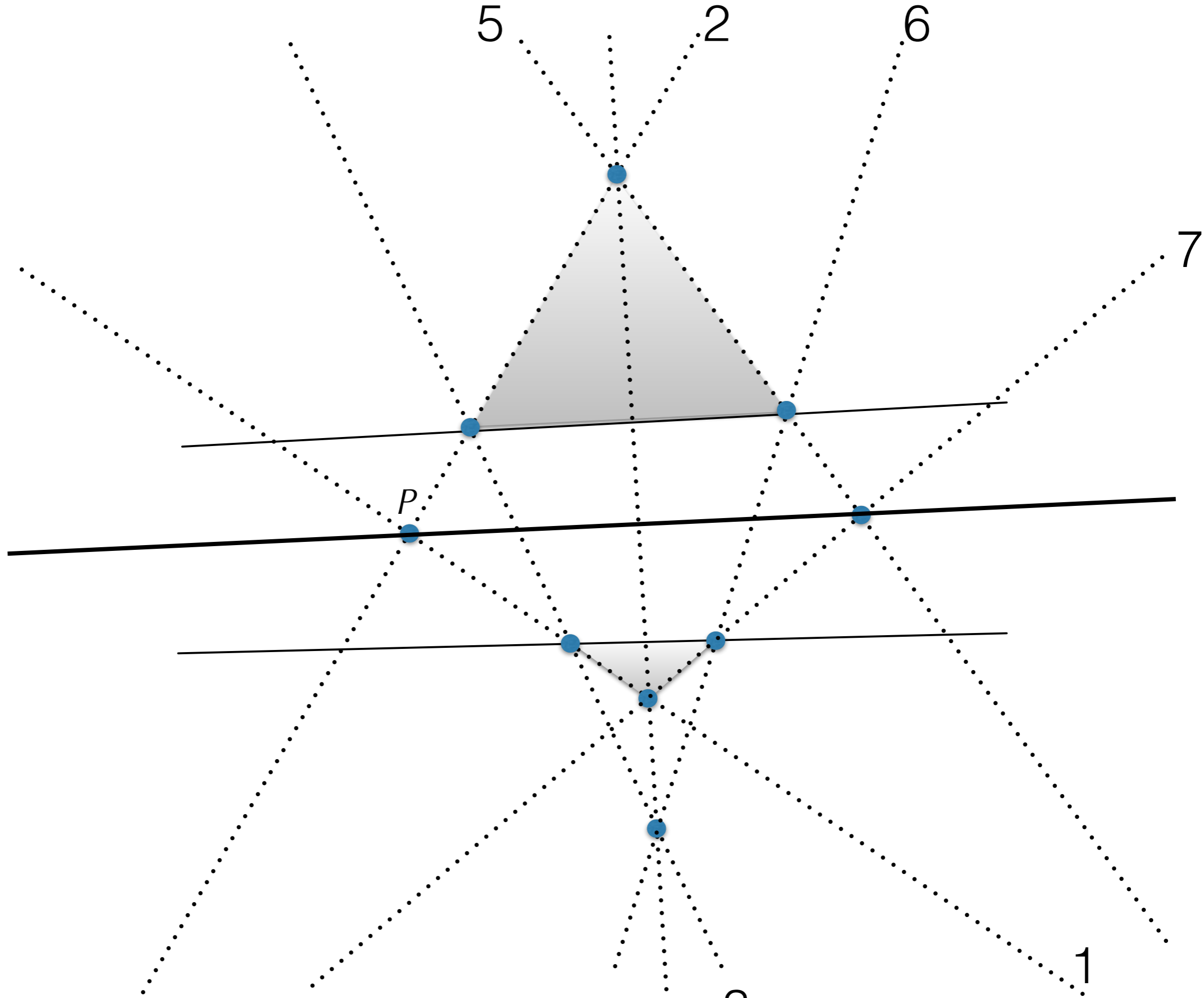
bending



free form deformations deformations



back to descriptive geometry



a typical problem