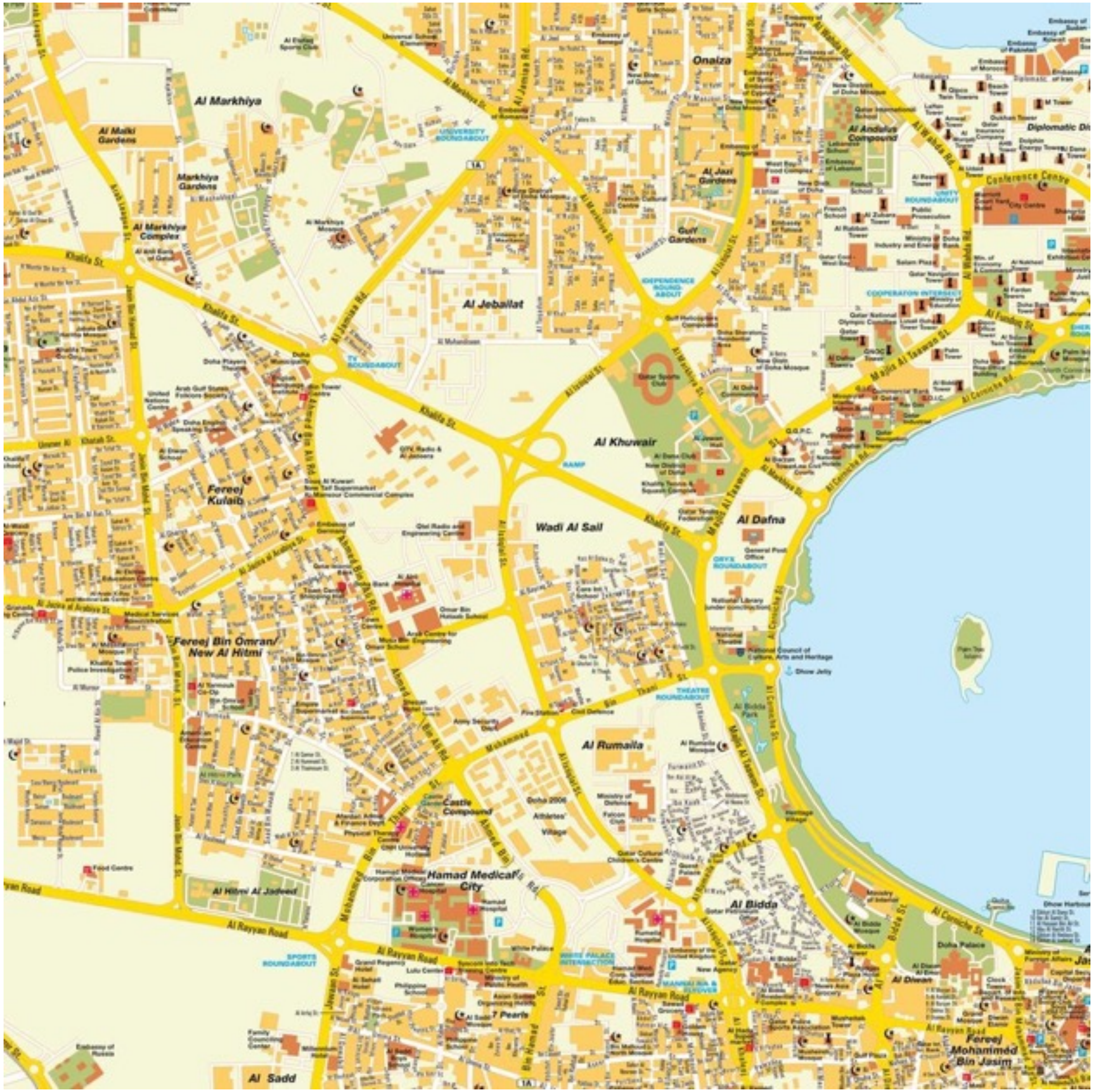
Descriptive Geometry

A typical problem

can you work out the area of the green area just using geometrical construction?

▶ a typical problem

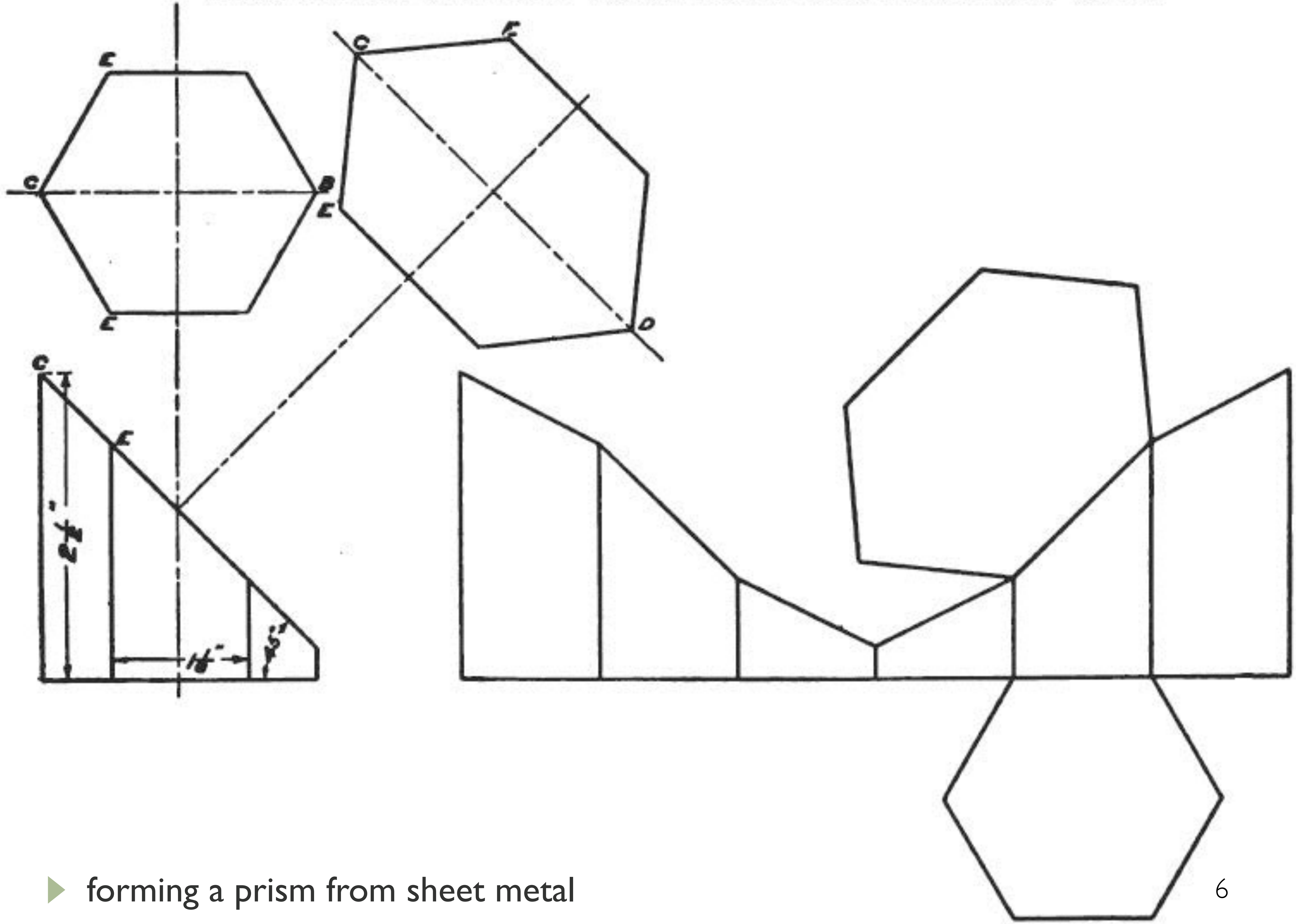
Or the green areas here?



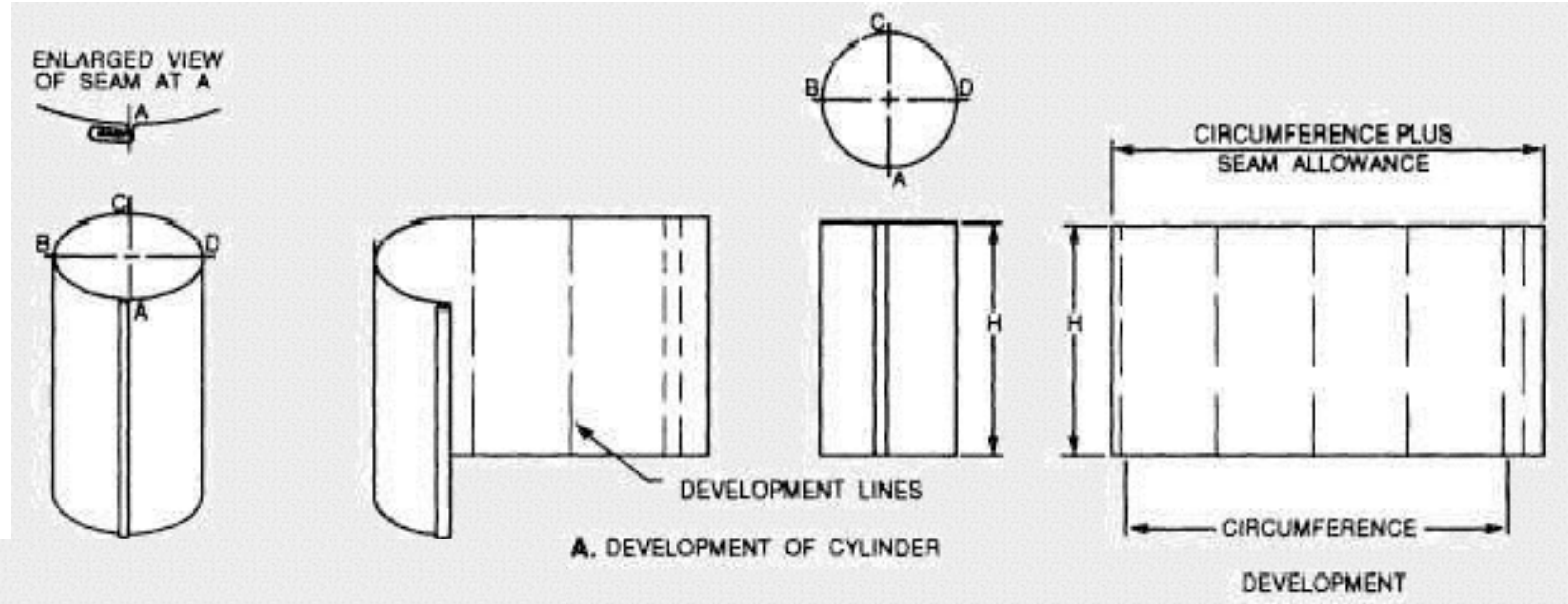
a typical problem

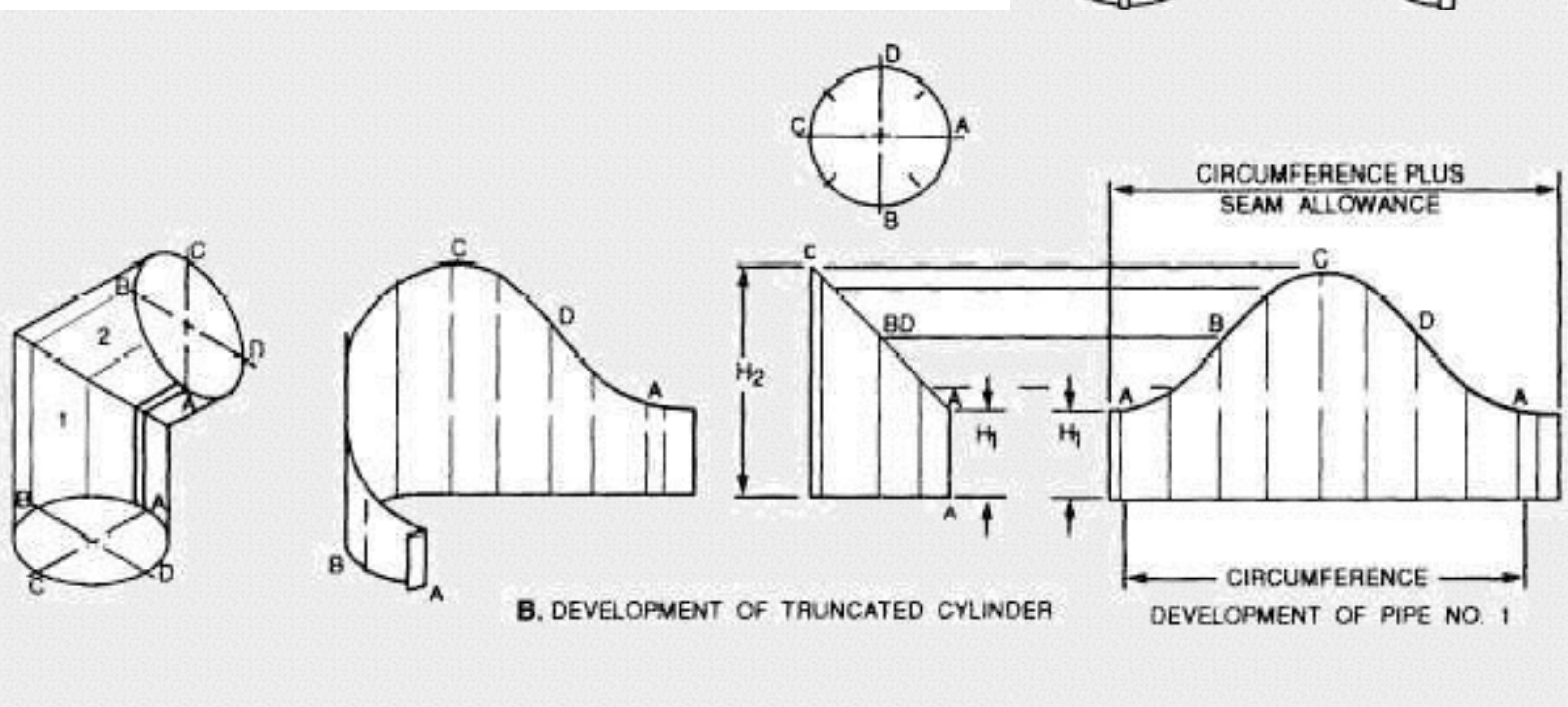
Development of an object

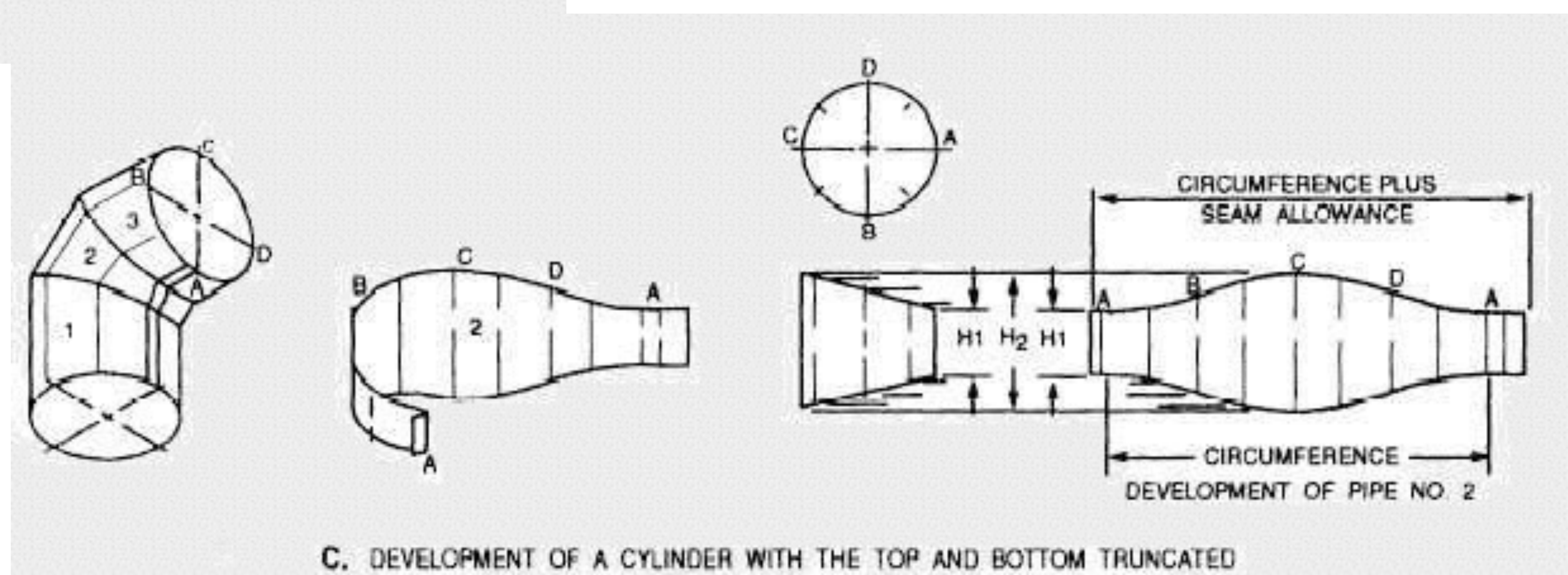
DEVELOPMENT OF A TRUNCATED HEXAGONAL PRISM



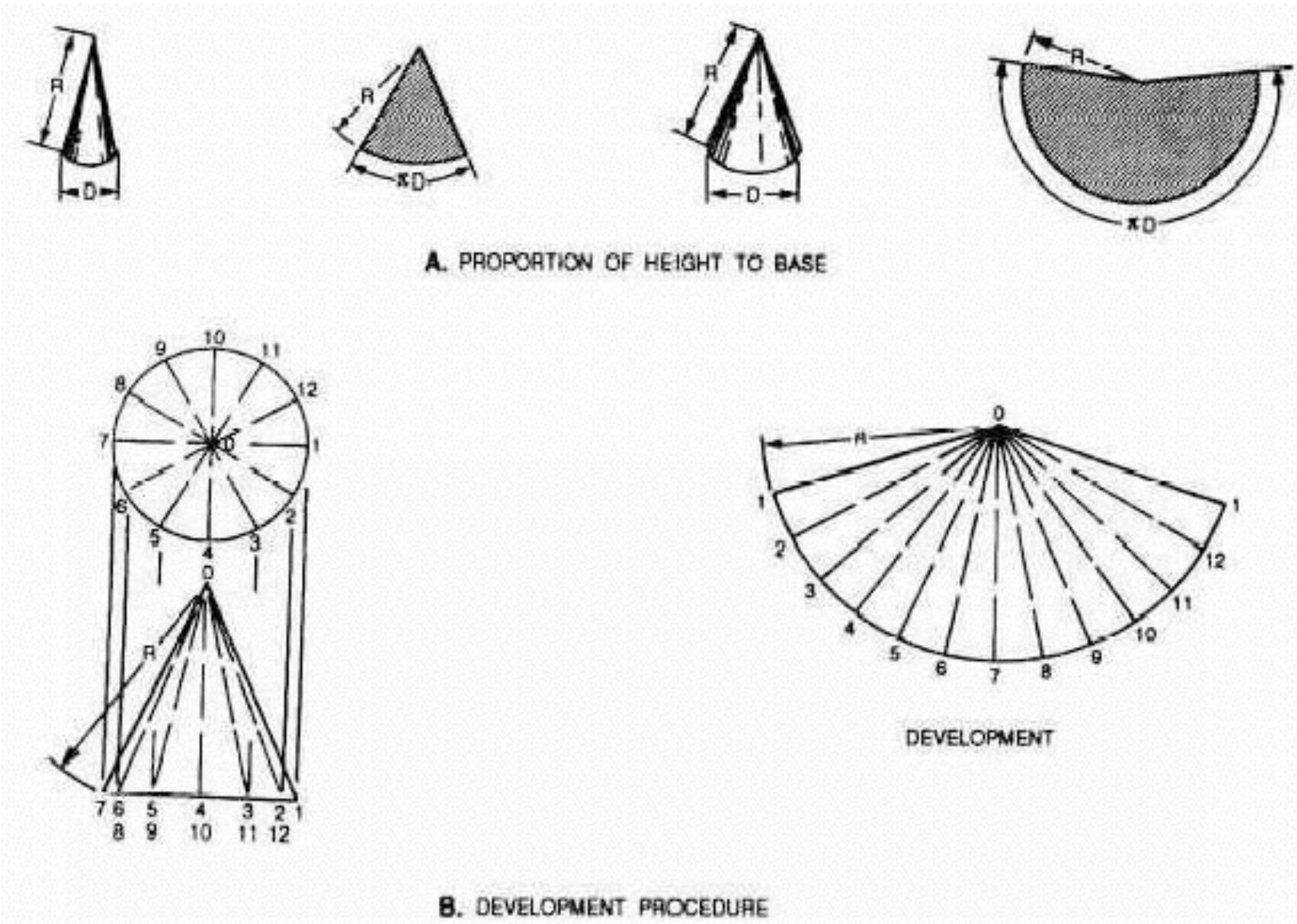
- forming a prism from sheet metal



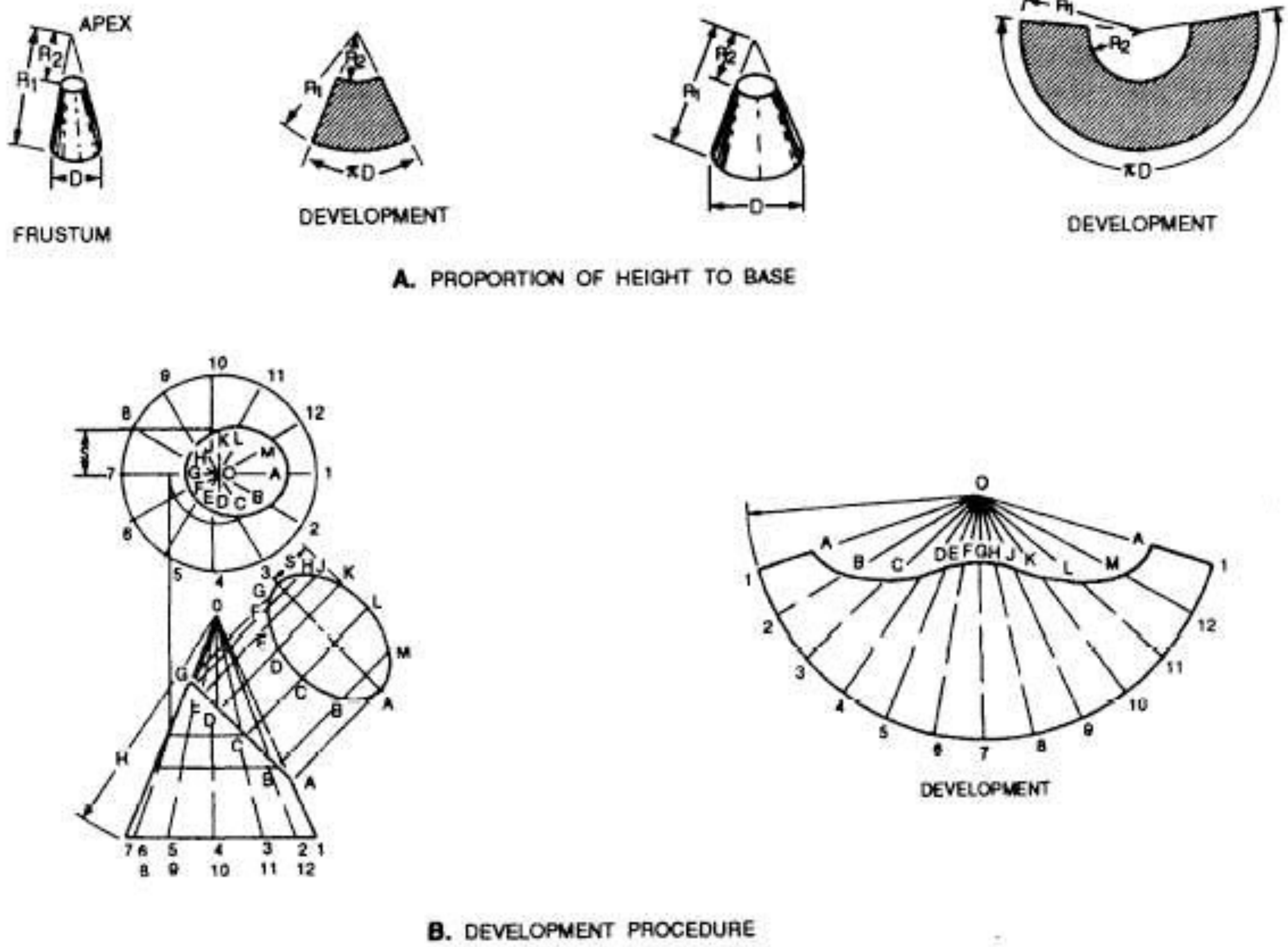




- development of a cylinder



- development of a cone

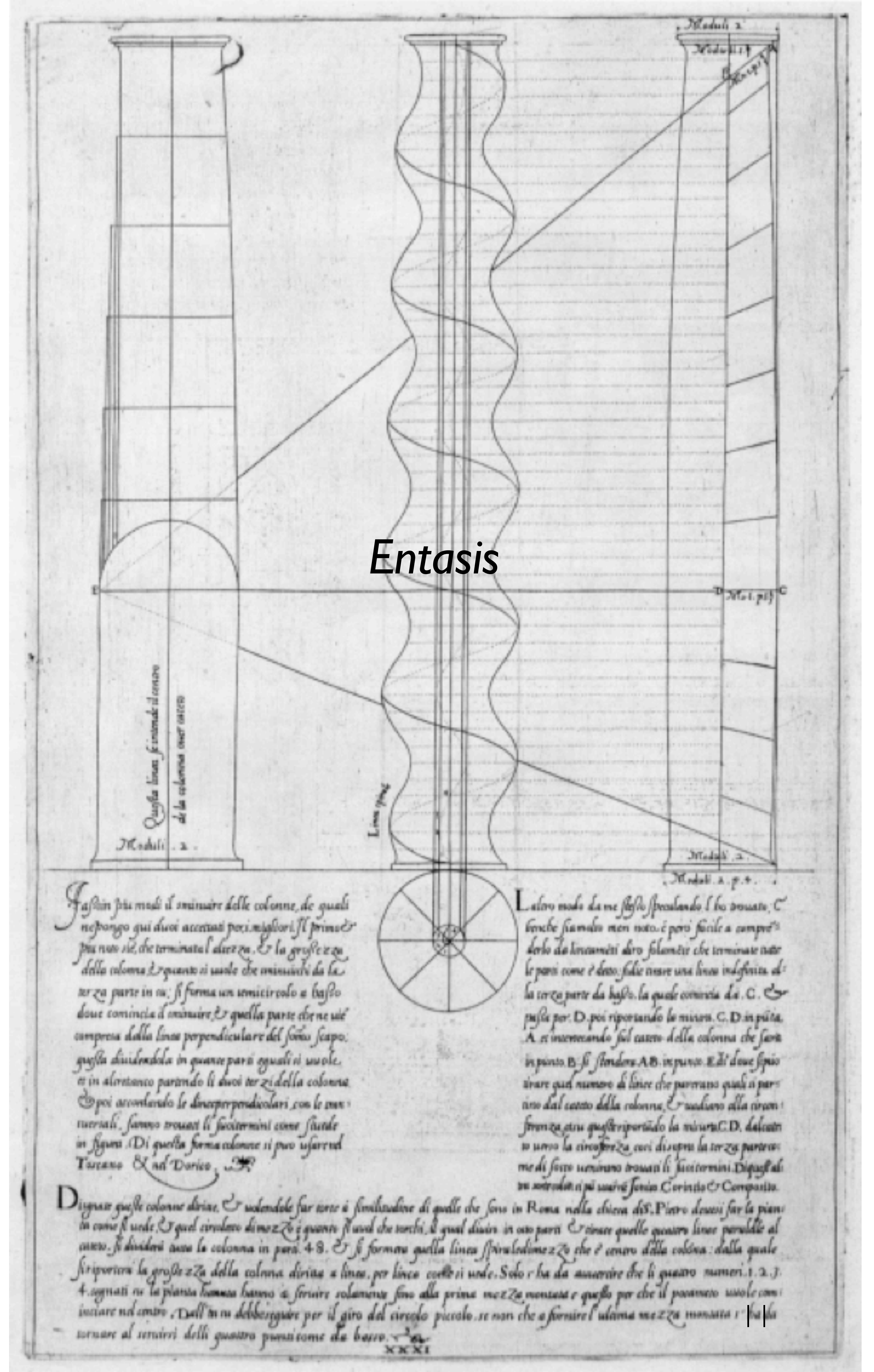


- development of a truncated cone

Canons of the Five Orders of Architecture

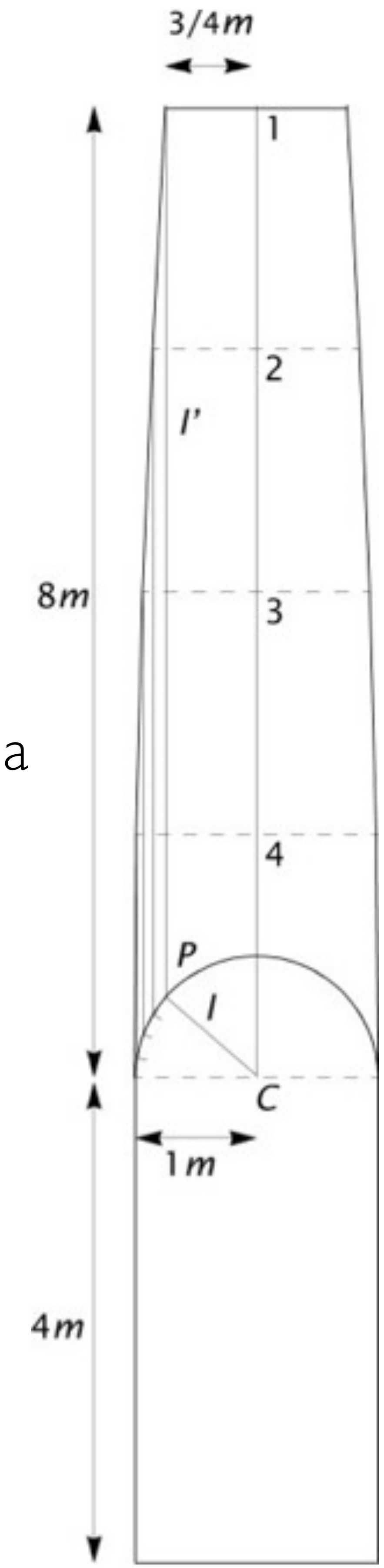
Giacomo Barozzi da Vignola

Canon of the Five Orders of Architecture



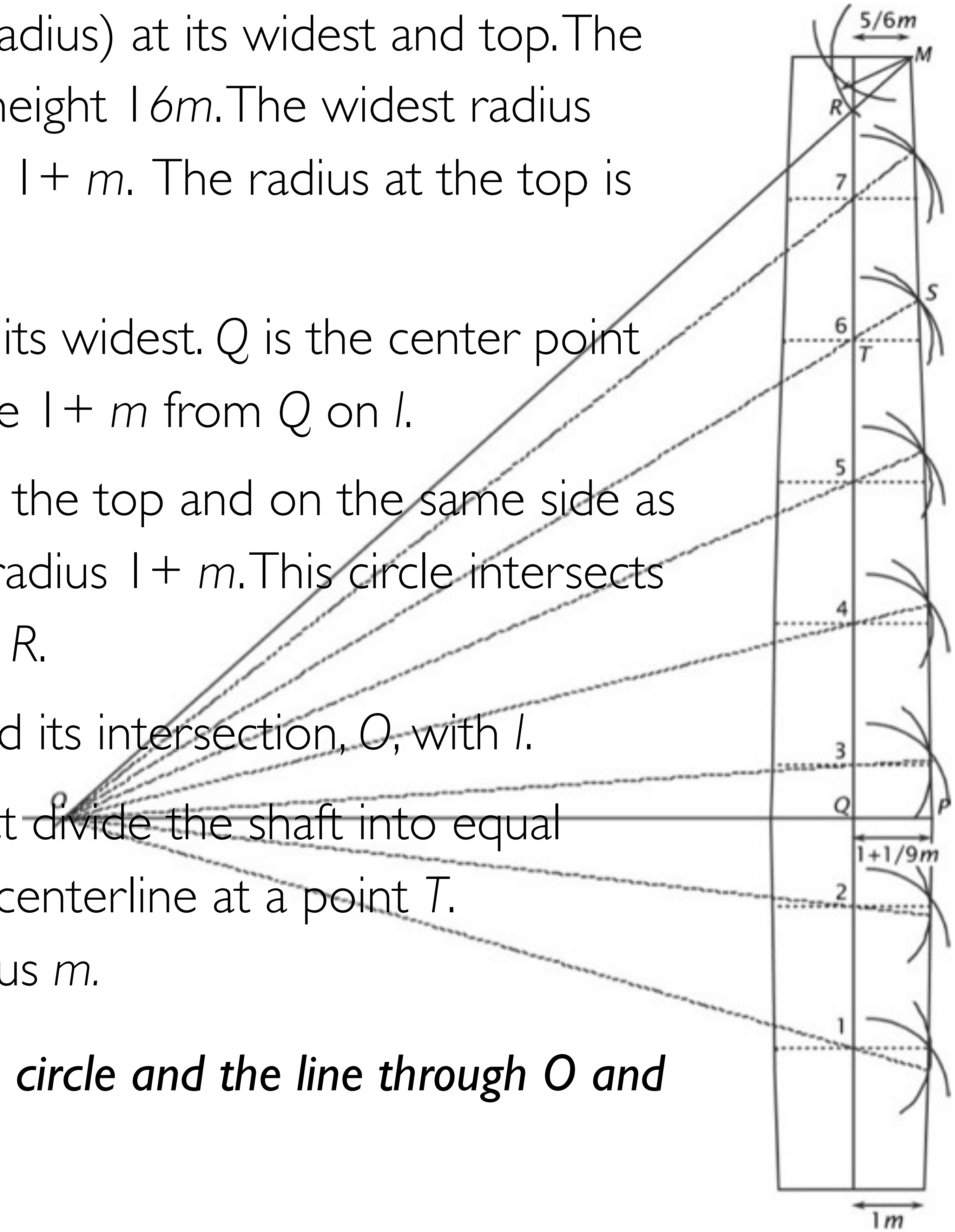
- the use of geometric tools

1. Determine height and largest diameter, *d*. These measures are normally integral multiples of a common module, *m*.
2. At $^1/_3$ the height, **draw a line**, *l*, across the shaft and draw a semi- circle, c, about the center point of *l*, *C*, with radius *d* (1*m*). The shaft has uniform diameter *d* below line *l*.
3. Determine smallest diameter at the top of the shaft (1.5*m* in our case). **Draw a perpendicular**, *l'*, through an end-point of the diameter. *l'* intersects *c* at a point *P*. The line through *P* and *C* defines together with *l* a segment of *c*.
4. **Divide the segment into segments of equal size** and divide the shaft above *l* into the same number of sections of equal height.
5. Each of these segments intersects *c* at a point. **Draw a perpendicular** line through each of these points and find the intersection point with the corresponding shaft division as shown. *Each intersection point is a point of the profile*.



- profile of a classical tapered column

1. Determine height and diameter (or radius) at its widest and top. The base is assumed to be 2*m* wide, the height 16*m*. The widest radius occurs at rd of the total height and is 1+ *m*. The radius at the top is *m*.
2. **Draw a line**, *l*, through the column at its widest. *Q* is the center point of the column on *l* and *P* is at distance 1+ *m* from *Q* on *l*.
3. *M is* at distance *m* from the center at the top and on the same side as *P*. **Draw a circle** centered at *M* with radius 1+ *m*. This circle intersects the centerline of the column at point *R*.
4. **Draw a line** through *M* and *R* and find its intersection, *O*, with *l*.
5. **Draw a series of horizontal lines** that divide the shaft into equal sections. Any such line intersects the centerline at a point *T*. **Draw a circle** about each *T* with radius *m.*

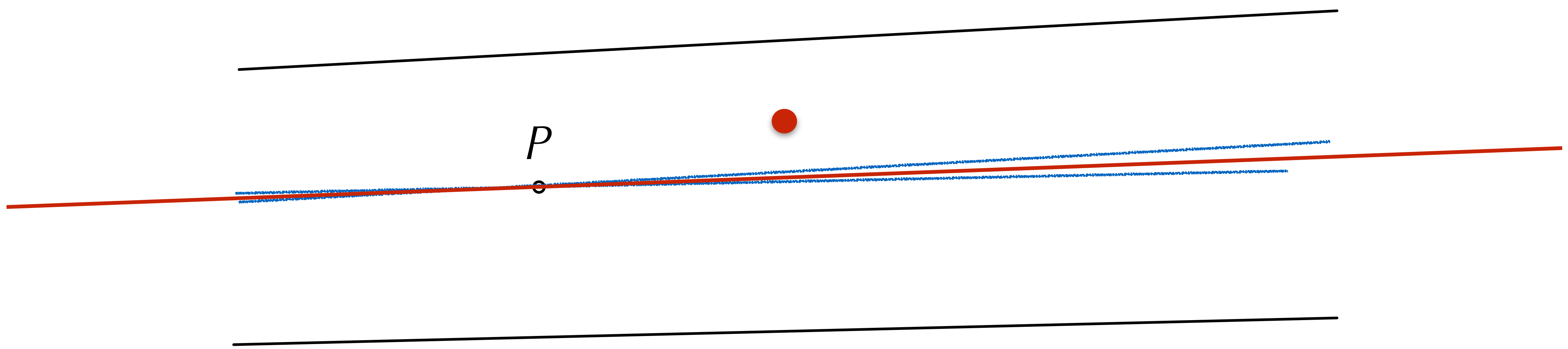


The point of intersection, S, between this circle and the line through O and T is a point on the profile.

- profile of a classical column with *entasis*

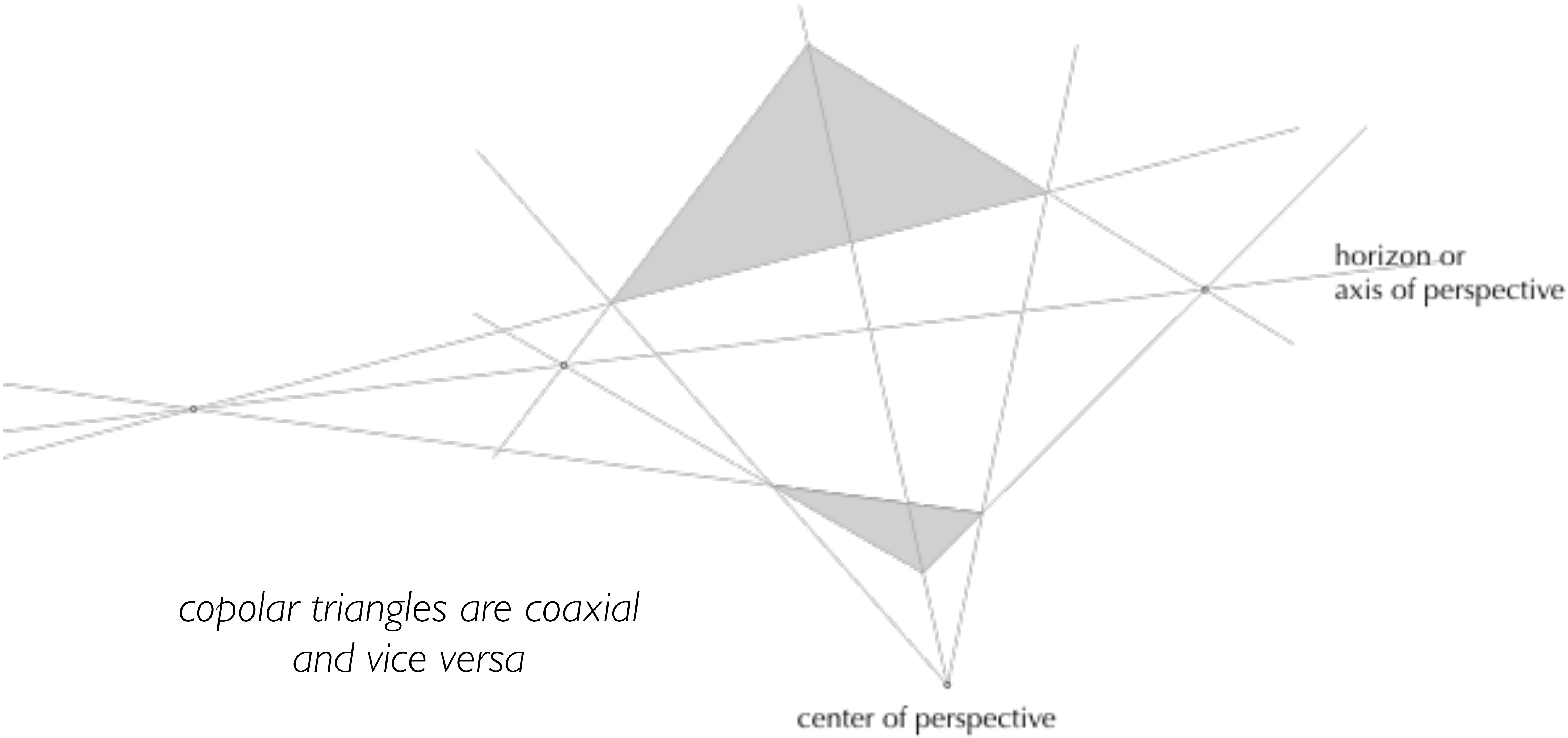
Another typical problem

draw a line through P that meets the intersection of the two lines?



a typical problem

hint



copolar triangles are coaxial and vice versa

Desargues configuration

Computation and Representations

Architecture

but also

Digital Fabrication

Engineering

Product design

CAM

Prototyping

Robot programming

Motion/sensor/… planning

Architecture

Mathematics

Engineering

Computer Science

areas where computation & representation is important

models are representations of physical artifacts, ideas, designs … of things in general

- *User interface*
- ***Geometrical** and algorithmic **level***
- *Arithmetic substratum*

models are representations of physical artifacts, ideas, designs …

of things in general

an **artificially constructed object** that makes the observation of another object easier

Levels of modeling

- World of physical objects
- **Geometrical** modeling space
- Representation space

a geometrical model is an abstraction
— an idealization of real 3D physical objects

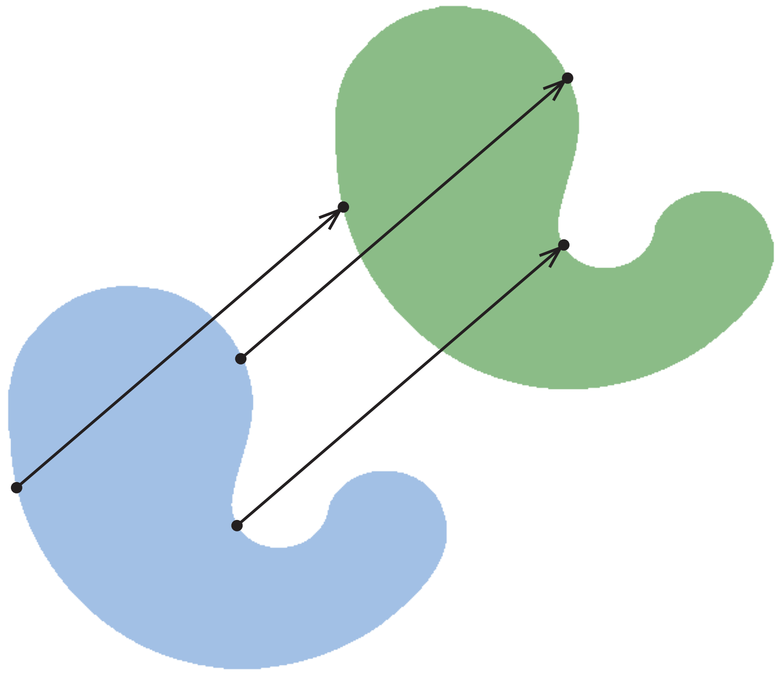
models are **representations** of physical artifacts, ideas, designs …
of things in general

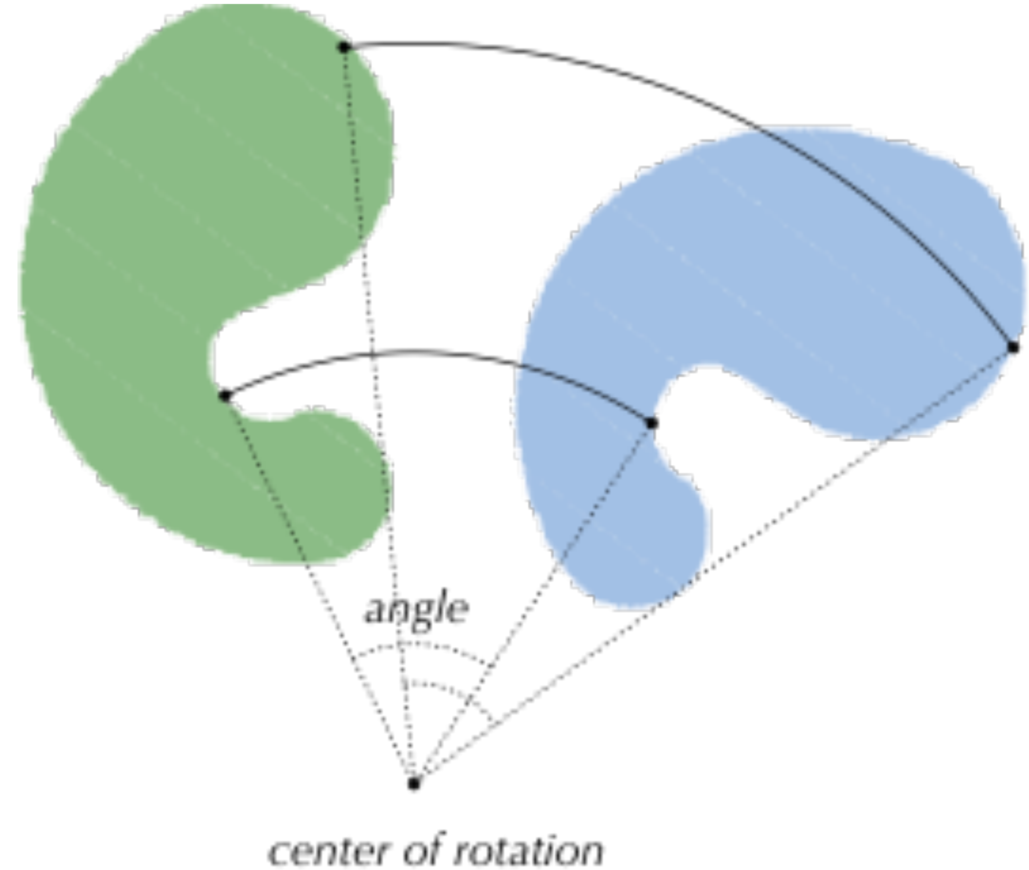
configurations of elements bearing relationships to one another to give an overall sense of structure

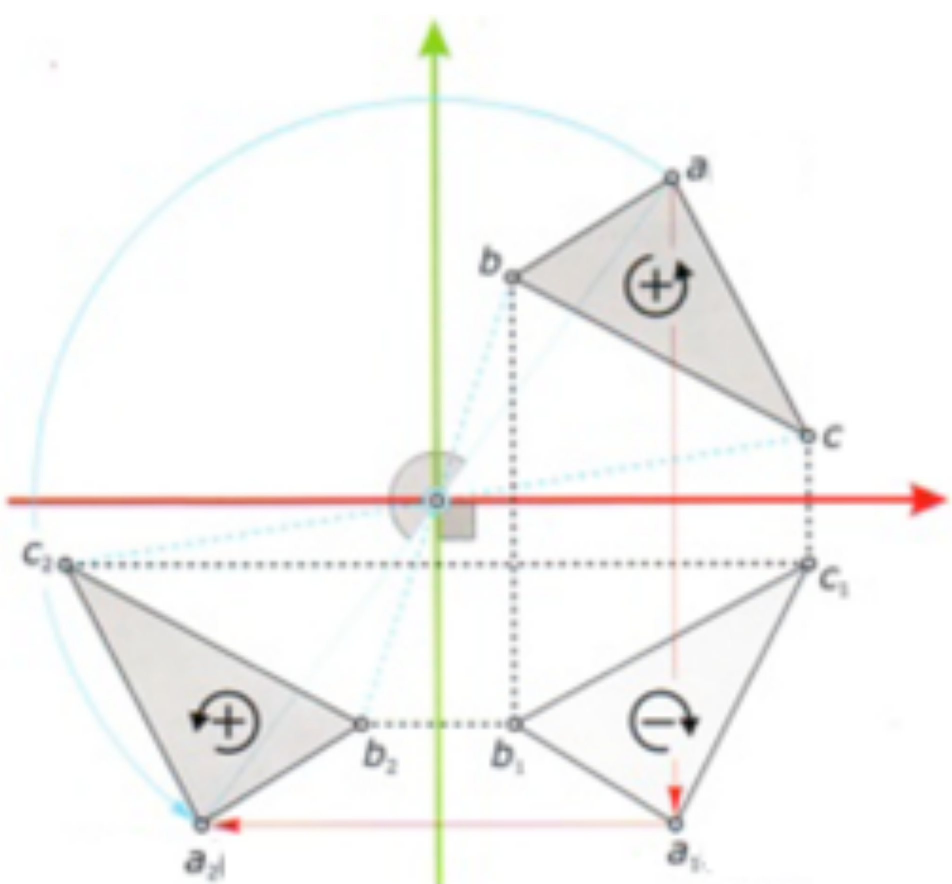
- **Elements**
- **Relationships between elements**
- **Structure**

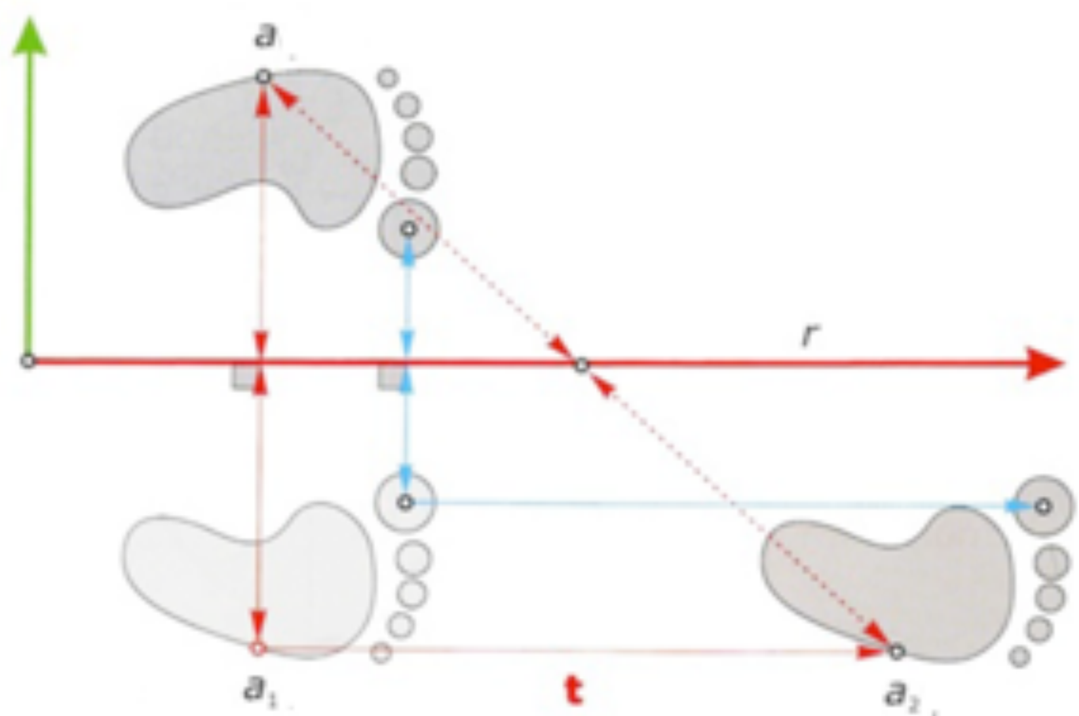
models and **representations**

http://www.designboom.com/architecture/ik-studio-conics-canopy/

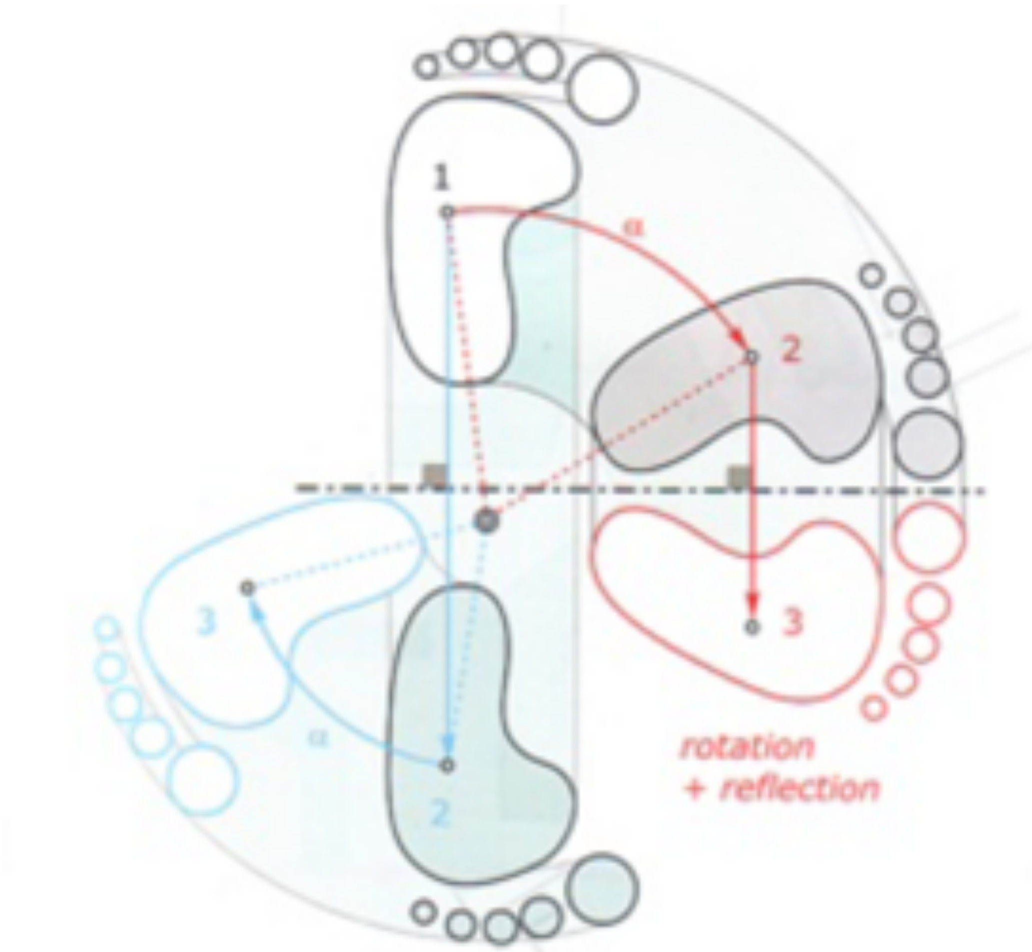








geometric transformations



Hint: all you need are mirrors!

rotating an object without using a compass

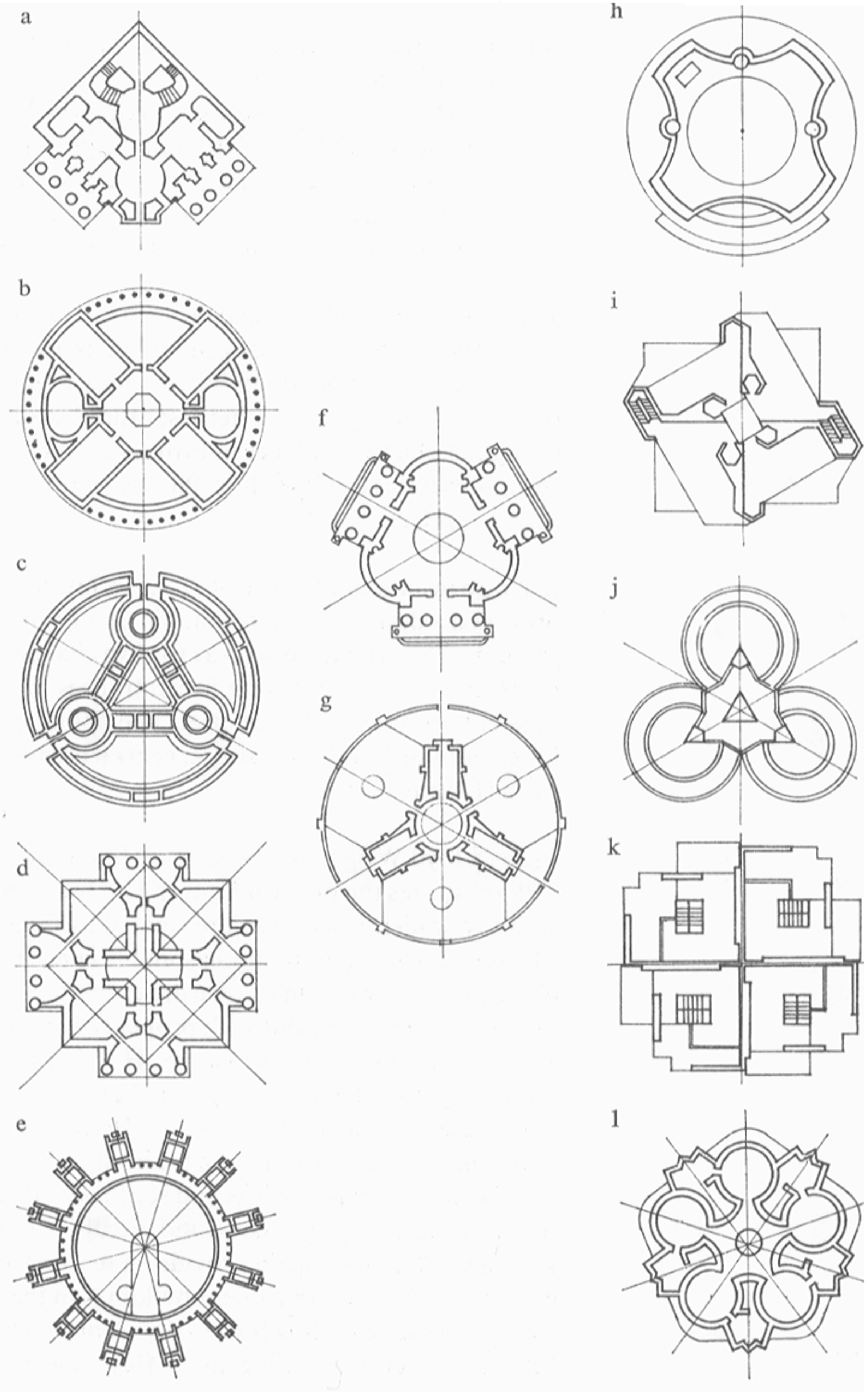


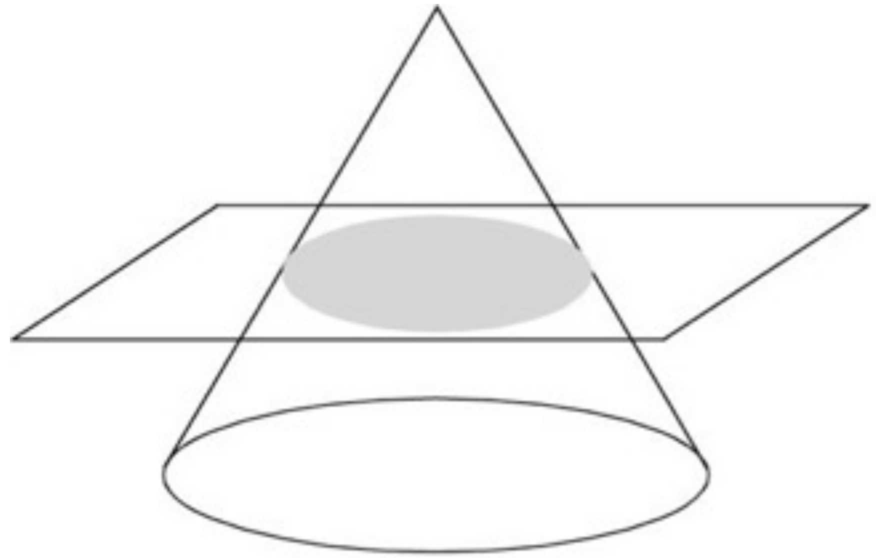
Figure 3.2
Examples of point groups in architectural plans:
a, Montmorency Palace, D_1
b, De Witt House, D_2
c, Inn St. Marceau, D_3
d, Barrière de Picpus, D_4
e, House of Entertainment (circular colonnade and pavilions), D_{12}
all projects and buildings by Claude-Nicolas Ledoux:

f, Sepulchral Church, D_3
g, Kennels, D_3
both designs by Sir John Soane:

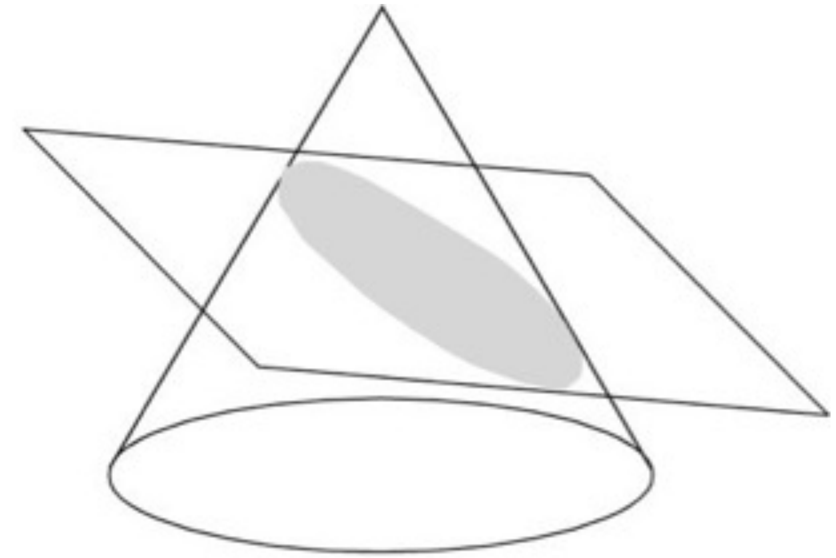
and projects and buildings by Frank Lloyd Wright
h, Greek Orthodox Temple, C_1
i, St Mark's Apartment Tower, C_2
j, Huntingdon Hartford Clubhouse, D_3
k, Suntop Homes, C_4
l, Daphne Mortuary, D_5

symmetry

Conic Sections

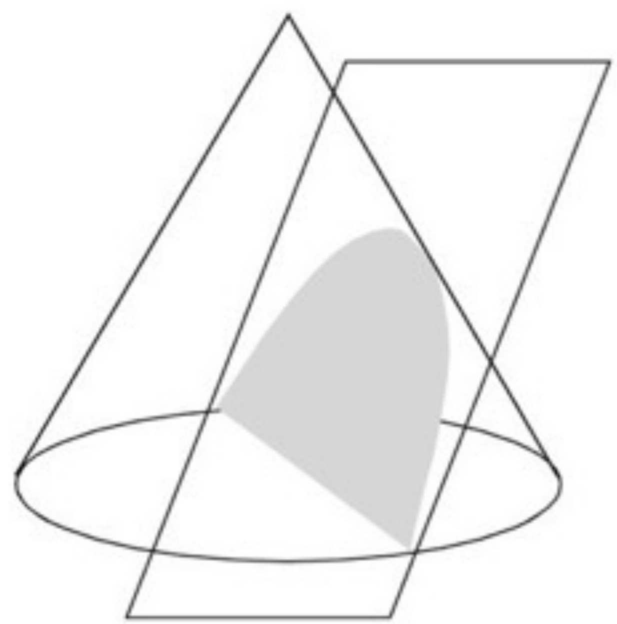


Circle

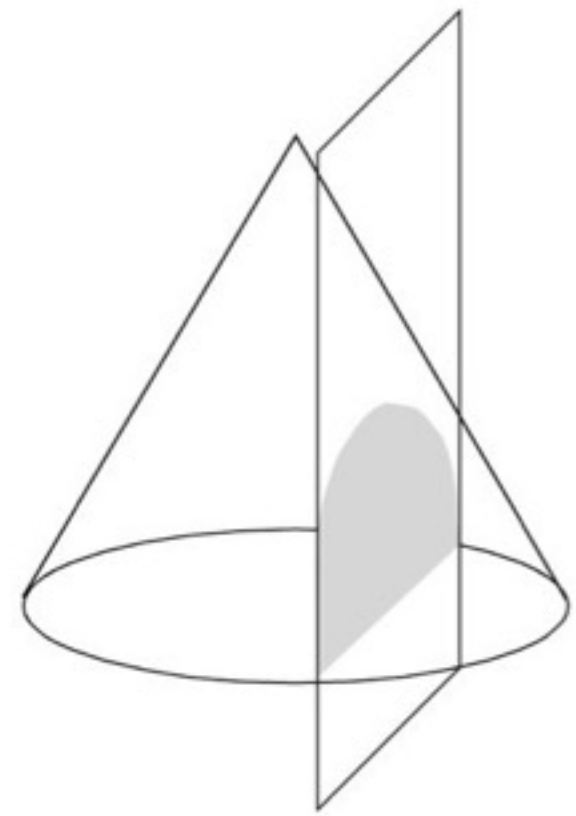


Ellipse

produced by slicing a cone by a cutting plane

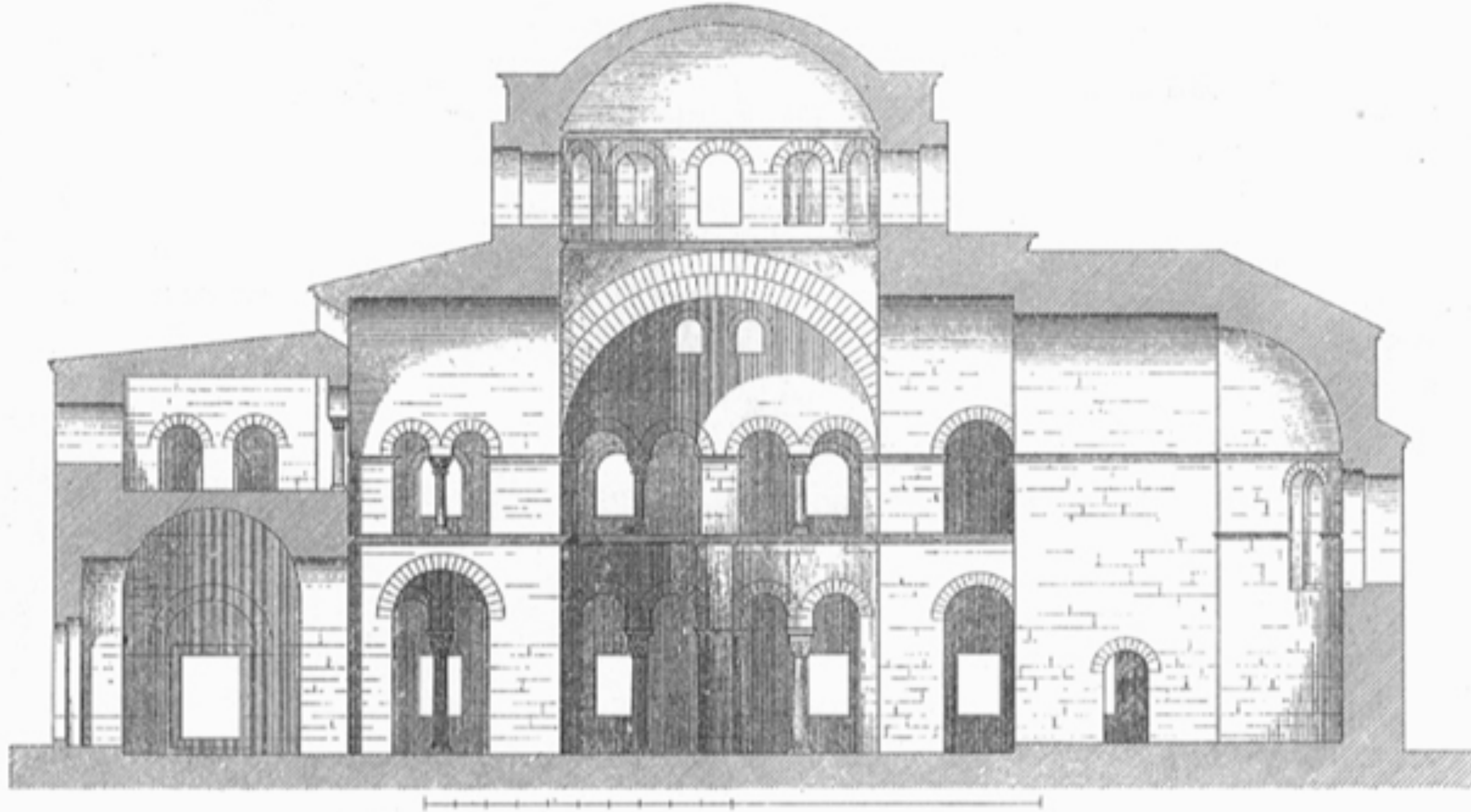


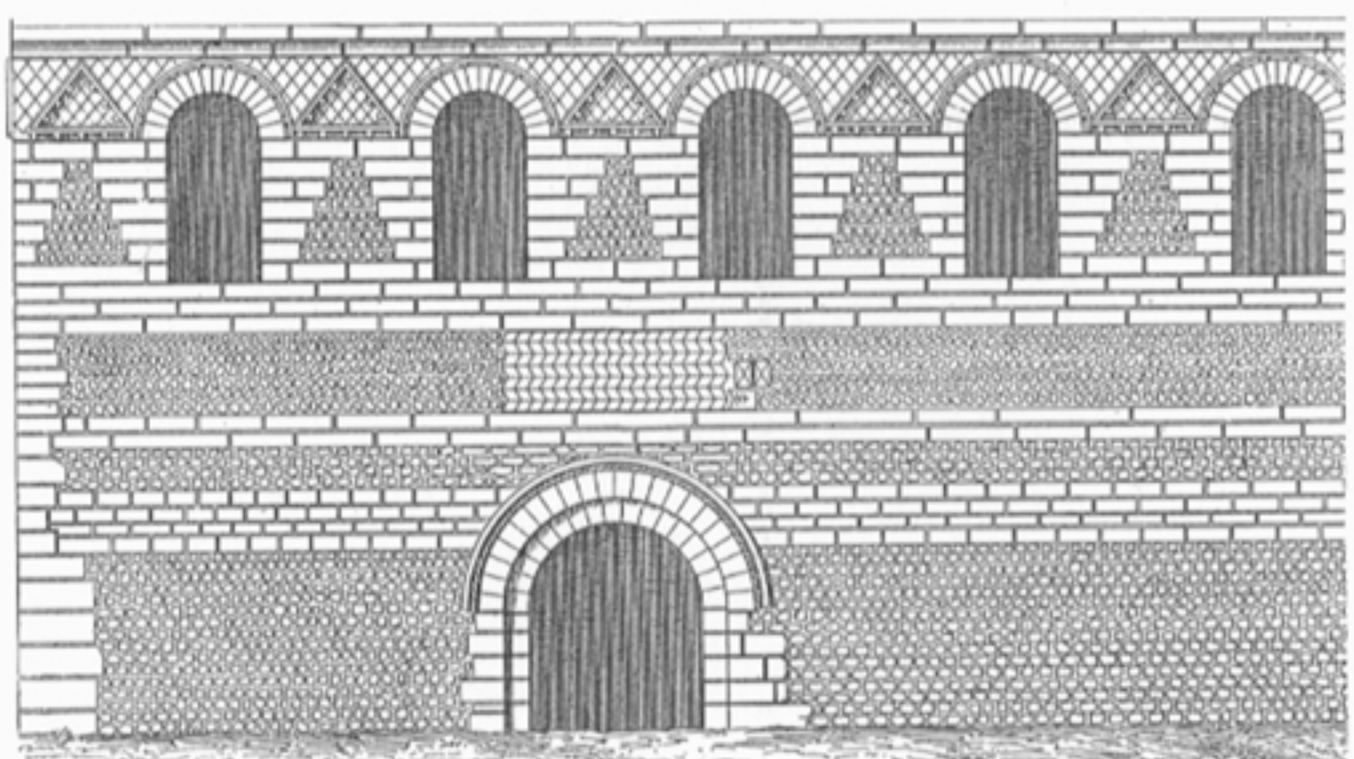
Parabola



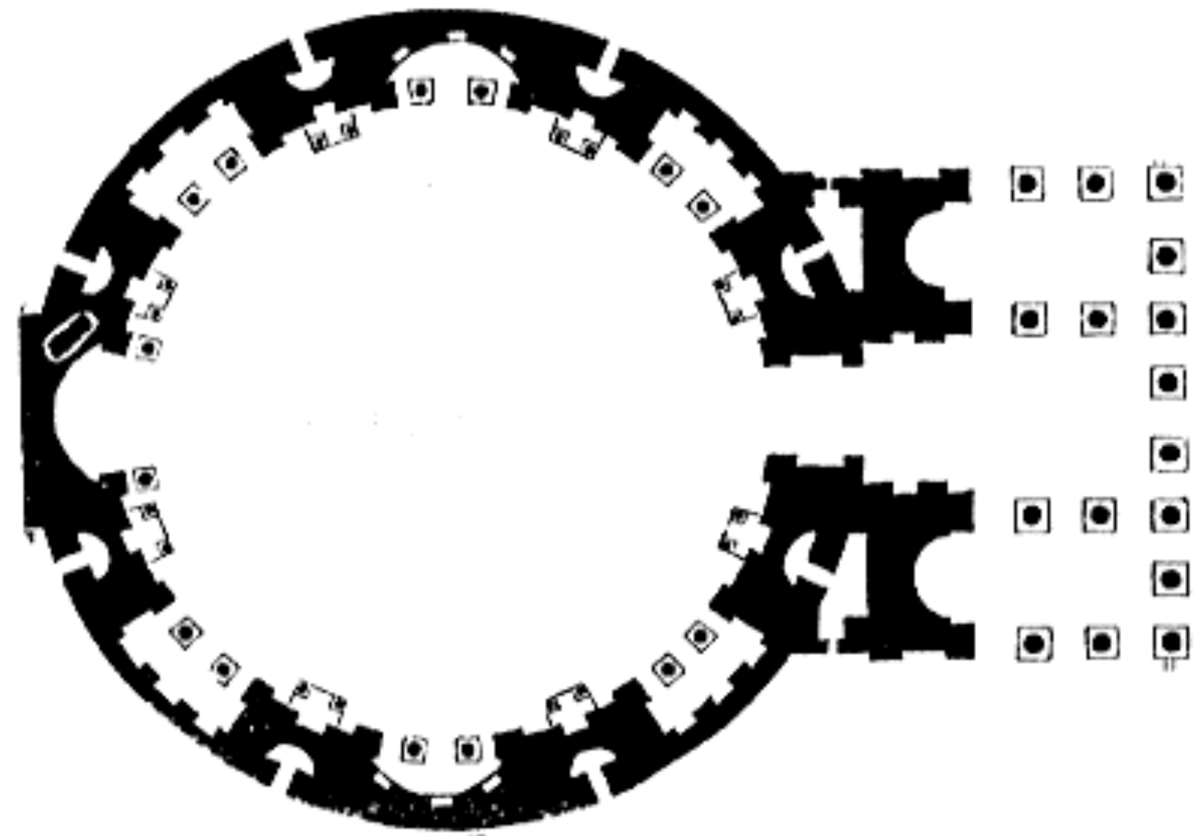
Hyperbola

conic sections

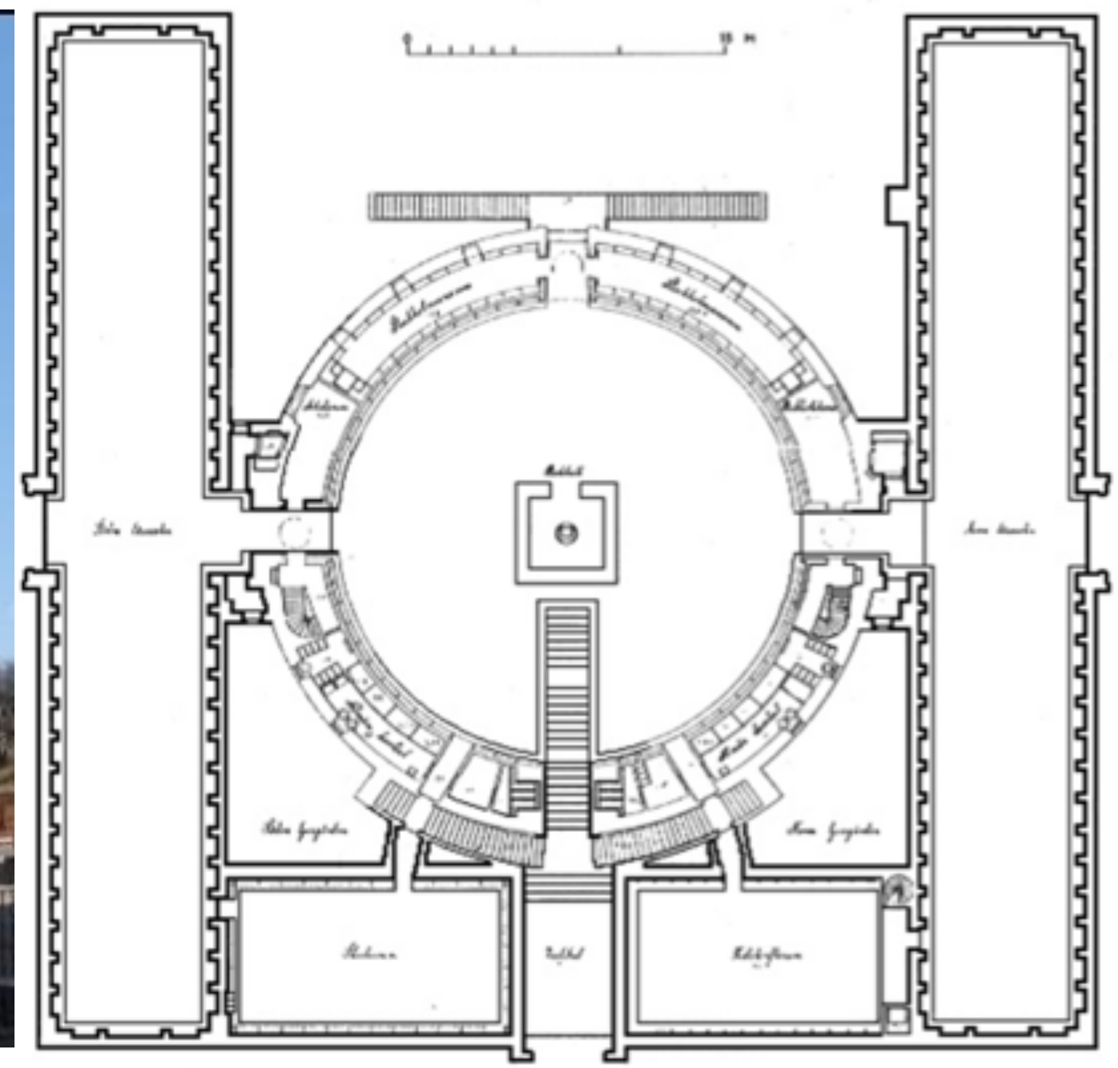




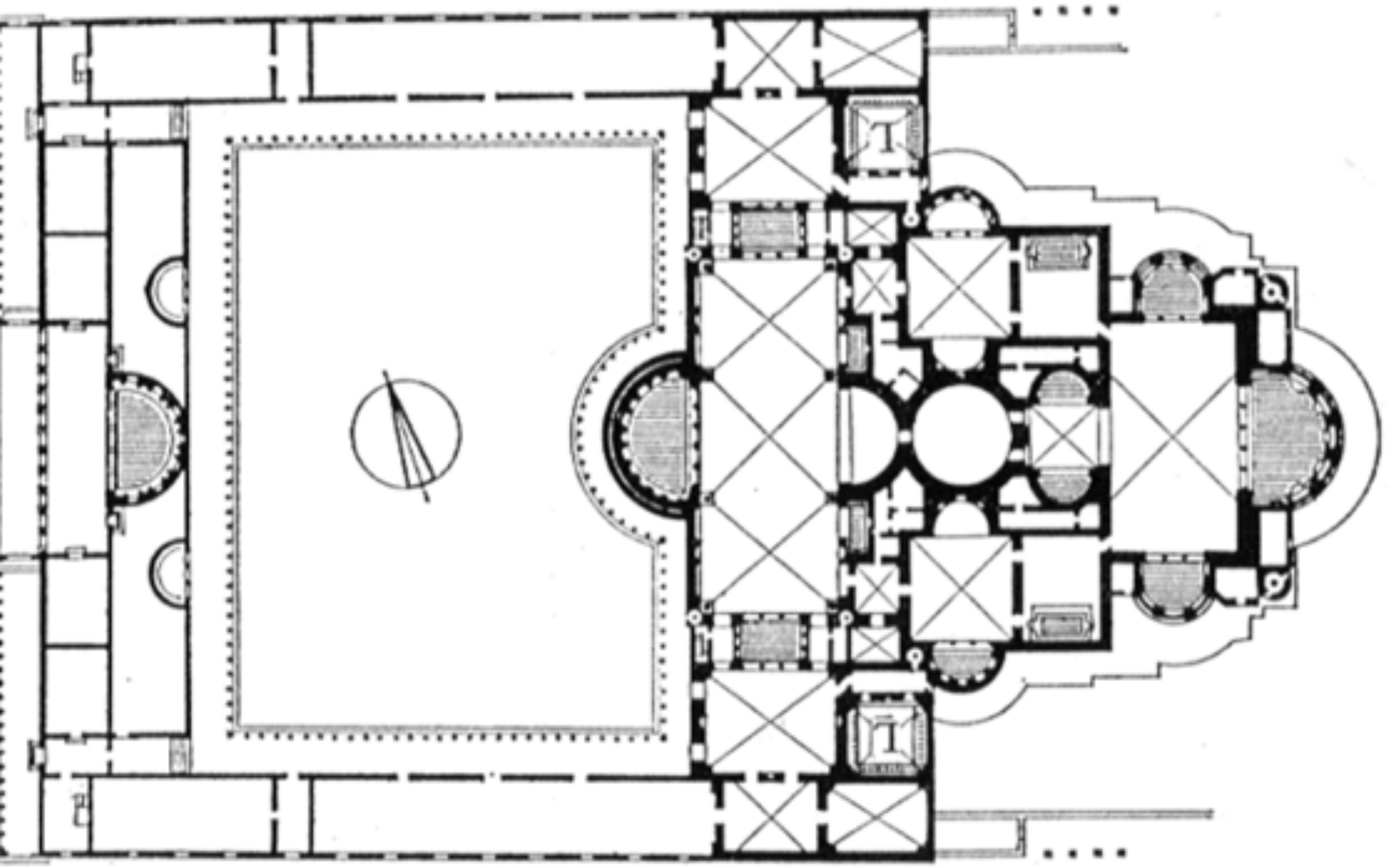
circle



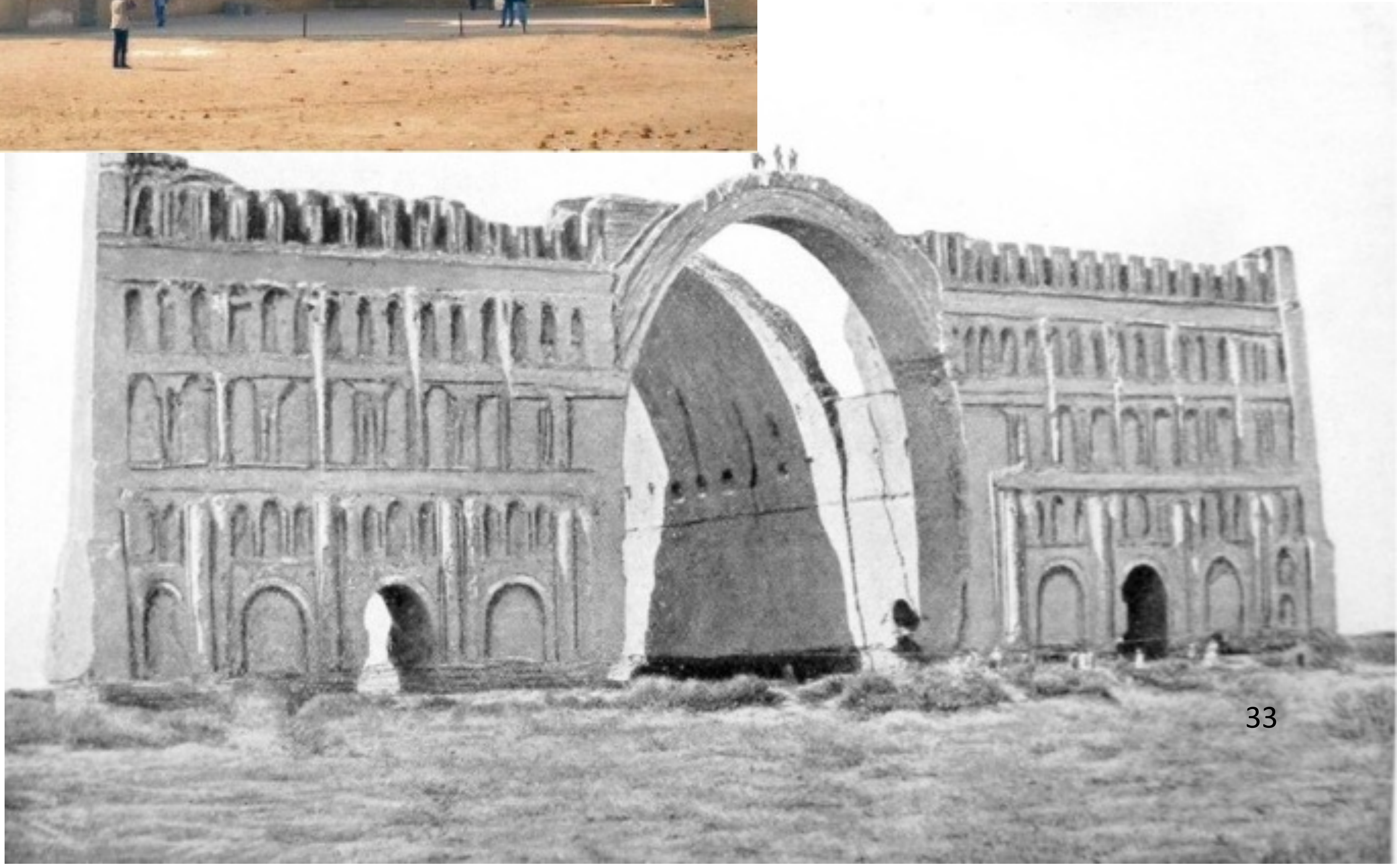
Pantheon



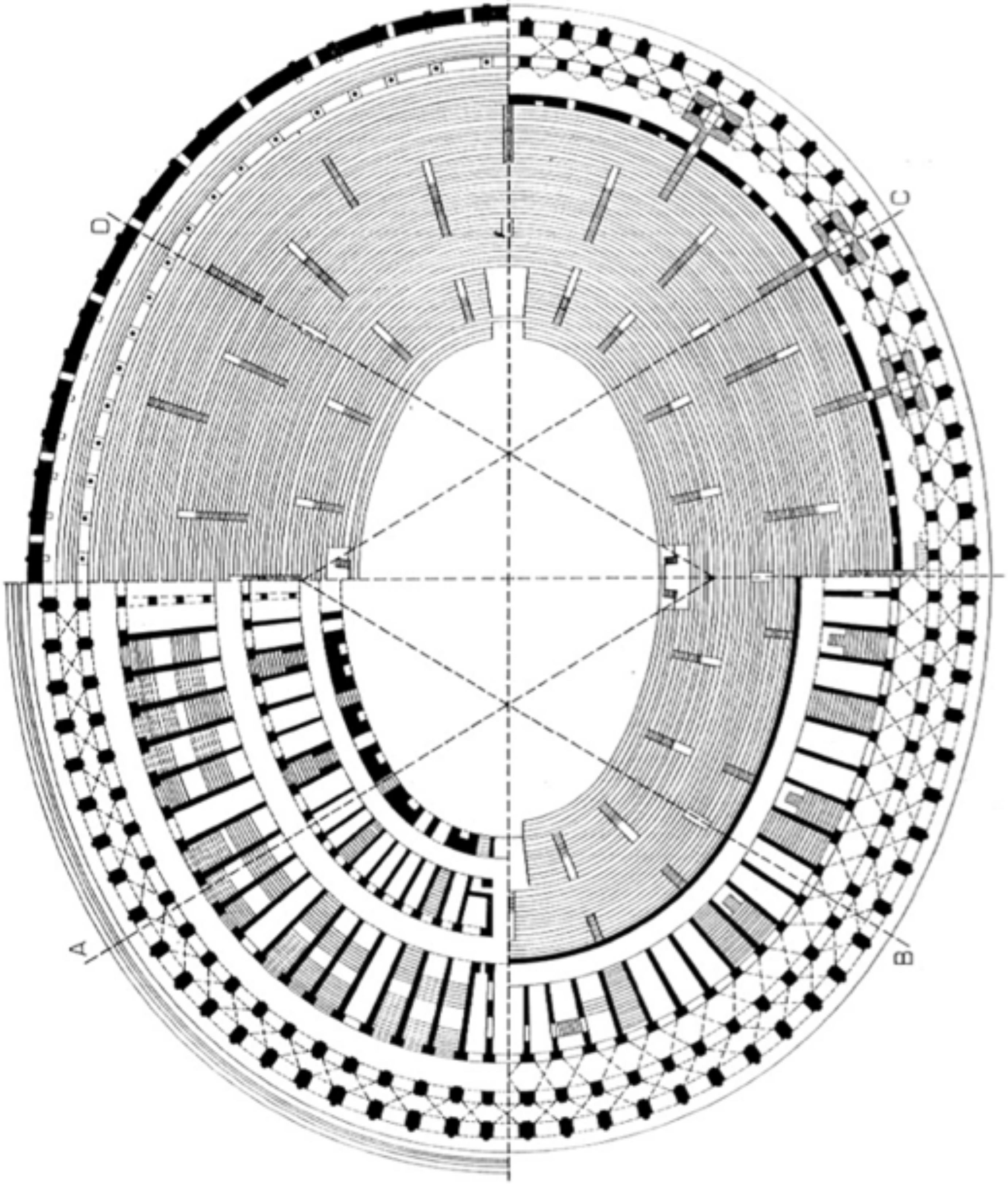
Stockholm Public Library

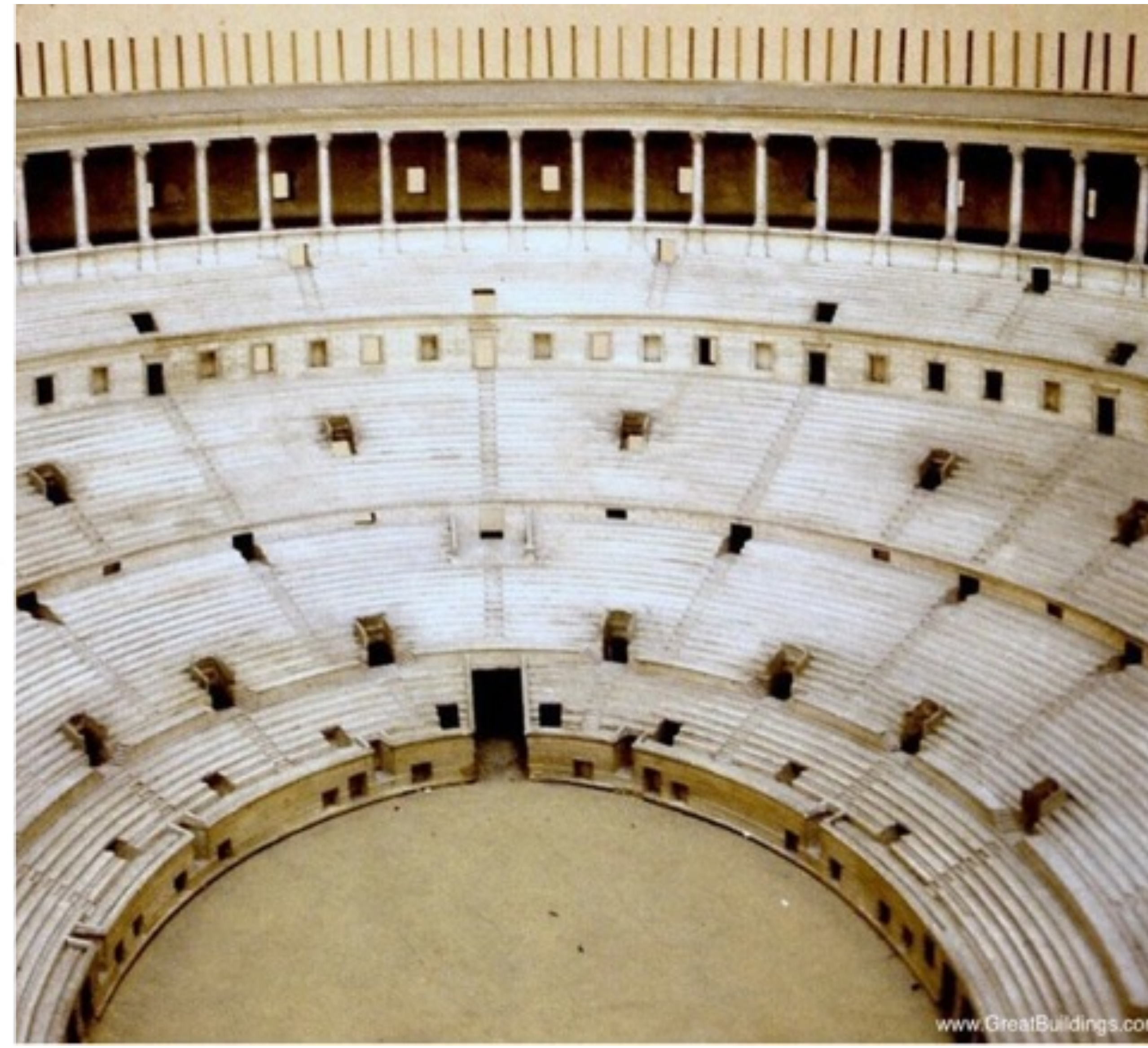


Imperial baths, Trier



Ctesiphon

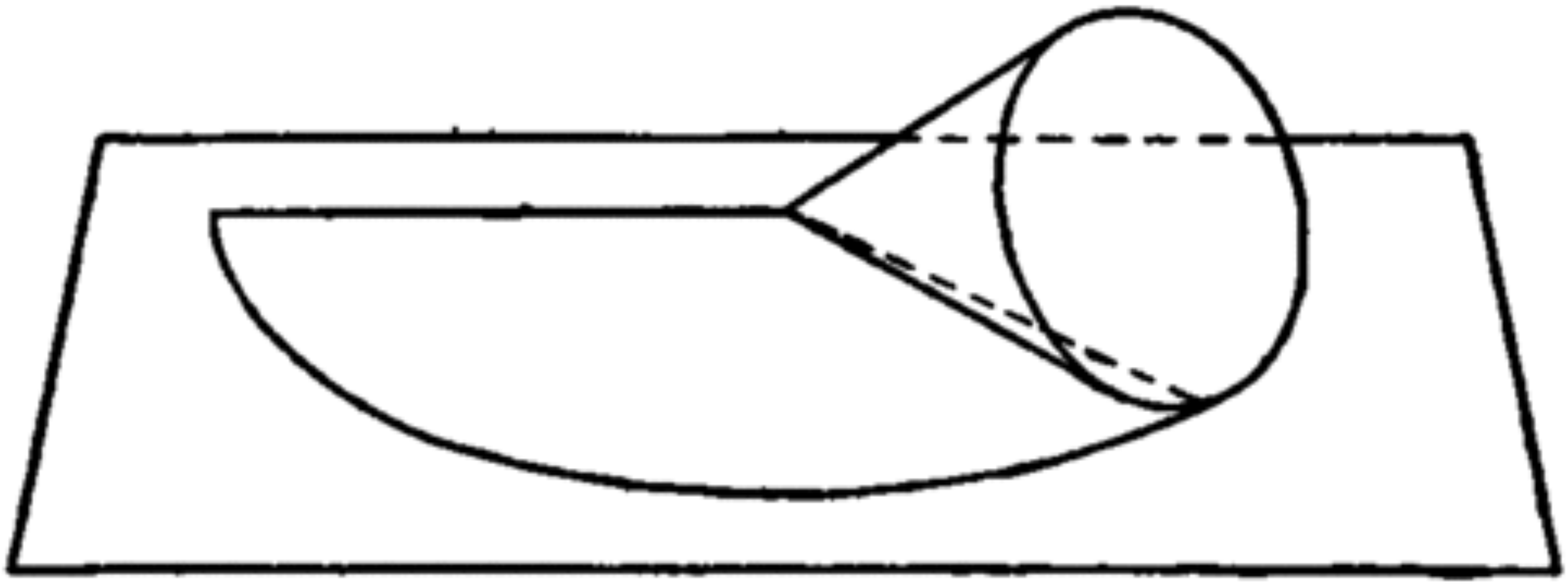




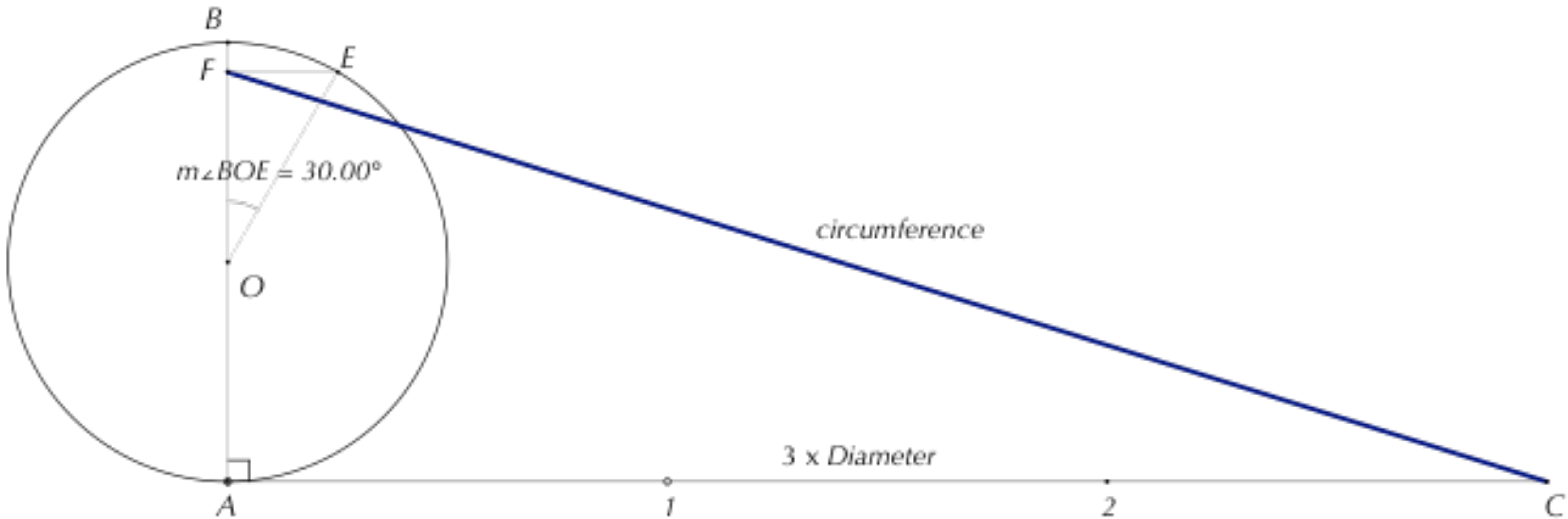
Colosseum

S.Vicente de Paul at Coyoacan

circle



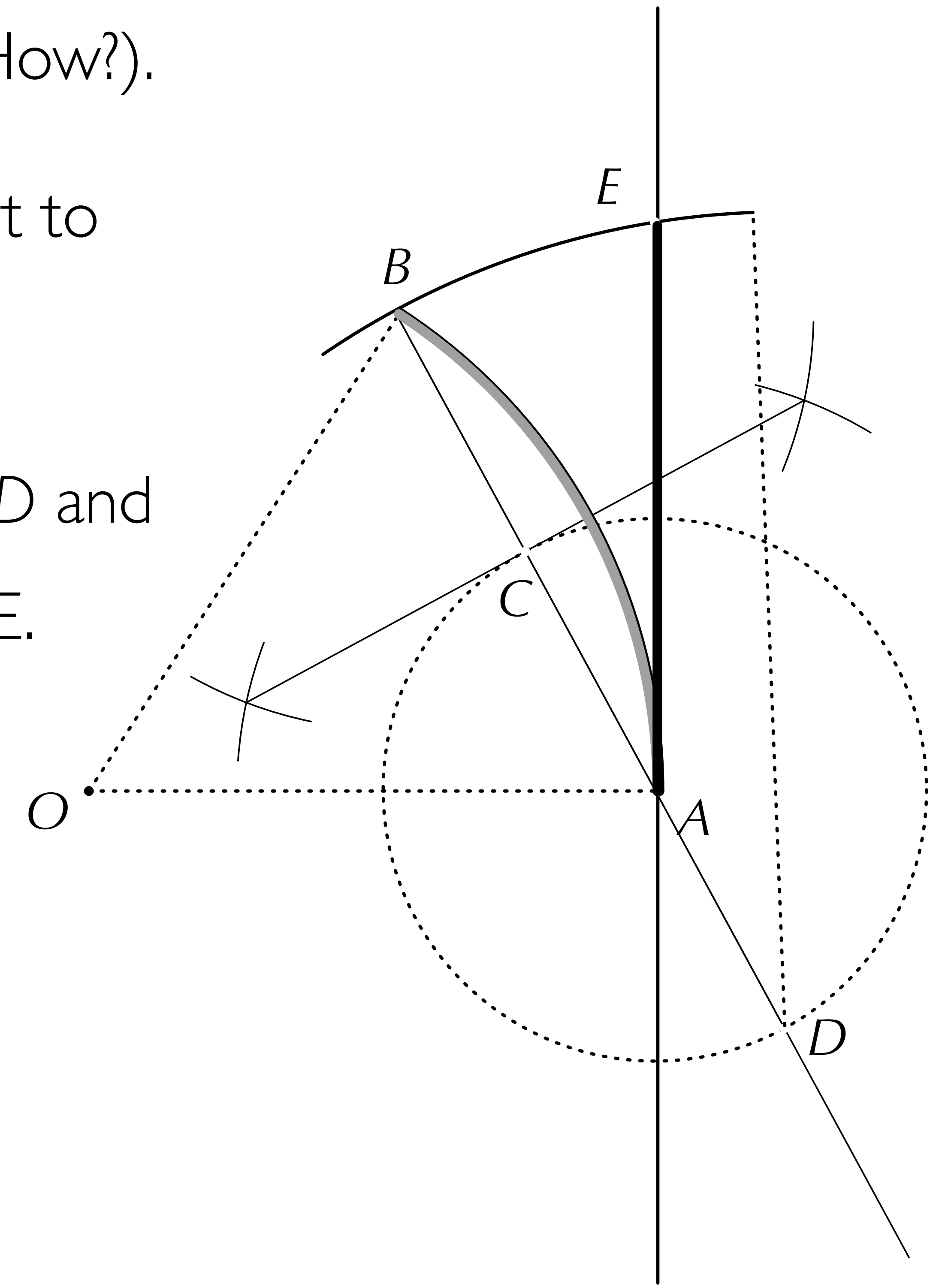
developing a cone



rectifying the circumference of a circle

rectification: approximate length of a circular arc

1. Draw a tangent to the arc at *A* (How?).
2. Join *A* and *B* by a line and extend it to produce *D* with $AD = \frac{1}{2}AB$.
3. Draw the circular arc with center *D* and radius *DB* to meet the tangent at *E*.



AE is the ***required length***

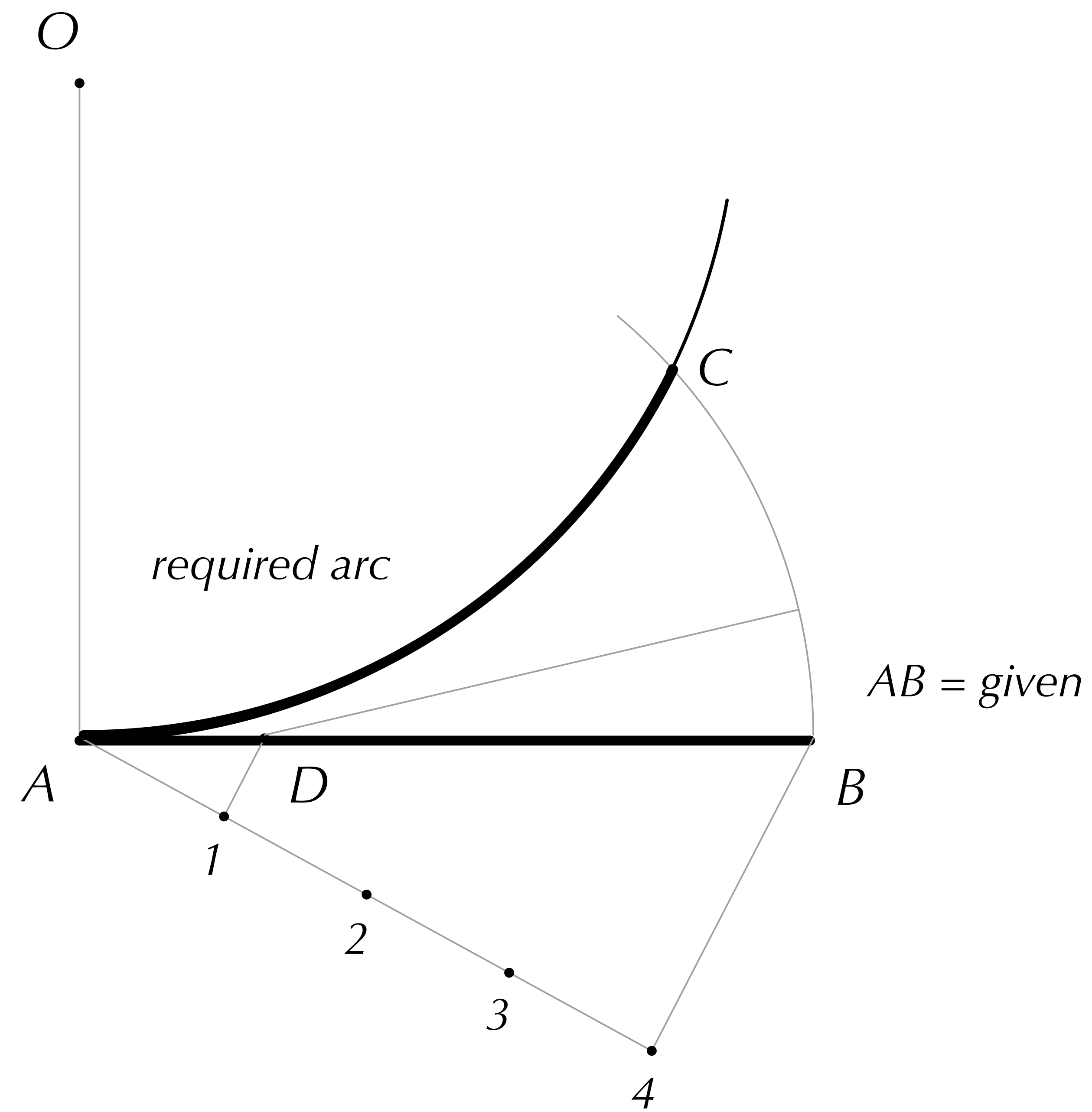
approximate circular arc of a given length

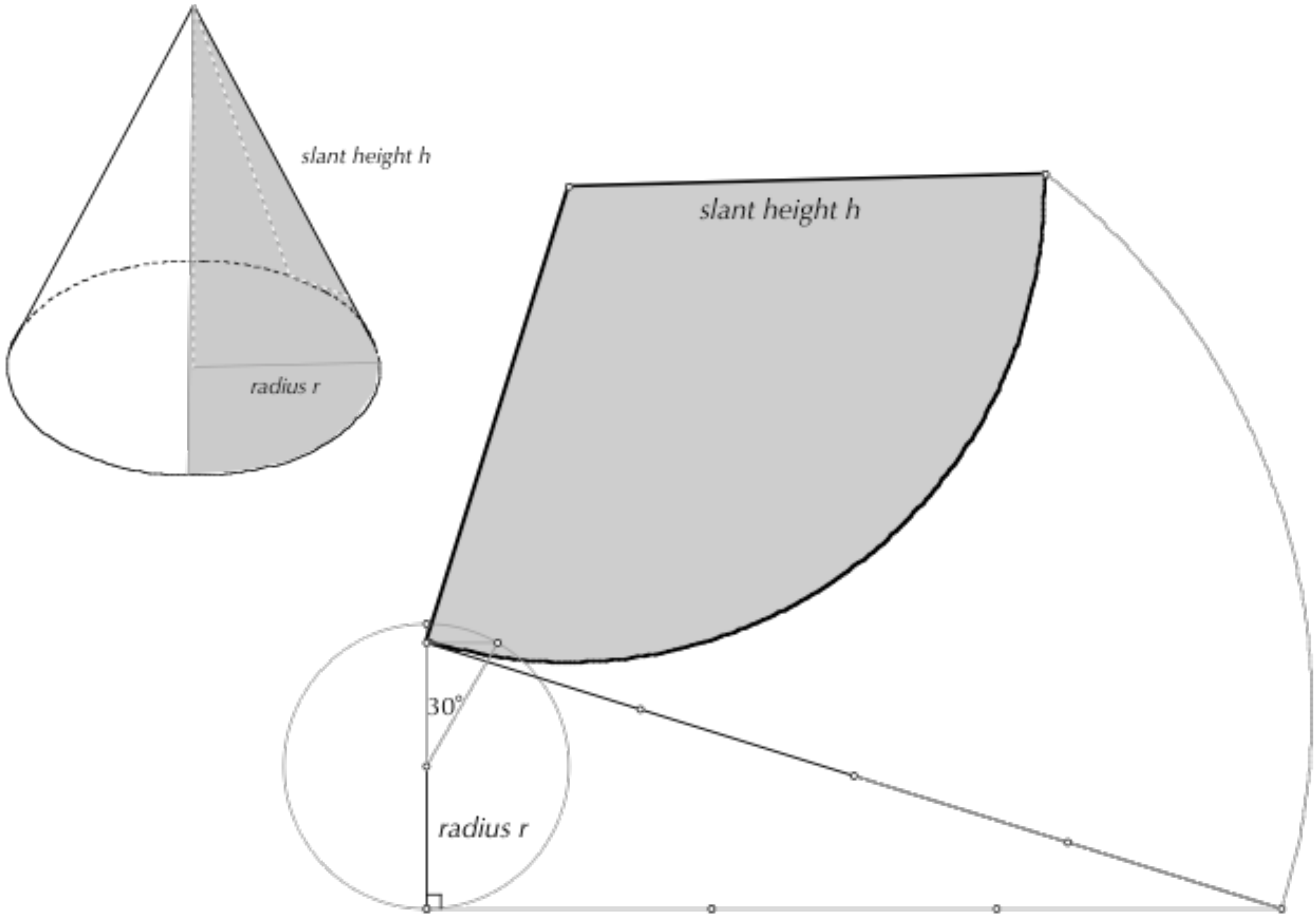
A be a point on the arc.

AB is the given length on the tangent at *A*.

1. Mark a point *D* on the tangent such that *AD*=¼*AB*.
2. Draw the circular arc with center *D* and radius *DB* to meet the original at *C*.

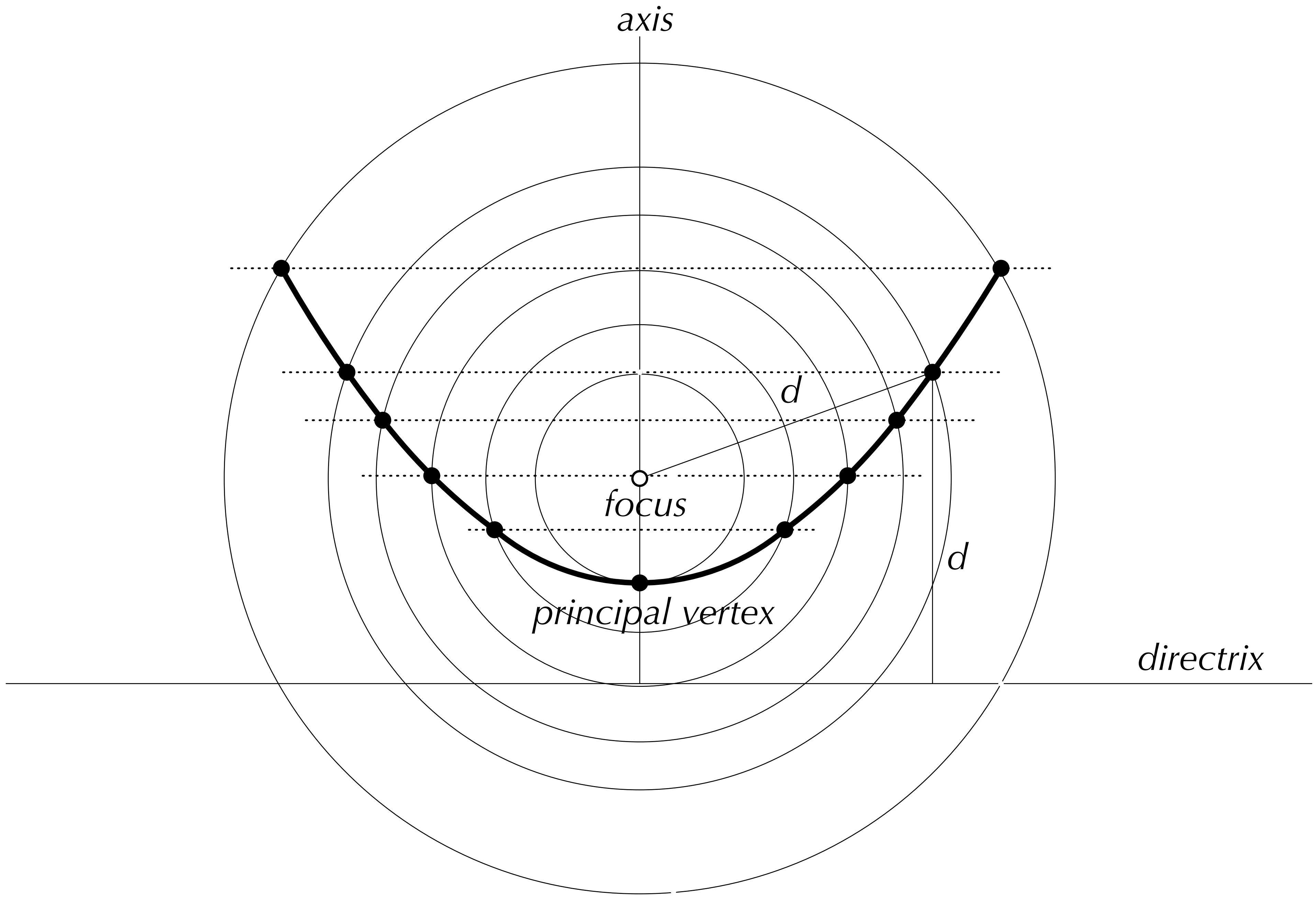
Arc *AC* is the ***required arc***



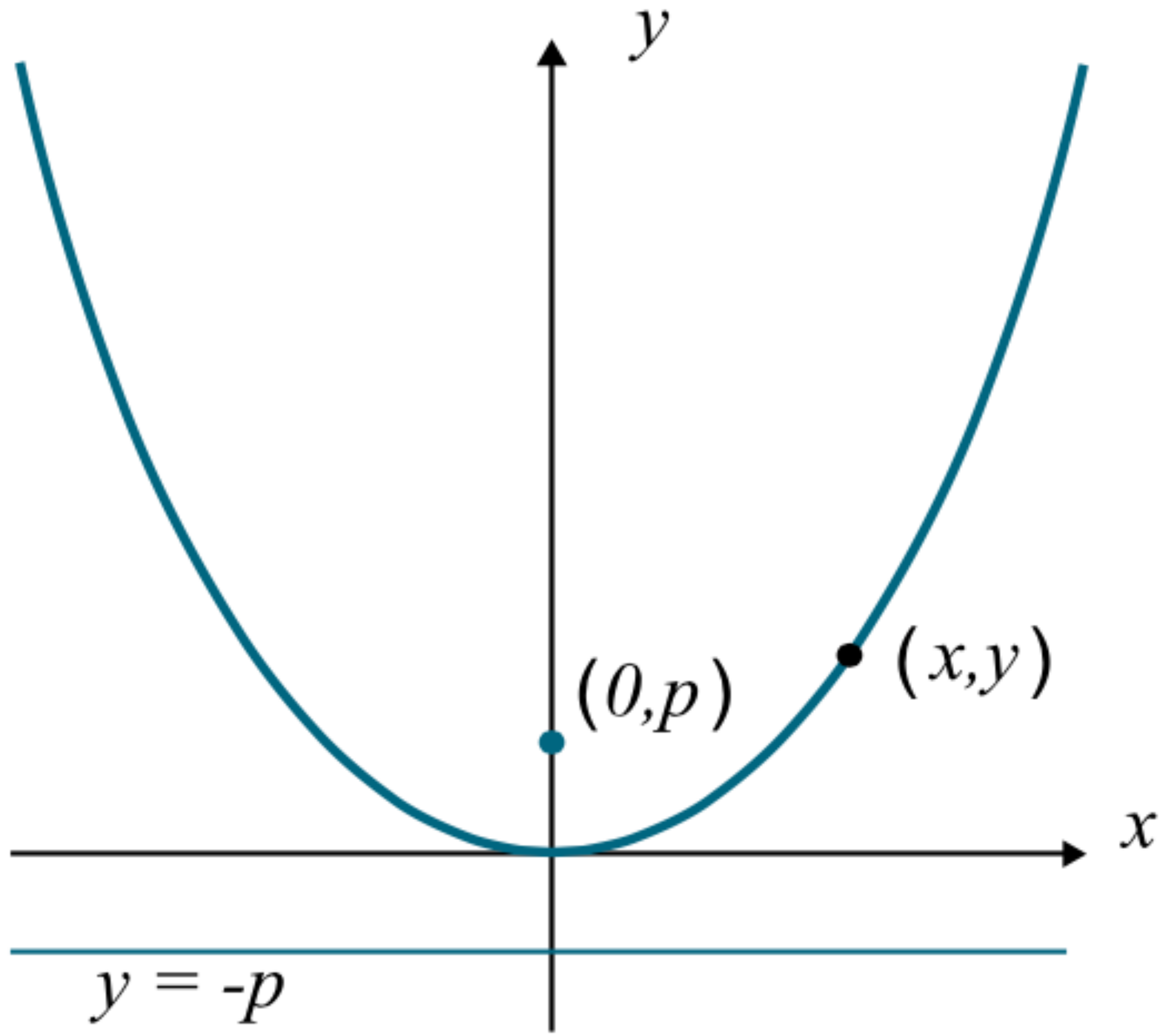


a practical application

parabola



parabola



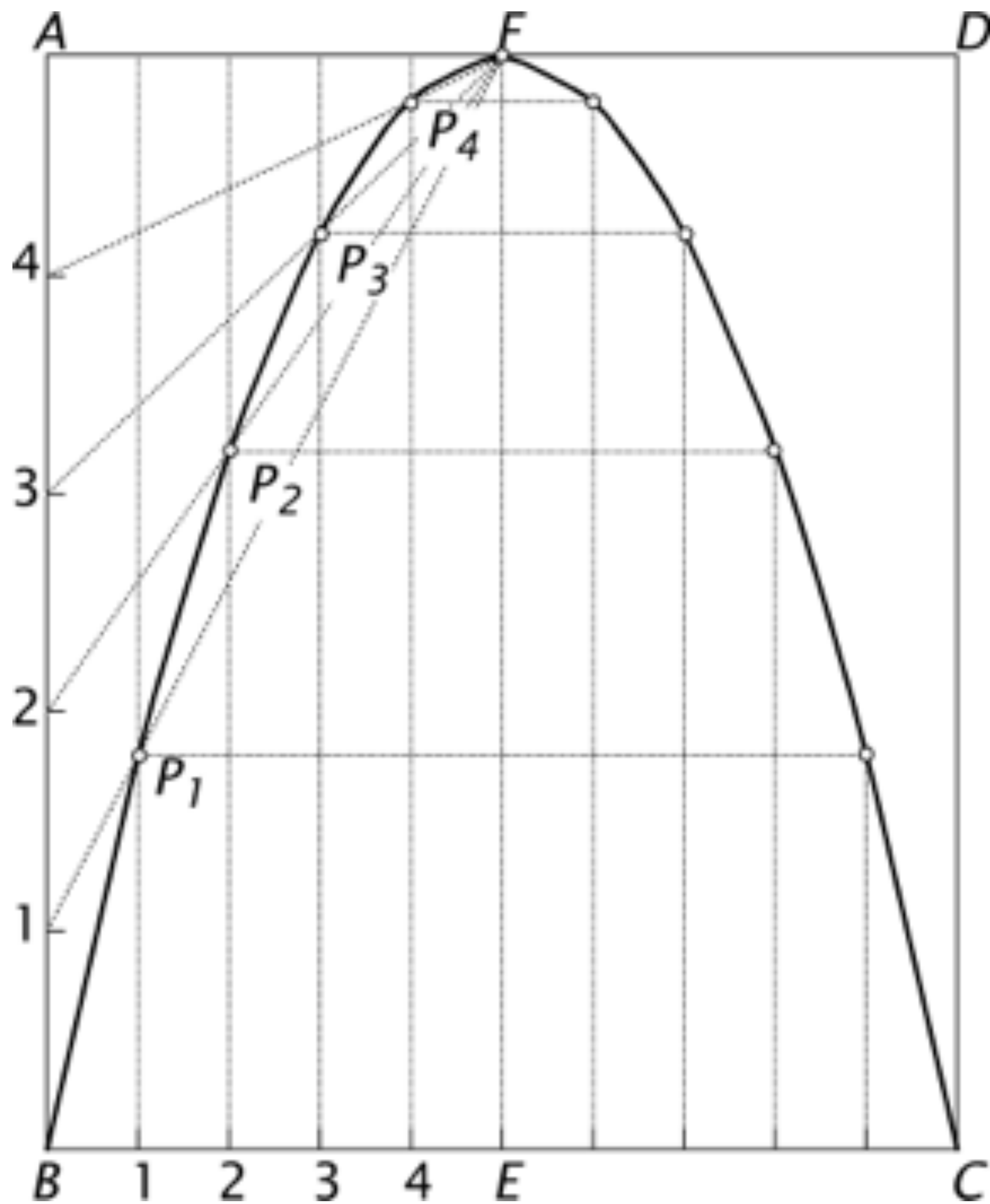
$$\sqrt{x^2 + (y-p)^2} = y + p$$

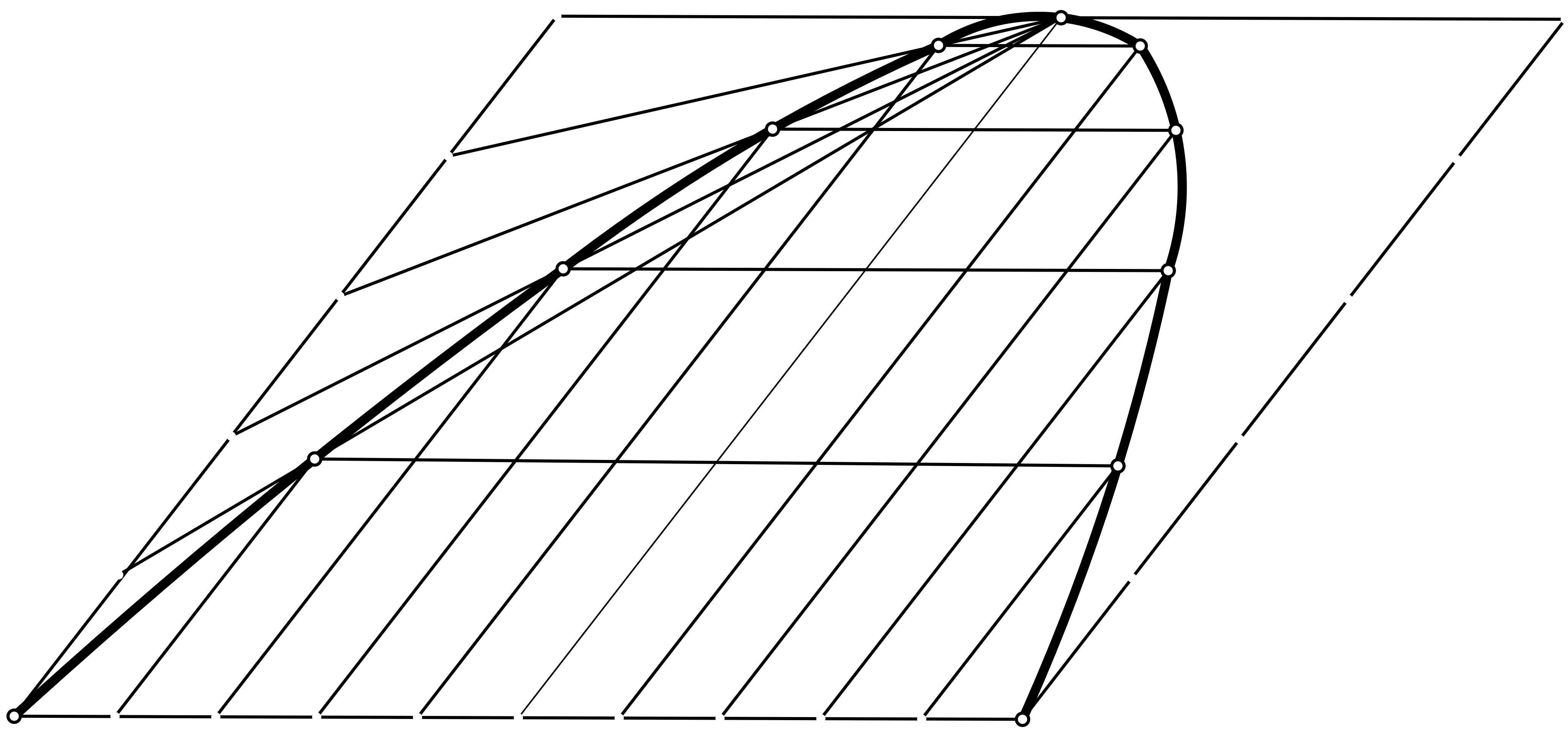
$$x^2 = 4py$$

analytic form

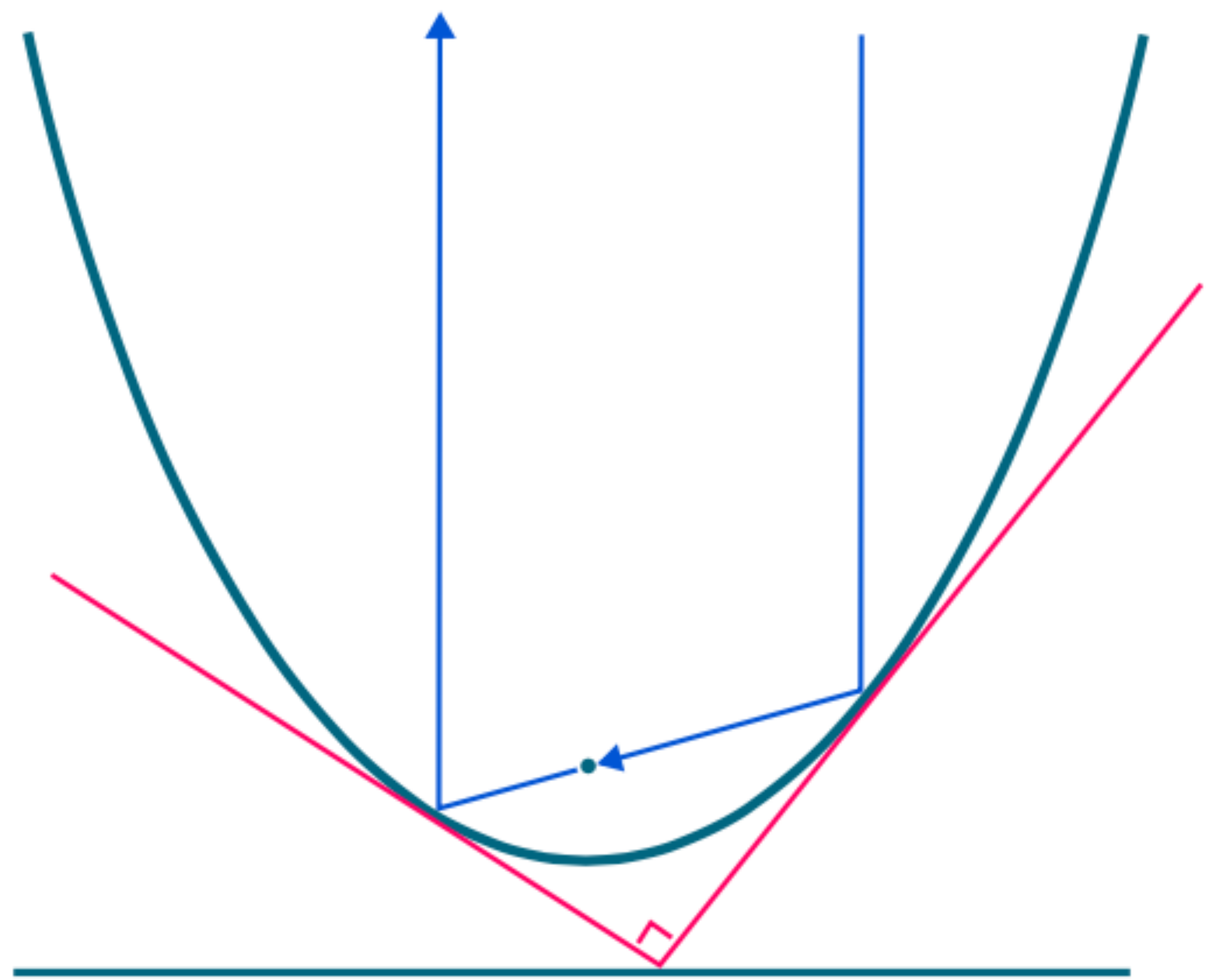
a parabola within a rectangle

- Bisect the sides and of the rectangle *ABCD* and join their midpoints, *E* and *F*, by a line segment.
- Divide segments and into the same number of equal parts, say $n = 5$, numbering them as shown.
- Join *F* to each of the numbered points on to intersect the lines parallel to through the numbered points on at points P_1, P_2, … P_{n-1} as shown.
- These points lie on the required parabola.





constructing an oblique parabola

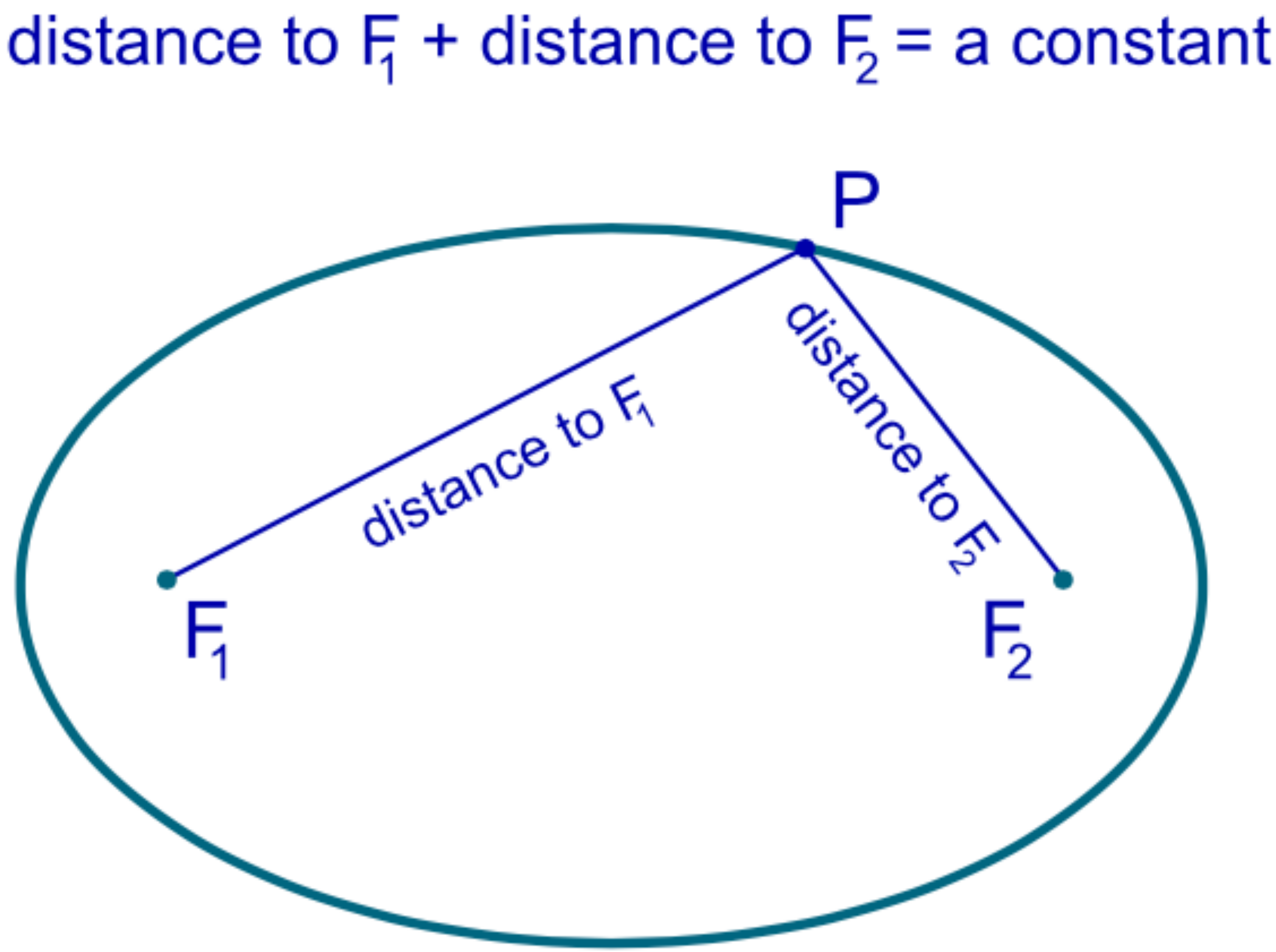


reflective property of a parabola

kraal in Namibia

Inuit igloo

ellipse

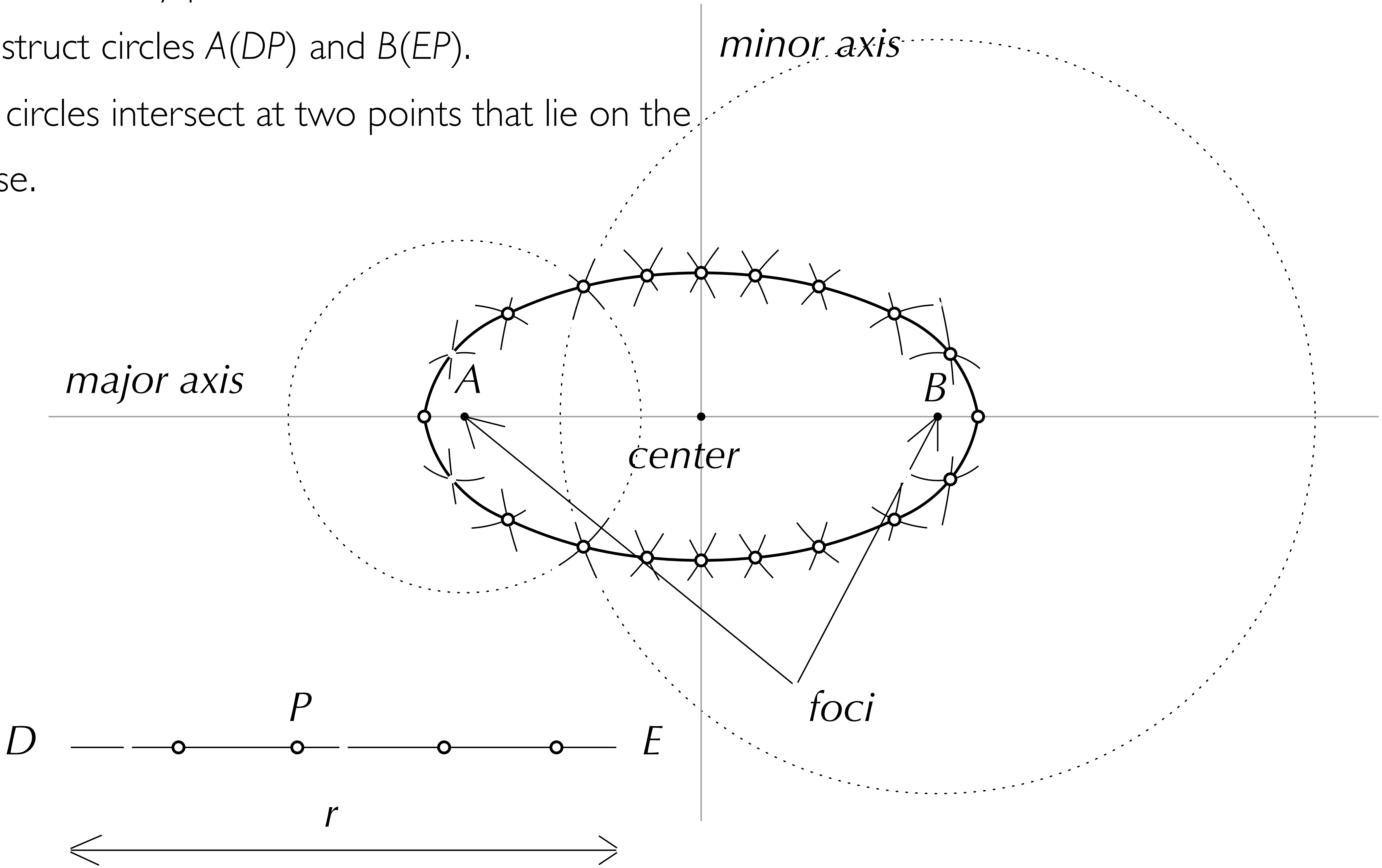


basic property of an ellipse

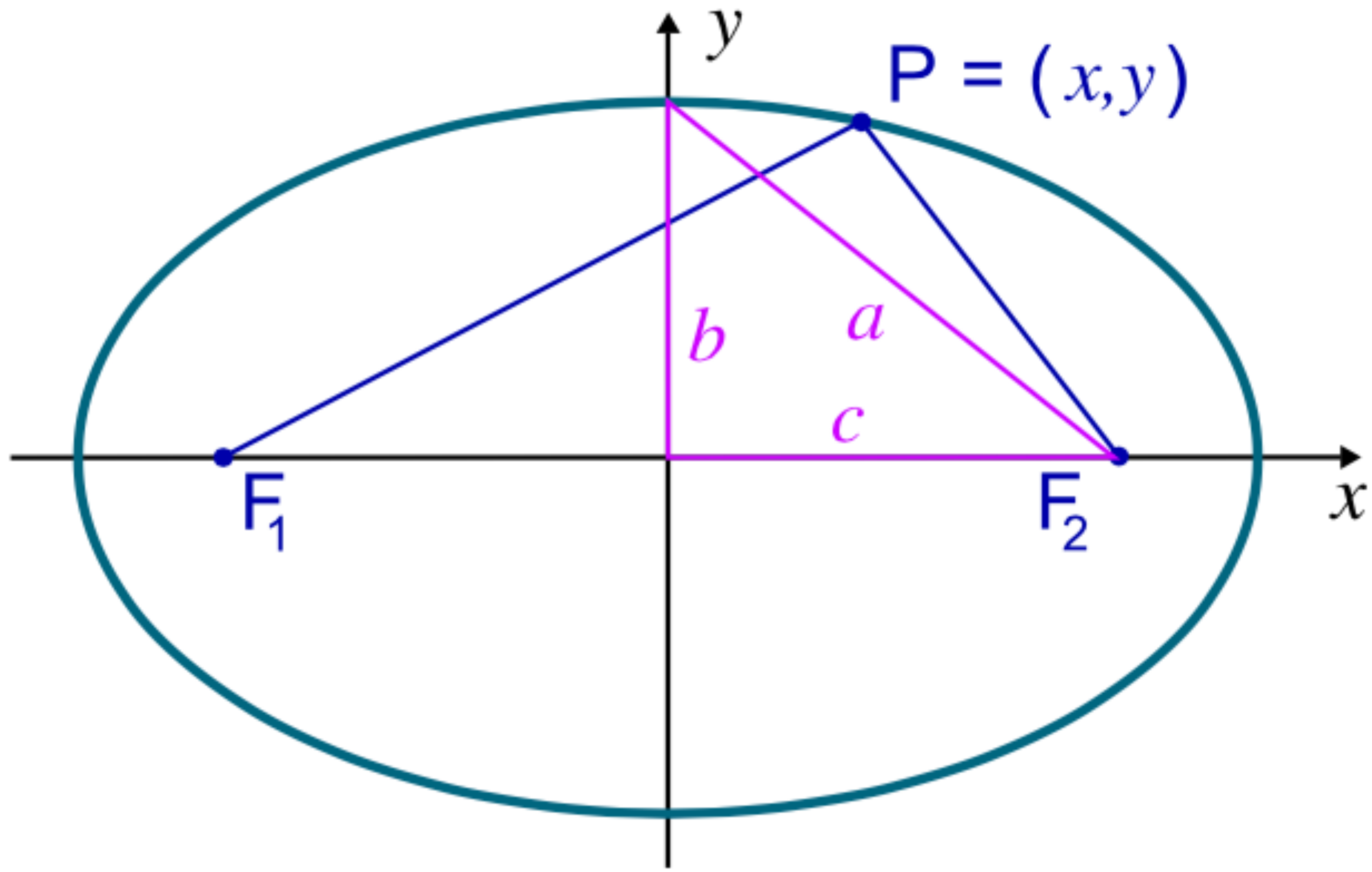
P is an arbitrary point between *D* and *E*.

Construct circles *A*(*DP*) and *B*(*EP*).

The circles intersect at two points that lie on the ellipse.



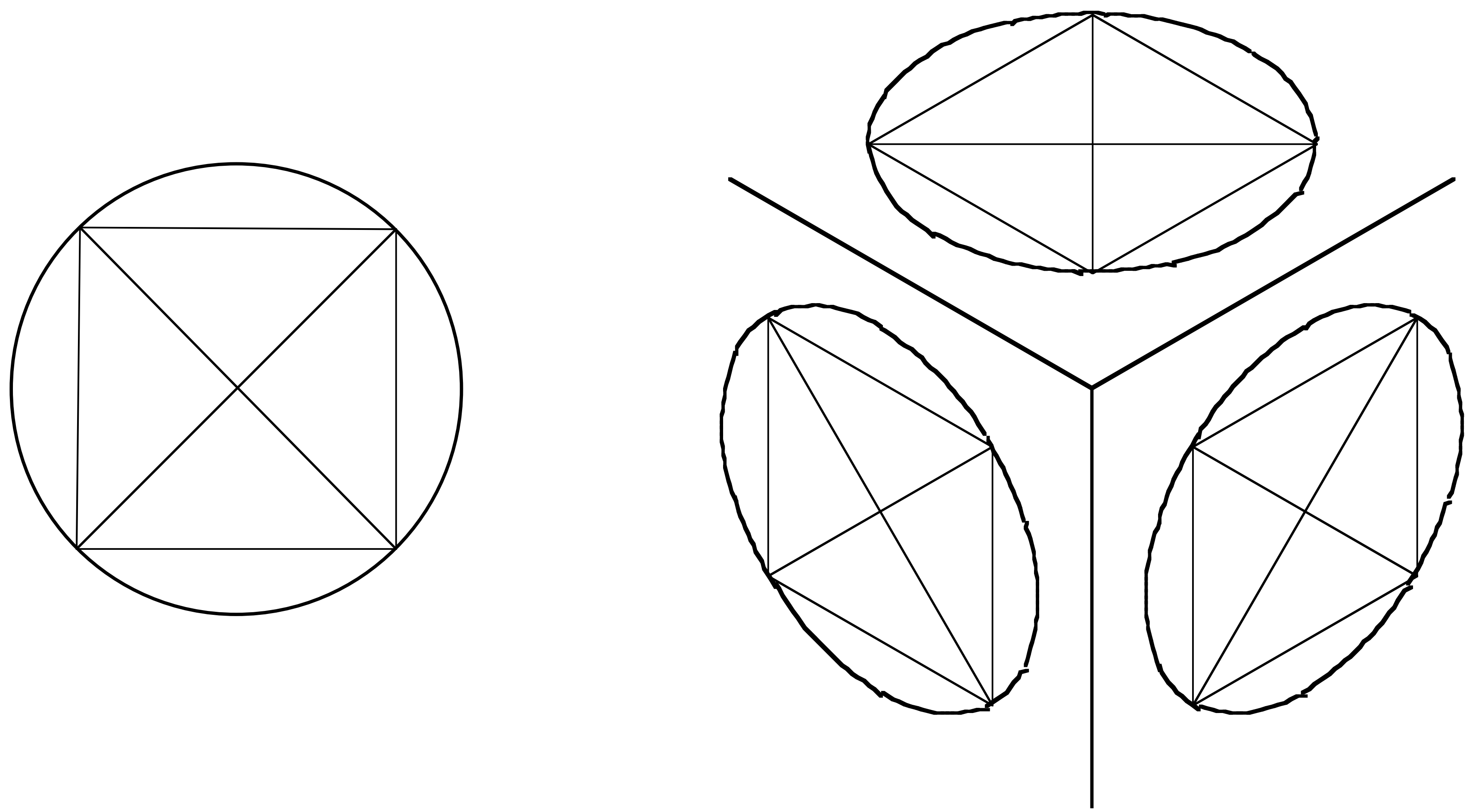
constructions involving ellipses



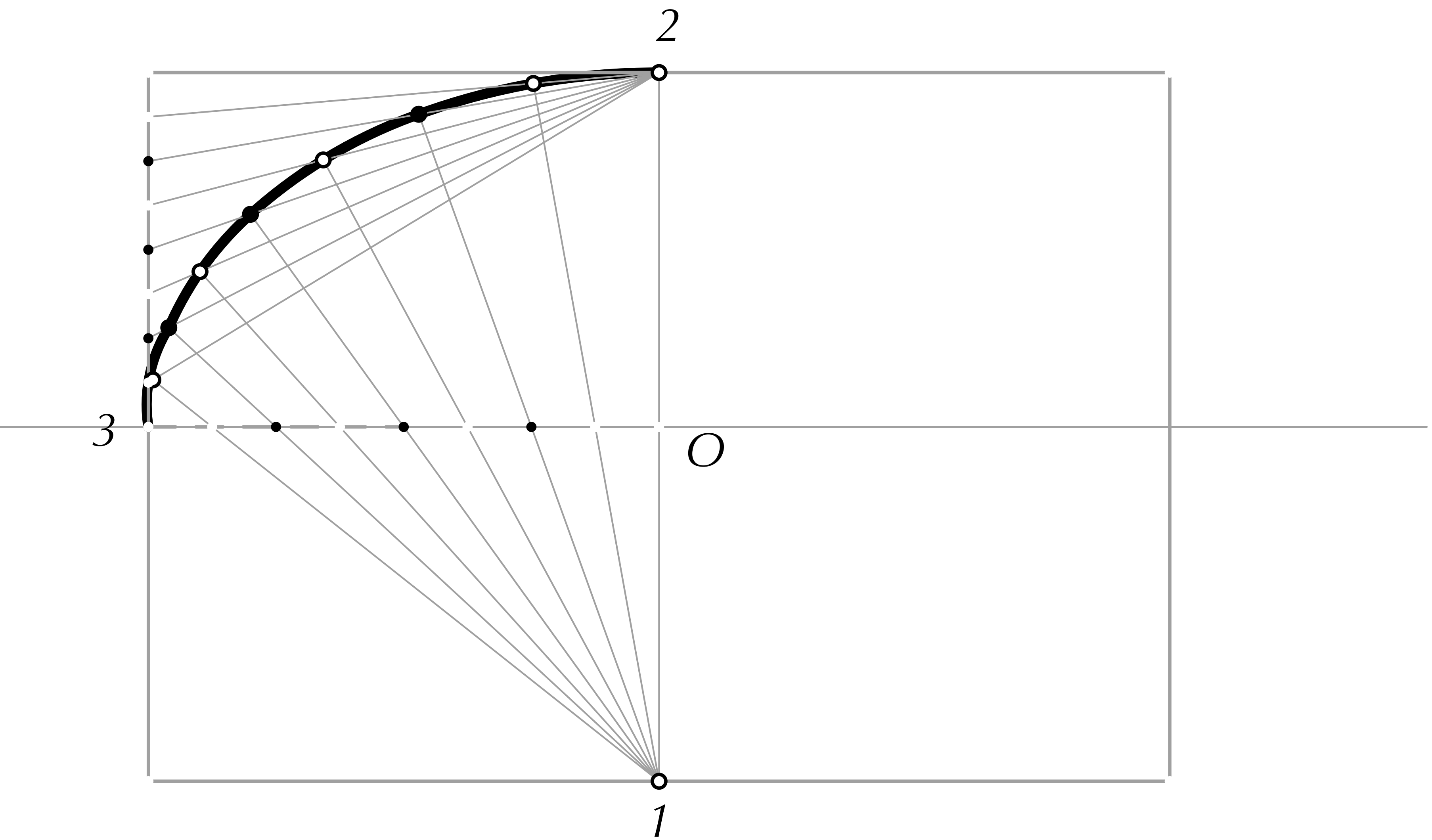
$$
\begin{aligned}
2a &= \overline{PF_1} + \overline{PF_2} \\
&= \sqrt{(x+c)^2 + y^2} + \sqrt{(x-c)^2 + y^2}
\end{aligned}
$$

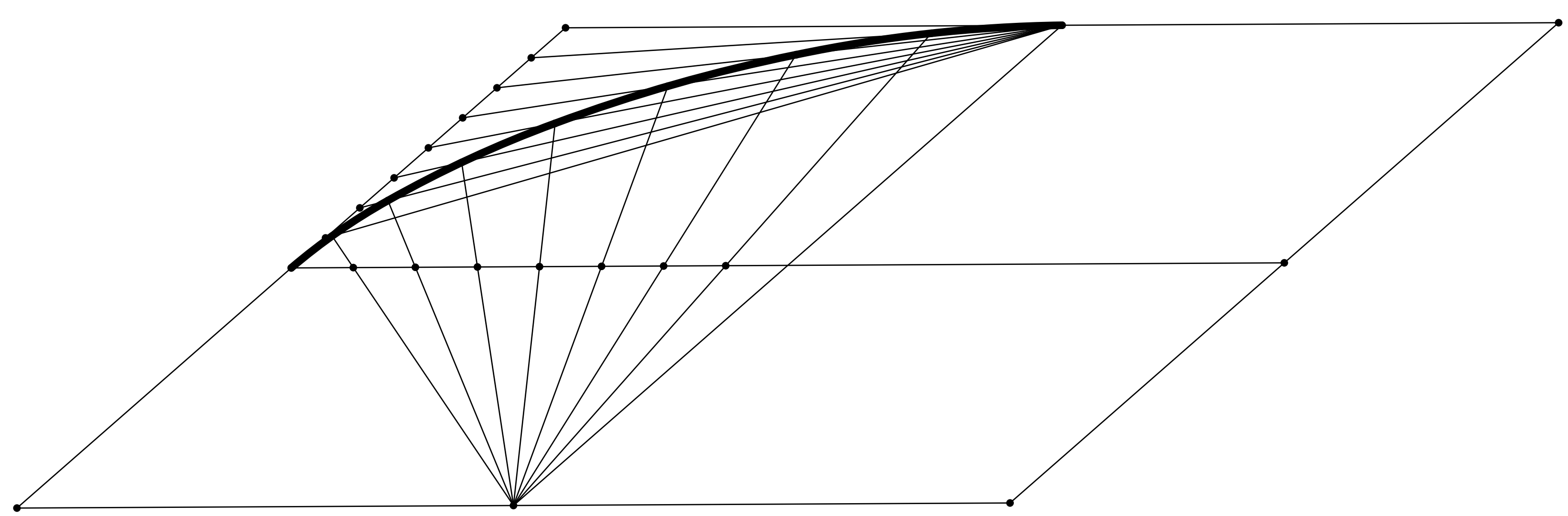
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

analytic form

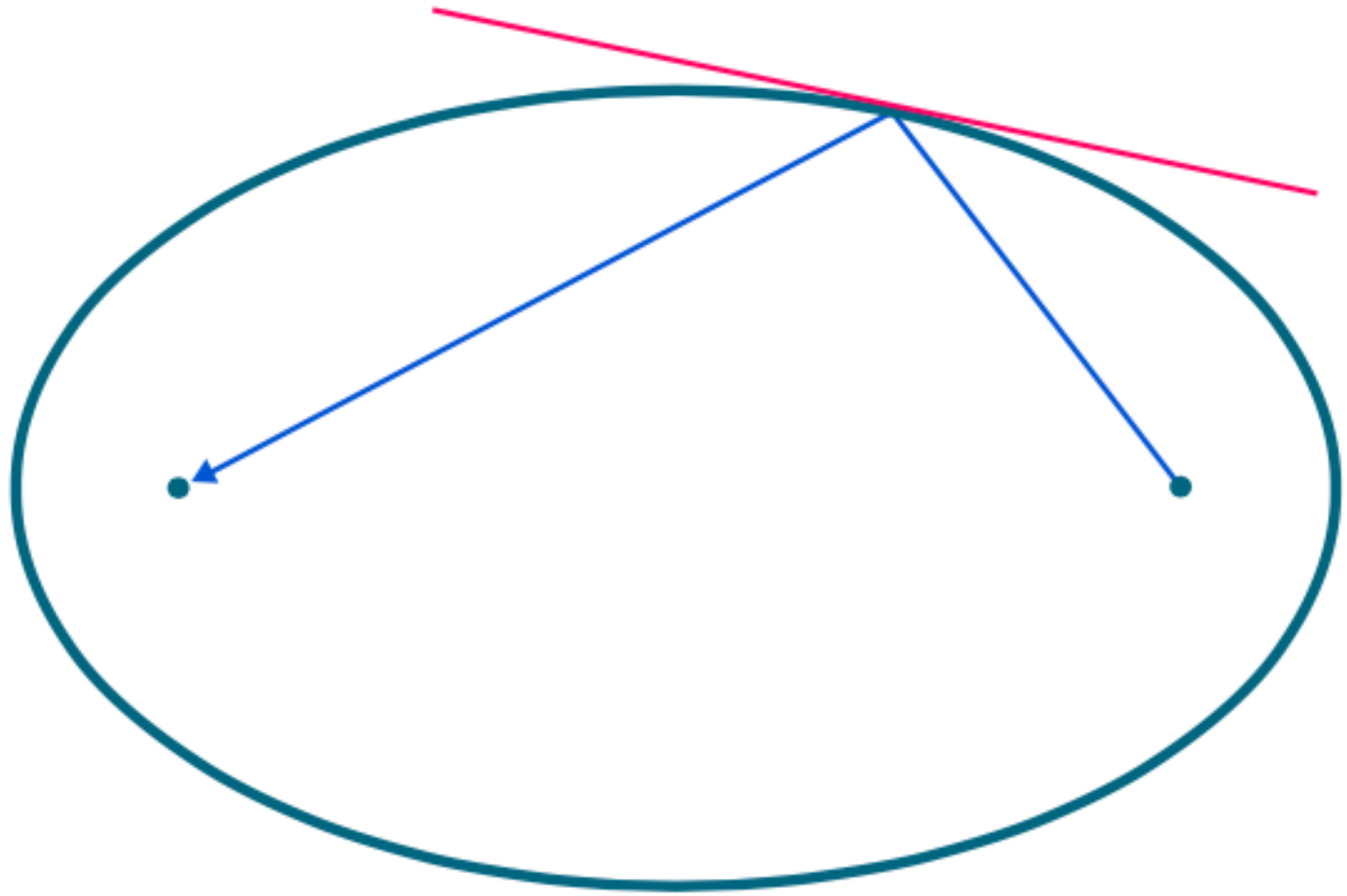


axonometric view of a circle is an ellipse

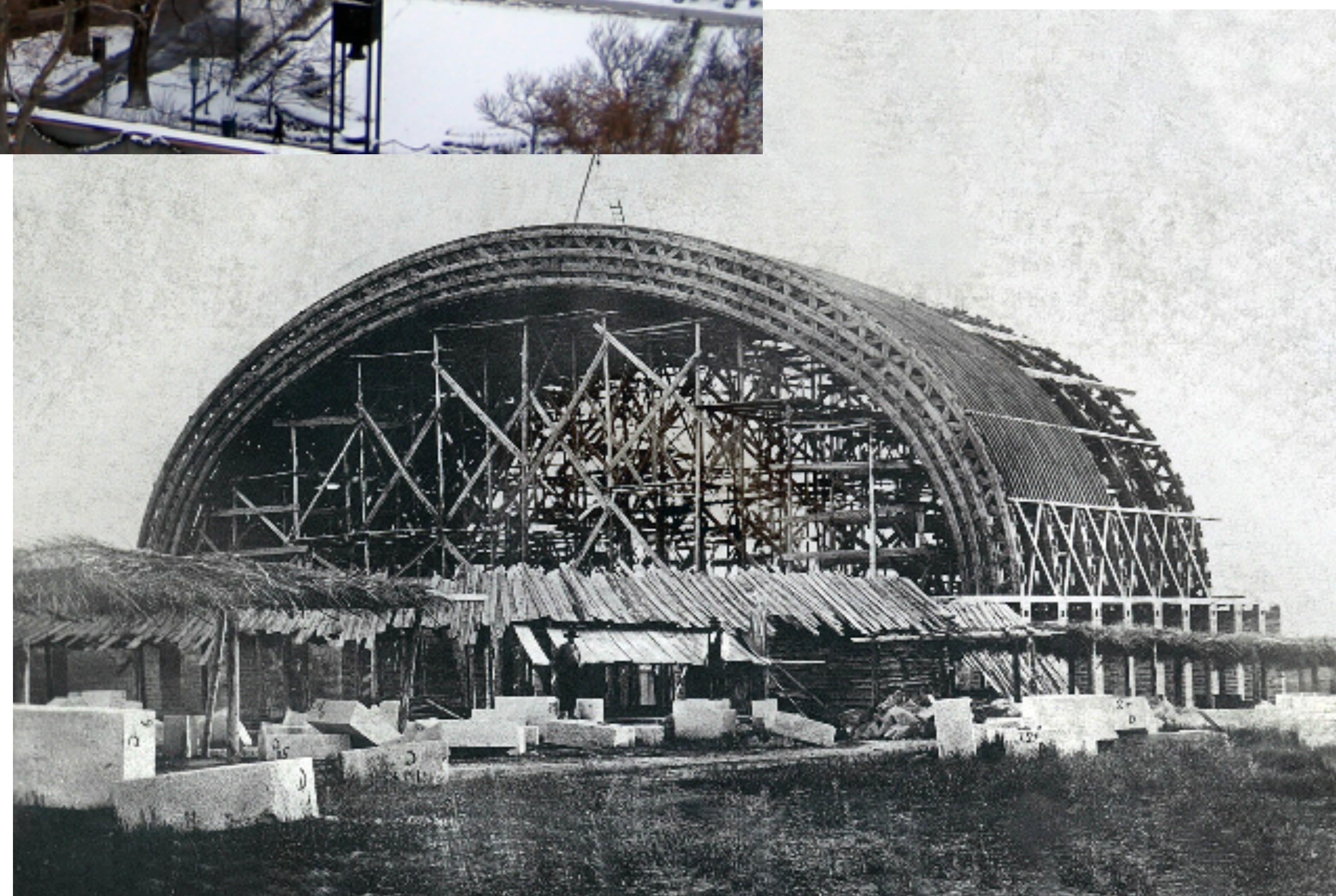




constructing an ellipse within a rectangle



reflective property of an ellipse

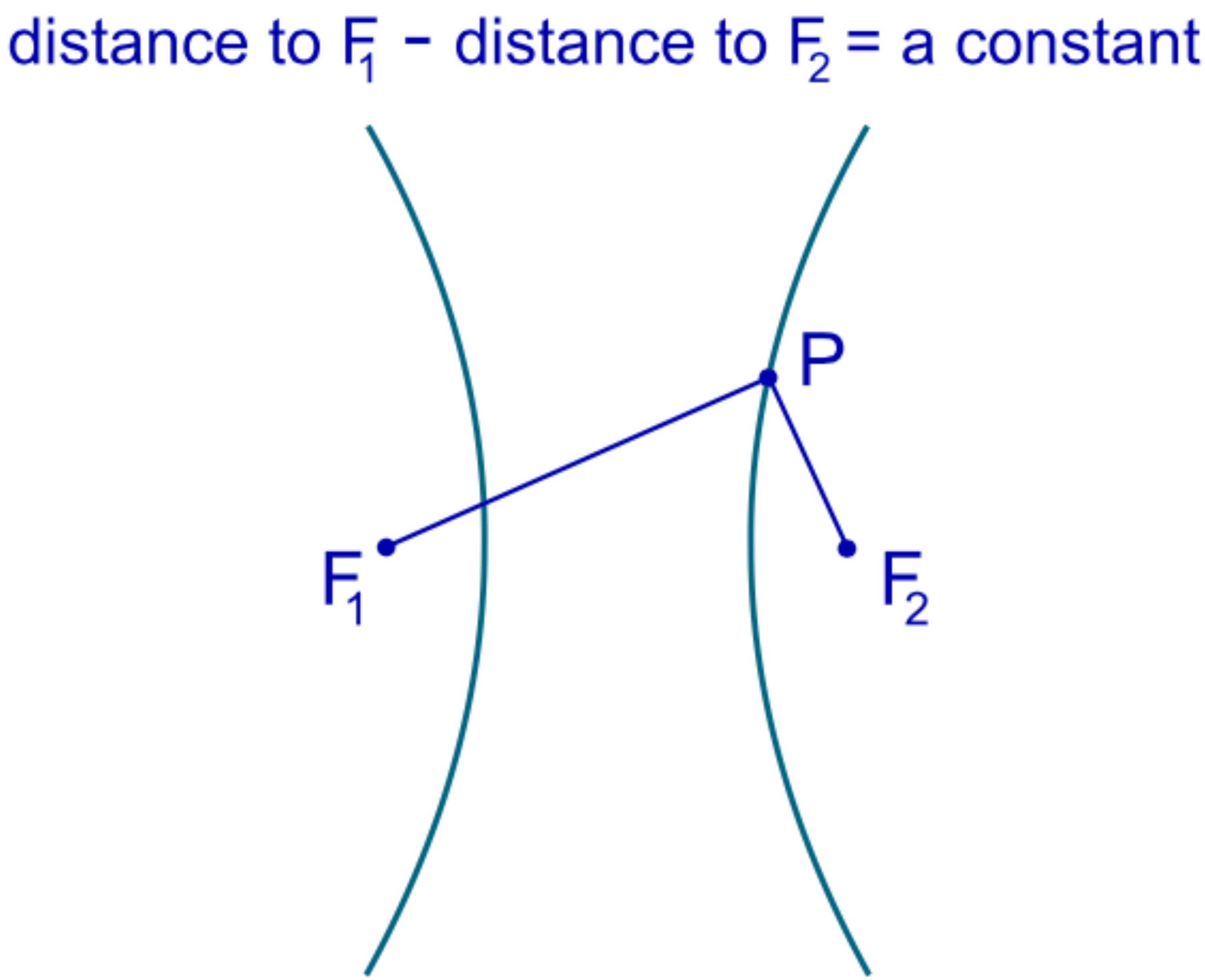


mormon tabernacle

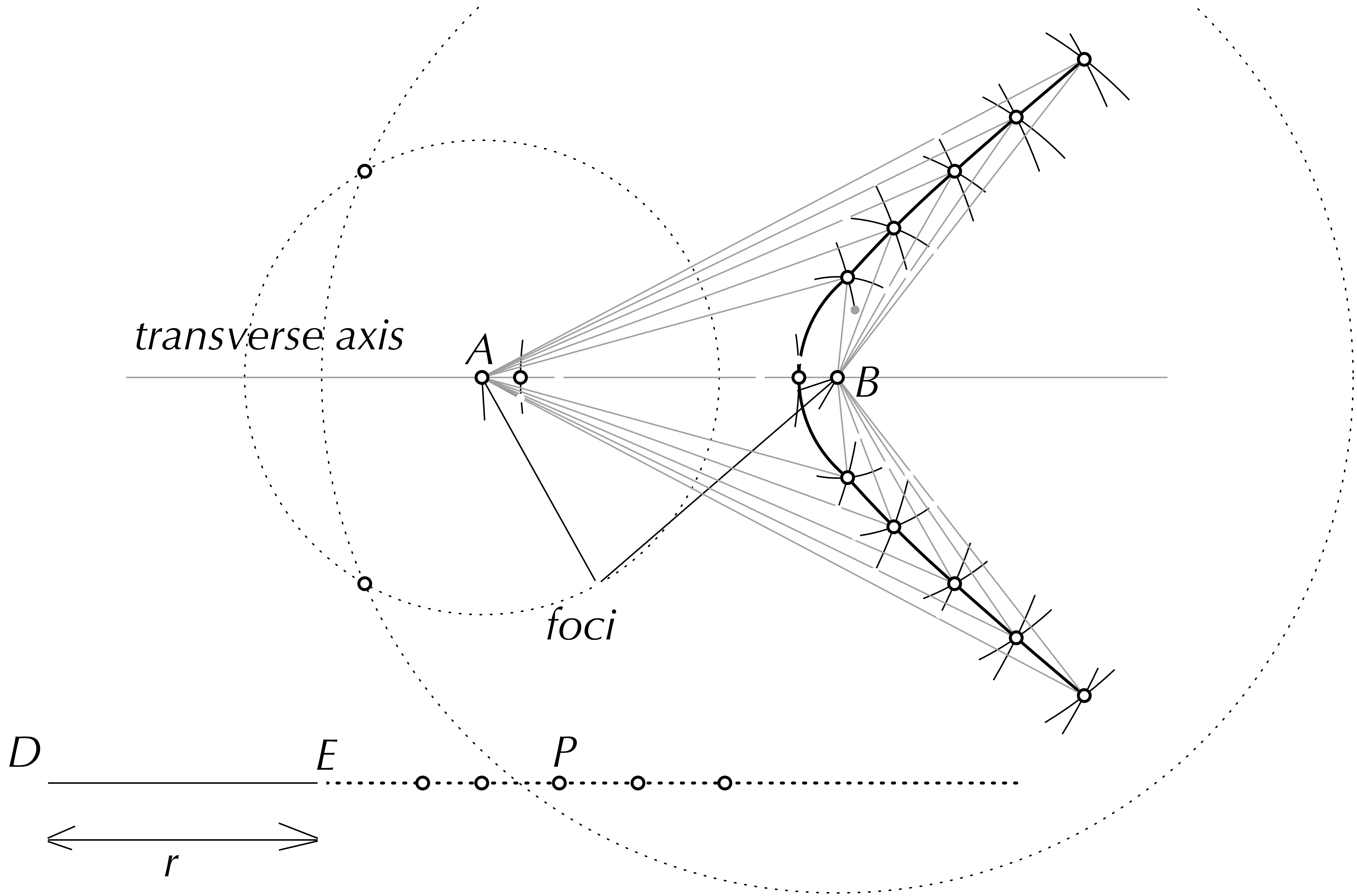
us capitol building

http://www.loop-the-game.com

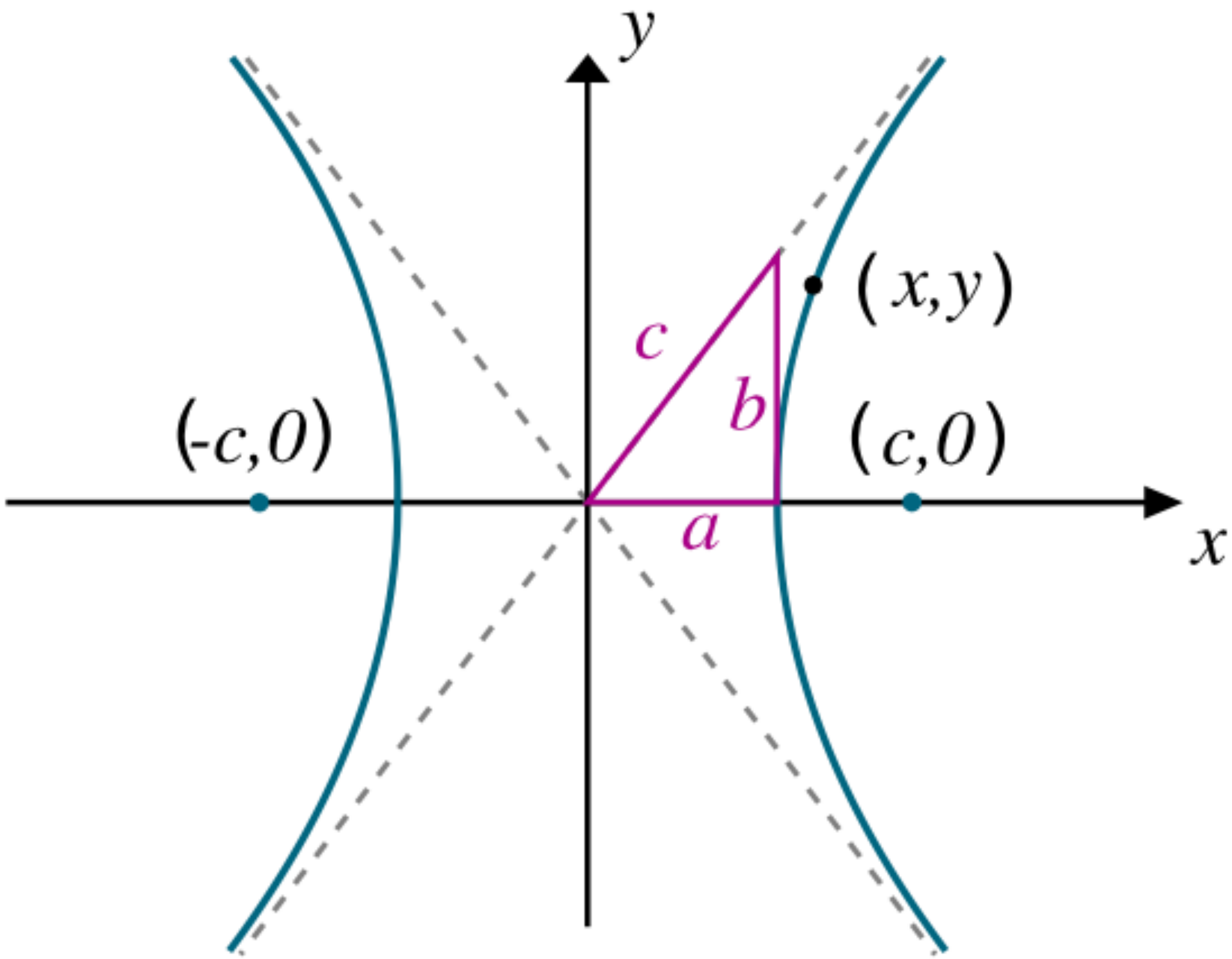
hyperbola



hyperbola



hyperbola



$$y = \pm\frac{b}{a}x$$

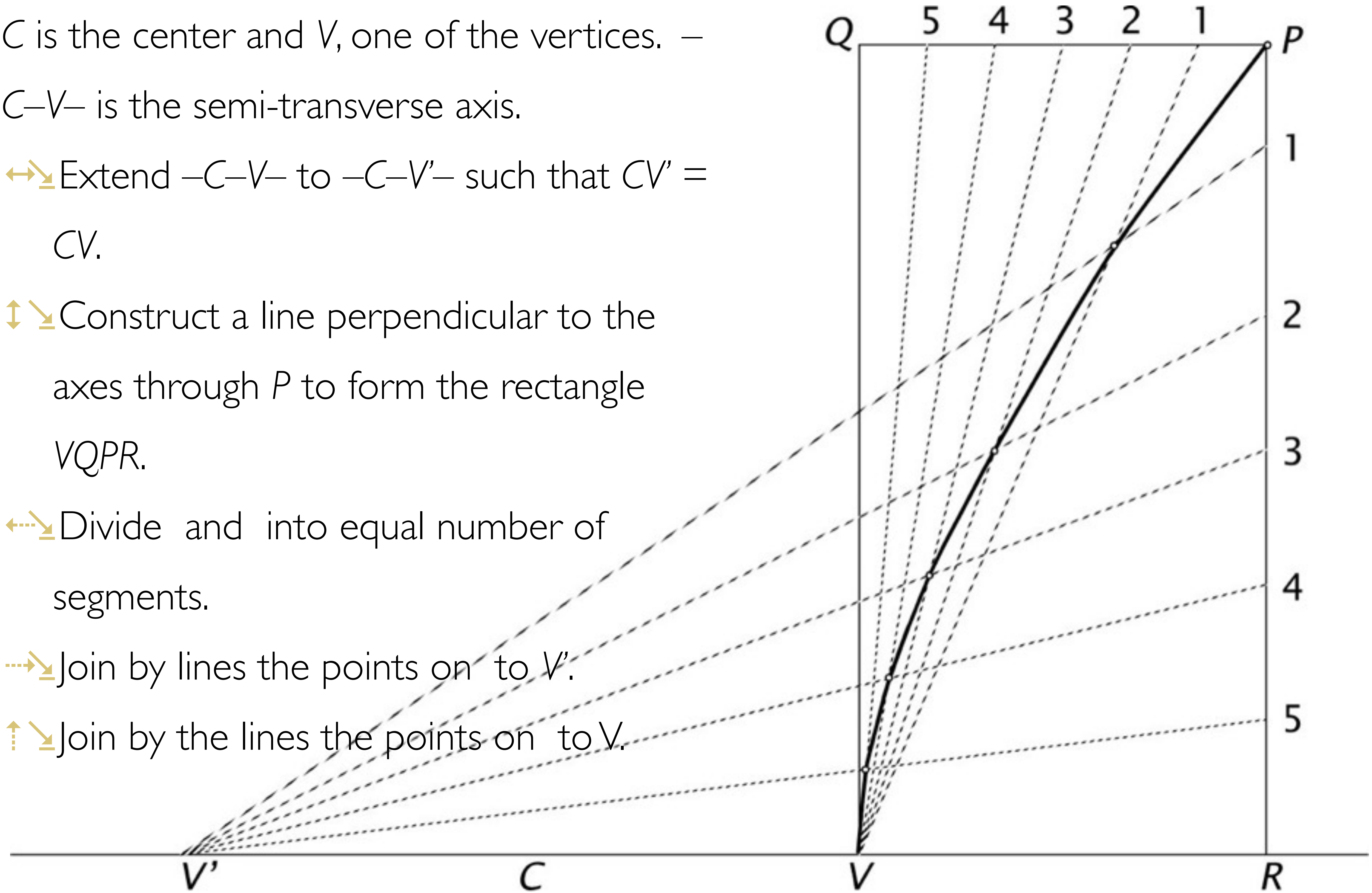
$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

analytic form

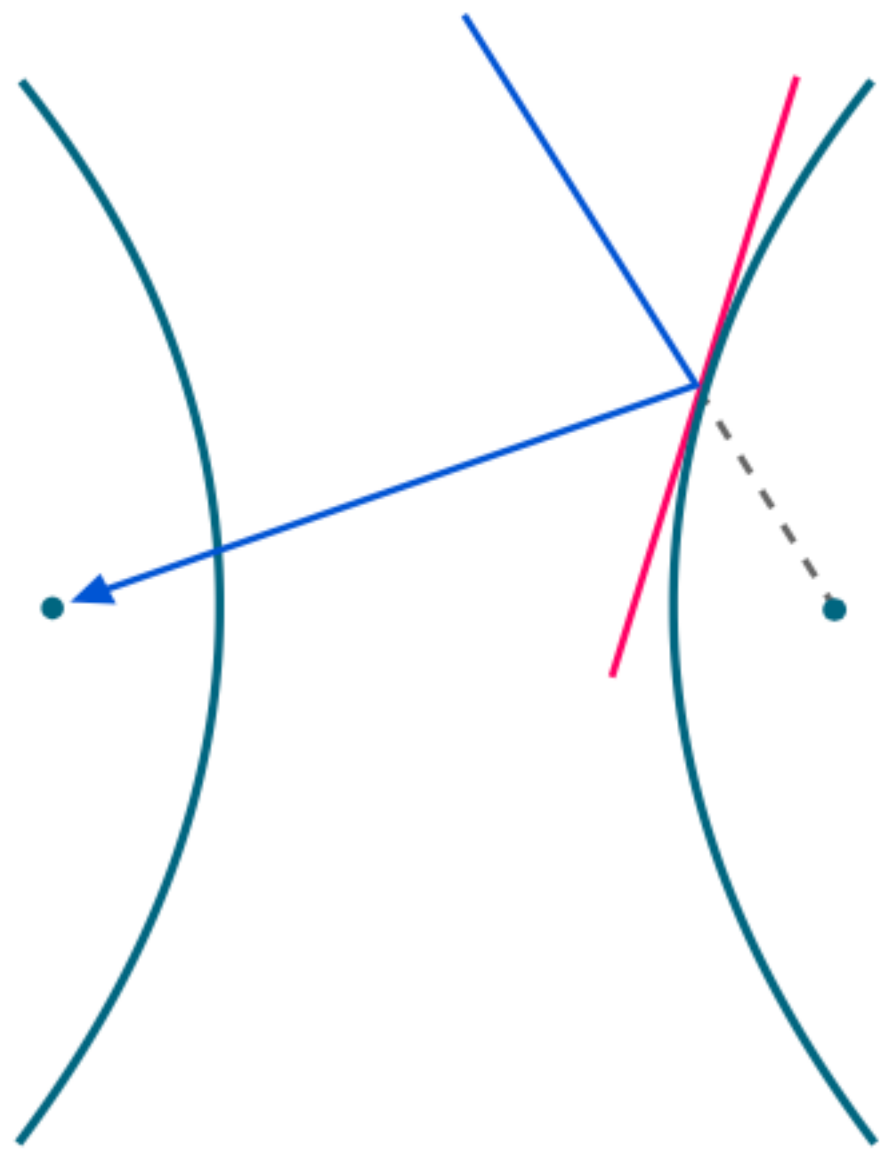
C is the center and *V*, one of the vertices. –

C–*V*– is the semi-transverse axis.

- Extend –*C*–*V*– to –*C*–*V*'– such that *CV*' = *CV*.
- Construct a line perpendicular to the axes through *P* to form the rectangle *VQPR*.
- Divide and into equal number of segments.
- Join by lines the points on to *V*'.
- Join by the lines the points on to V.



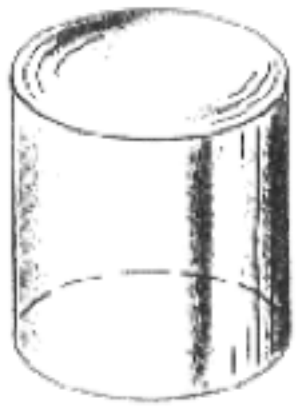
hyperbola given semi-transverse axis and a point



reflective property of a hyperbola

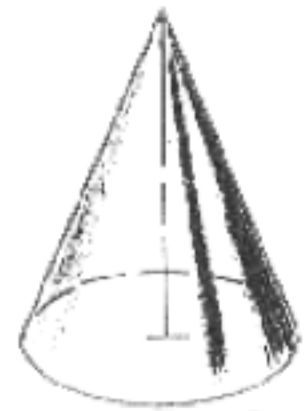
oscar neimeyer

creating surfaces from conic curves

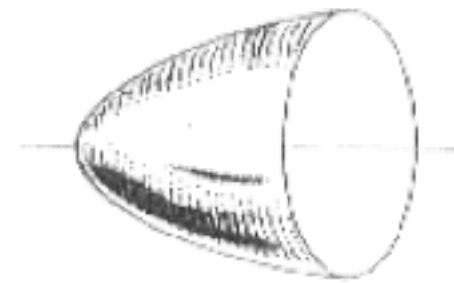


Right Cyl

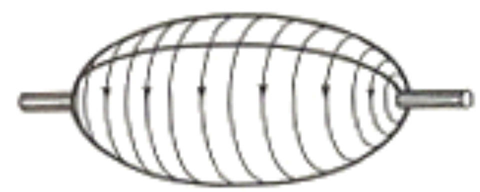
Sphere



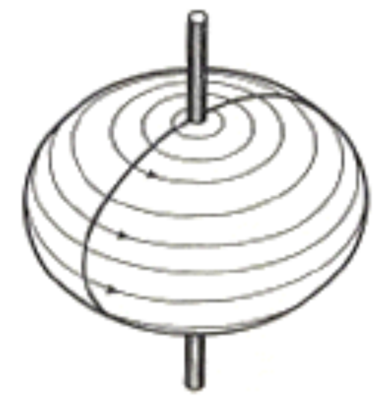
Right Cone



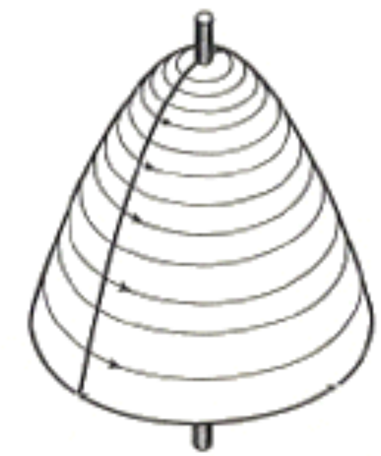
Paraboloid



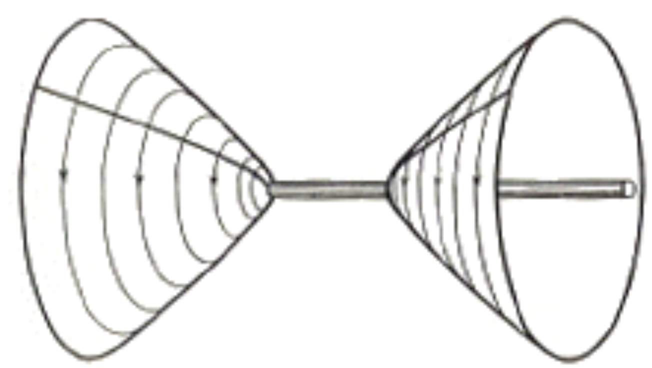
PROLATE SPHEROID



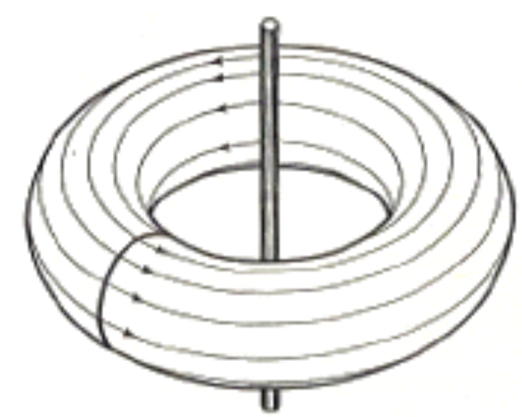
OBLATE SPHEROID



PARABOLOID



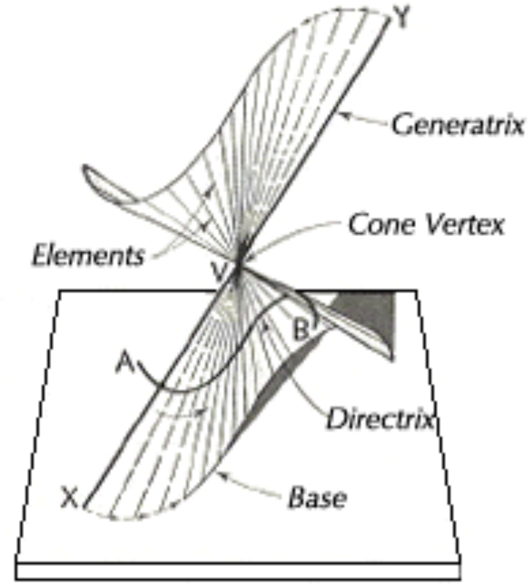
HYPERBOLOID OF TWO NAPPES



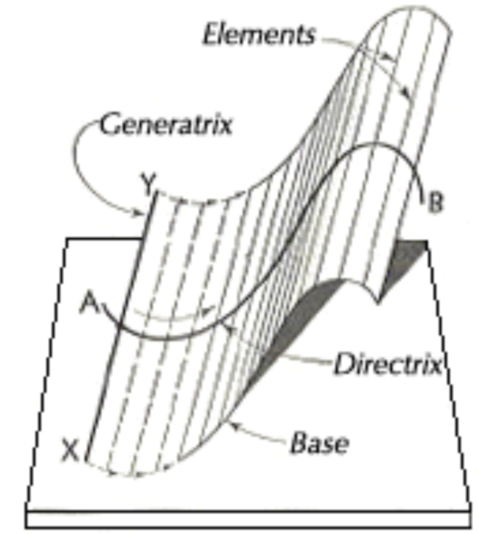
ANNULAR TORUS

by revolving

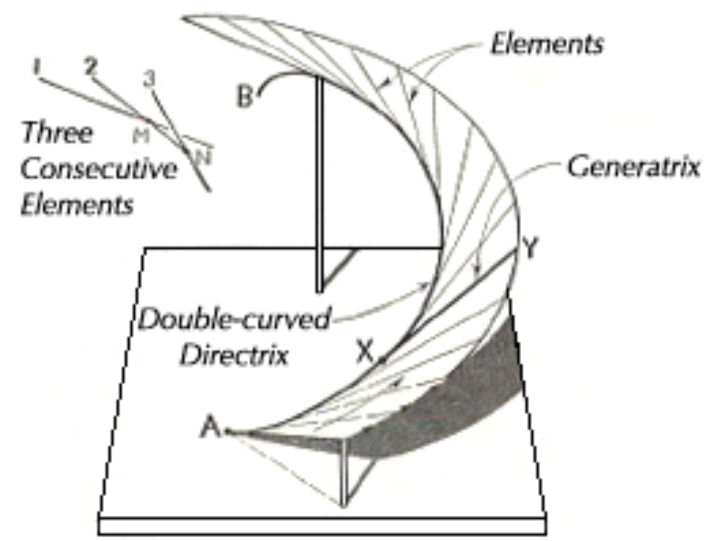
- Is produced when a line is moved in contact with a curve (directrix) in the plane to produce a surface



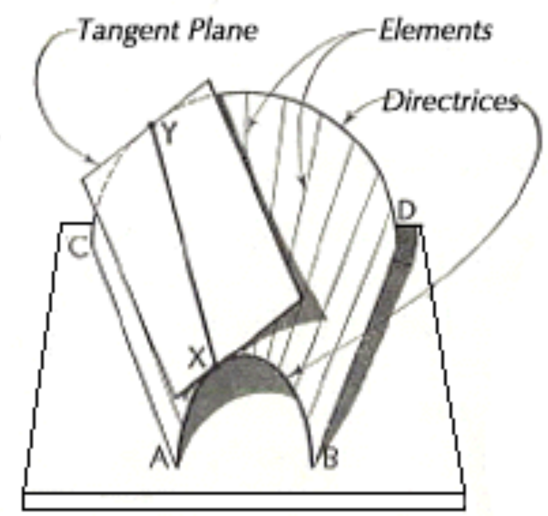
Generating a conical surface



Generating a cylindrical surface



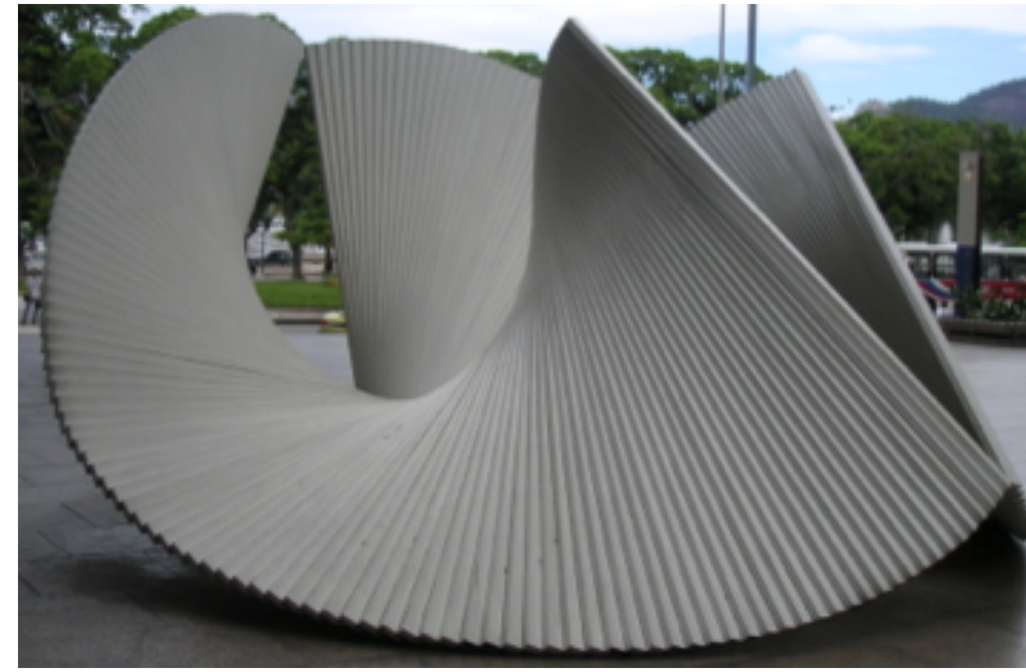
Generating a convolute
(Tangent line)

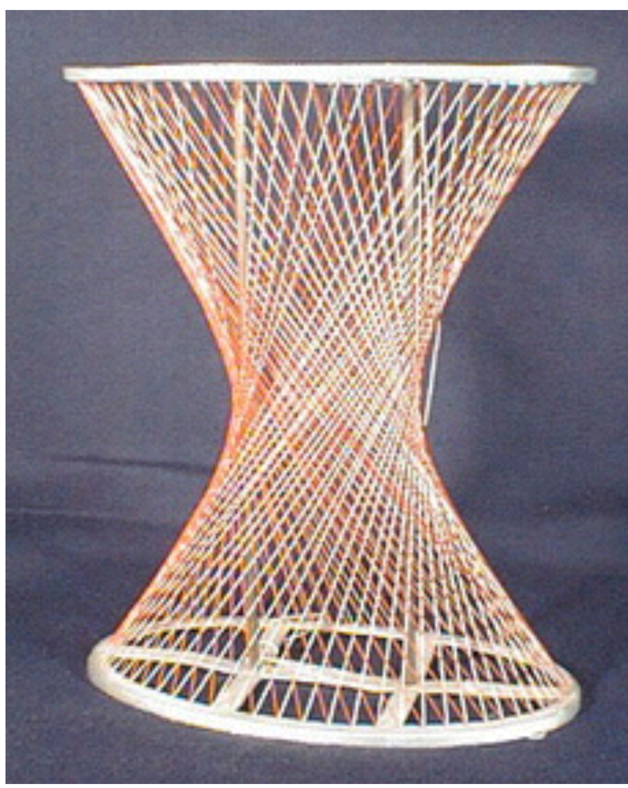


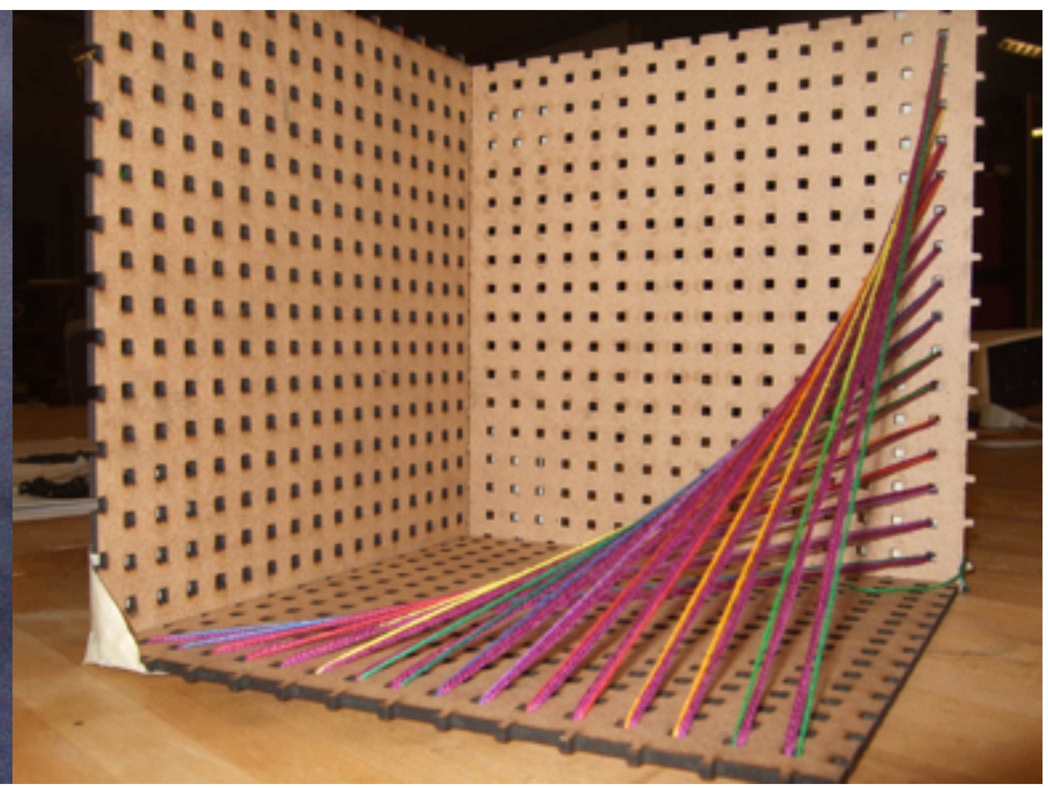
Generating a convolute
(Tangent plane)

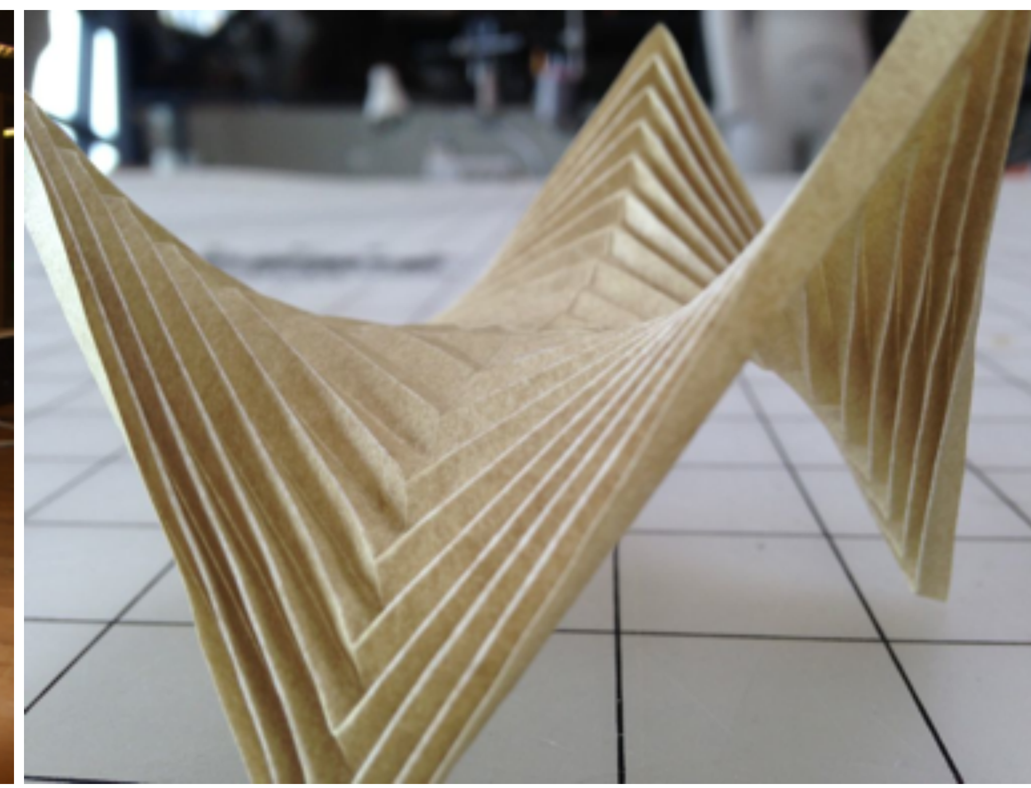
ruled surface

A ruled surface has the property that a straight line on the surface can be drawn through any point on the surface.



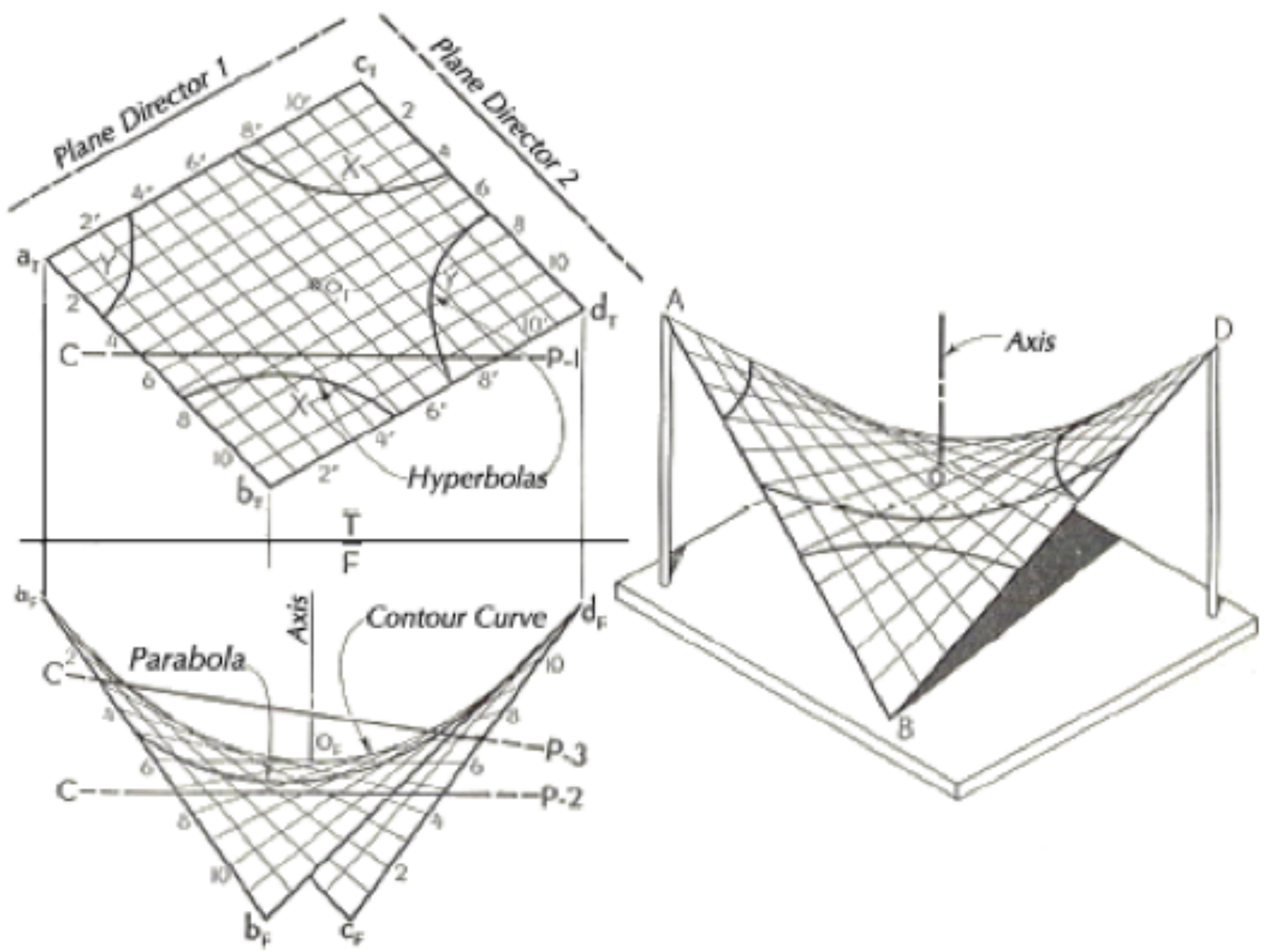




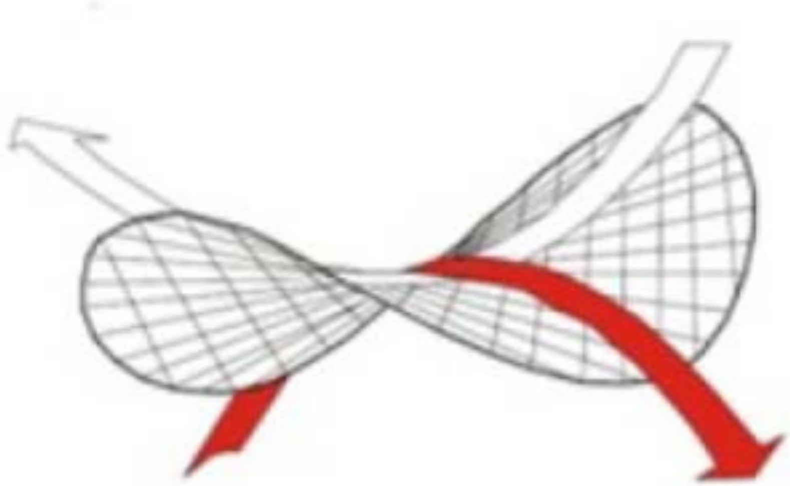


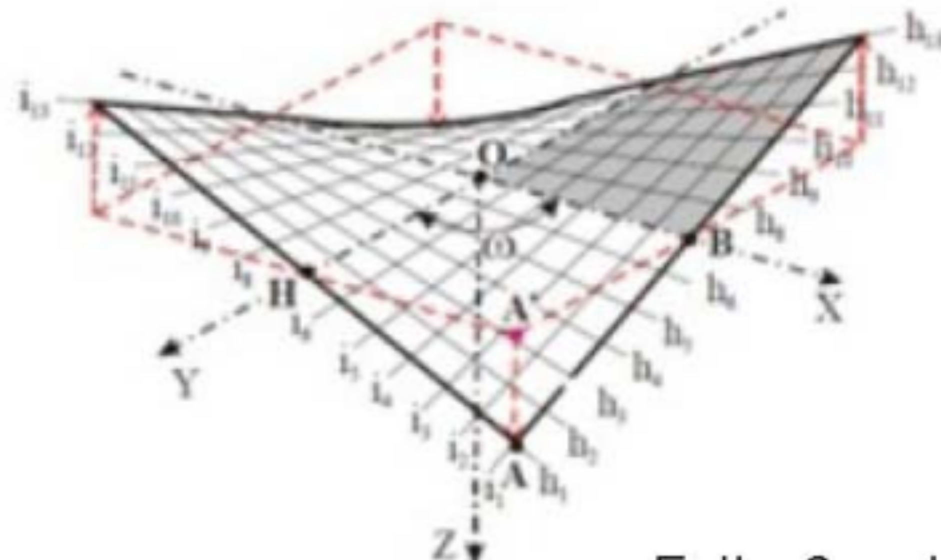
ruled surfaces

- Is a ruled surface for which two successive elements are neither parallel nor pass through a common point



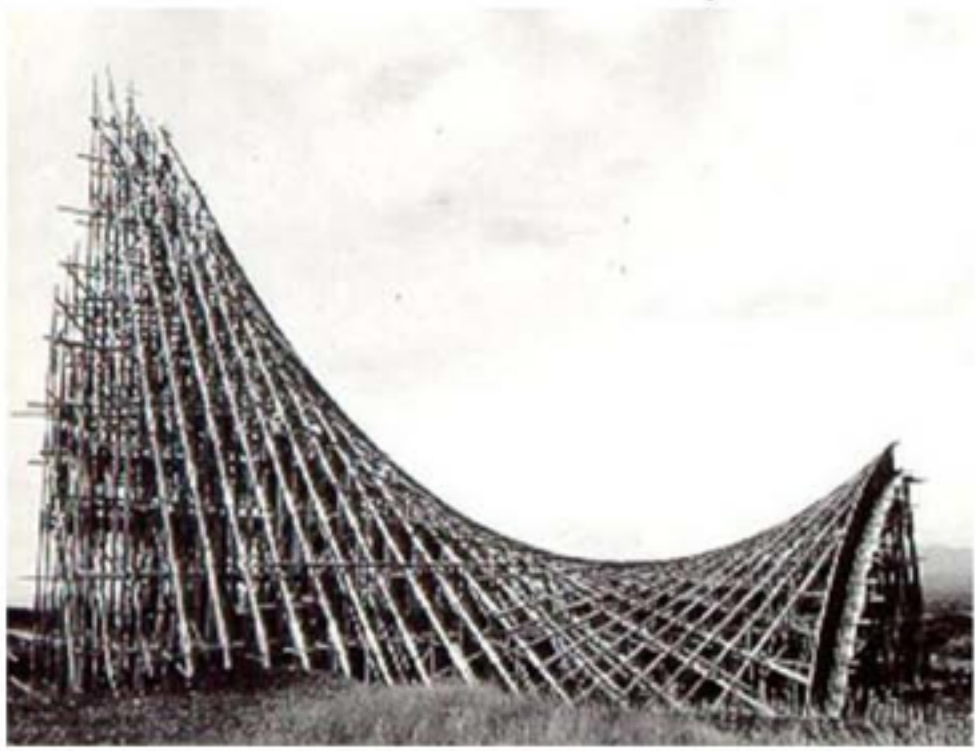
warped surface



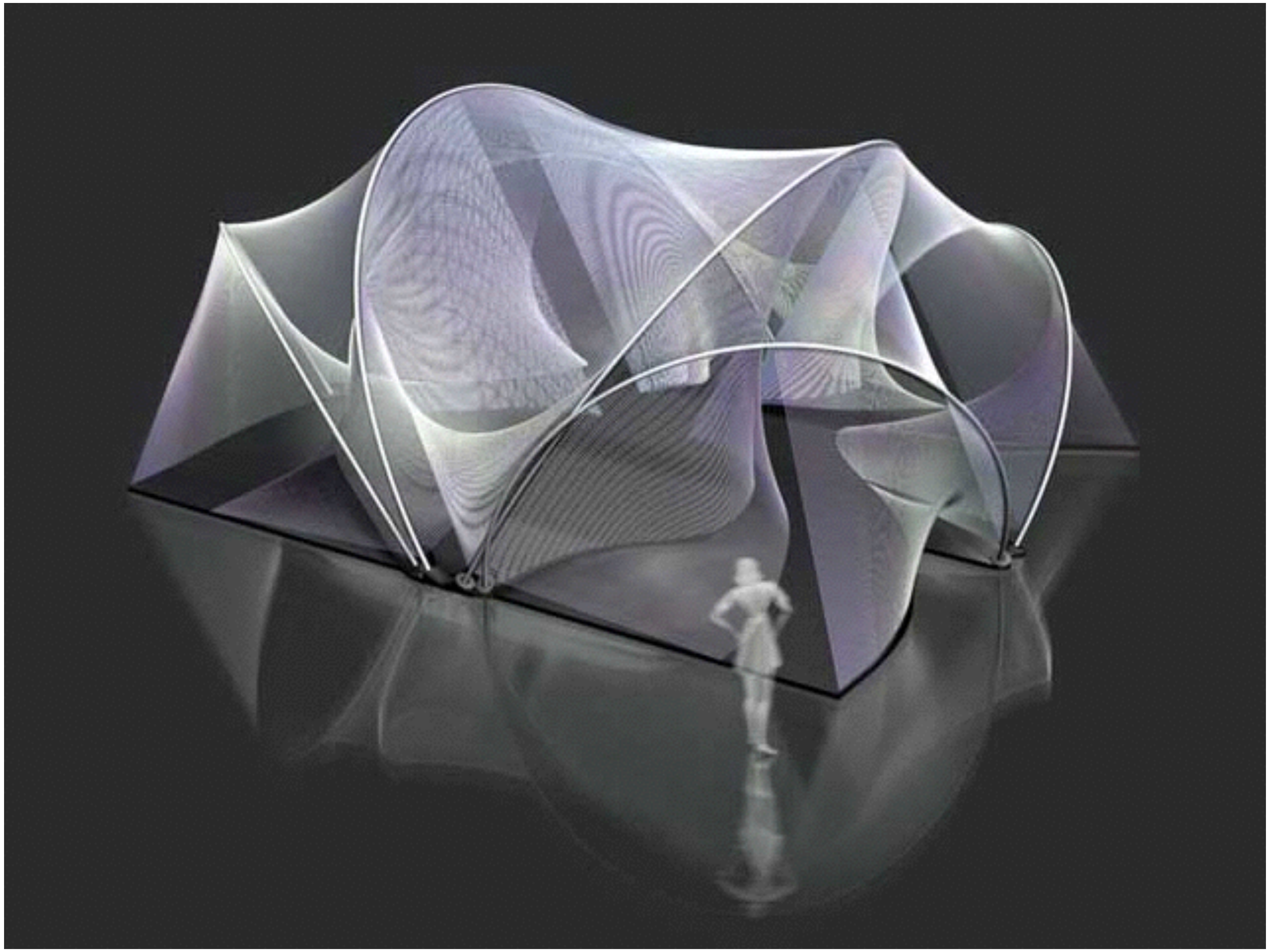


Hyperbolic Paraboloid
Curves constructed from a doubly ruled surface

Felix Candela
Capilla de Palmira, Cuernavaca, Mexico

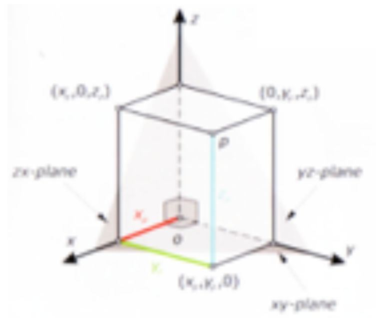


doubly-curved surface

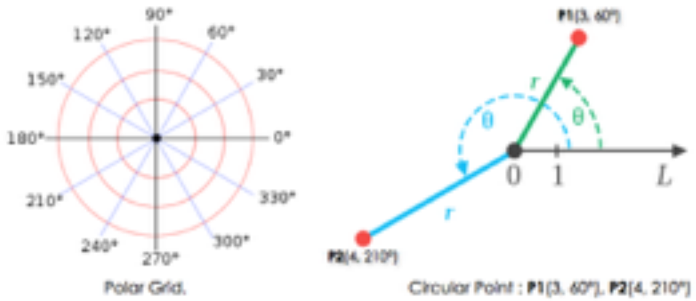


http://www.achimmenges.net

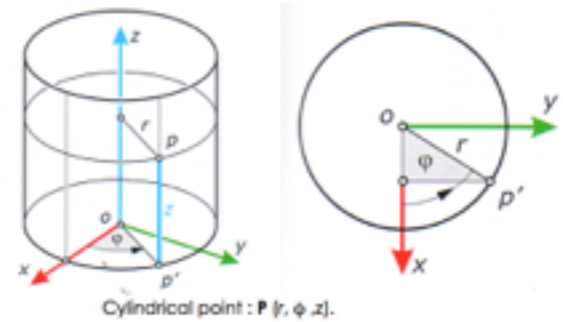
coordinates and transformations



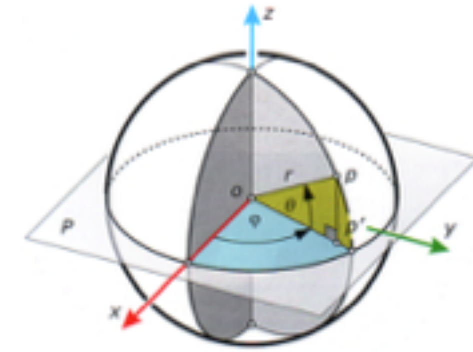
cartesian coordinate system



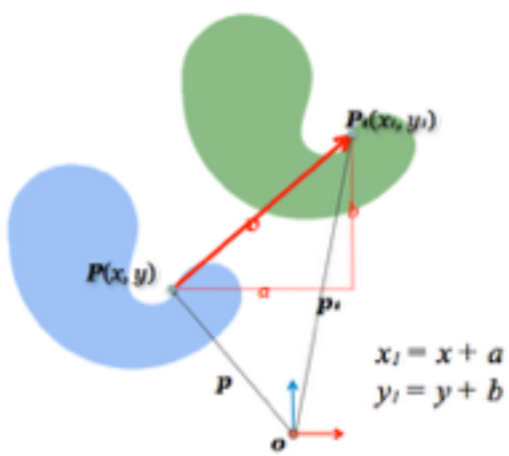
polar coordinate system



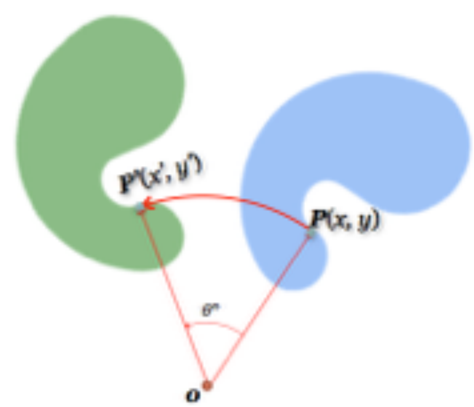
cylindrical coordinate system



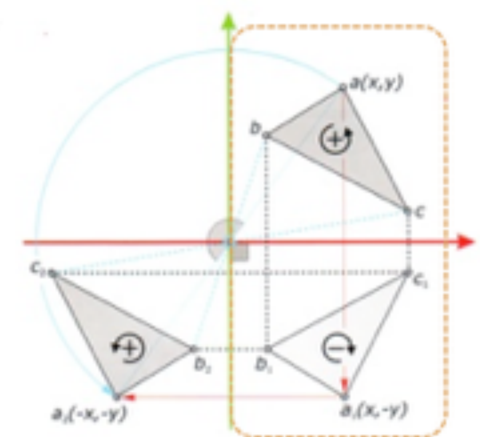
spherical coordinate system



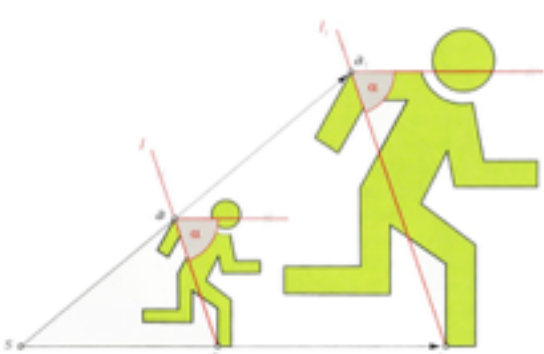
translation



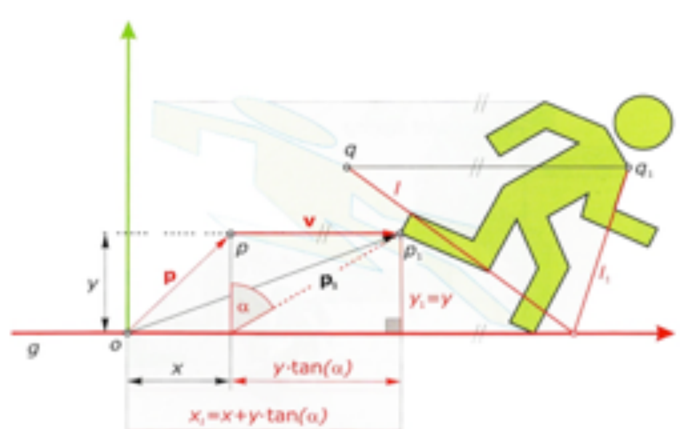
rotation



reflection

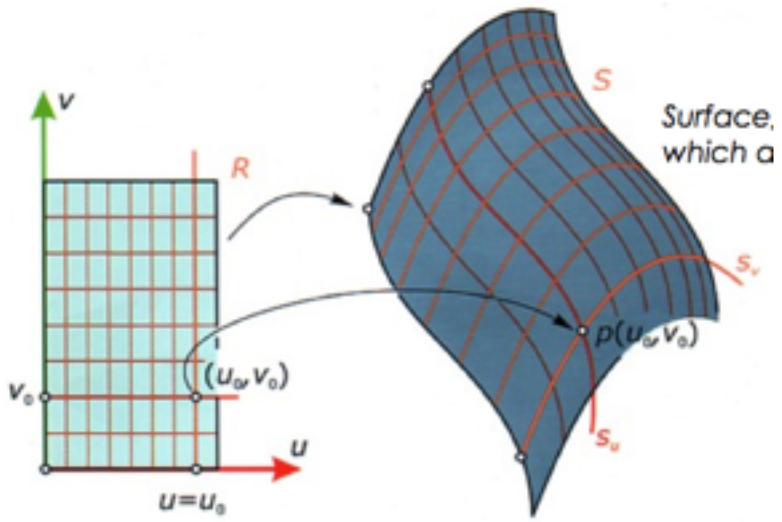


scale transformation

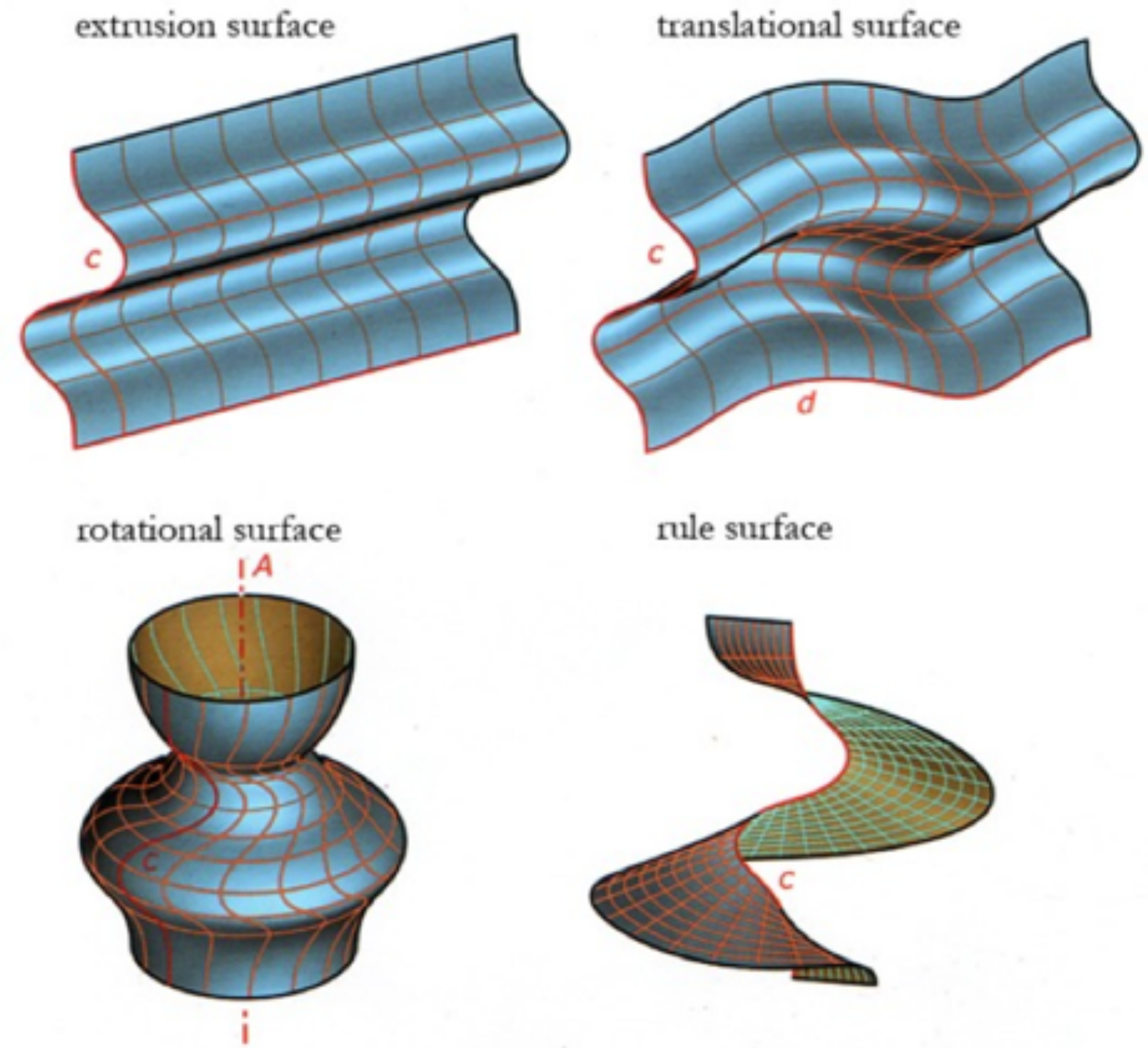


shear transformation

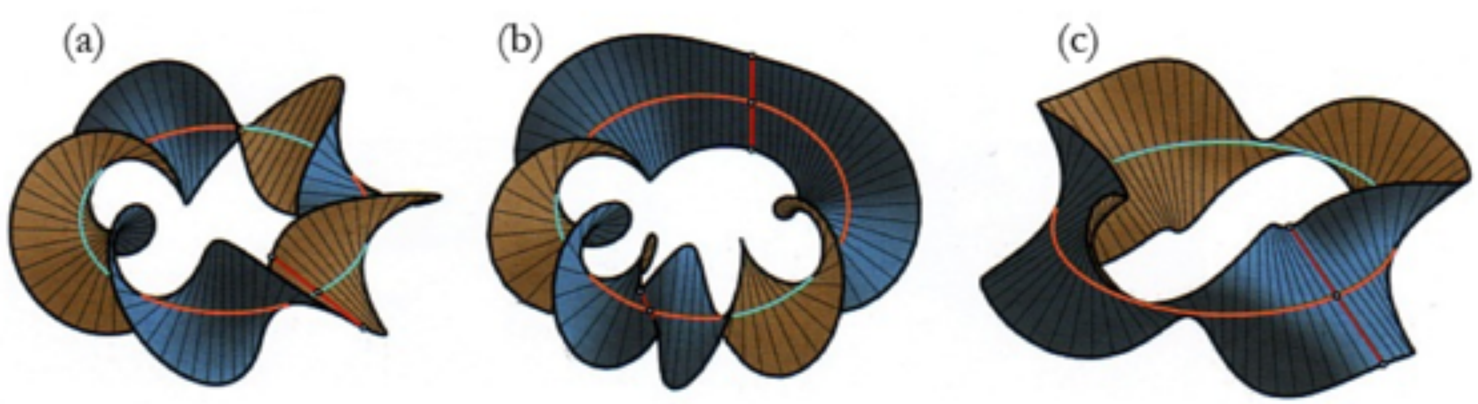
freeform curves to surfaces



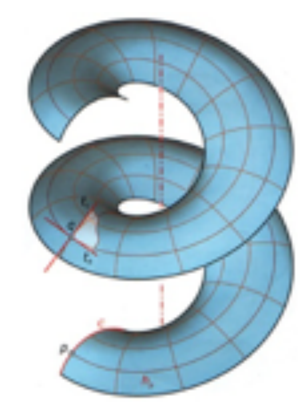
surface parameterization



surface classes

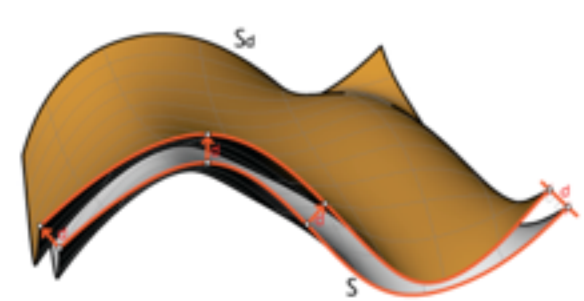


mobius strip

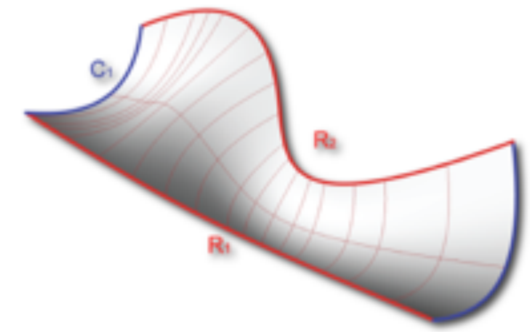

helical surface


pipe surface

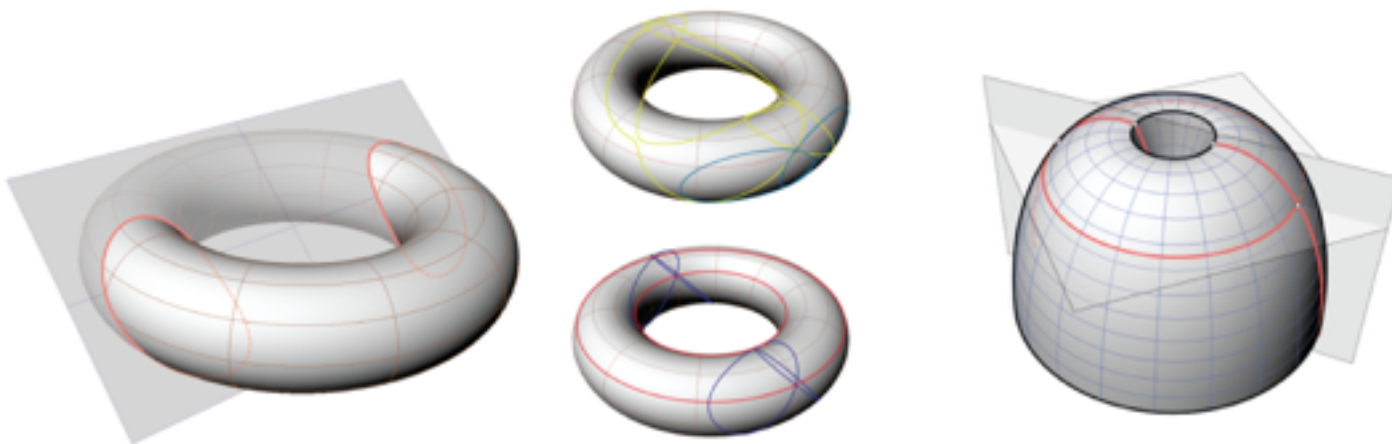
surface constructions



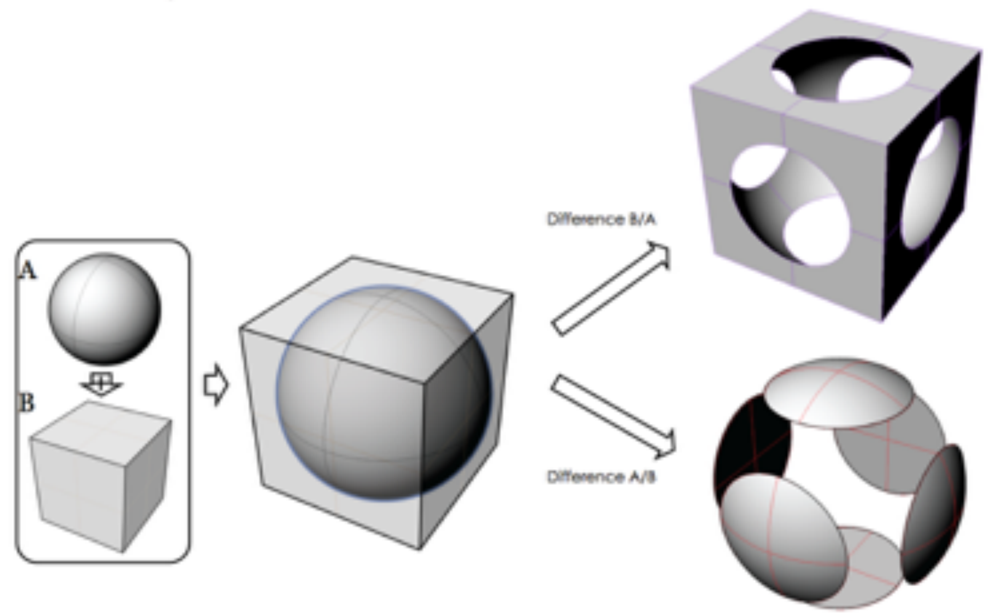
offset surface



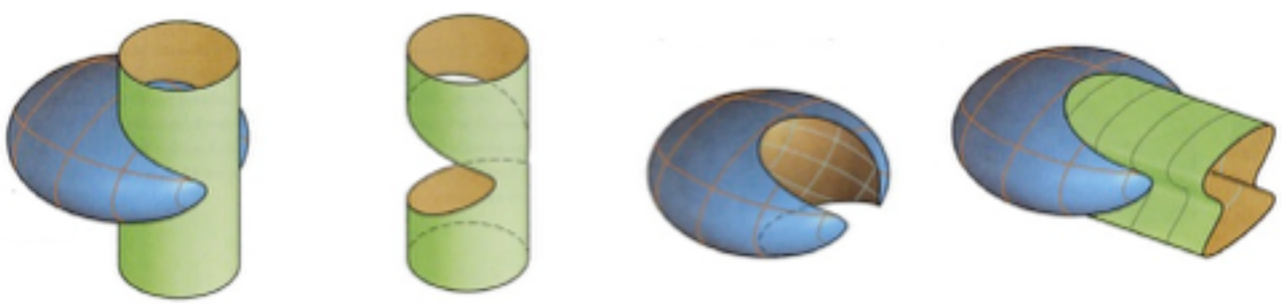
swept surface



intersection curves of surfaces

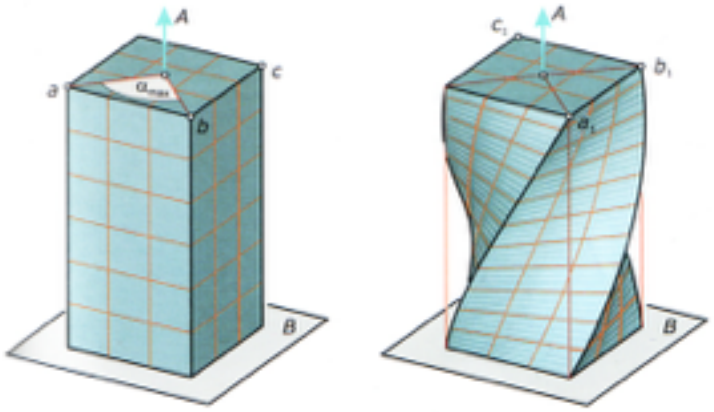


boolean operations

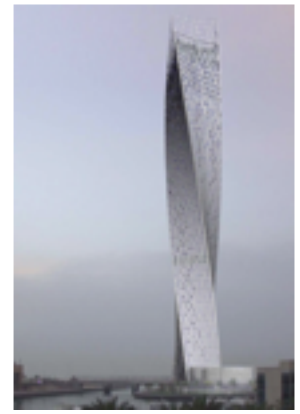


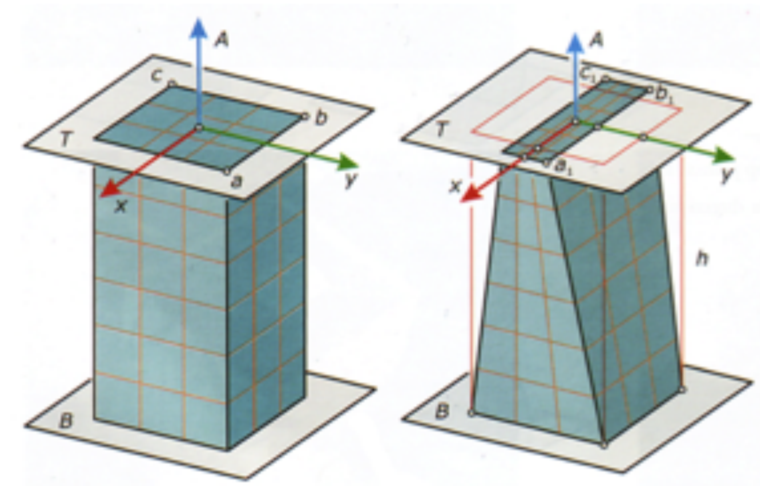
trim and split

deformations

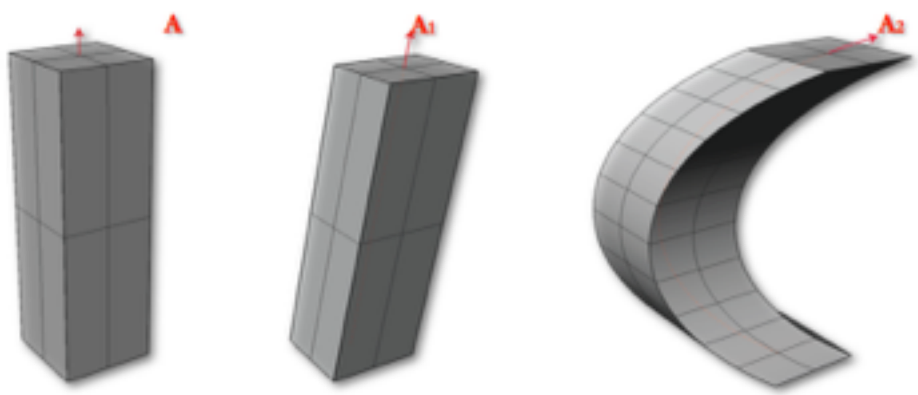


twisting

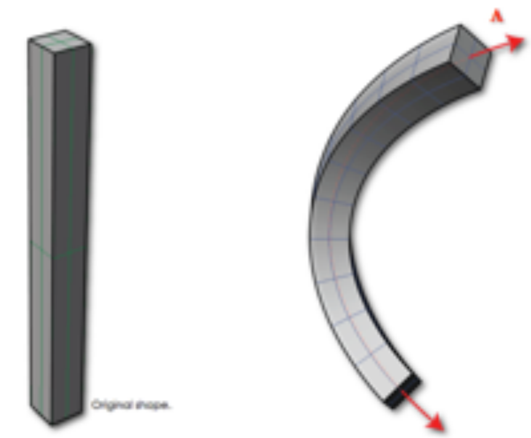




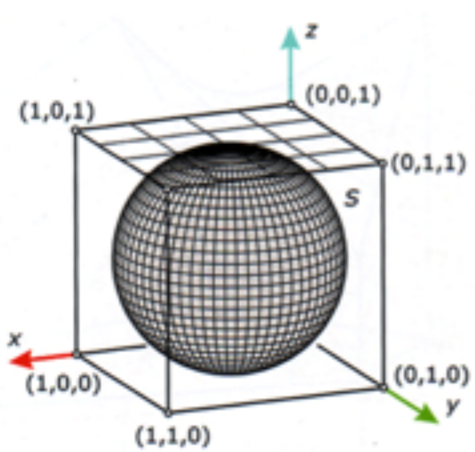
tapering

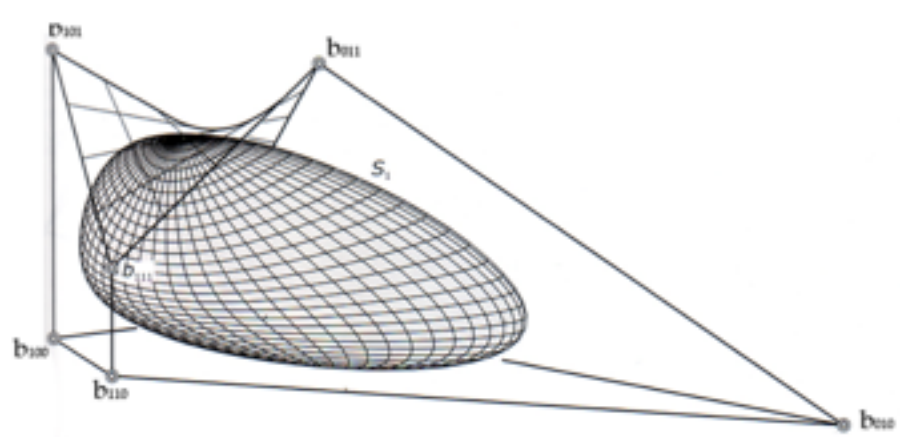


shear deformations



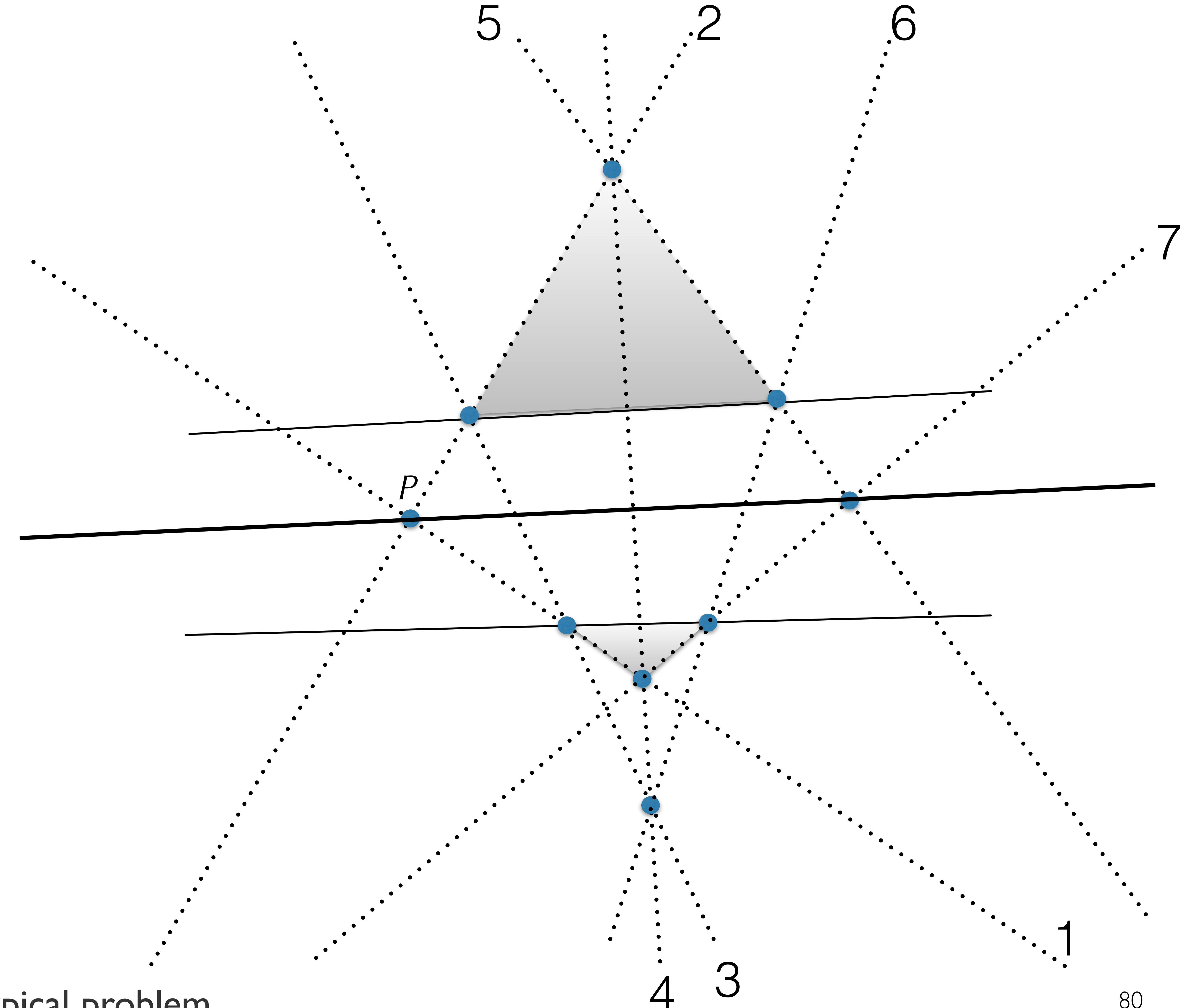
bending





free form deformations deformations

back to descriptive geometry



a typical problem