48-175
Descriptive Geometry

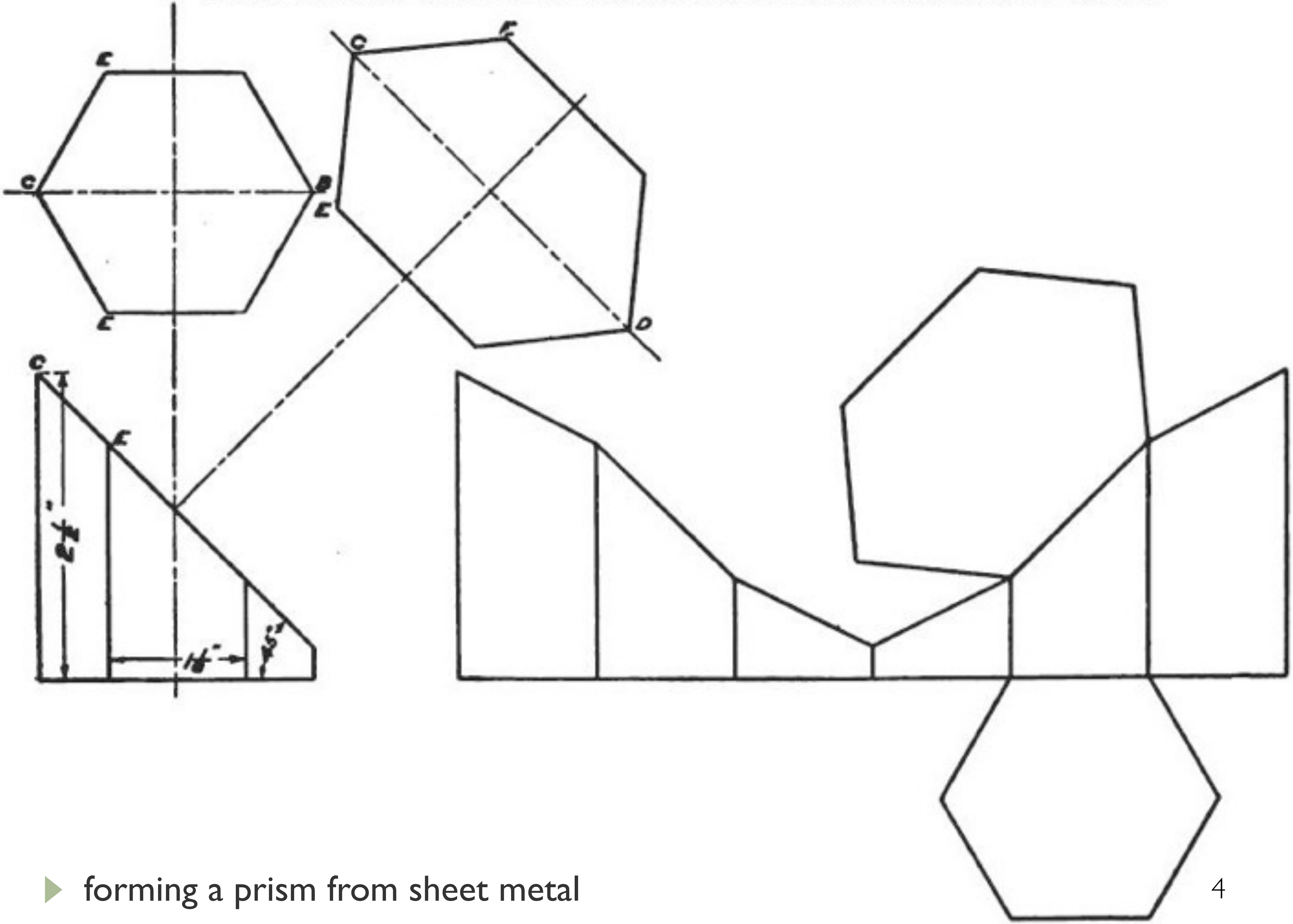
Introduction to Geometric Constructions

can you work out the area of the green area just using geometrical construction?

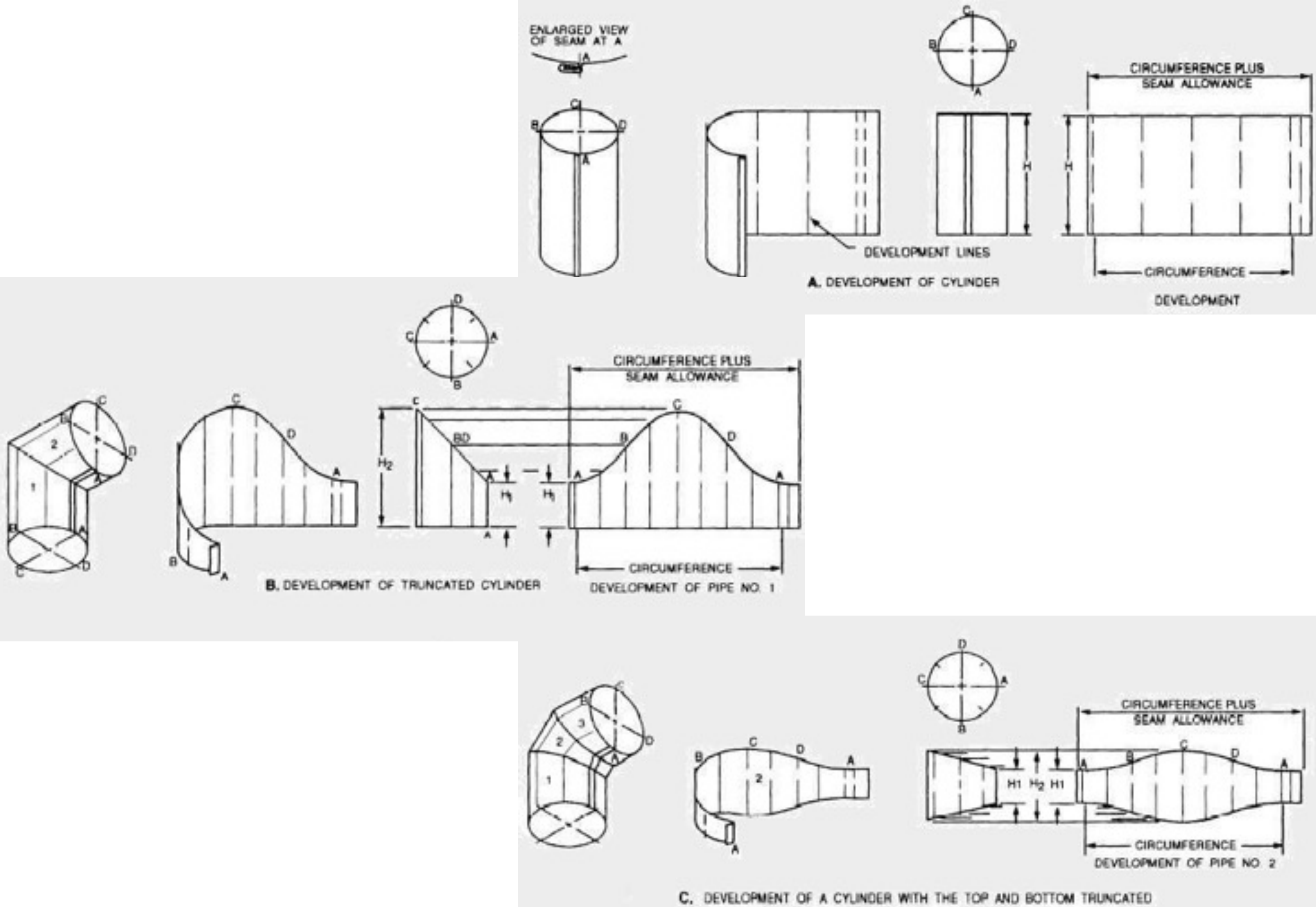
a typical problem

Development of an object

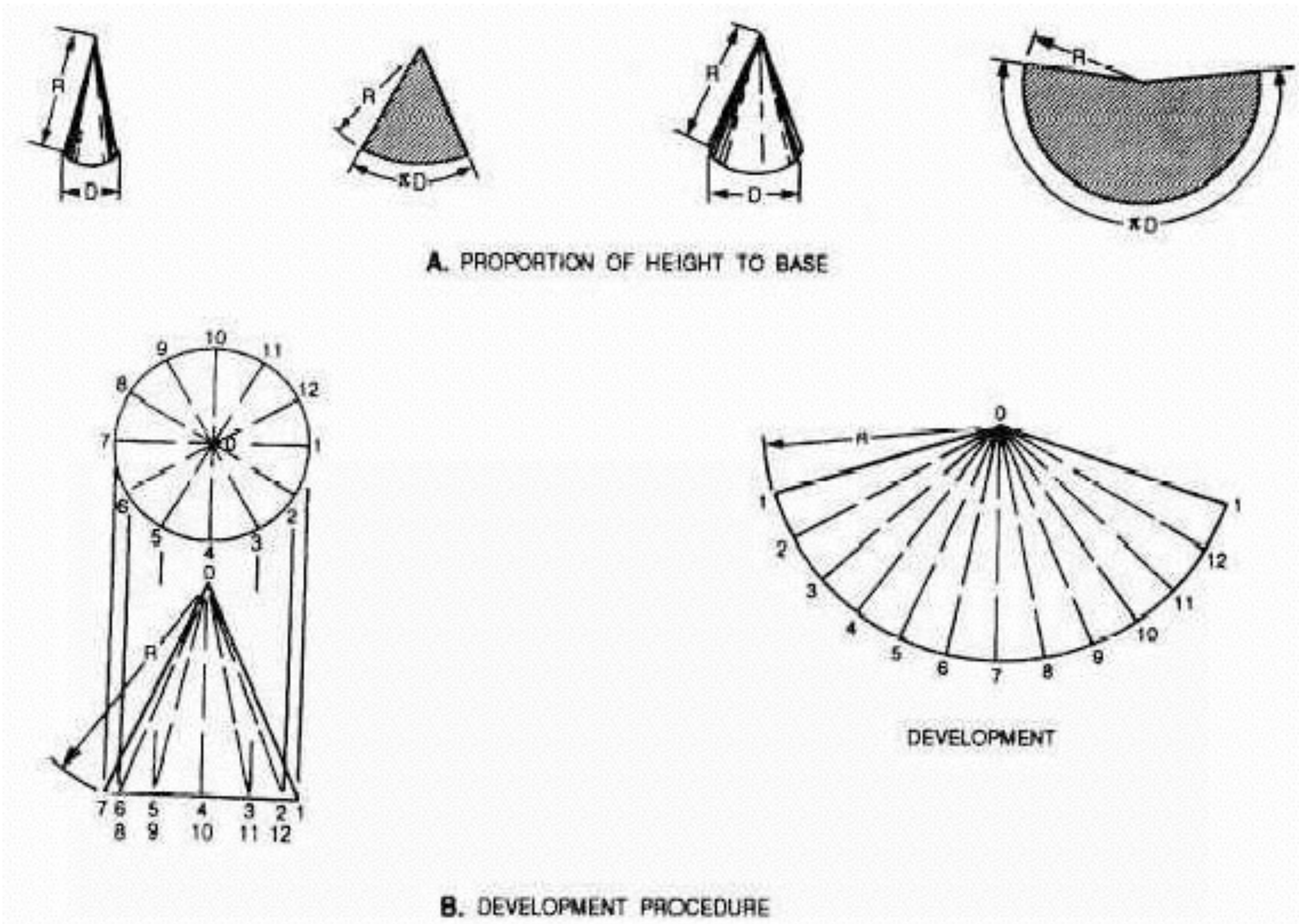
DEVELOPMENT OF A TRUNCATED HEXAGONAL PRISM



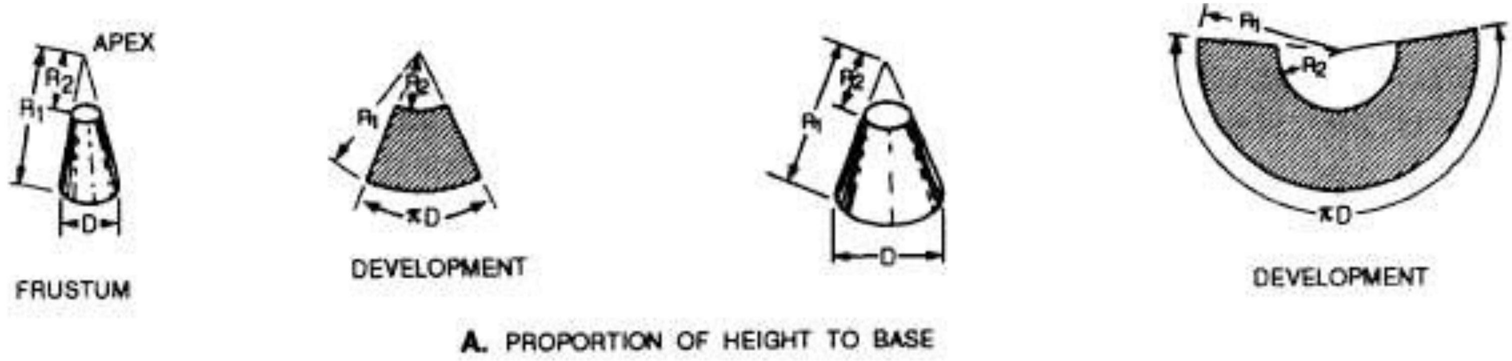
- forming a prism from sheet metal

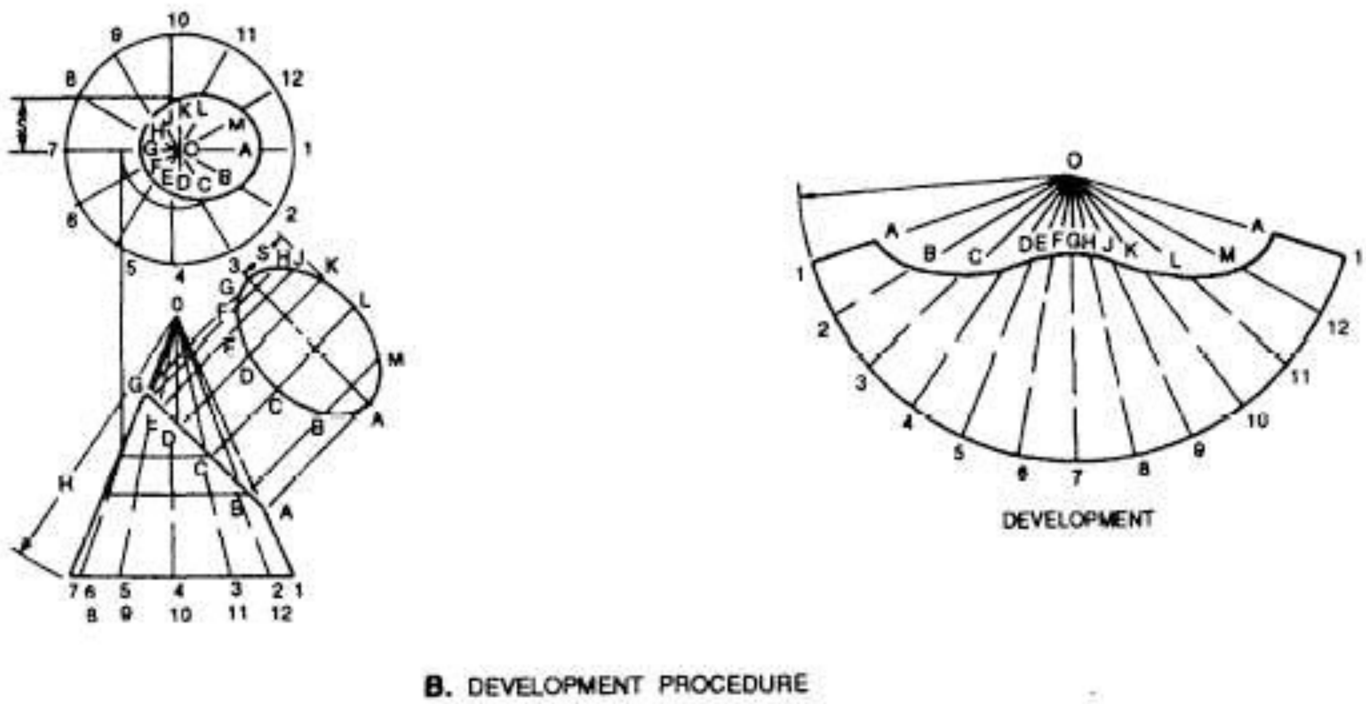


- development of a cylinder



- development of a cone





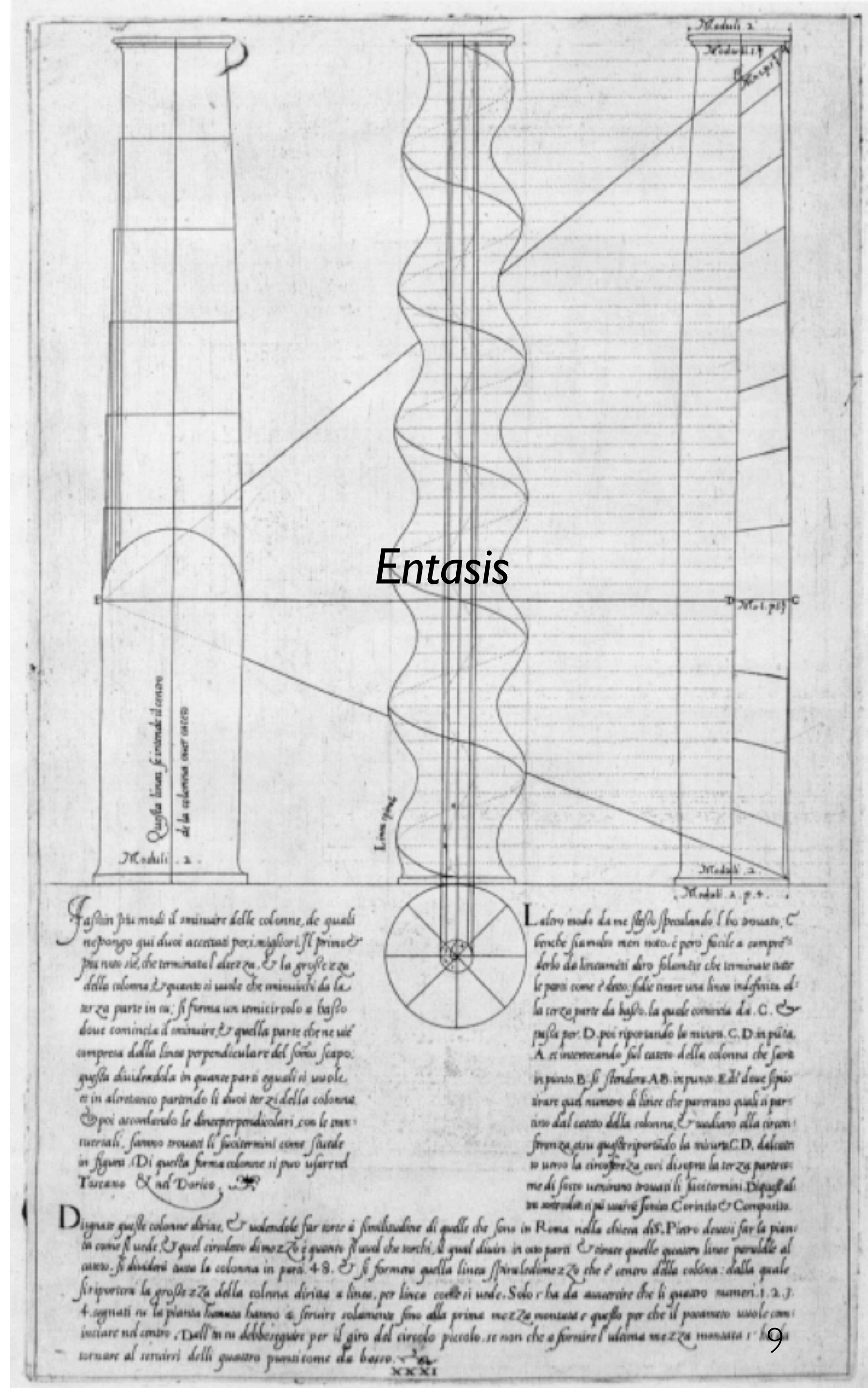
- development of a truncated cone

Canons of the Five Orders of Architecture

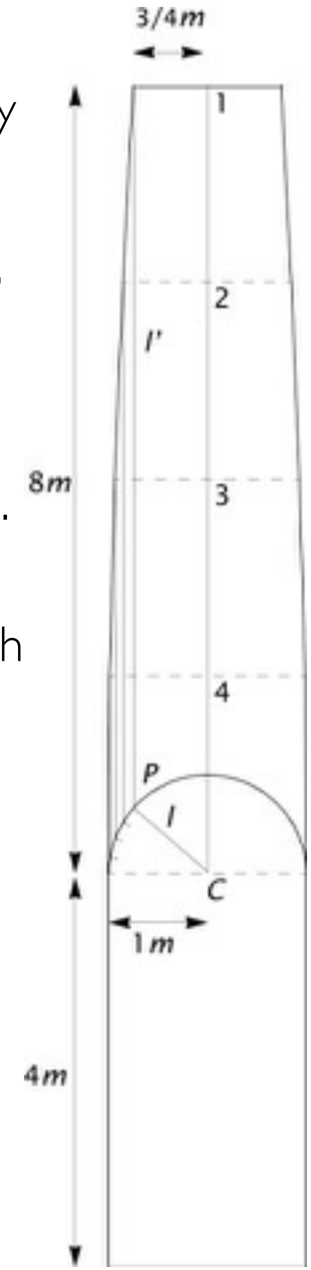
Giacomo Barozzi da Vignola

Canon of the Five Orders of Architecture

- the use of geometric tools



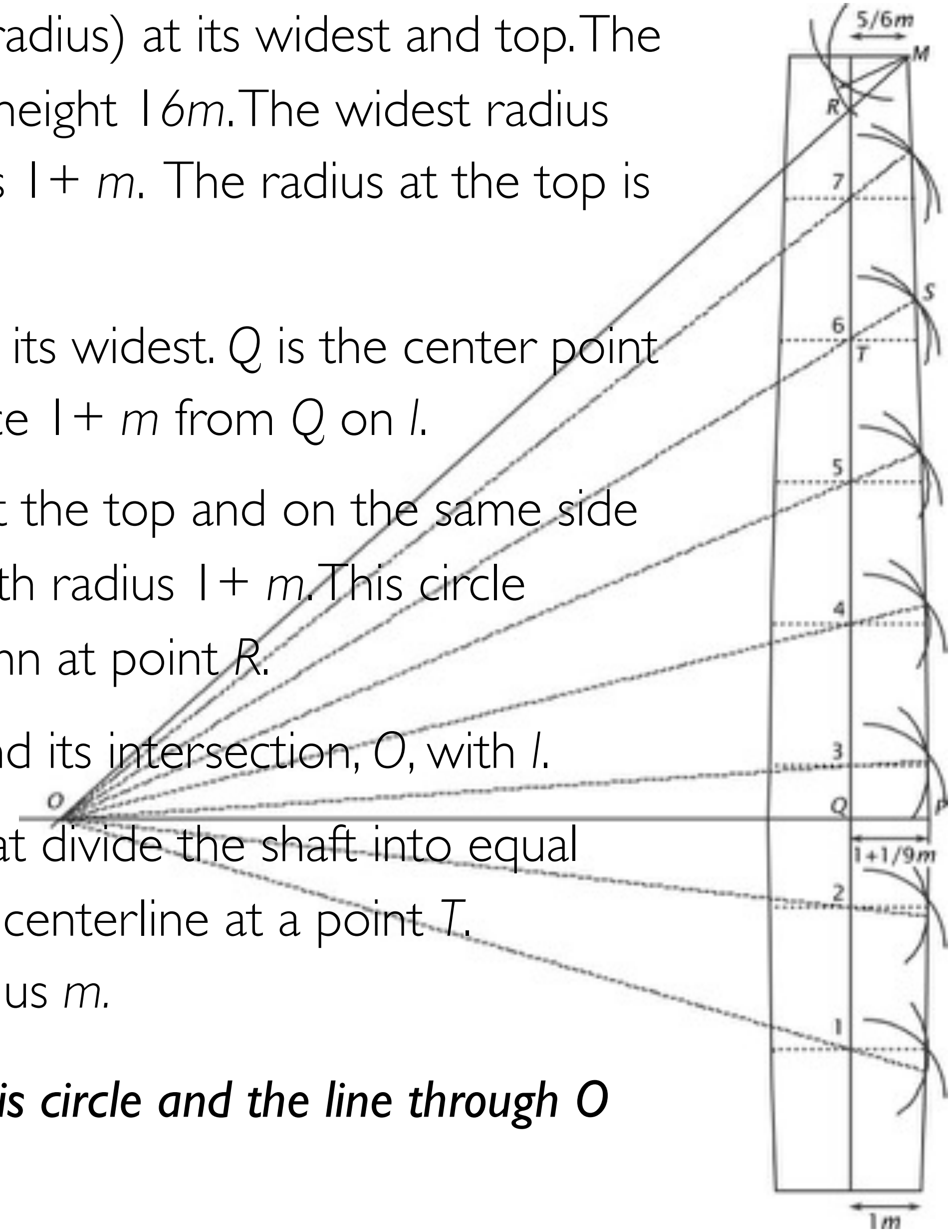
1. Determine height and largest diameter, *d*. These measures are normally integral multiples of a common module, *m*.
2. At $^{1}/_{3}$ the height, **draw a line**, *l*, across the shaft and draw a semi- circle, c, about the center point of *l*, *C*, with radius *d* (1*m*). The shaft has uniform diameter *d* below line *l*.
3. Determine smallest diameter at the top of the shaft (1.5*m* in our case). **Draw a perpendicular**, *l'*, through an end-point of the diameter. *l'* intersects c at a point *P*. The line through *P* and *C* defines together with *l* a segment of c.
4. **Divide the segment into segments of equal size** and divide the shaft above *l* into the same number of sections of equal height.
5. Each of these segments intersects *c* at a point. **Draw a perpendicular** line through each of these points and find the intersection point with the corresponding shaft division as shown. *Each intersection point is a point of the profile.*



profile of a classical tapered column

1. Determine height and diameter (or radius) at its widest and top. The base is assumed to be 2*m* wide, the height 16*m*. The widest radius occurs at rd of the total height and is 1+ *m*. The radius at the top is *m*.
2. **Draw a line**, *l*, through the column at its widest. *Q* is the center point of the column on *l* and *P* is at distance 1+ *m* from *Q* on *l*.
3. *M is* at distance *m* from the center at the top and on the same side as *P*. **Draw a circle** centered at *M* with radius 1+ *m*. This circle intersects the centerline of the column at point *R*.
4. **Draw a line** through *M* and *R* and find its intersection, *O*, with *l*.
5. **Draw a series of horizontal lines** that divide the shaft into equal sections. Any such line intersects the centerline at a point *T*. **Draw a circle** about each *T* with radius *m.*

The point of intersection, S, between this circle and the line through O and T is a point on the profile.



- profile of a classical column with *entasis*

Measurements

- width = 1,

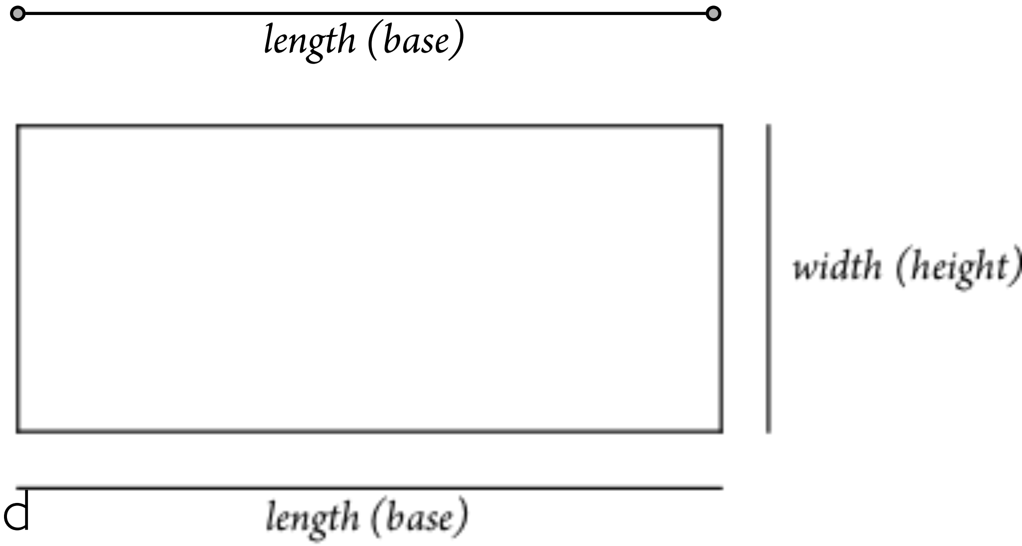
  then ***area*** = ***length***

- width = 10,

  then ***area*** = ***length*** +

  positionally add a zero at the end

  or move decimal point to the right by one position



- width = 100,

  area = ***length*** +

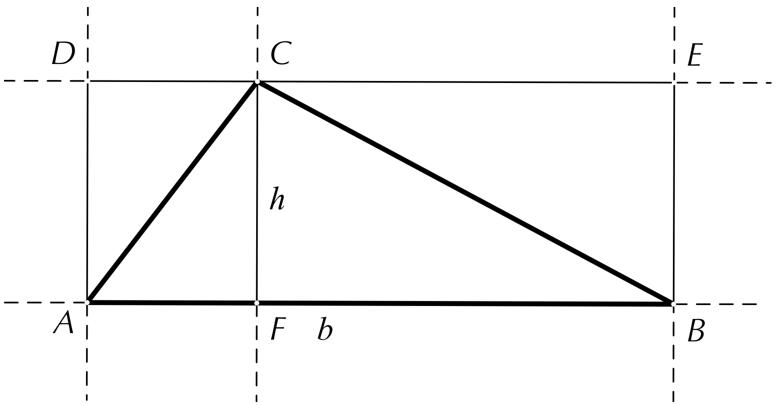
  positionally add two zeroes at the end or

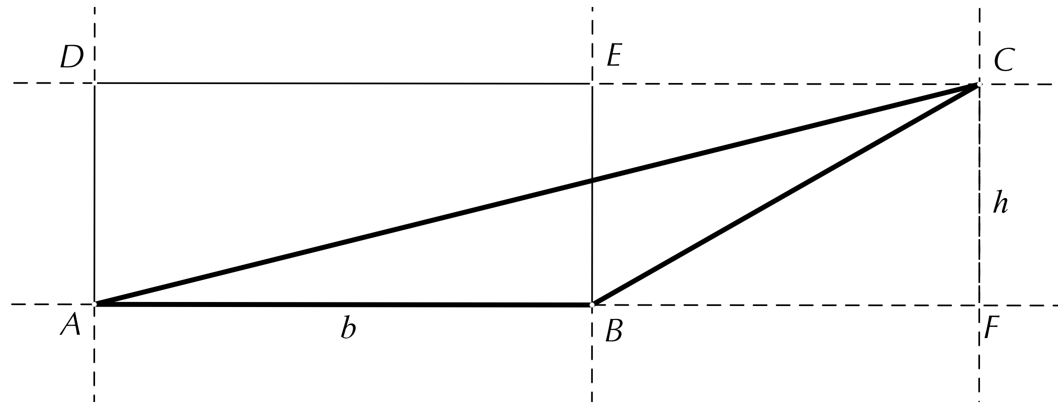
  move the decimal point to the right by two positions

- and so on …

- length can represent area

diagonal divides a rectangle into identical triangles



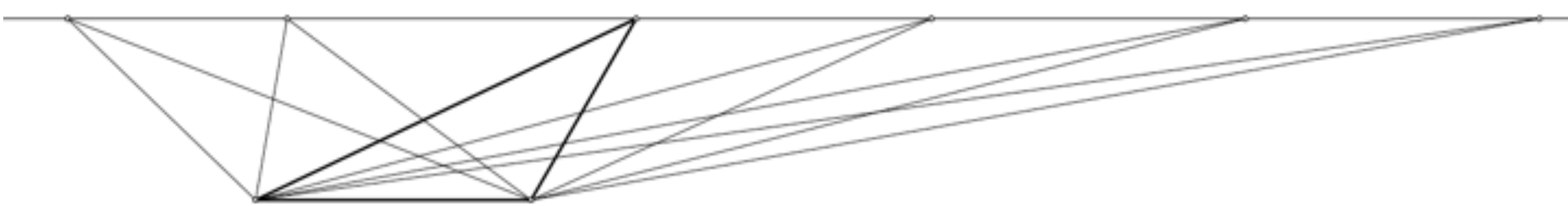


$\triangle_a ABC = \triangle_a ACF + \triangle_a CFB$

$\triangle_a ABC = ½ \square_a ADCF + ½ \square_a CEBF$
$= ½ \square_a ABED$

$\triangle_a ABC = \triangle_a ACF - \triangle_a BCF$

$\triangle_a ABC = ½ \square_a ADCF - ½ \square_a CEBF$
$= ½ \square_a ABED$



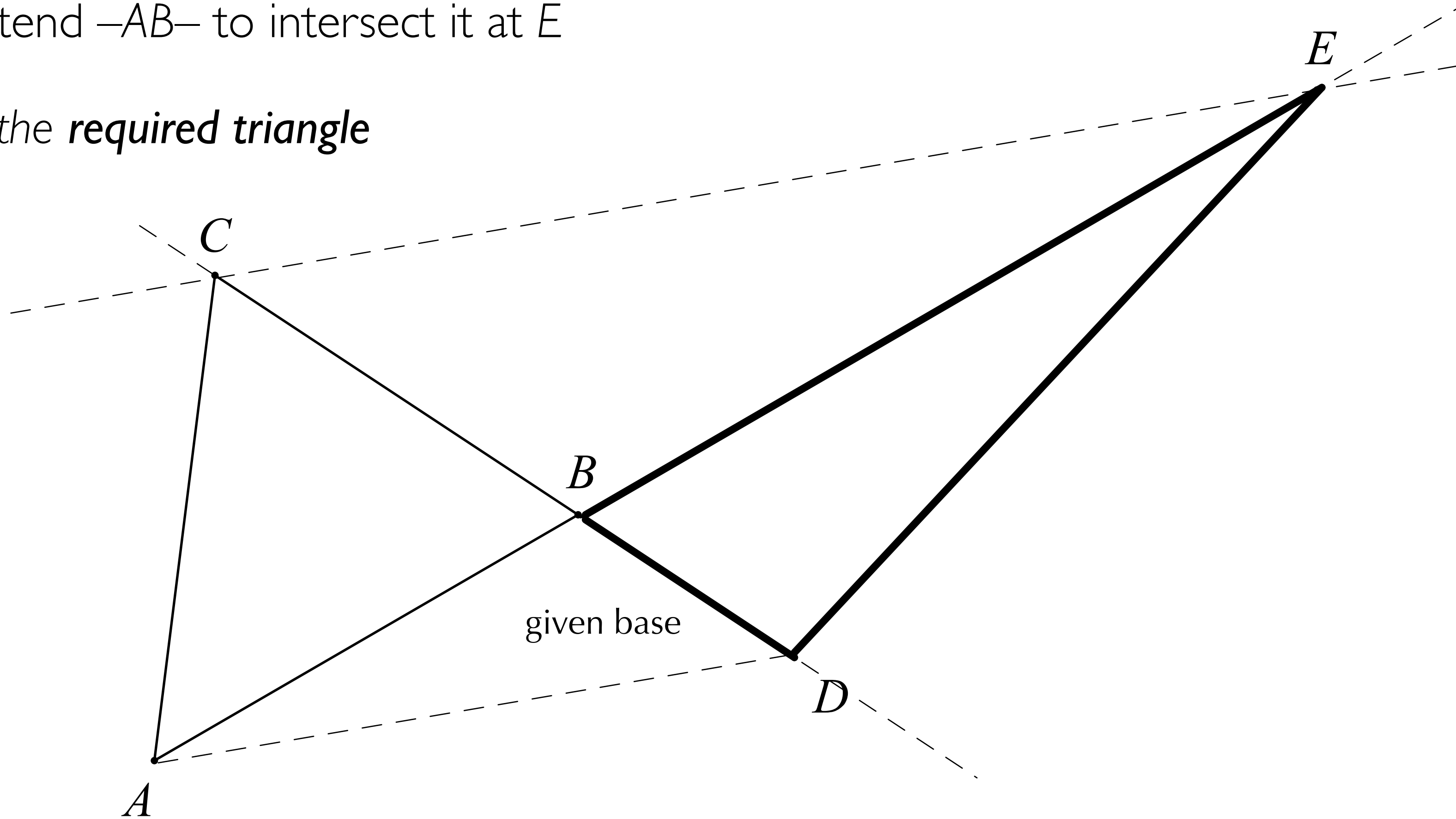
–*A*–*B*–, –*AB*–	A *line* passing through points *A* and *B*.
–*A*–	A *line* passing through point A.
A–	A *ray* emanating from point *A*
AB	The *line segment* between points *A* and *B*
AB	Length of the line segment between points *A* and *B*
$\|AB\|$	The signed length of the segment *AB*. $\|AB\| = -\|BA\|$
$\perp$	Is perpendicular, e.g., –*AB*– $\perp$ –*C*–
$\parallel$	Is parallel, e.g., –*AB*– $\parallel$ –*C*–
$O(r)$	Circle centered at point O with radius *r*.
$O(AB)$	Circle centered at O with radius equal to the length $\|AB\|$
$O(P)$	Circle centered at *O* with *P* a point on its circumference, or radius = *OP*
$\triangle ABC$	Triangle with corners *A*, *B* and *C*
$\square\ ABCD$	Quadrilateral with corners *A*, *B*, *C* and *D*
▫*ABCDEF* ...	Polygon with corners *A*, *B*, *C*, *D*, *E*, *F*, ...
$\angle BAC$	Angle at *A* defined by sides *AB* and *AC*
$\triangle_a, \square_a, \ldots$	Area of triangle, etc.

notation

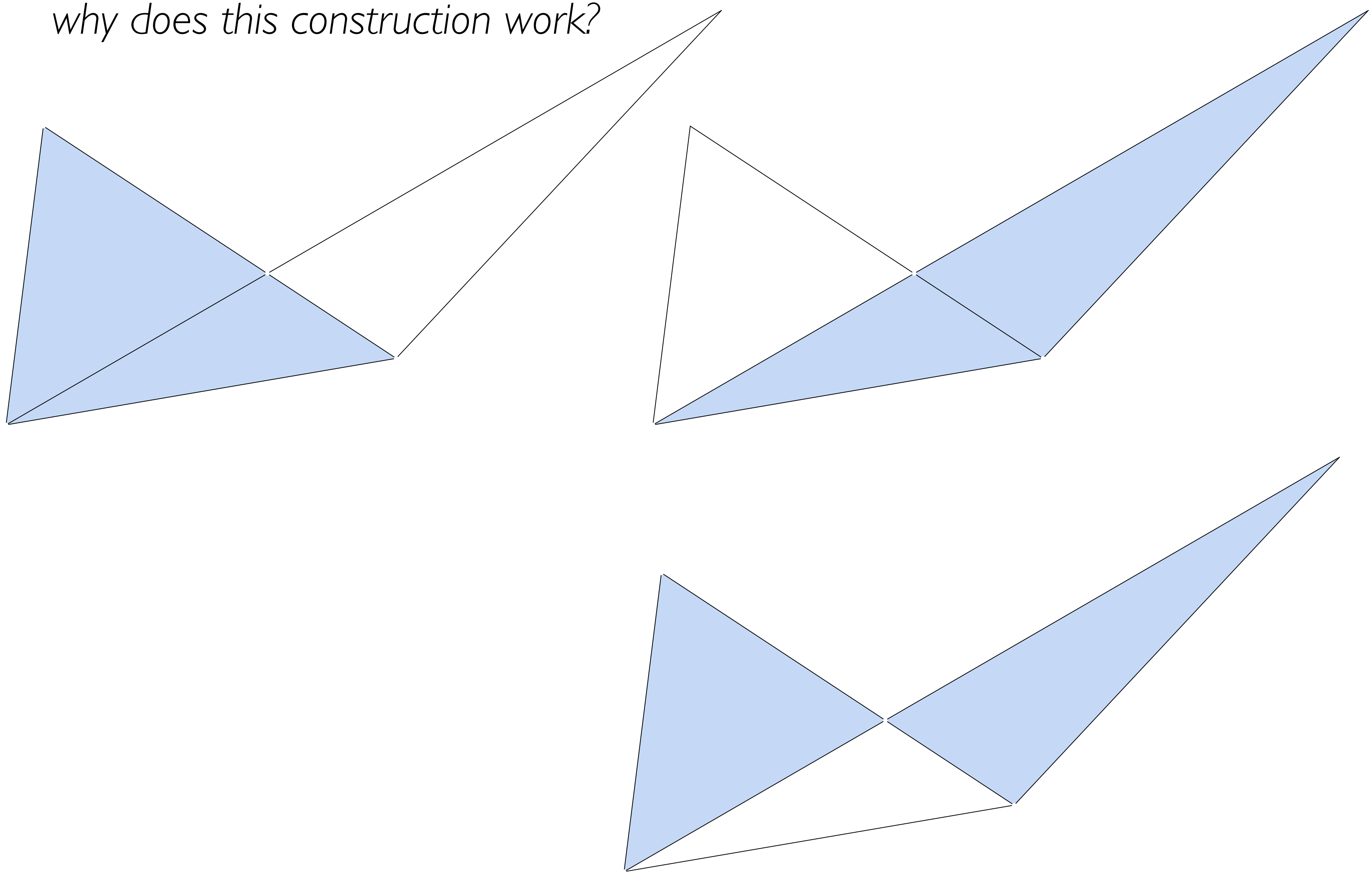
1. Extend –CB– to –CBD– so that **BD = given base**

2. Draw a line –*C*– parallel to –*AD*–, that is, –*C*– || –*AD*–; and extend –*AB*– to intersect it at *E*

△ *BED is the* ***required triangle***

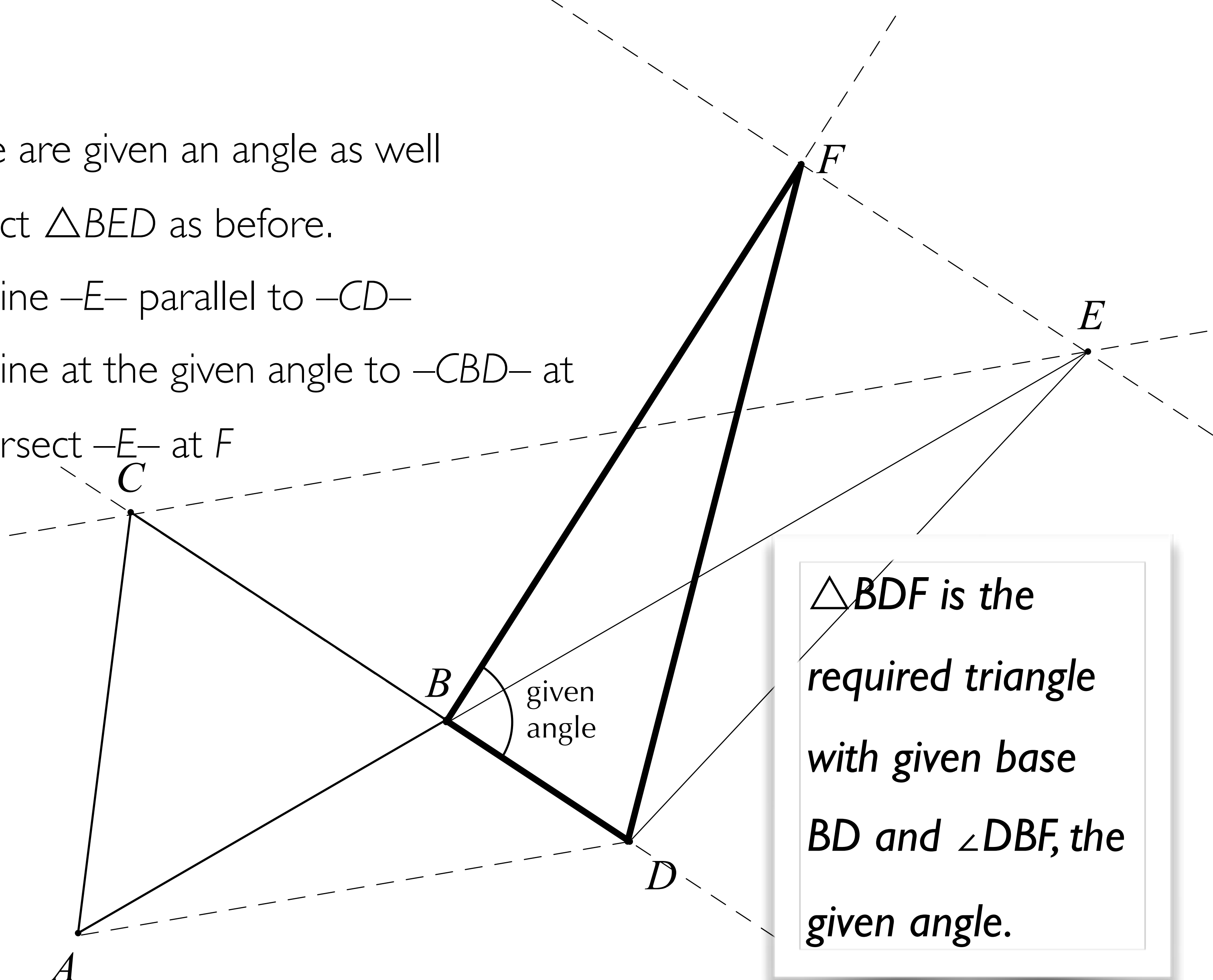


triangle of given base of equal area to another *why does this construction work?*



Suppose we are given an angle as well

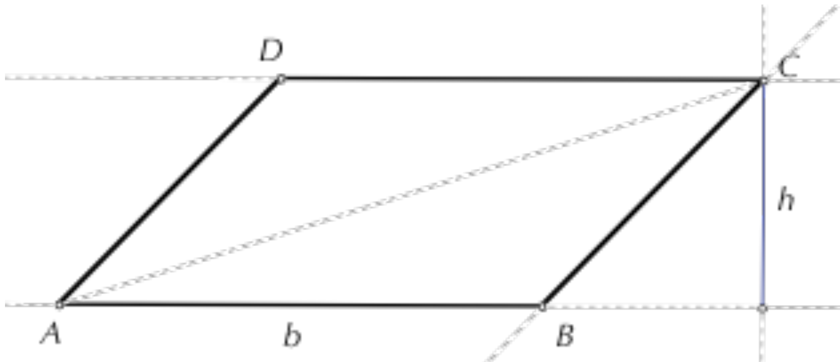
1. Construct $\triangle BED$ as before.
2. Draw a line –*E*– parallel to –*CD*–
3. Draw a line at the given angle to –*CBD*– at *B* to intersect –*E*– at *F*



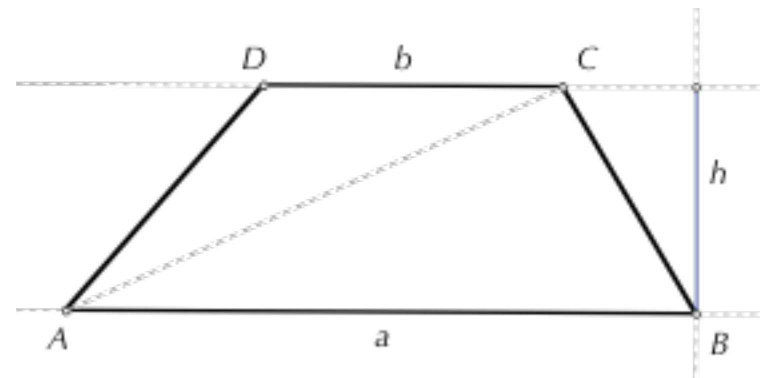
$\triangle BDF$ is the required triangle with given base BD and $\angle DBF$, the given angle.

▶ triangle of given base & angle of equal area to another

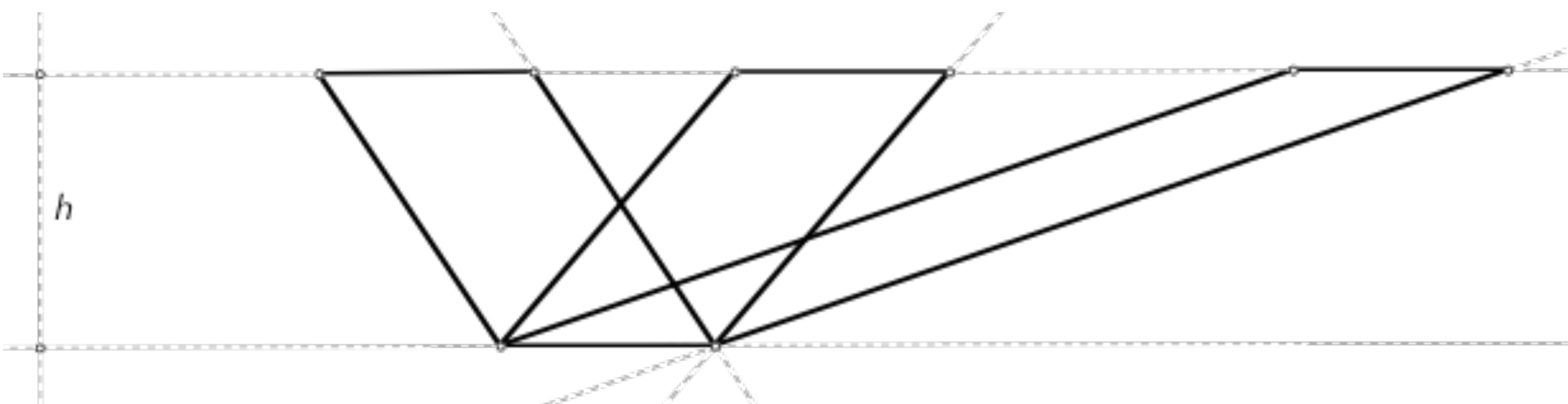
Can you find a ***single line*** *whose length equals the area of a triangle based on what we have done so far?*



$$\square_a ABCD = \triangle_a ABC + \triangle_a ADC$$
$$= \tfrac{1}{2}\, bh + \tfrac{1}{2}\, bh = bh$$



$$\square_a ABCD = \triangle_a ABC + \triangle_a ADC$$
$$= \tfrac{1}{2}\, bh + \tfrac{1}{2}\, bh = bh$$

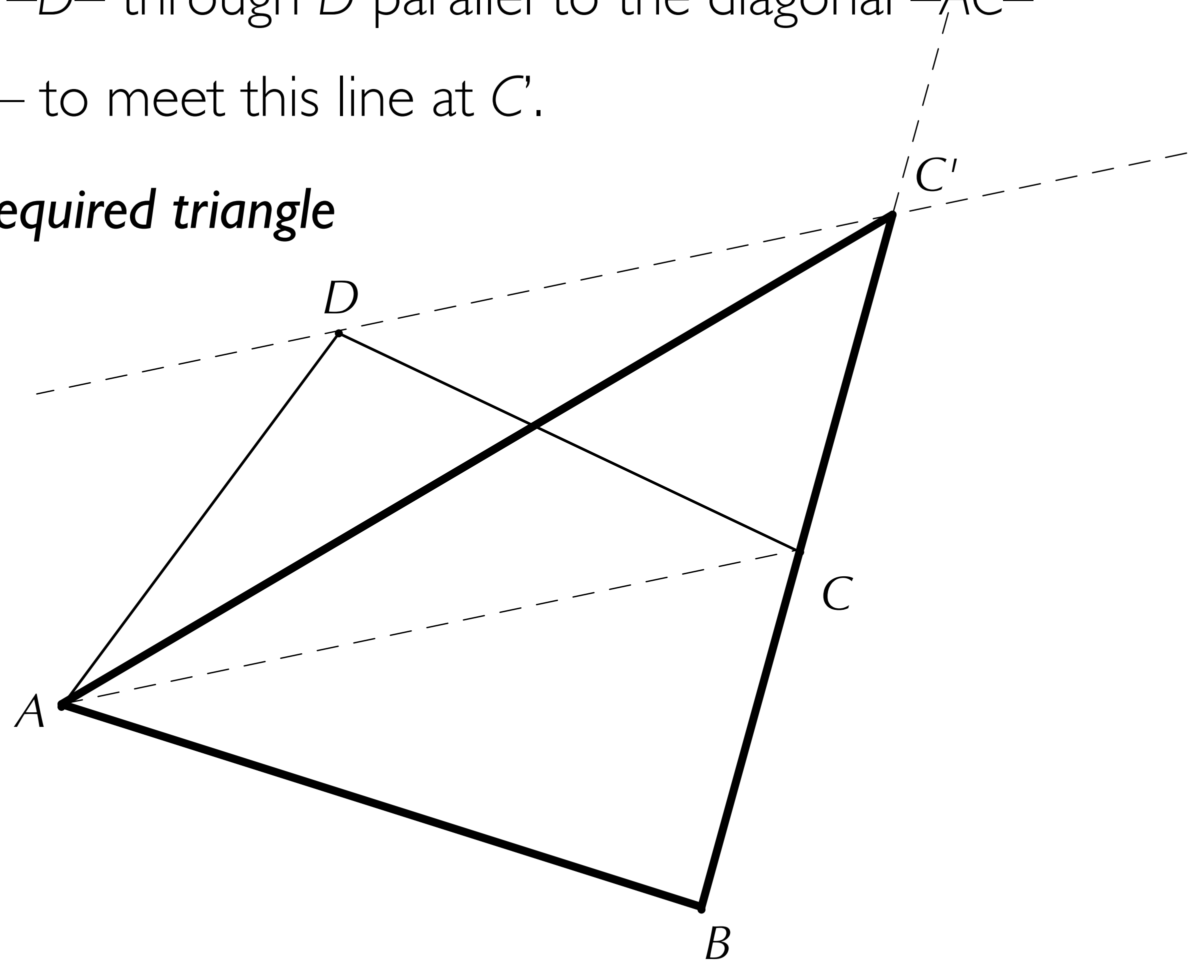


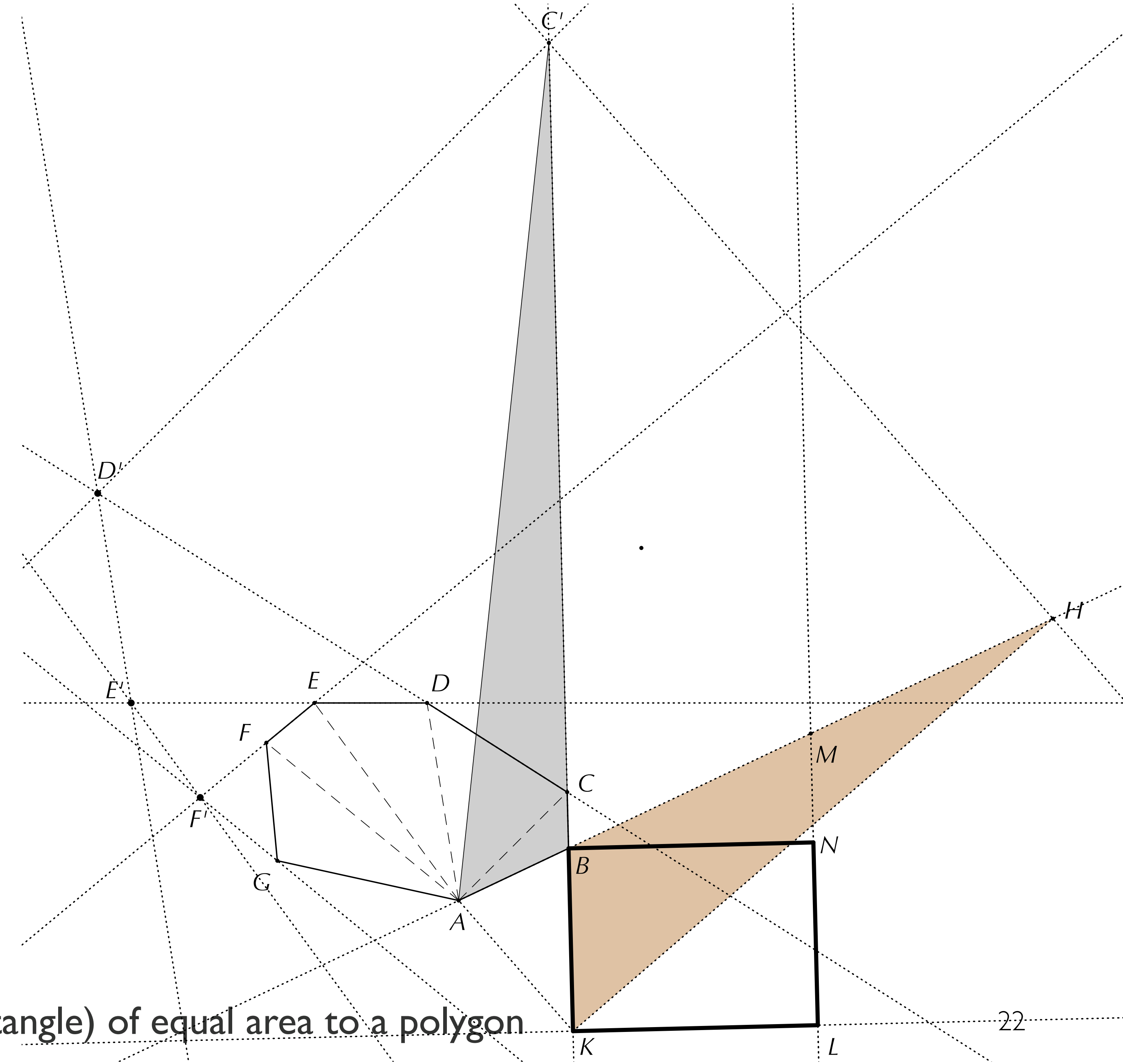
parallelograms & trapeziums

Let ▱ *ABCD* be the given quadrilateral

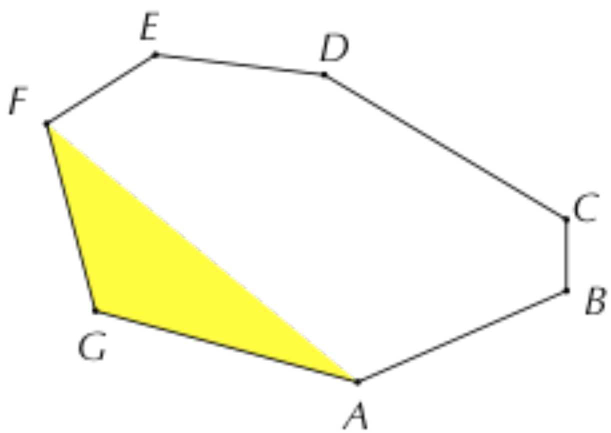
1. Draw a line –*D*– through *D* parallel to the diagonal –*AC*–
2. Extend –*BC*– to meet this line at *C*'.

△ *ABC' is the* ***required triangle***

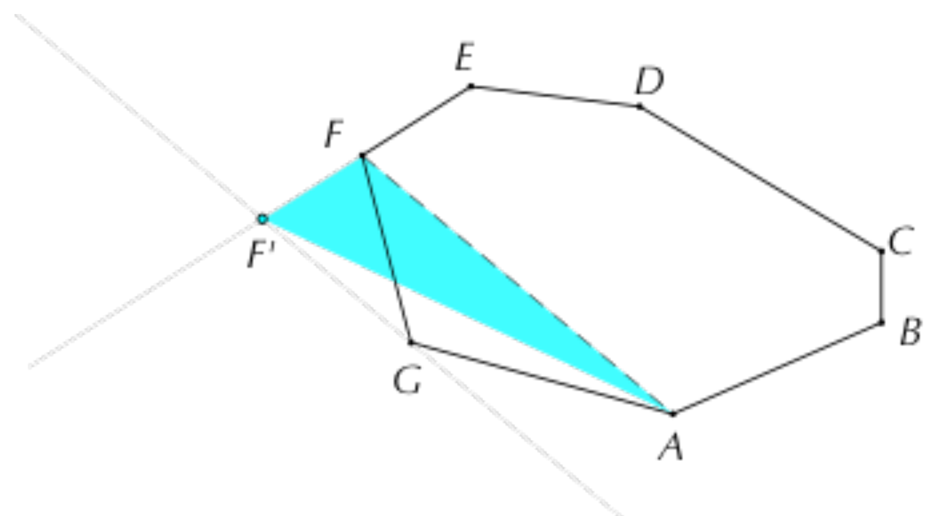




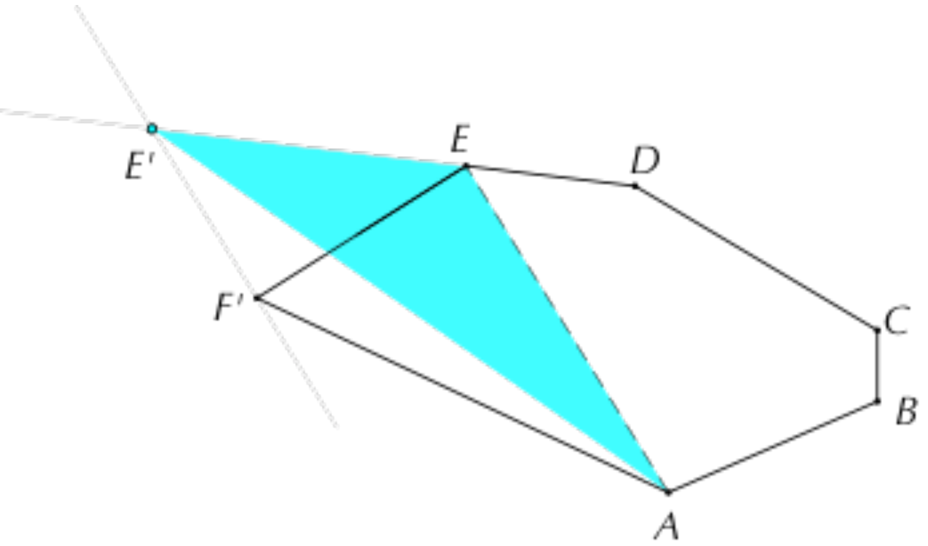
- triangle(rectangle) of equal area to a polygon

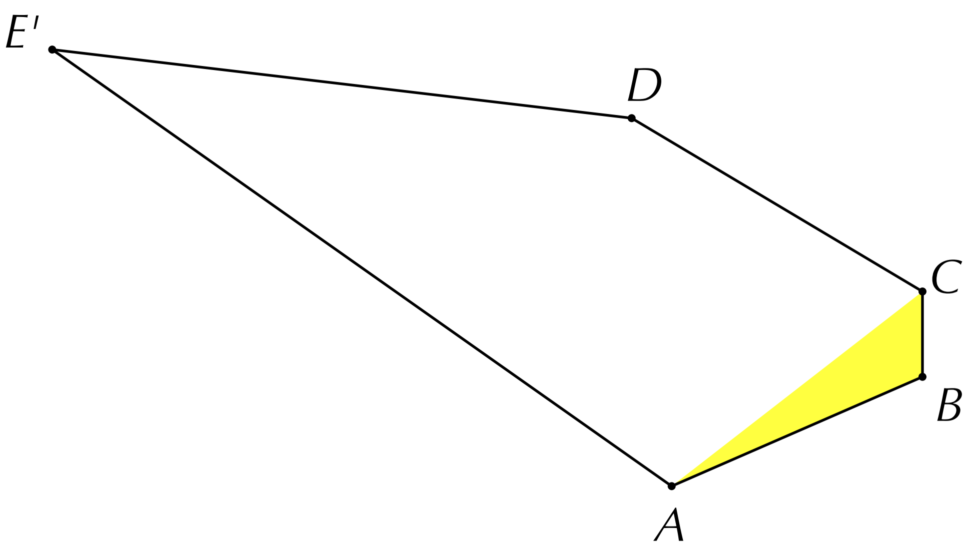


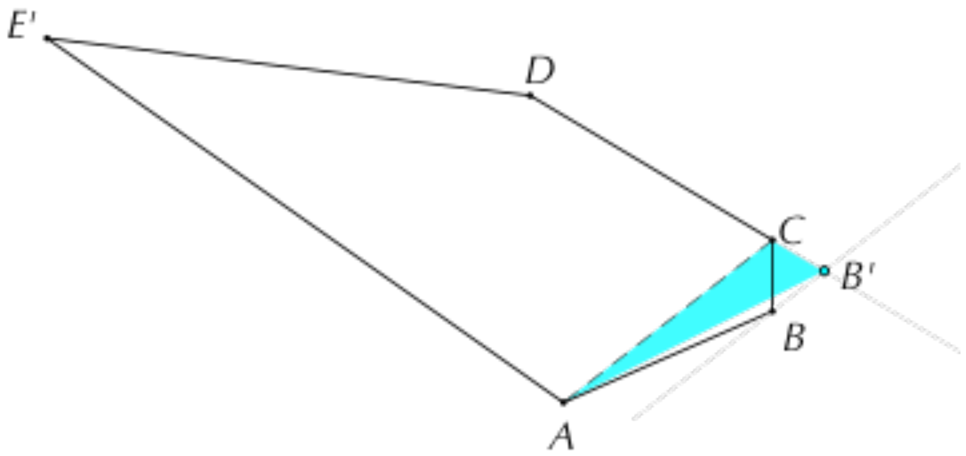
E
D
F
C
B
G
A

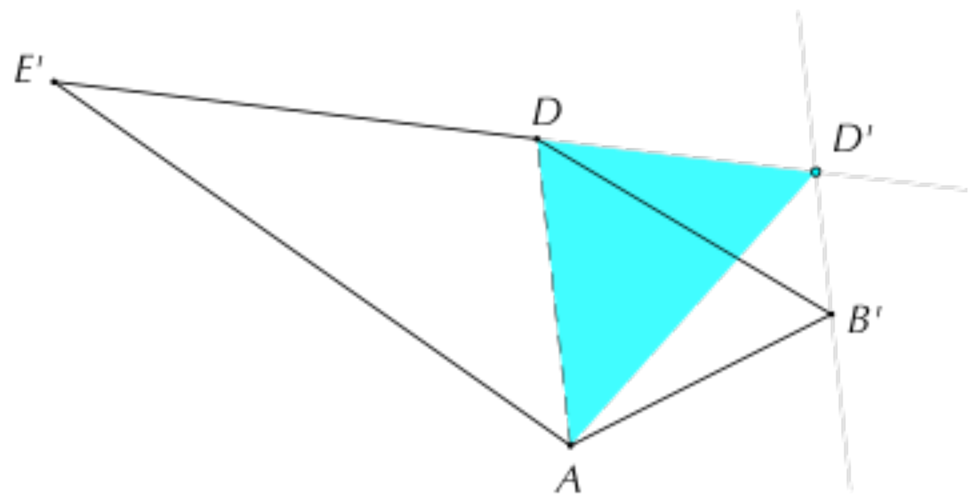


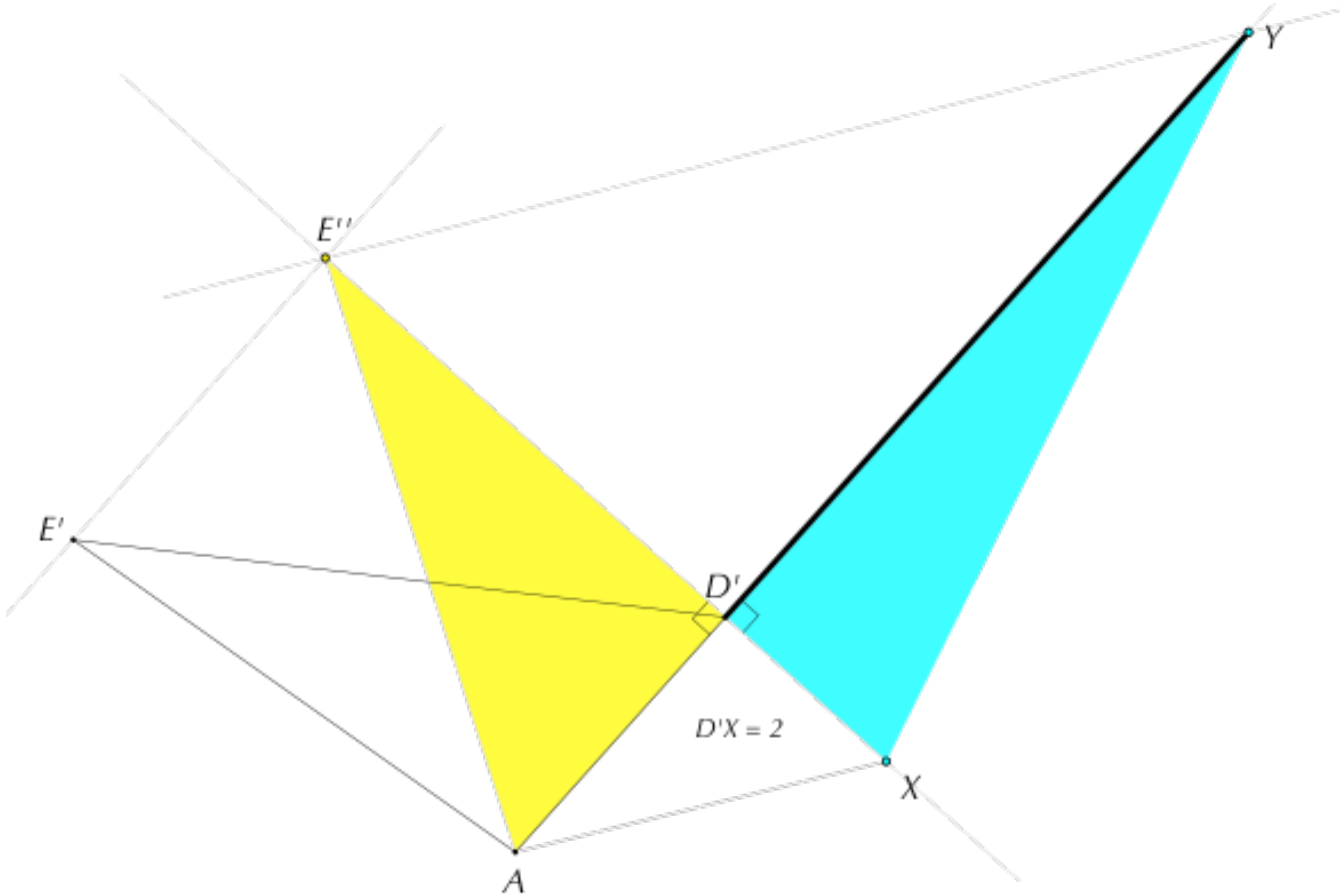
E
D
F
C
F'
B
G
A

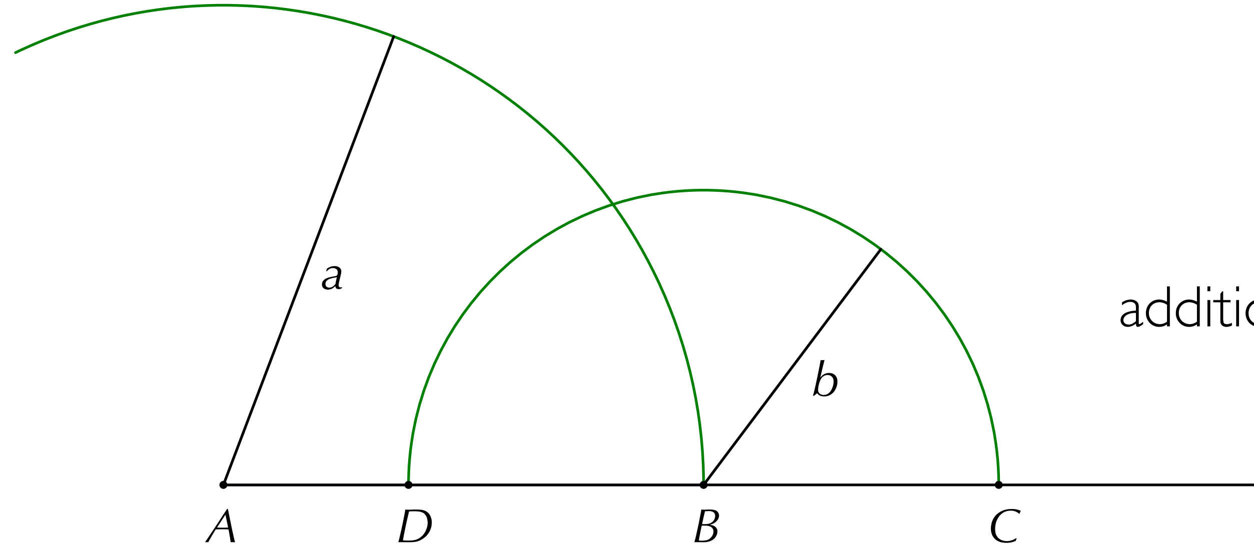

E
D
E'
F'
C
B
A


E'
D
C
B
A


E'
D
C
B'
B
A

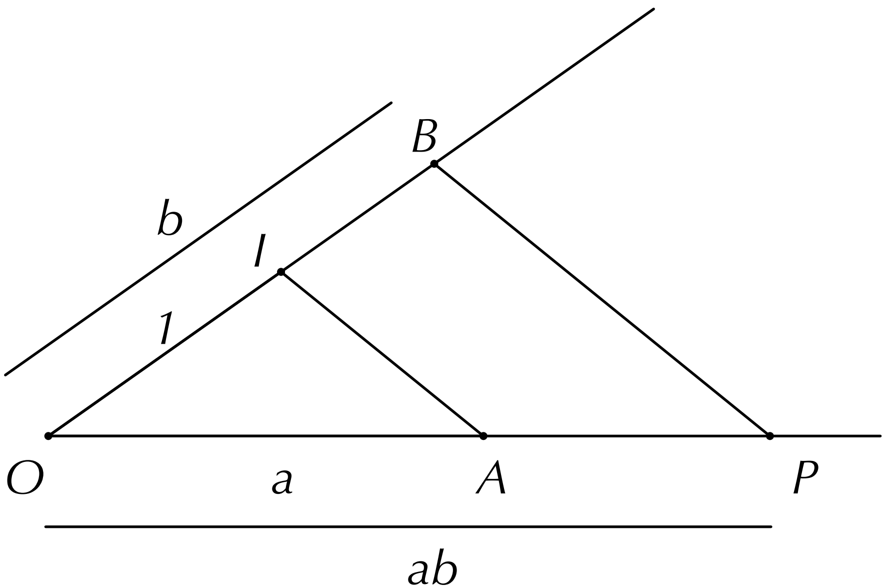

E'
D
D'
B'
A

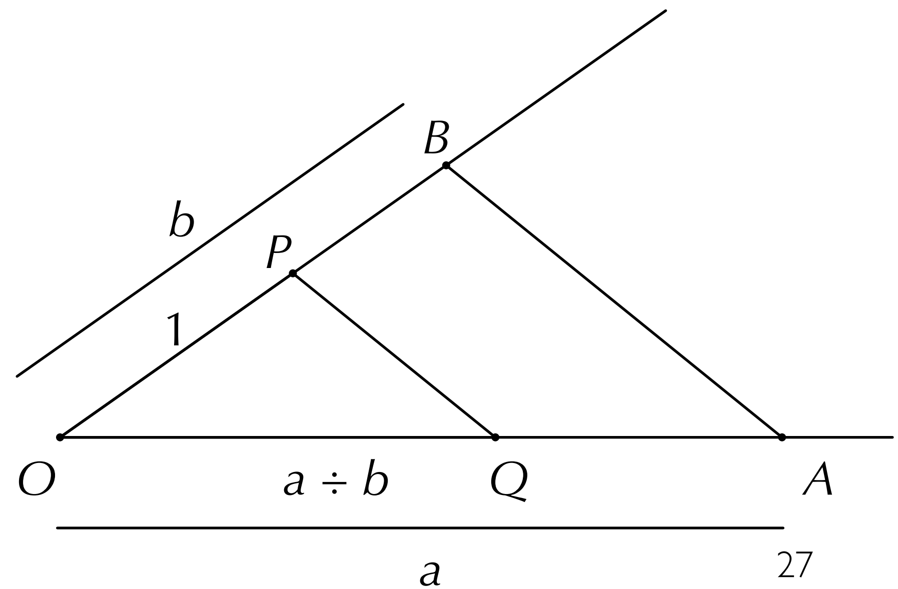

Y
E''
E'
D'
D'X = 2
X
A



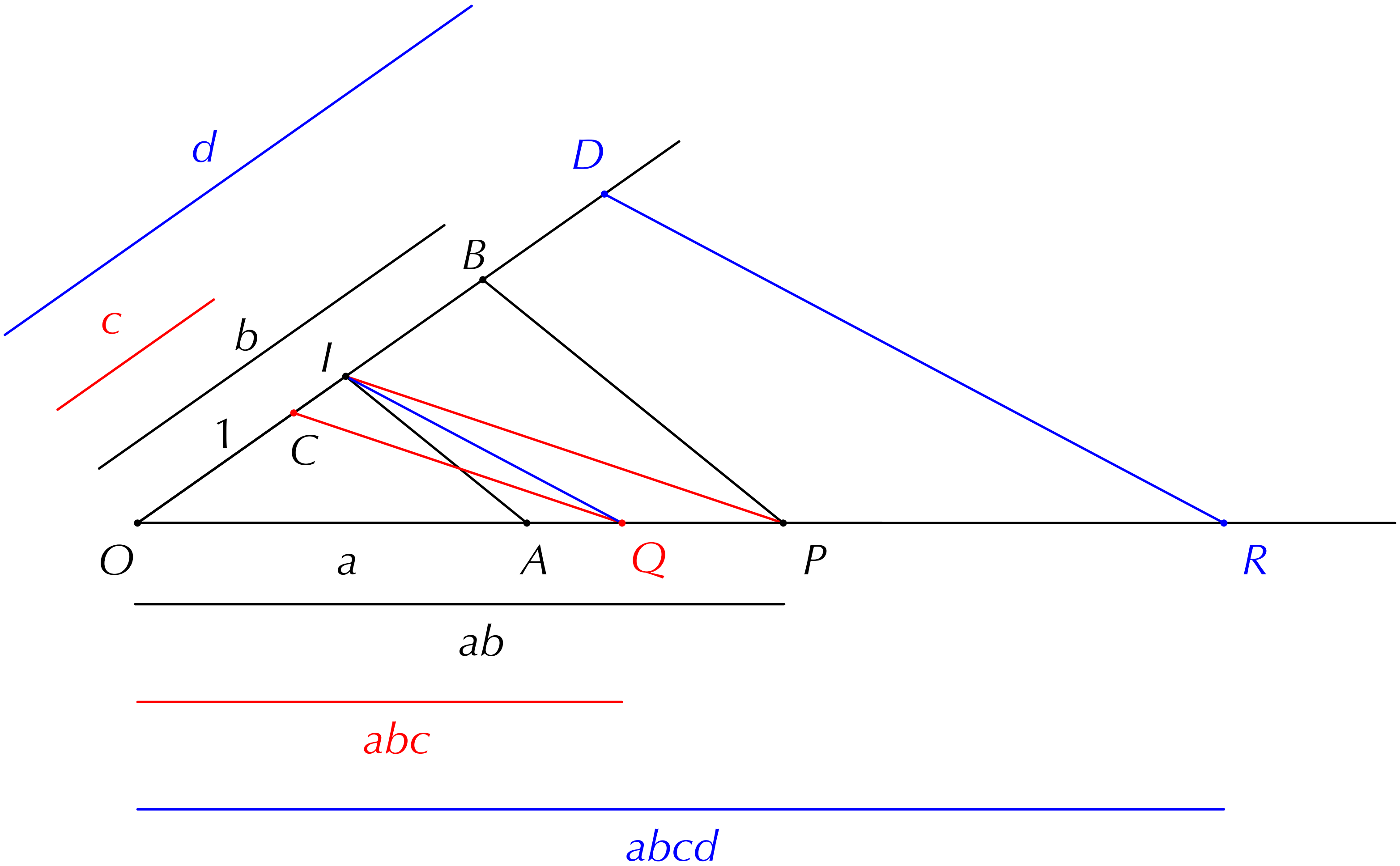
addition & subtraction

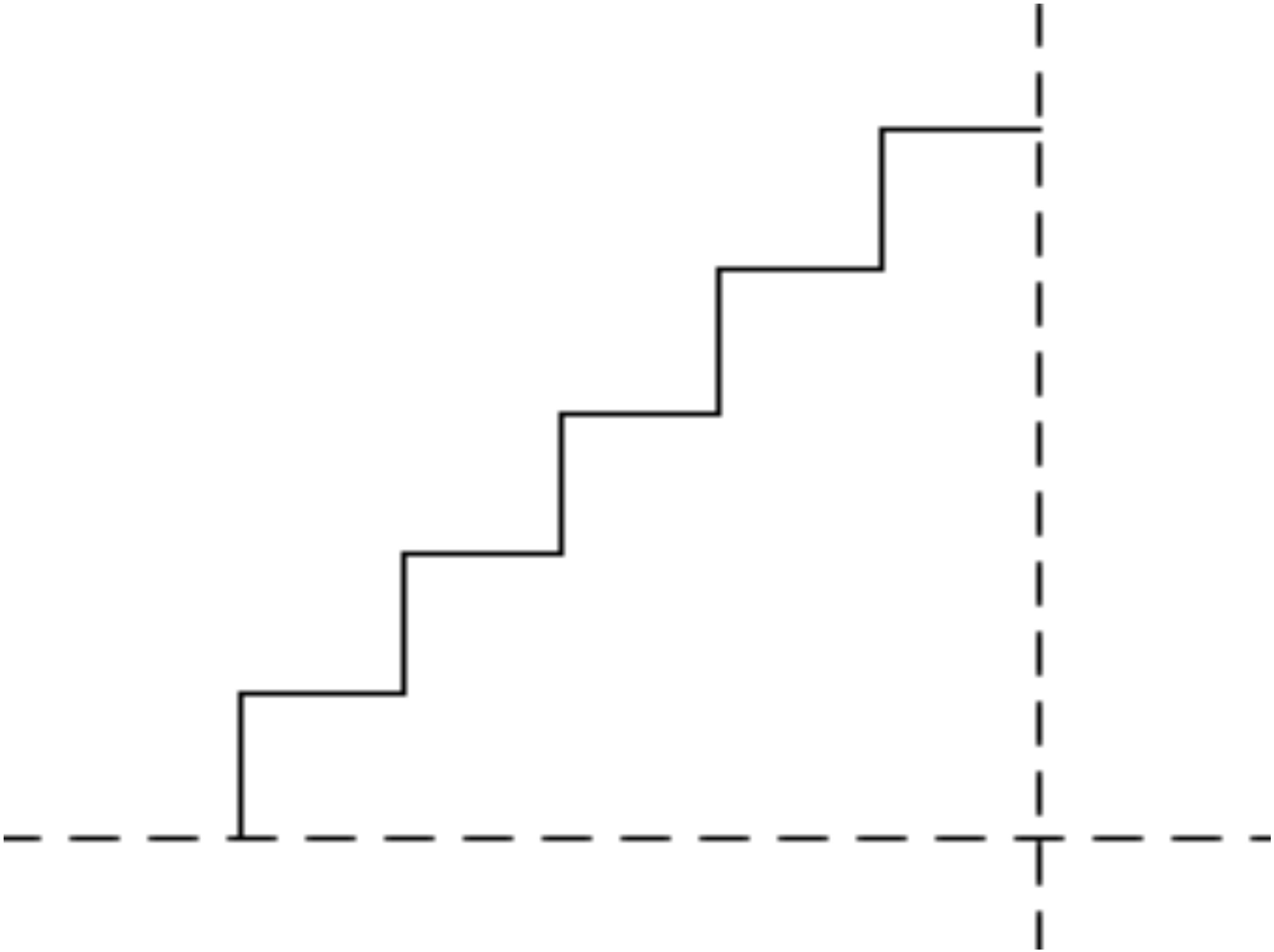
multiplication & division



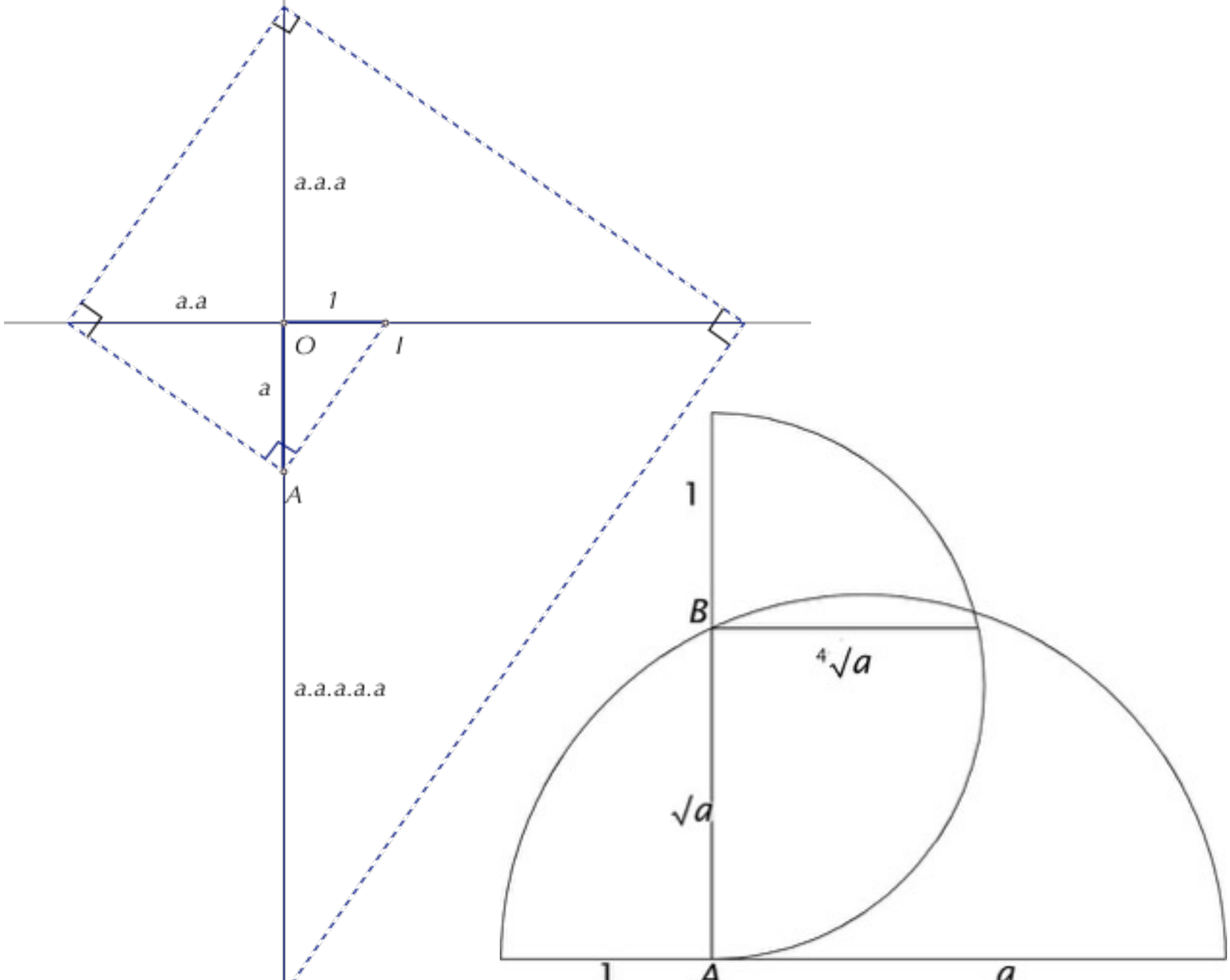


- constructible numbers


d
D
B
c
b
I
1
C
O
a
A
Q
P
R
ab
abc
abcd

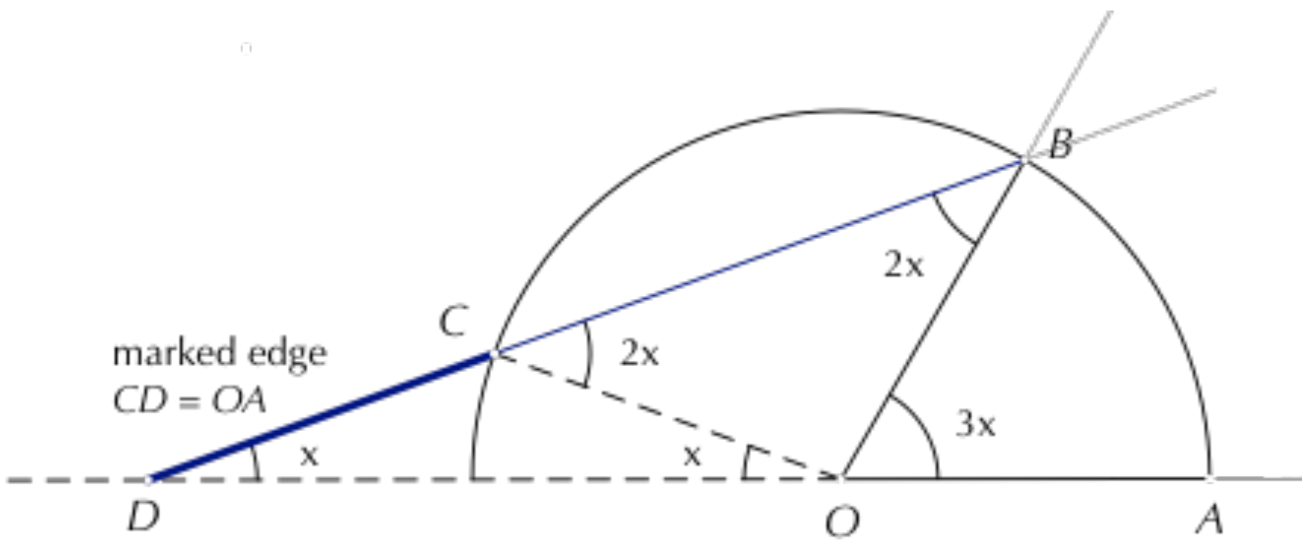


- can you construct this staircase?

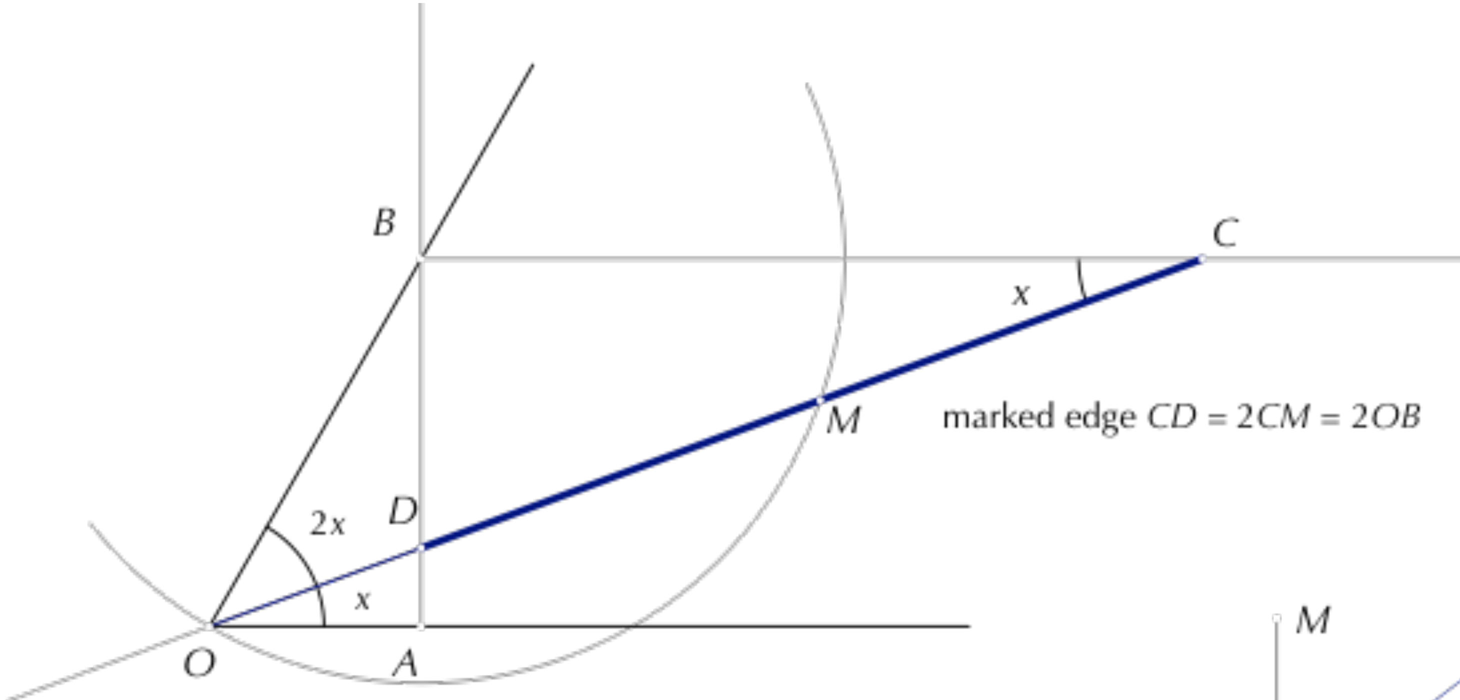


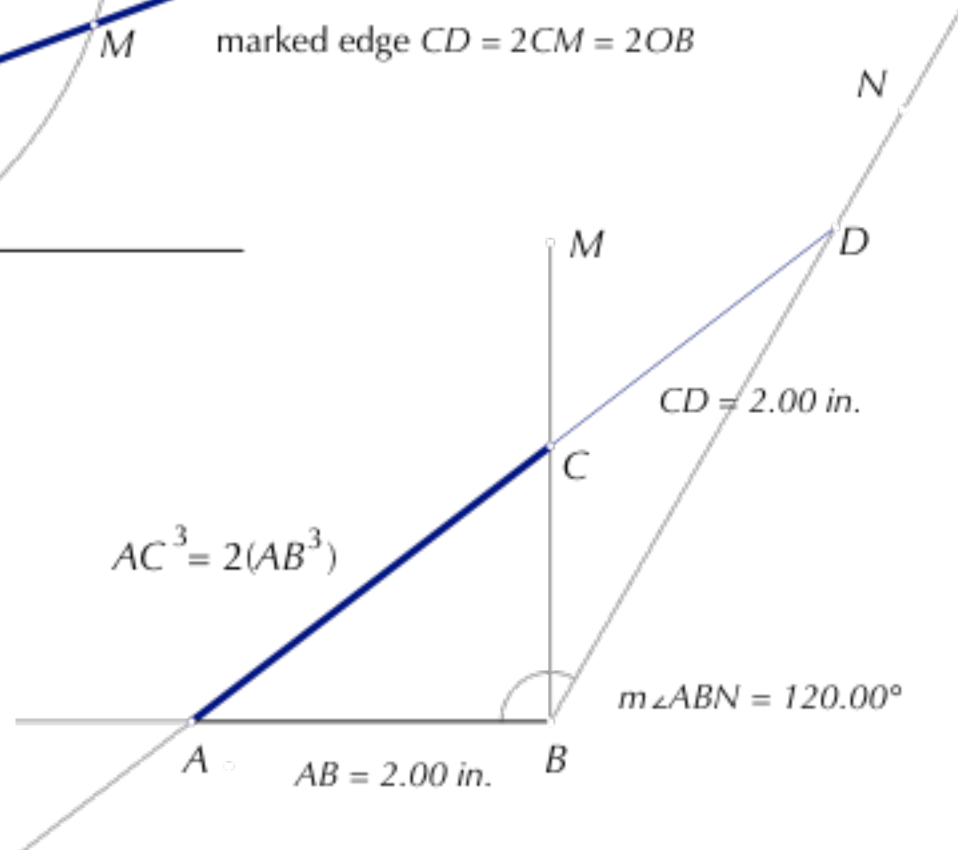
- powers and roots

Given	∠*AOB* (without loss in generality, let *OA* = *OB*)		
Draw	*O*(*OA*), –*BCD*–, *CD*=*OA*, *OC*=*OA*	–*OA*–, –*BD*–	∠*ADB*
Points of intersection	*C*	*D*	

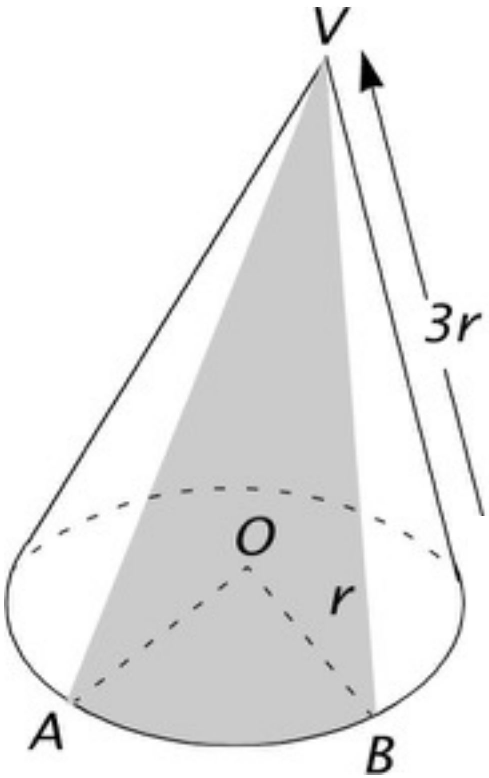


- 'impossible' constructions

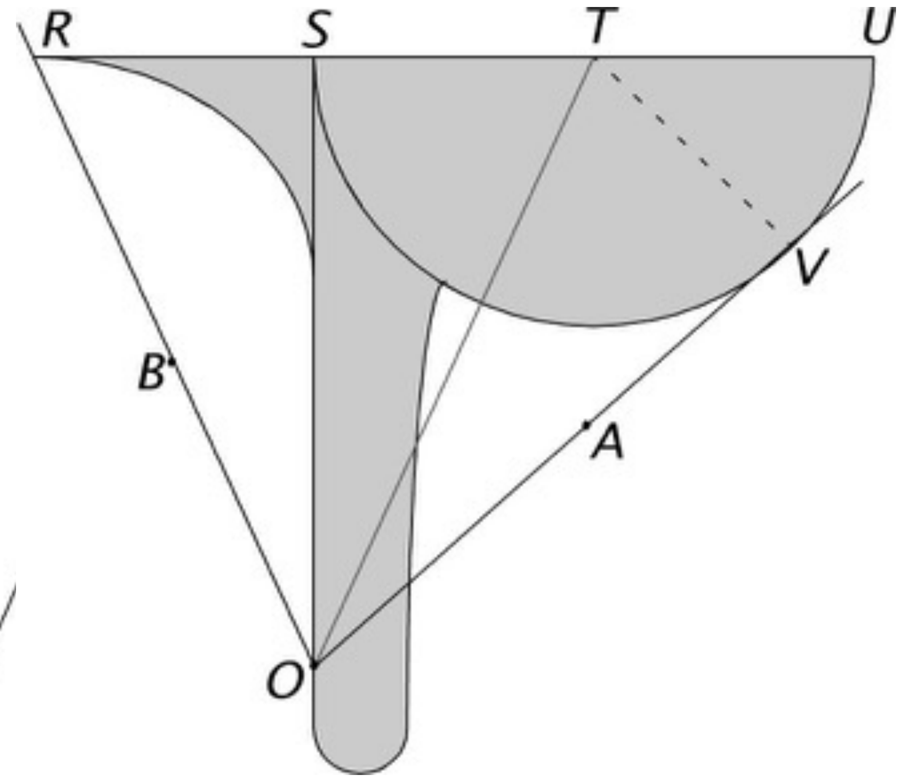


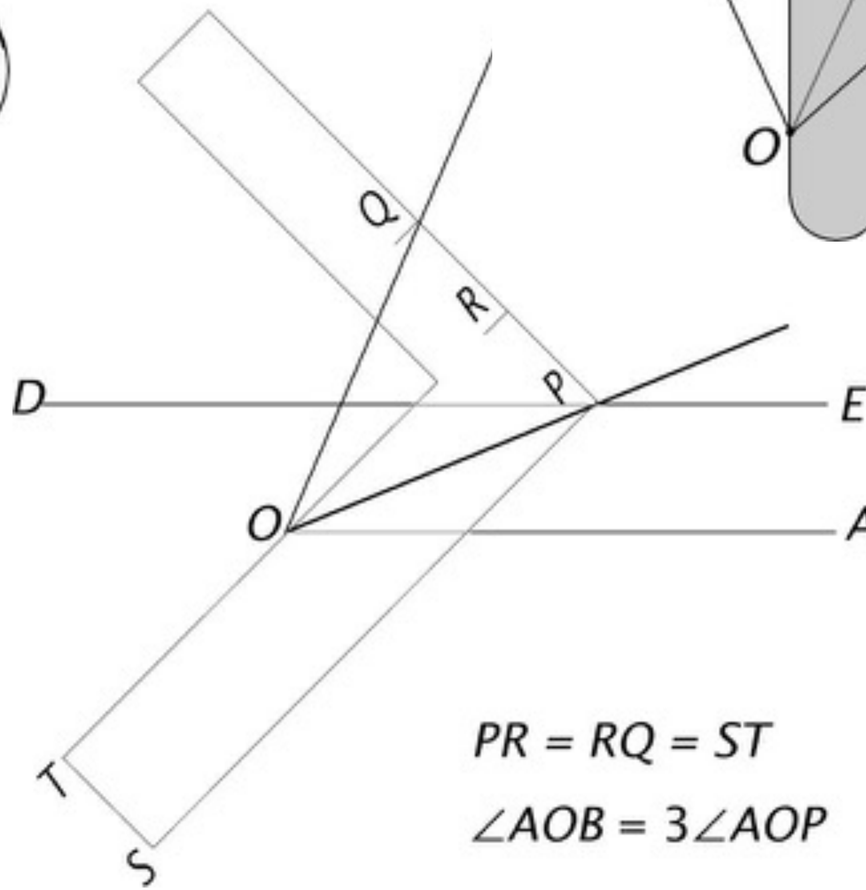


- more ‘impossible’ construction



$$AB = r\angle AOB = 3r\angle AVB$$

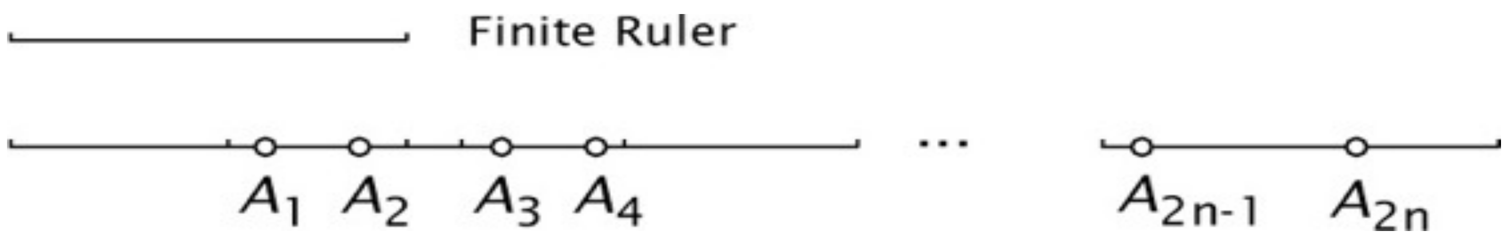


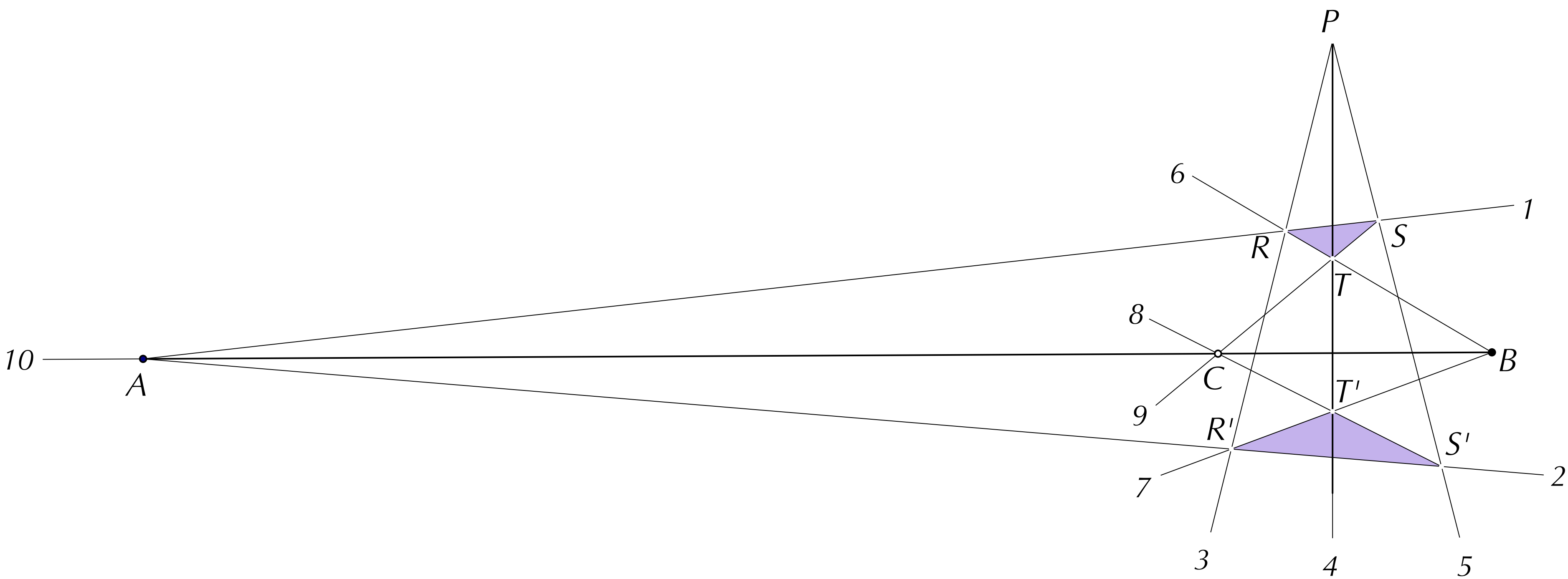


$$PR = RQ = ST$$

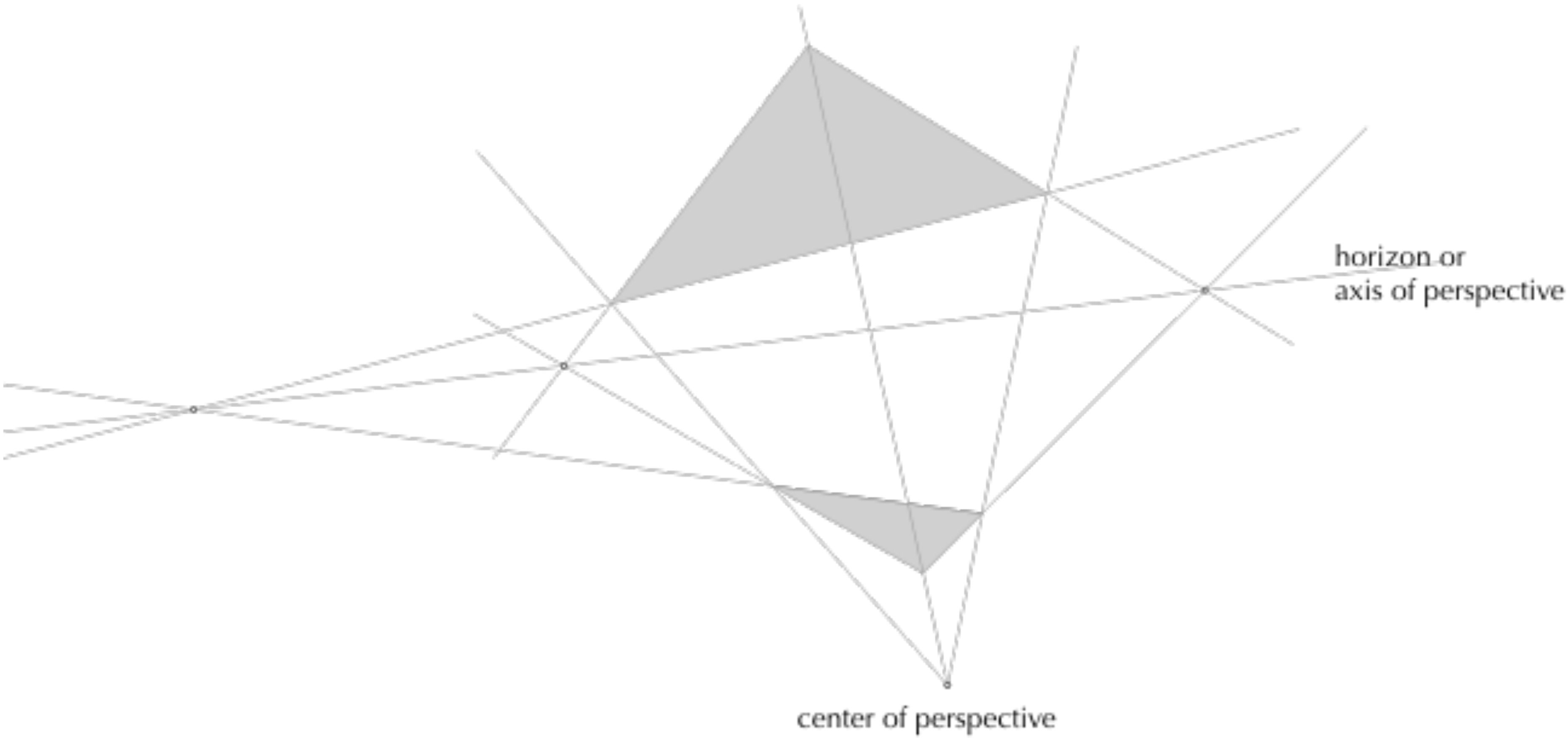
$$\angle AOB = 3\angle AOP$$

- practical tools

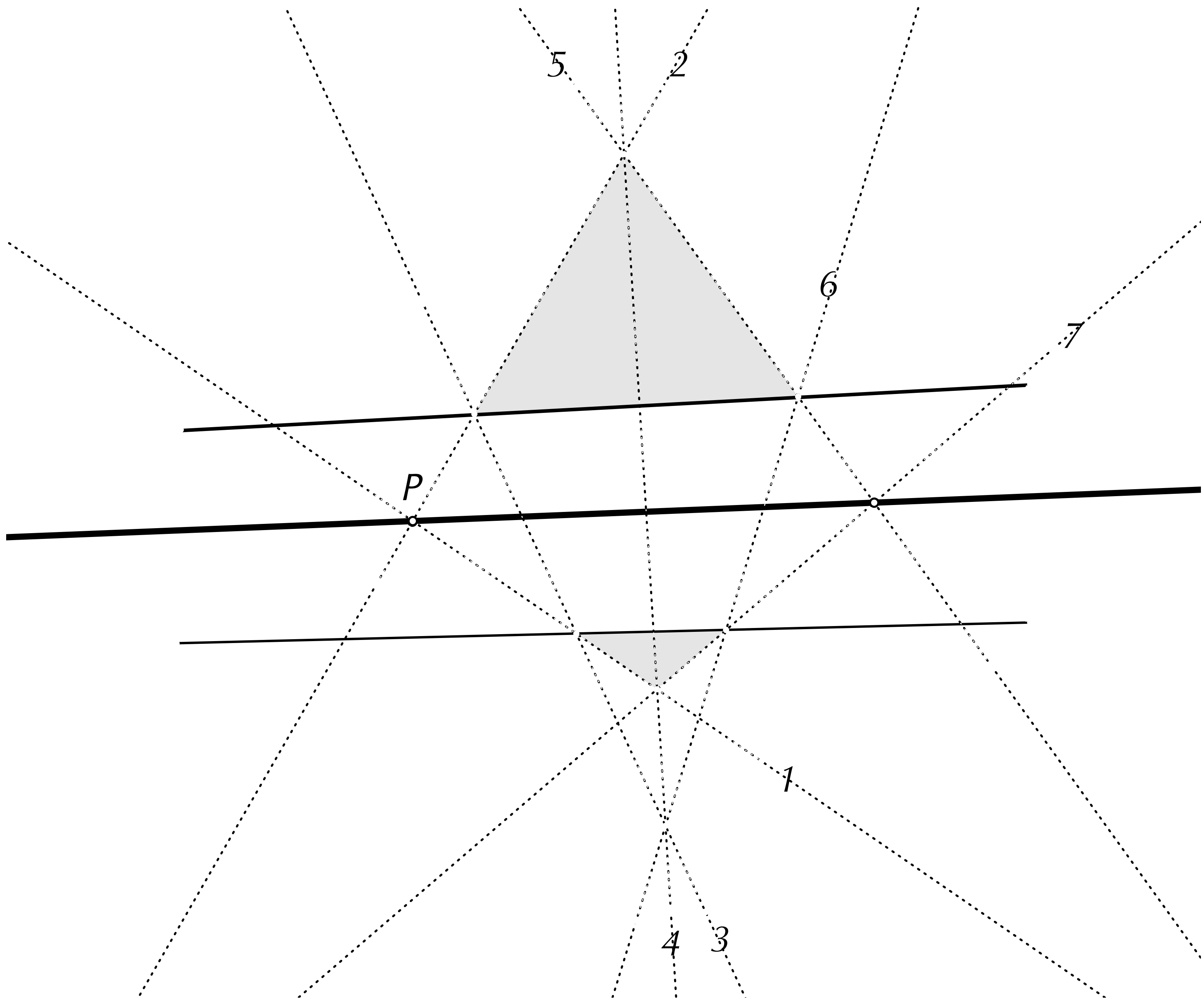




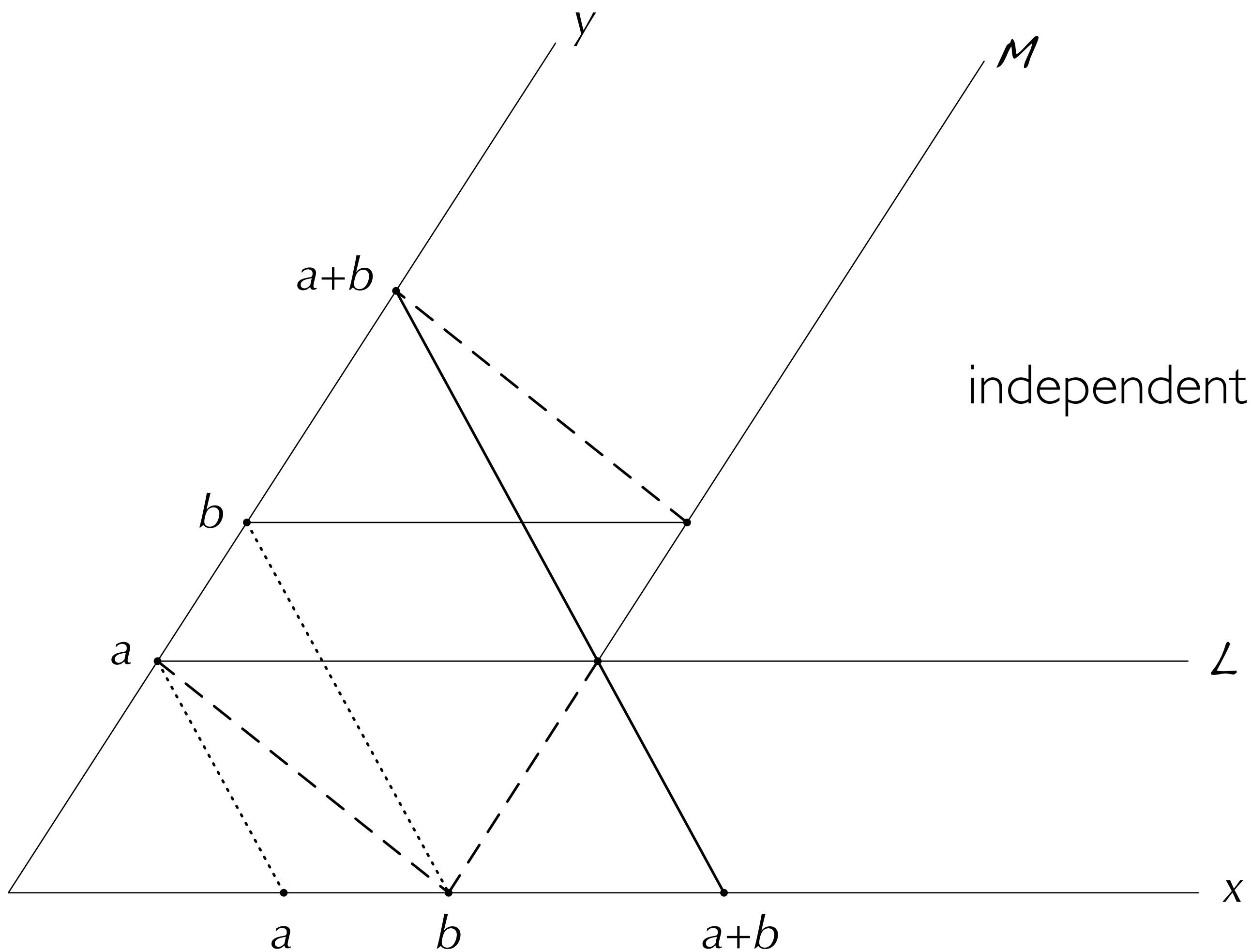
- small rulers



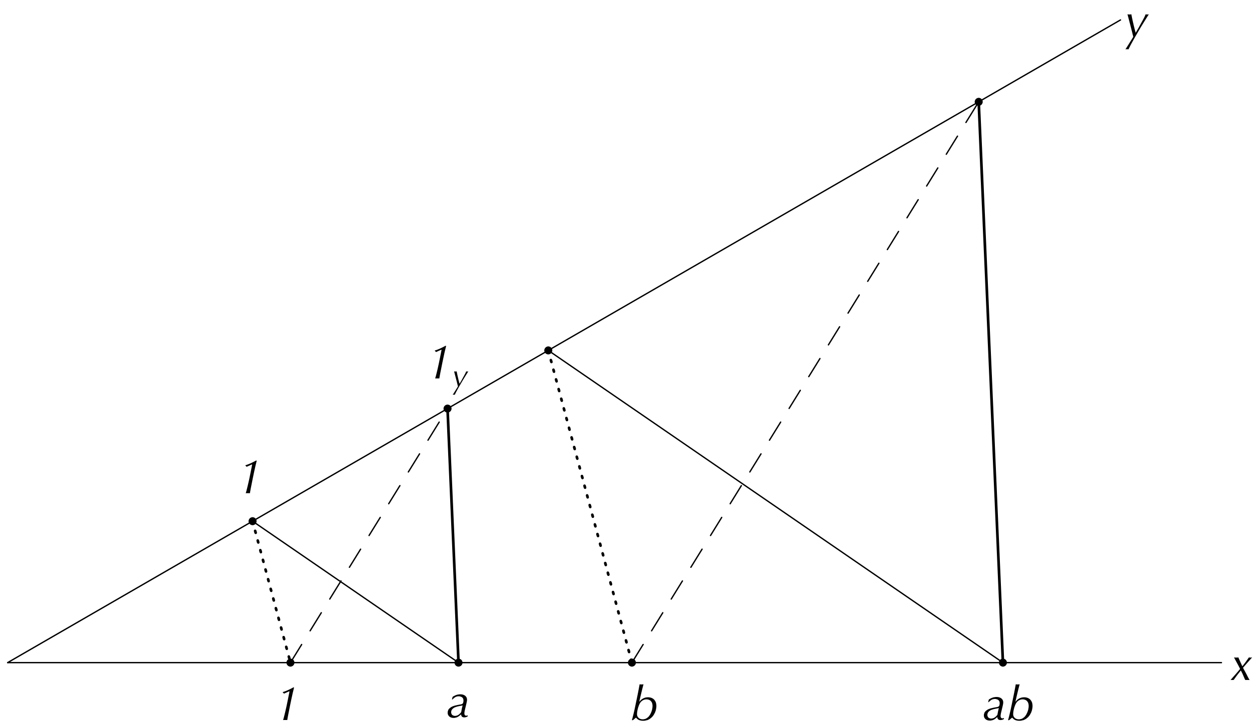
desargues configuration



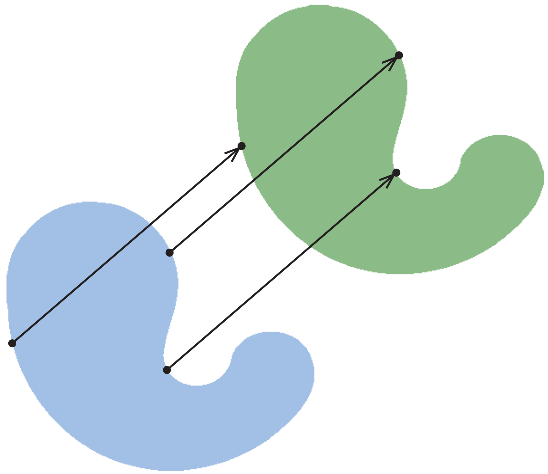
- a typical problem

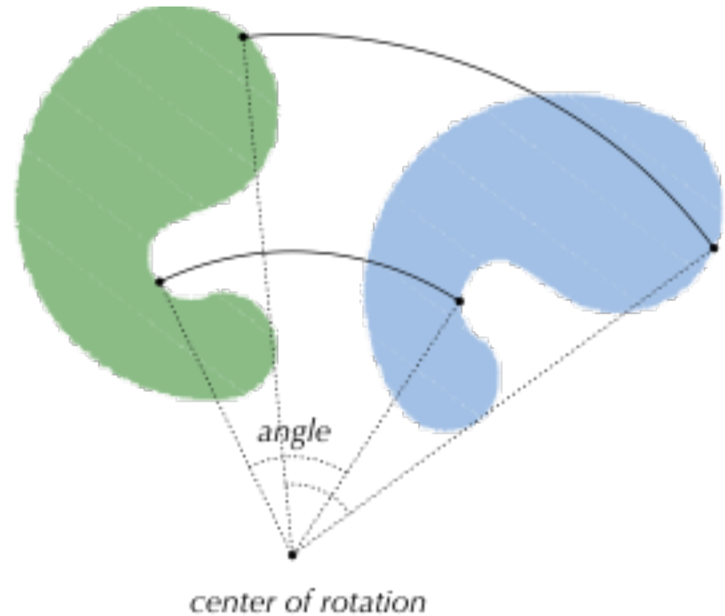


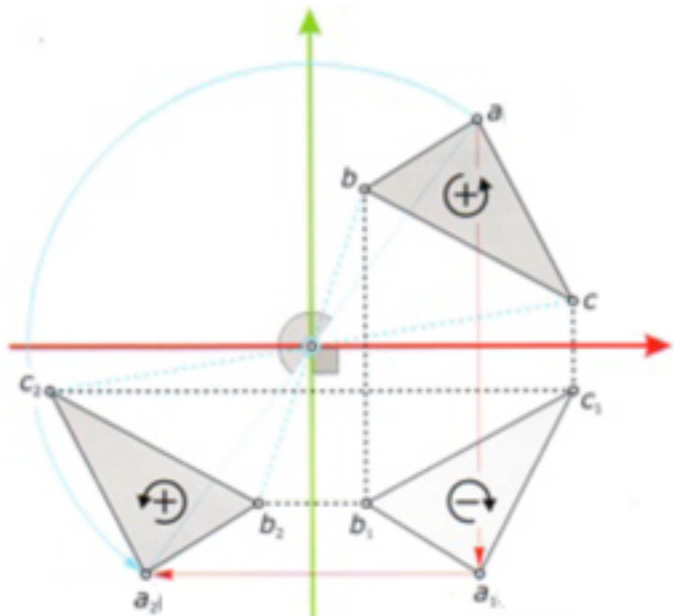
independent of *L*

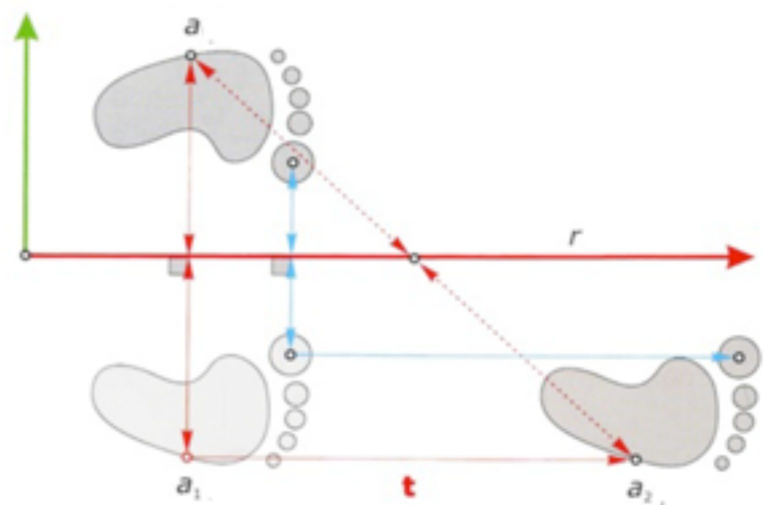

y
1
1_y
1
a
b
ab
x

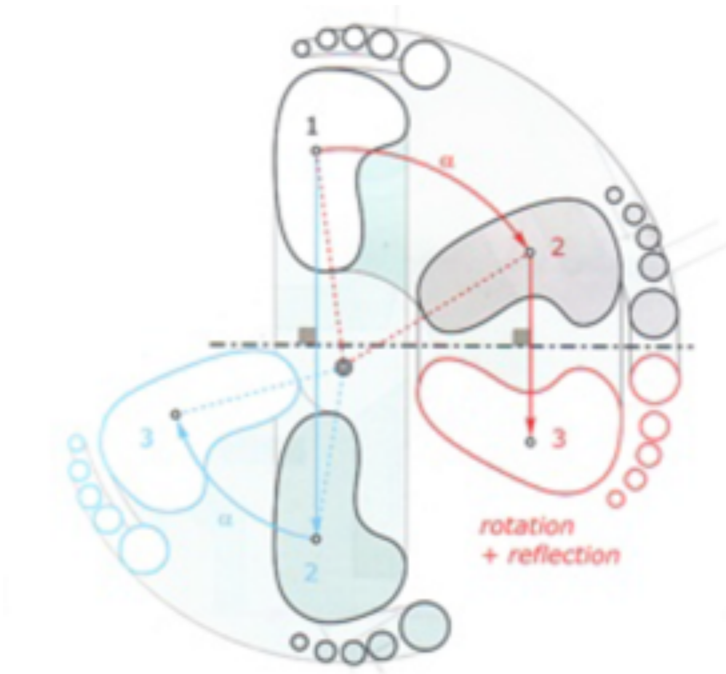
Geometric Transformations




angle
center of rotation


a
b
c
c2
c1
b2
b1
a2
a1


a
r
a1
t
a2



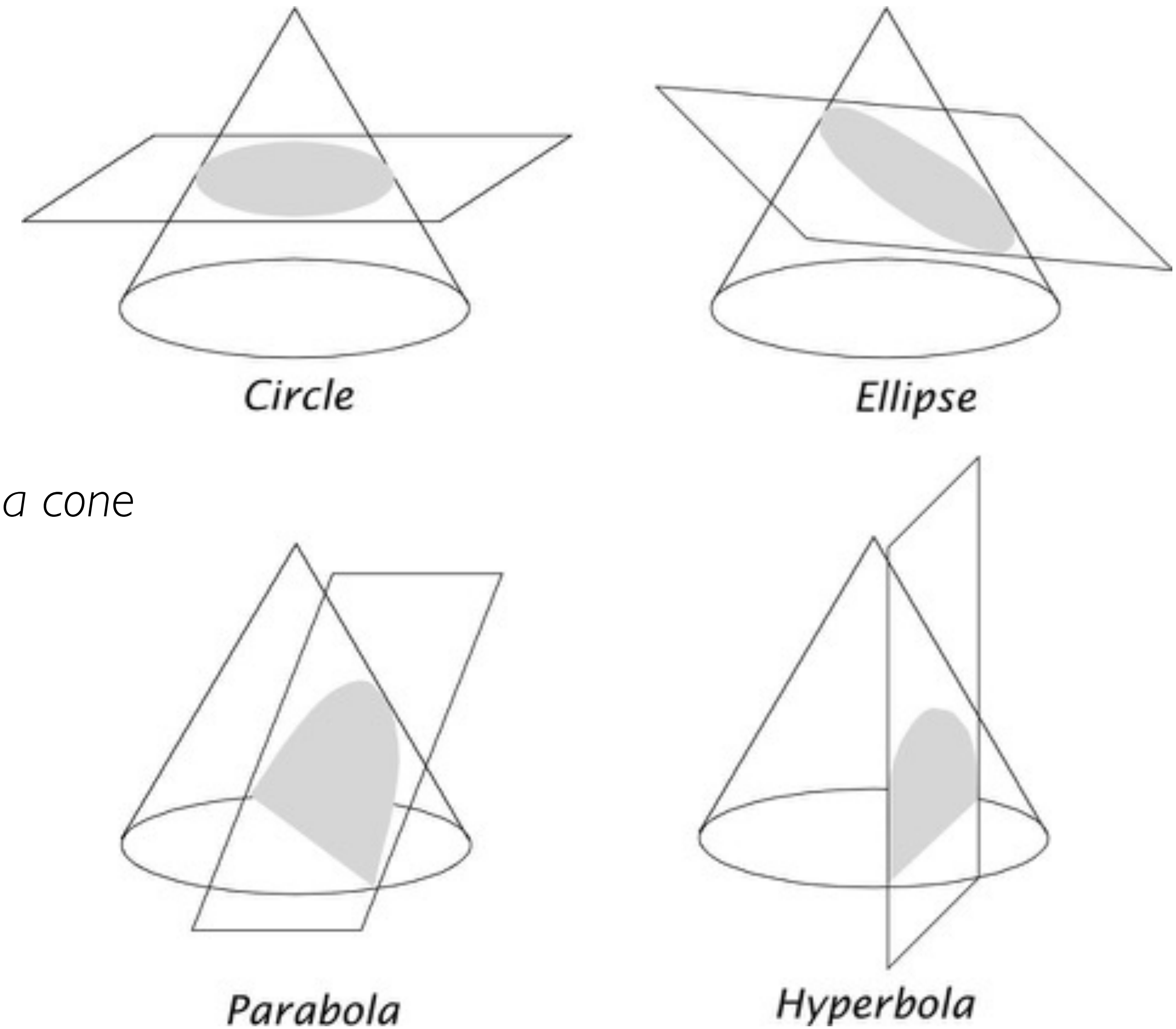
Hint: what you need are mirrors!

- rotating an object without using a compass

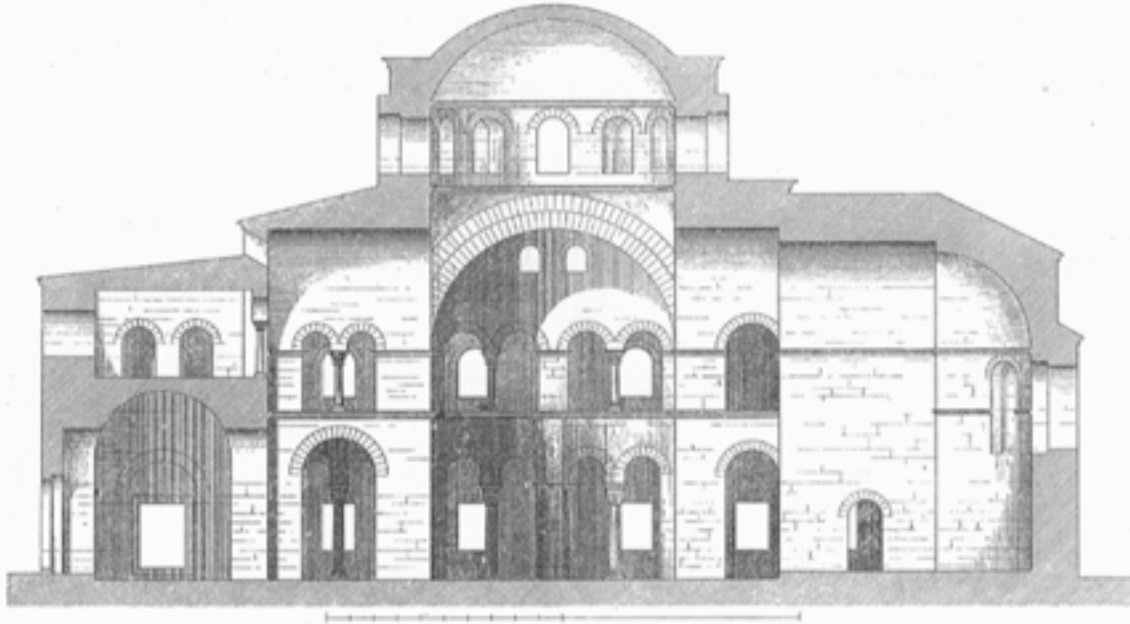
Conic Sections

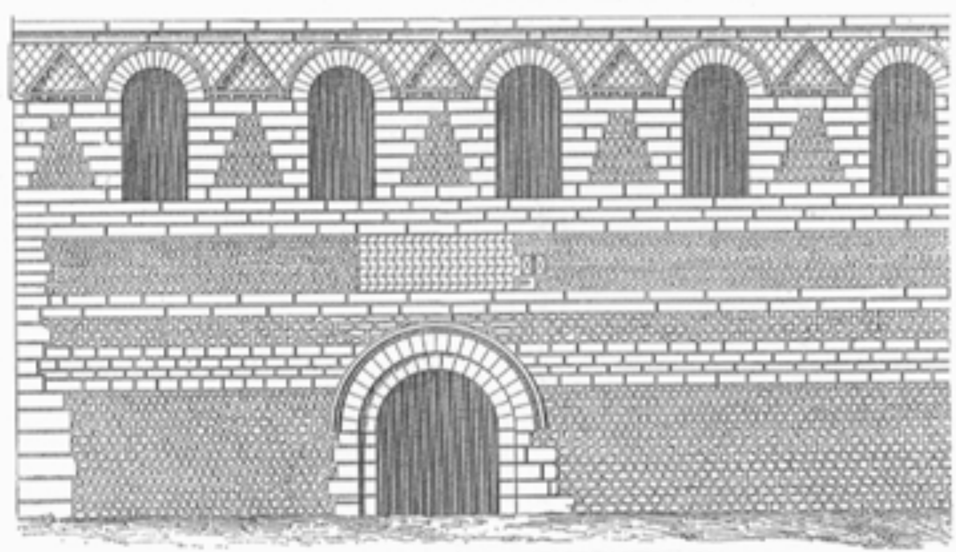
produced by slicing a cone

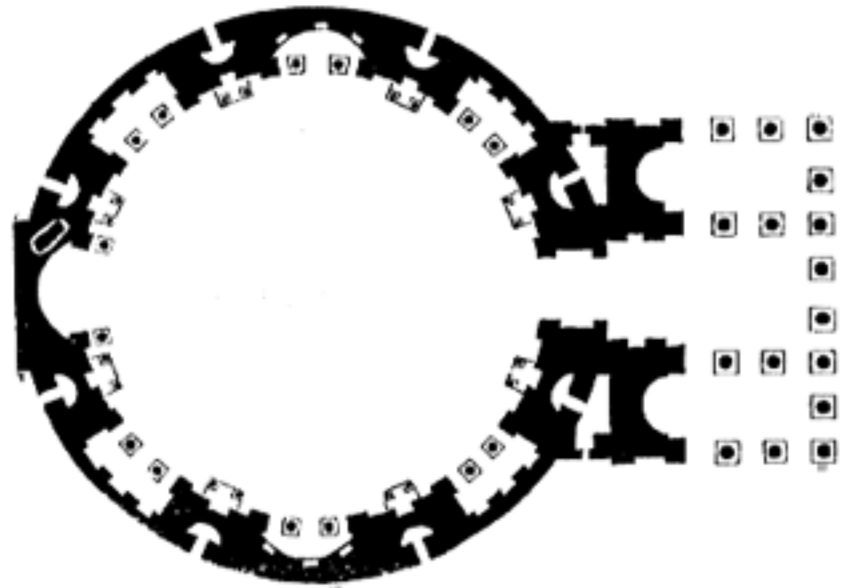
by a cutting plane



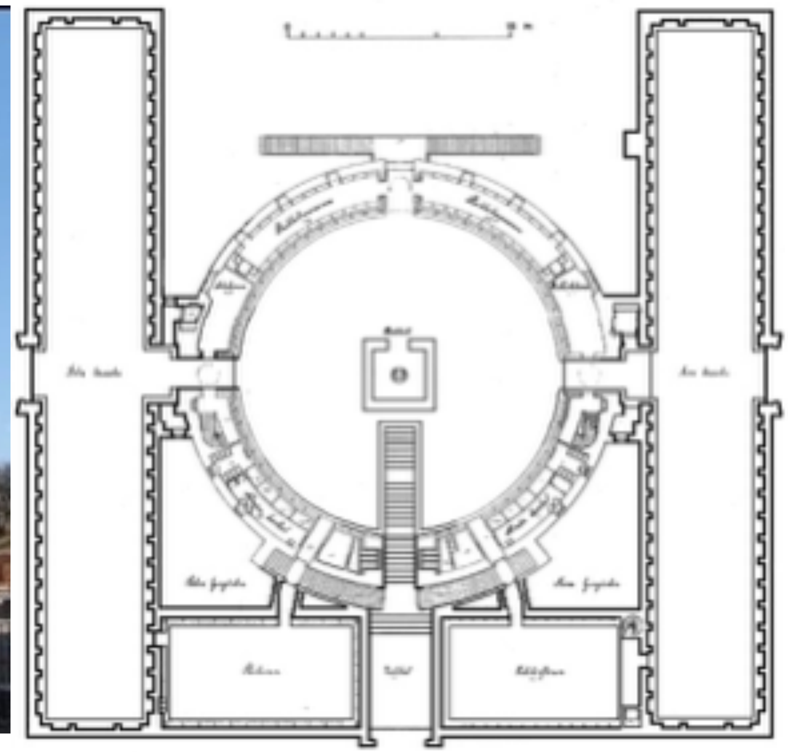
conic sections



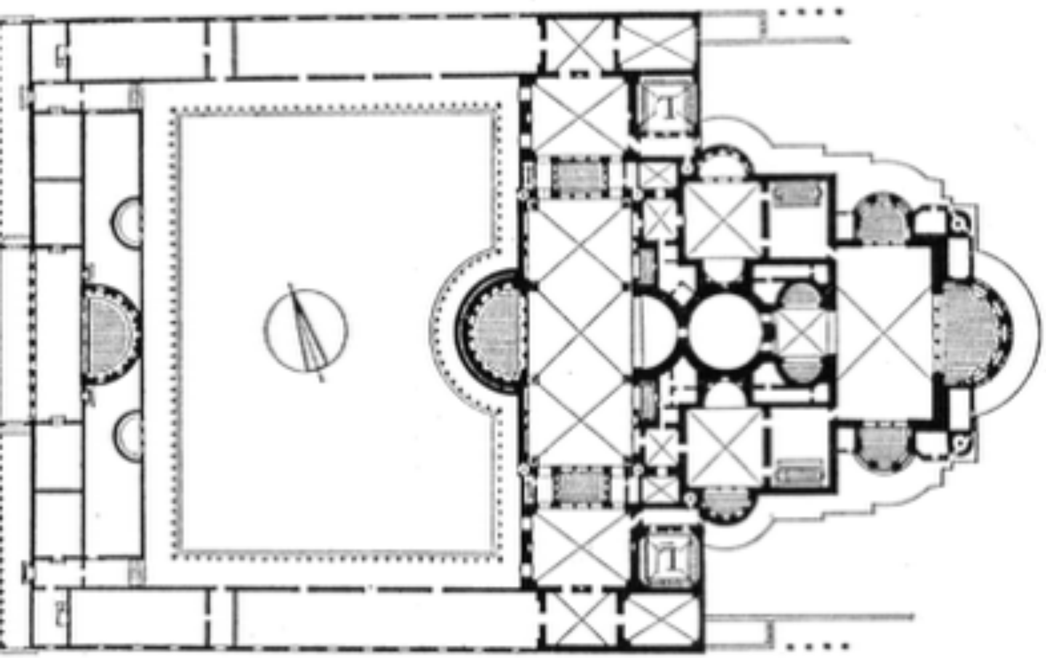




- Pantheon

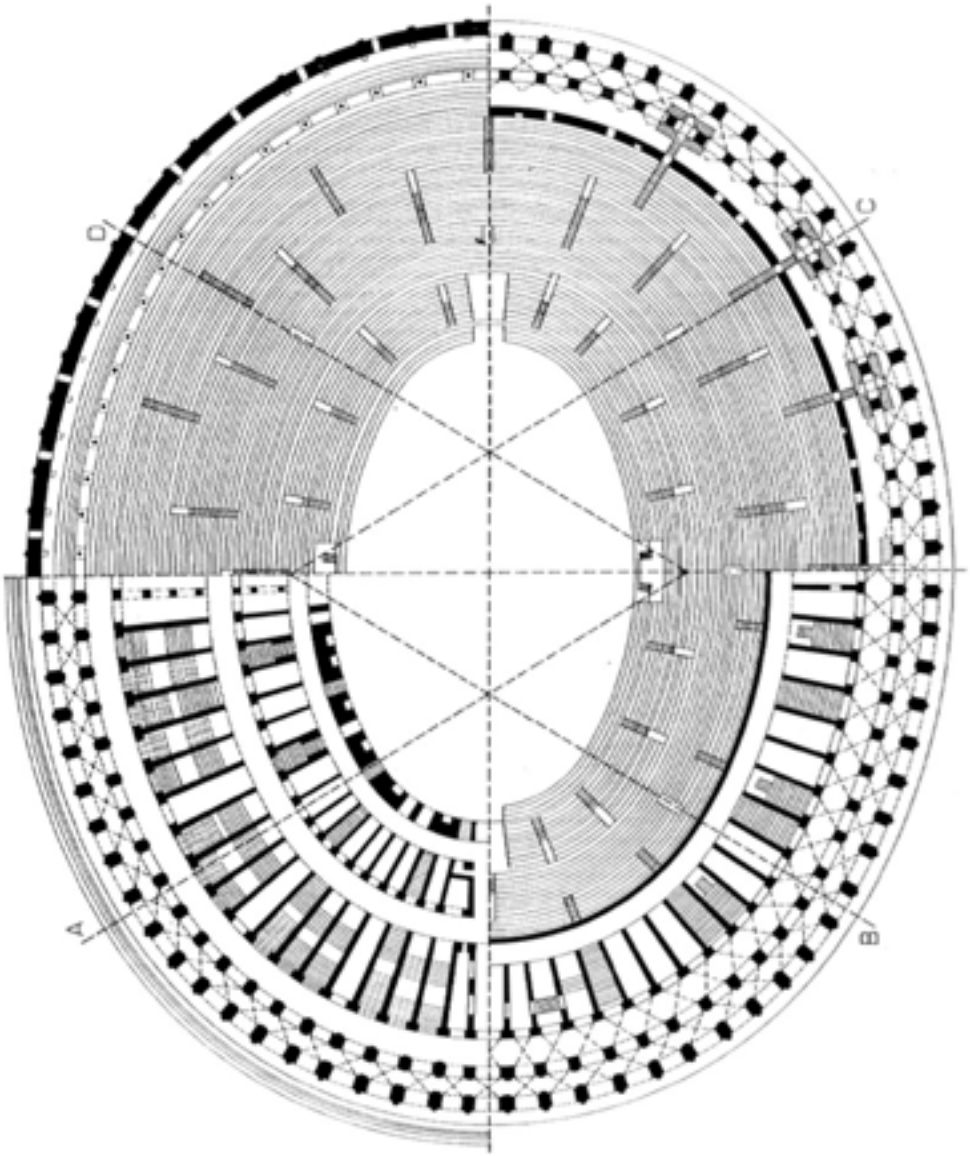


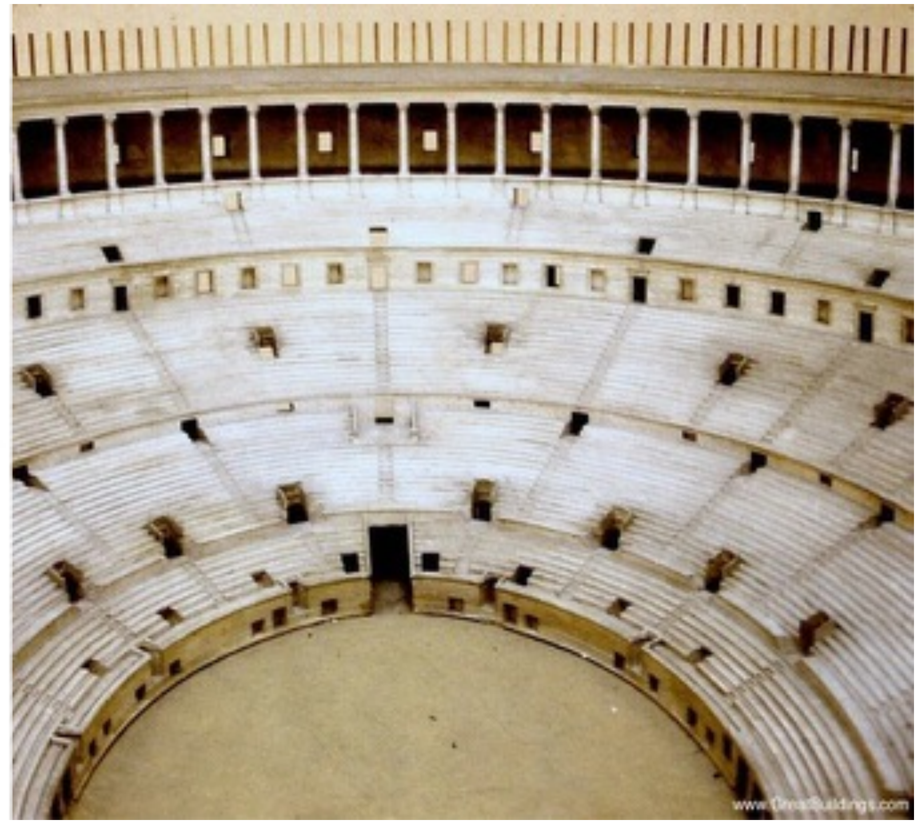
- Stockholm Public Library



- Imperial baths, Trier

- Ctesiphon



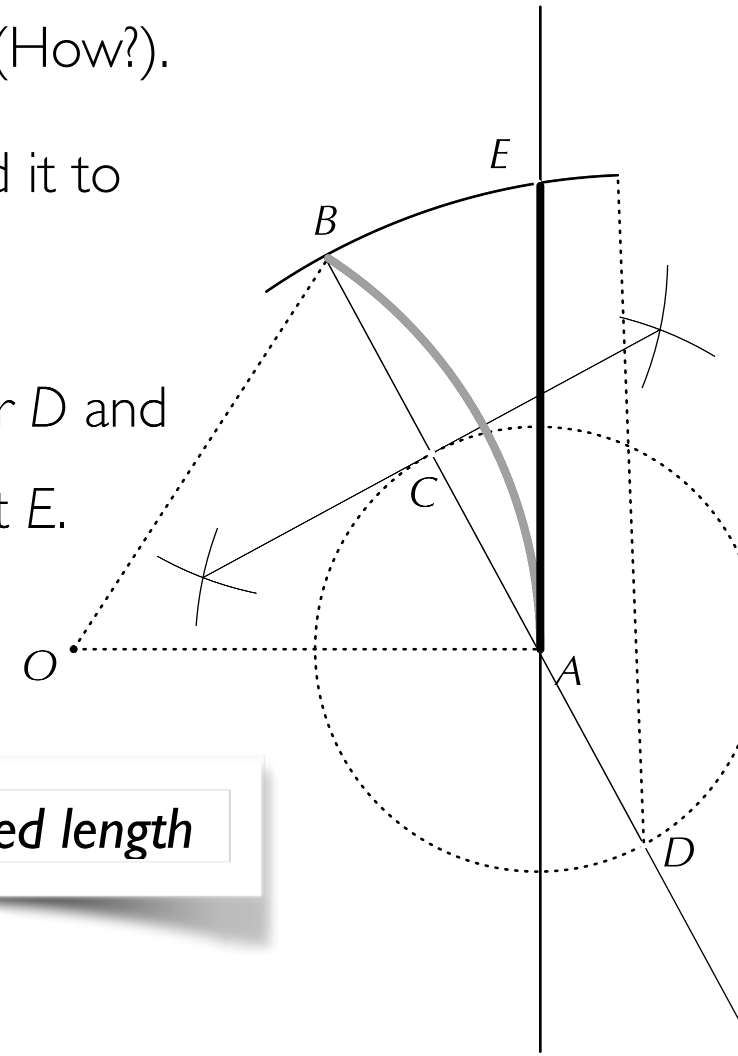


- Colosseum

S.Vicente de Paul at Coyoacan

rectification: approximate length of a circular arc

1. Draw a tangent to the arc at *A* (How?).
2. Join *A* and *B* by a line and extend it to produce *D* with *AD* = ½*AB*.
3. Draw the circular arc with center *D* and radius *DB* to meet the tangent at *E*.



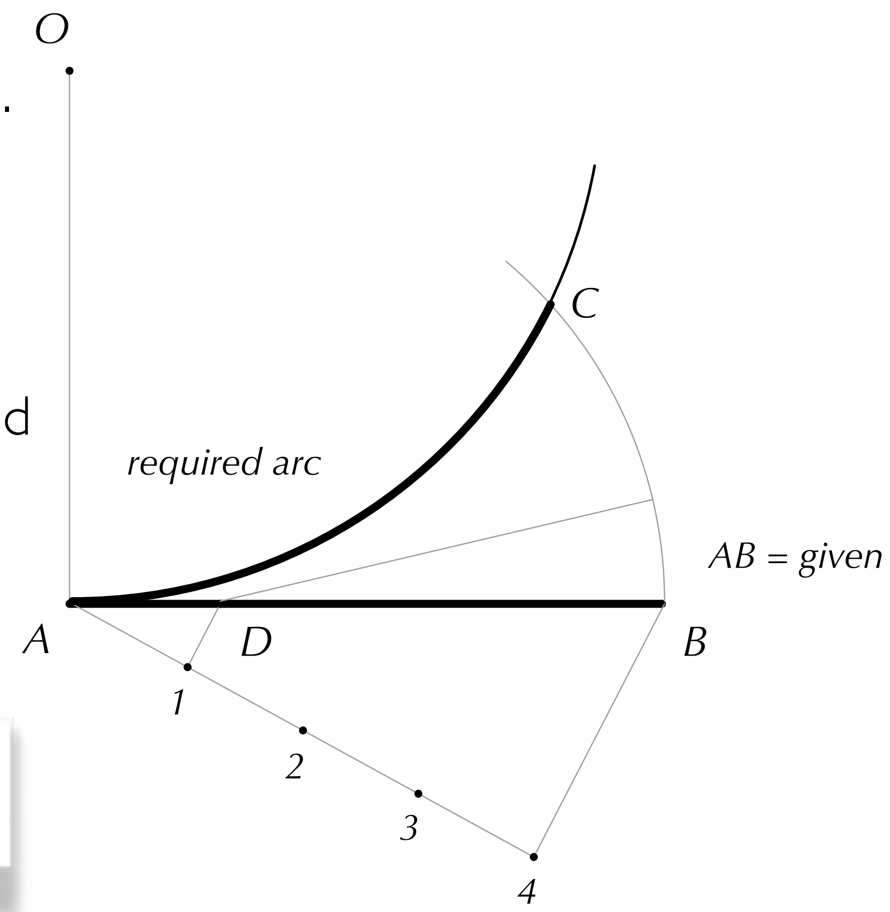
AE is the ***required length***

approximate circular arc of a given length

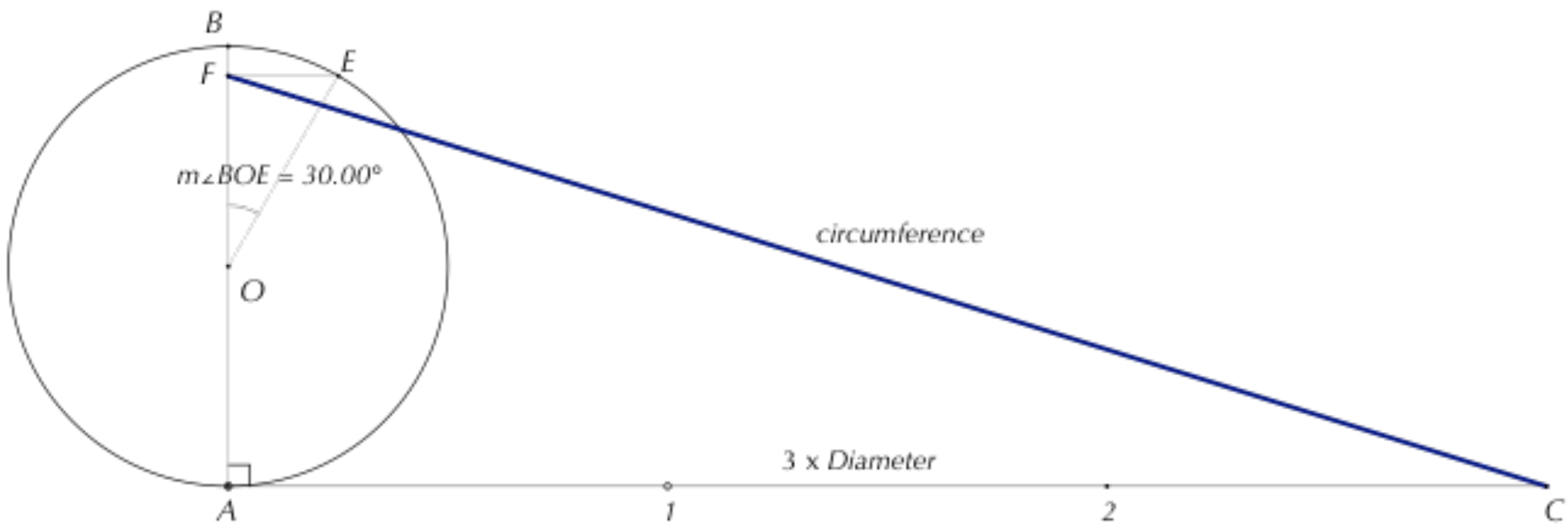
A be a point on the arc.

AB is the given length on the tangent at *A*.

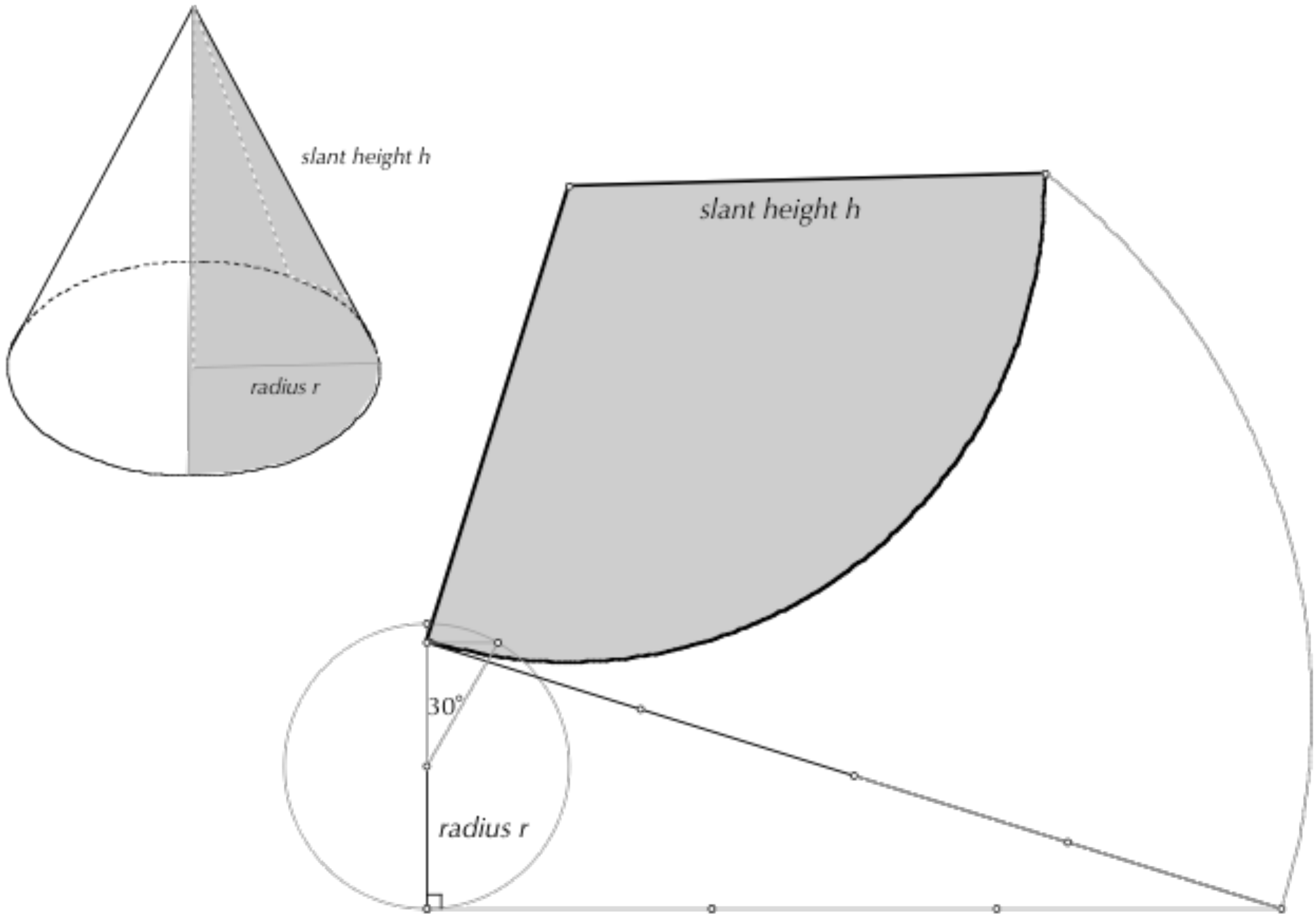
1. Mark a point *D* on the tangent such that *AD*=¼*AB*.
2. Draw the circular arc with center *D* and radius *DB* to meet the original at *C*.



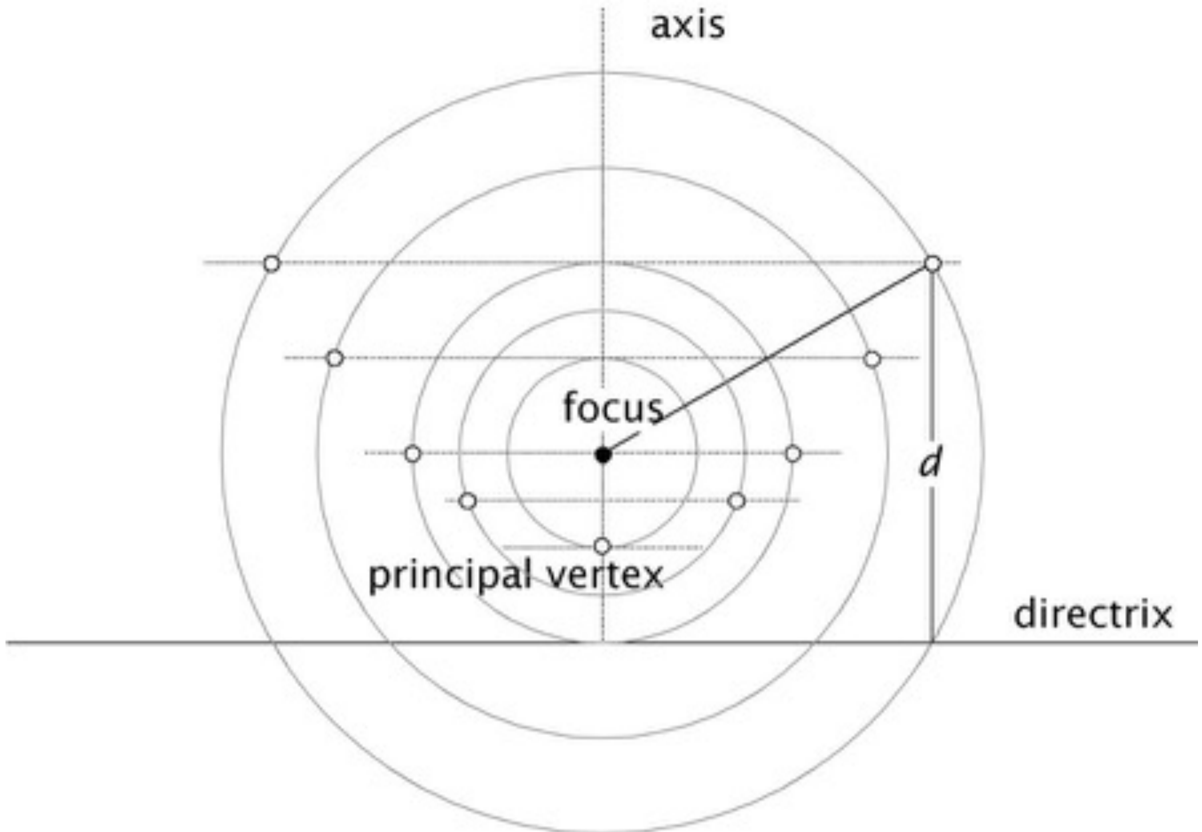
- Arc *AC* is the ***required arc***



- rectifying the circumference of a circle



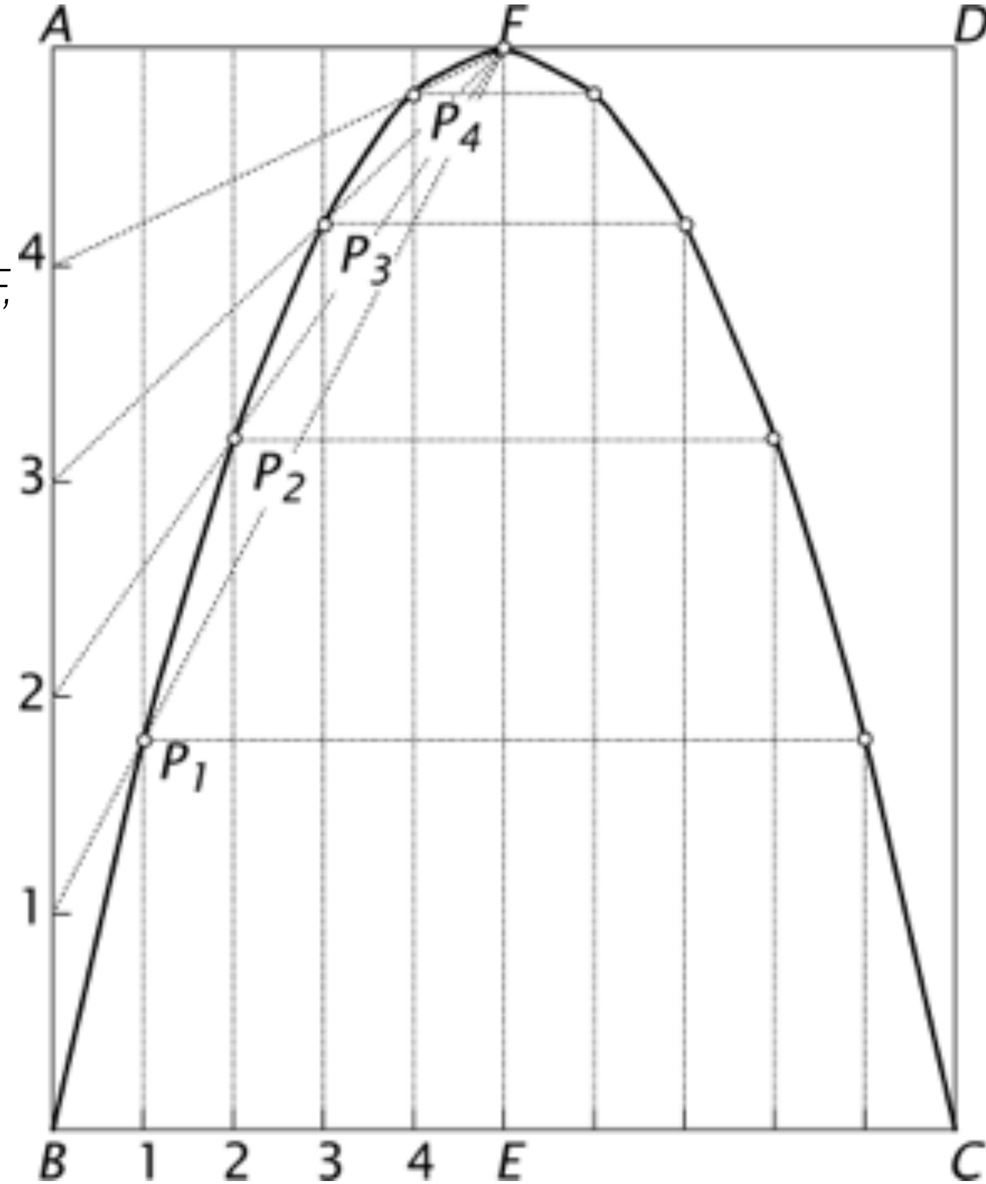
- a practical application



parabola

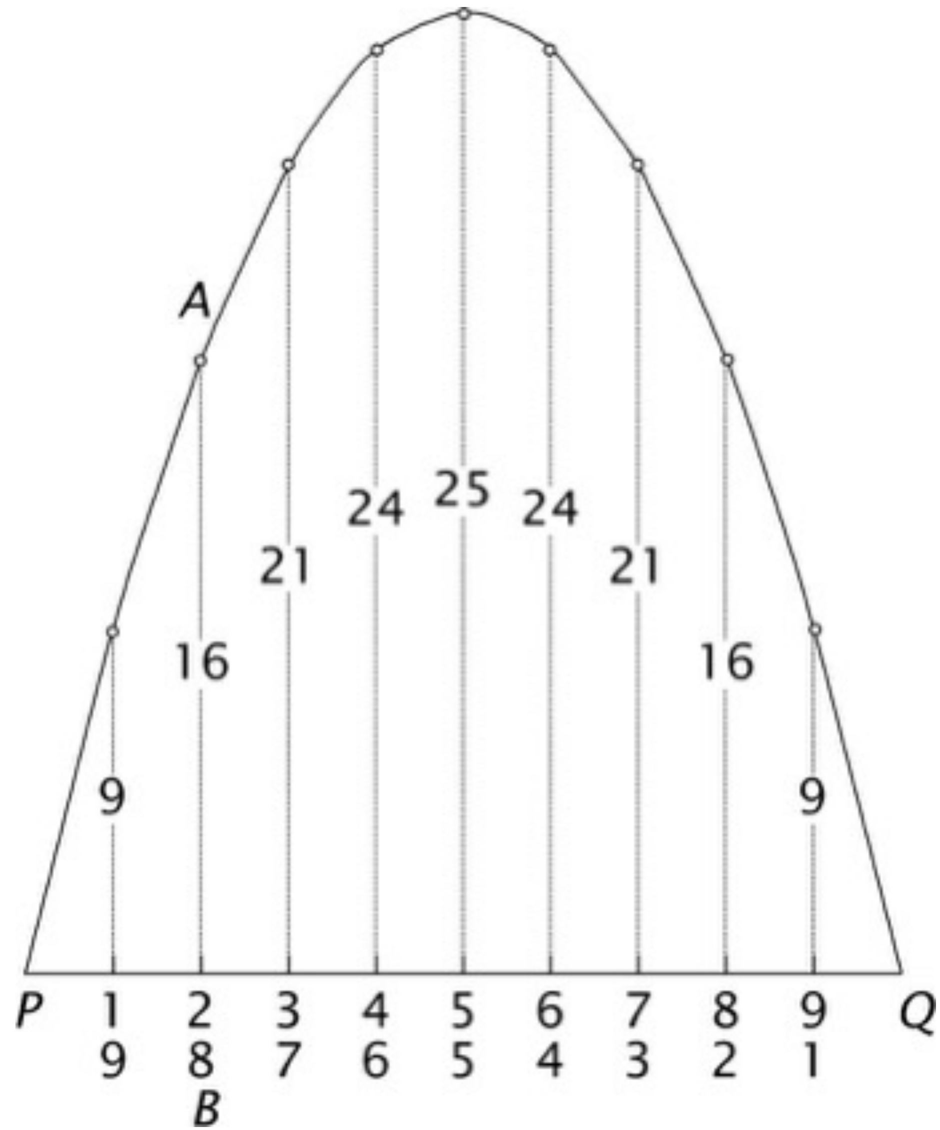
a parabola within a rectangle

1. Bisect the sides and of the rectangle *ABCD* and join their midpoints, *E* and *F*, by a line segment.
2. Divide segments and into the same number of equal parts, say $n = 5$, numbering them as shown.
3. Join *F* to each of the numbered points on to intersect the lines parallel to through the numbered points on at points P_1, P_2, … P_{n-1} as shown.
4. These points lie on the required parabola.



a parabola by abscissae

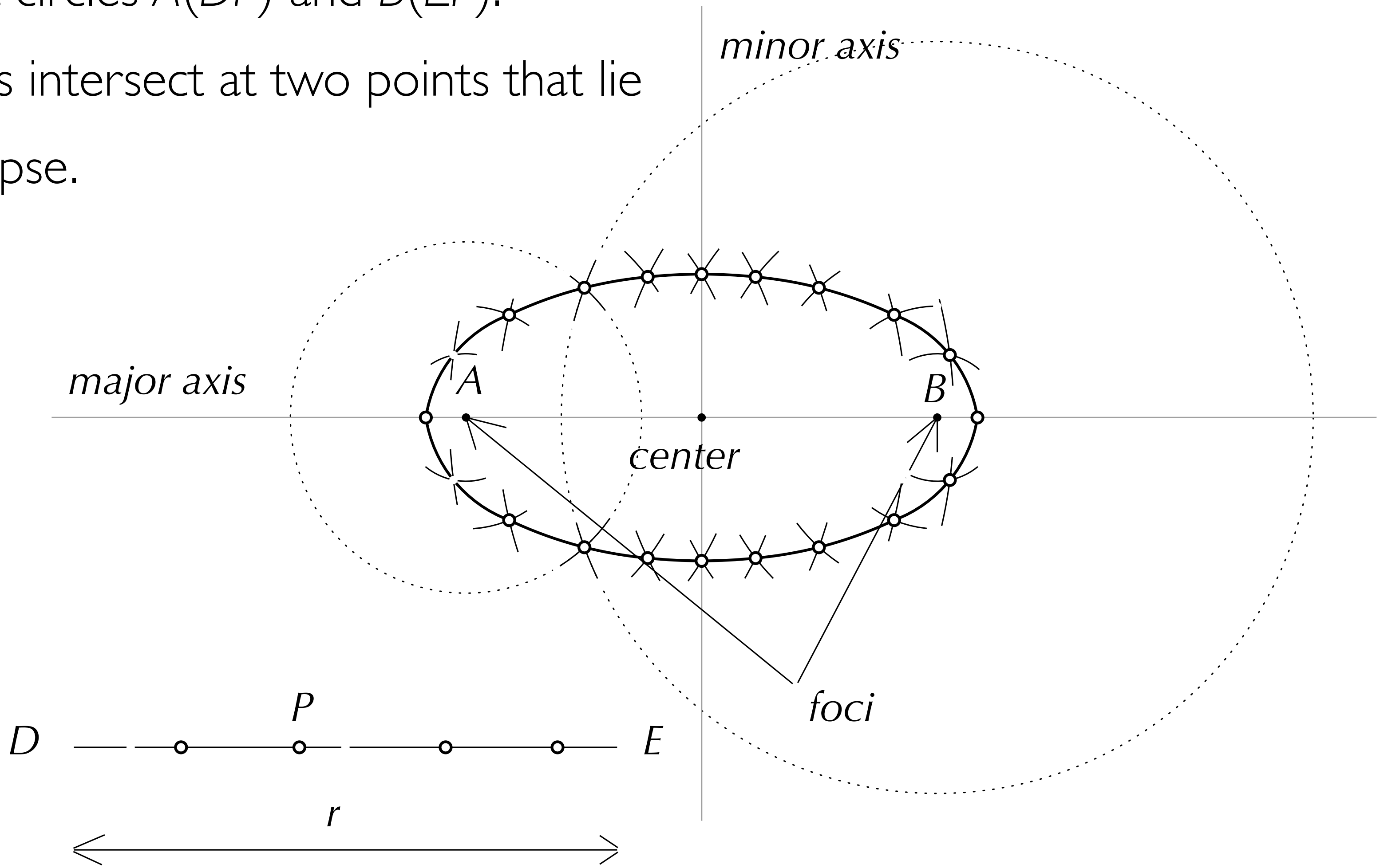
an **abscissa** is related to any of its double ordinate by the ratio, *AB*: (*PB* × *BQ*), which is always a constant. That is, the abscissa is a scaled multiple of the parts into which it divides the double ordinate.



P is an arbitrary point between *D* and *E*.

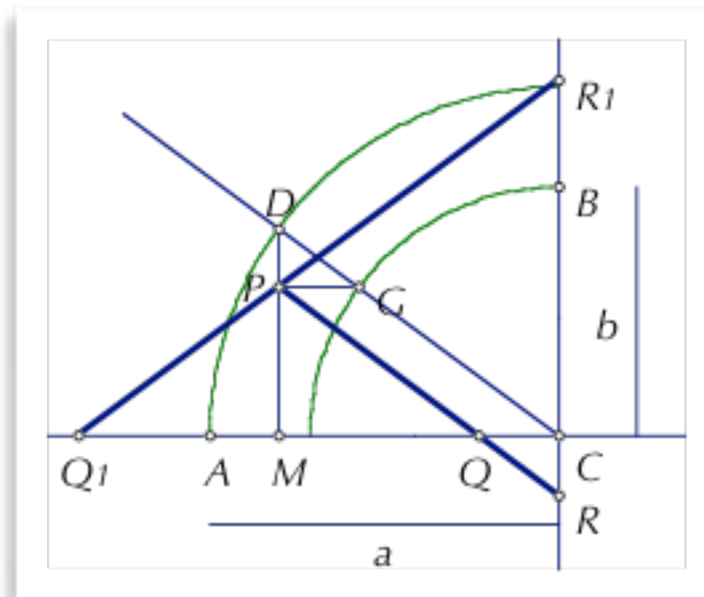
Construct circles *A*(*DP*) and *B*(*EP*).

The circles intersect at two points that lie on the ellipse.



The Trammel Method

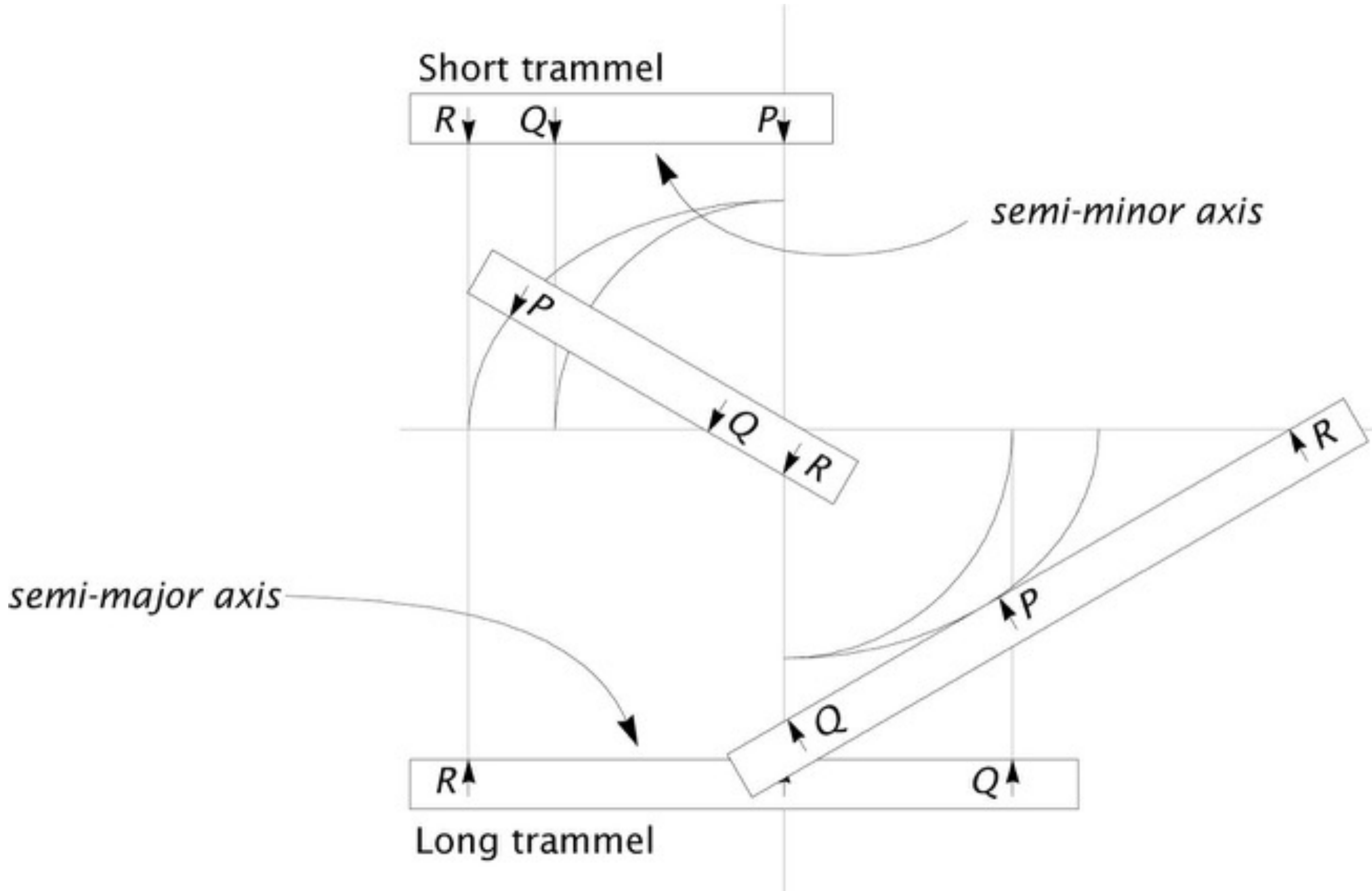
Draw the axes and mark off along a straight strip of card-board the distances *PQ* and *PR*. Apply the trammel so that *Q* lines up with the major axis and *R* lines up with the minor axis; *P* is a point on the ellipse. More points *P* can be plotted, by moving the trammel so that *Q* and *R* slide along their respective axes.



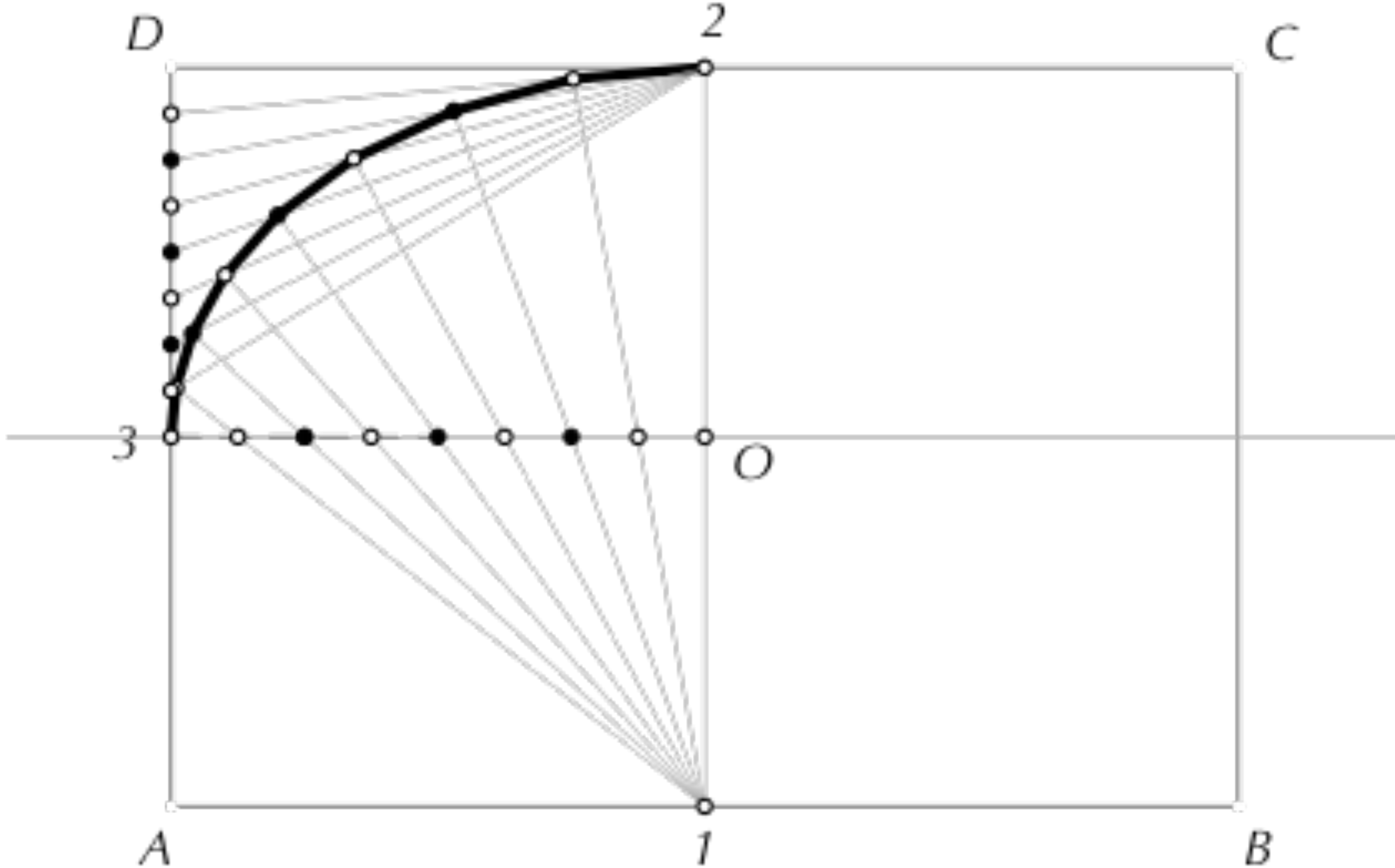
W Abbott

Practical Geometry and Engineering Graphics

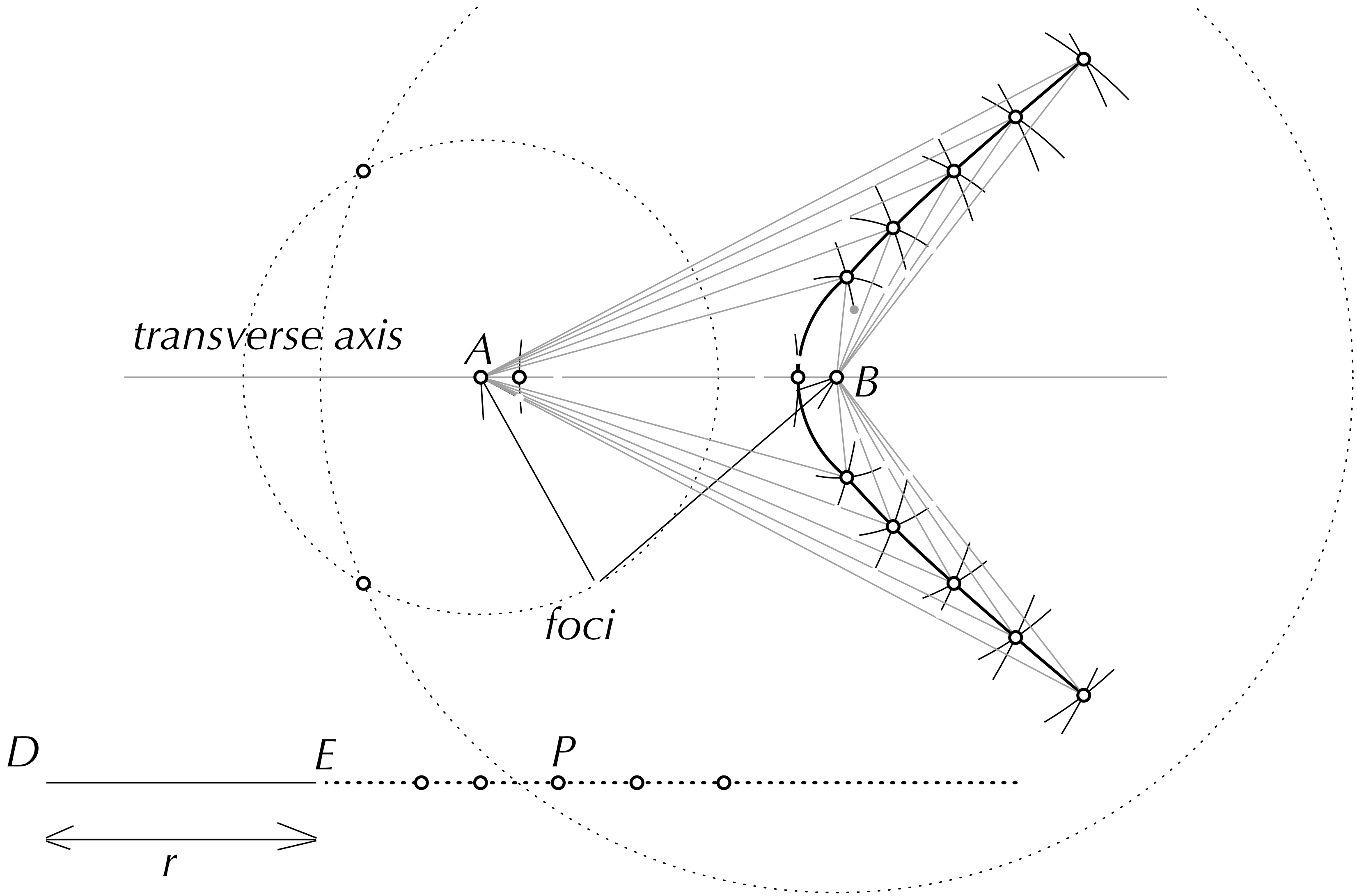
Blackie & Son Ltd, Glasgow, 1971.



- the trammel method

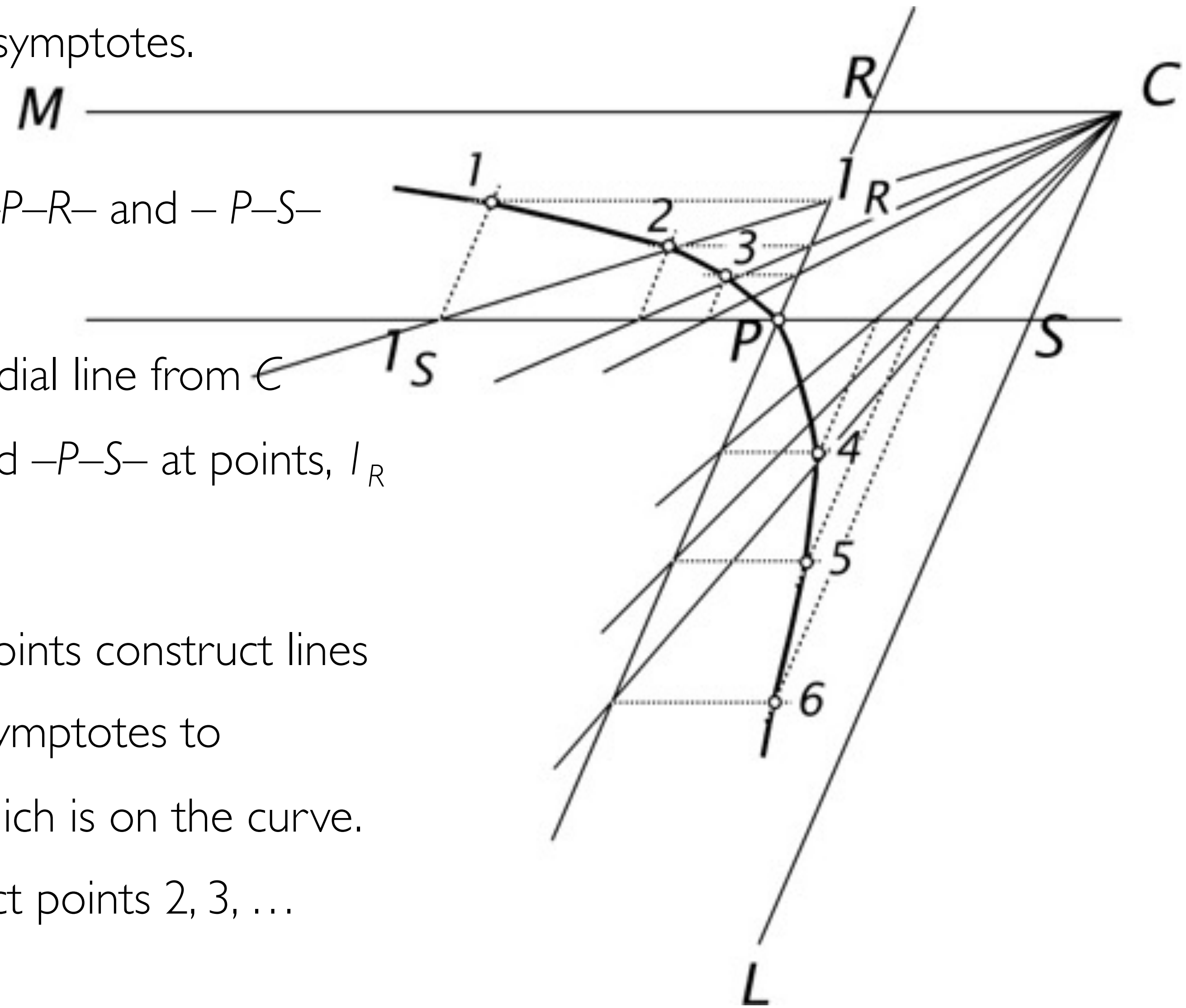


- constructing an ellipse within a rectangle



- hyperbola

CL and *CM* are the asymptotes.



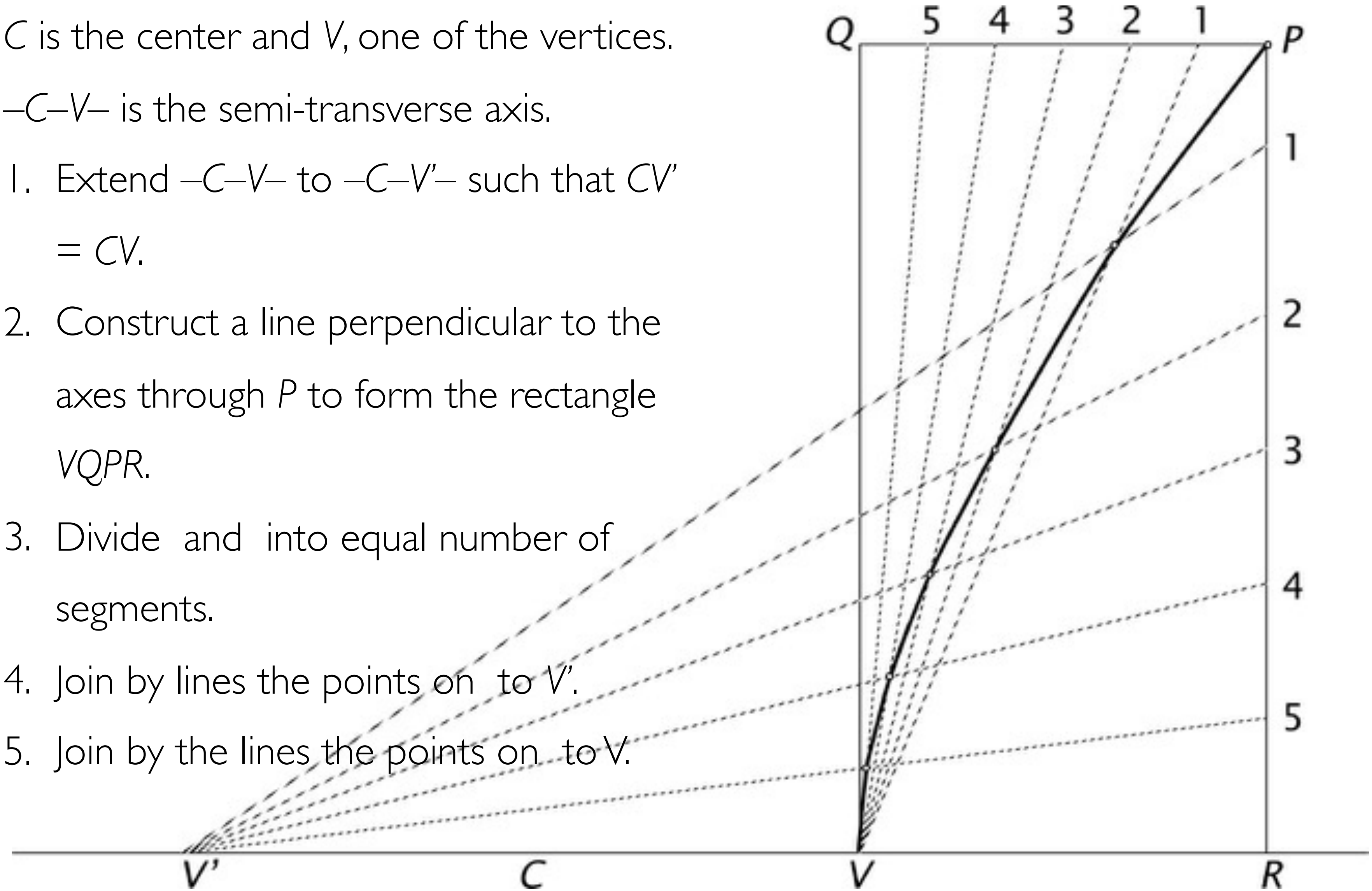
1. Construct lines –*P*–*R*– and – *P*–*S*– parallel to them.
2. Construct any radial line from *C* cutting –*P*–*R*– and –*P*–*S*– at points, I_R and I_S.
3. Through these points construct lines parallel to the asymptotes to intersect at *I*, which is on the curve.
4. Similarly construct points 2, 3, …

- **hyperbola given asymptotes and a point**

C is the center and *V*, one of the vertices.

–*C*–*V*– is the semi-transverse axis.

1. Extend –*C*–*V*– to –*C*–*V'*– such that *CV'* = *CV*.
2. Construct a line perpendicular to the axes through *P* to form the rectangle *VQPR*.
3. Divide and into equal number of segments.
4. Join by lines the points on to *V'*.
5. Join by the lines the points on to V.



hyperbola given semi-transverse axis and a point

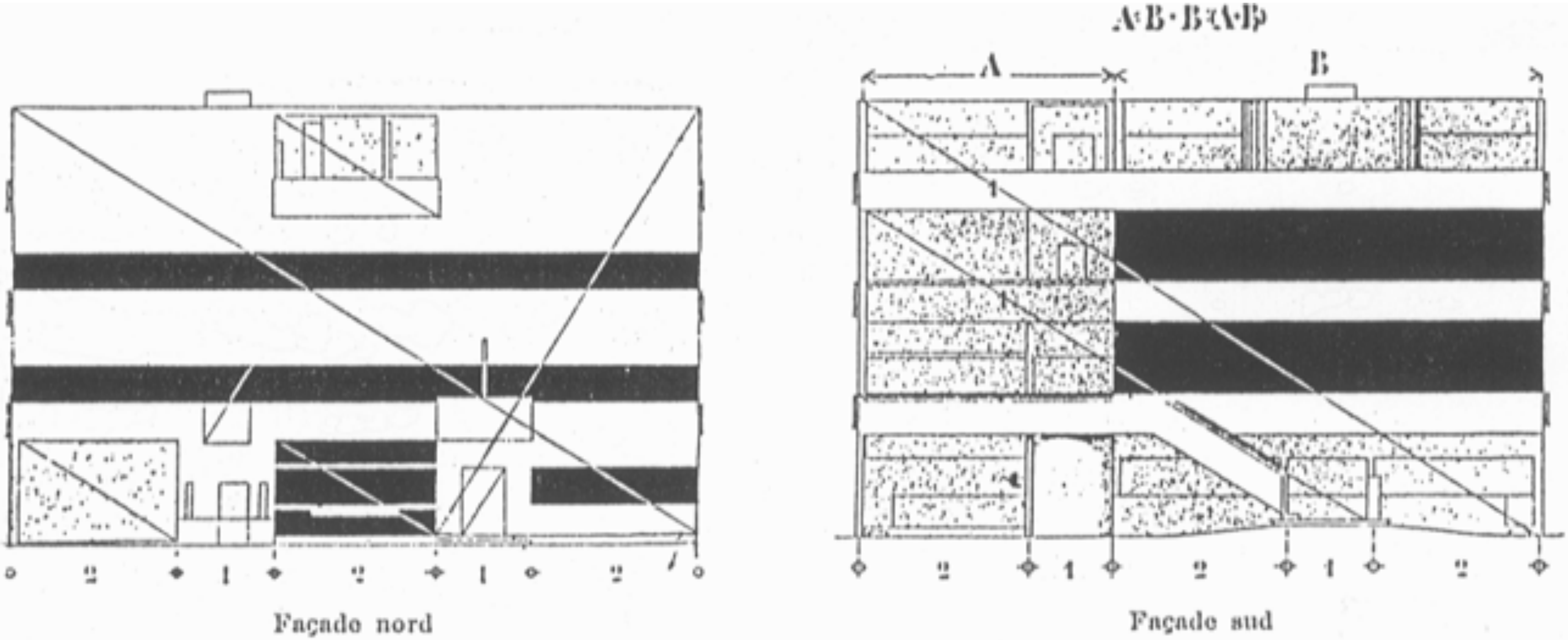
Golden Section

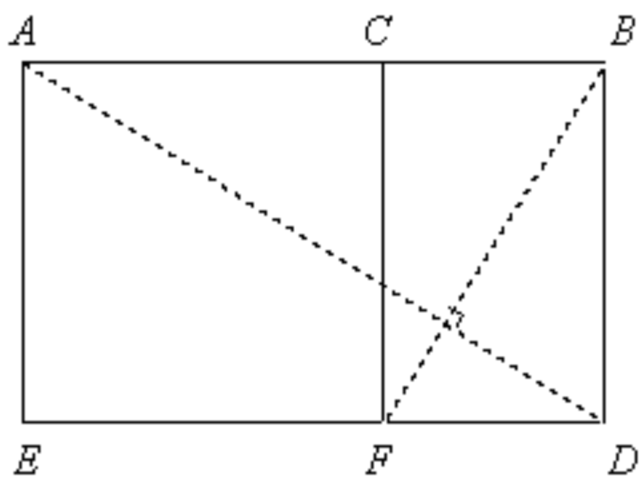
AB is a segment and *C* a point so that *A-C-B*.

C divides AB in the **golden ratio** if *AB:AC* = *AC:CB*

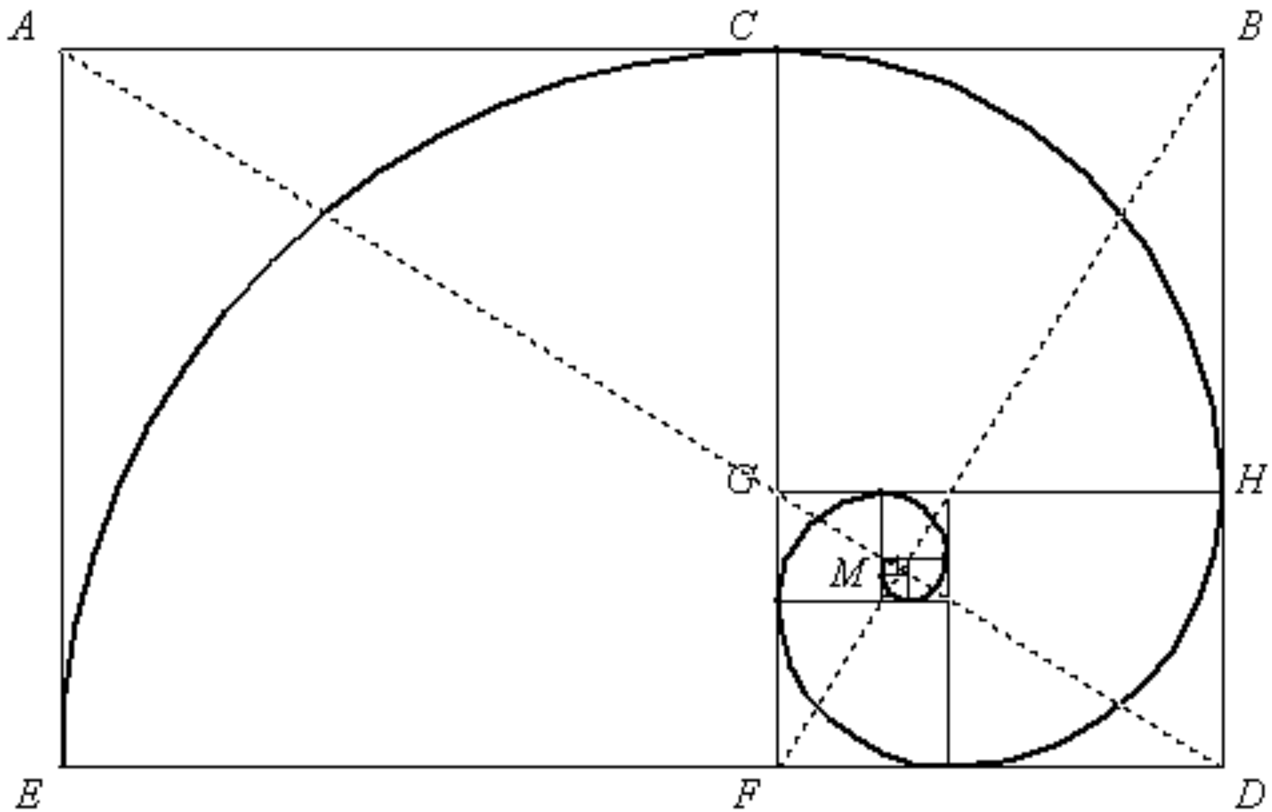
Any division that satisfies the golden ratio is called a **golden section =** $\frac{1}{2}(1+\sqrt{5})$

Le Corbusier's Villa Stein at Garches

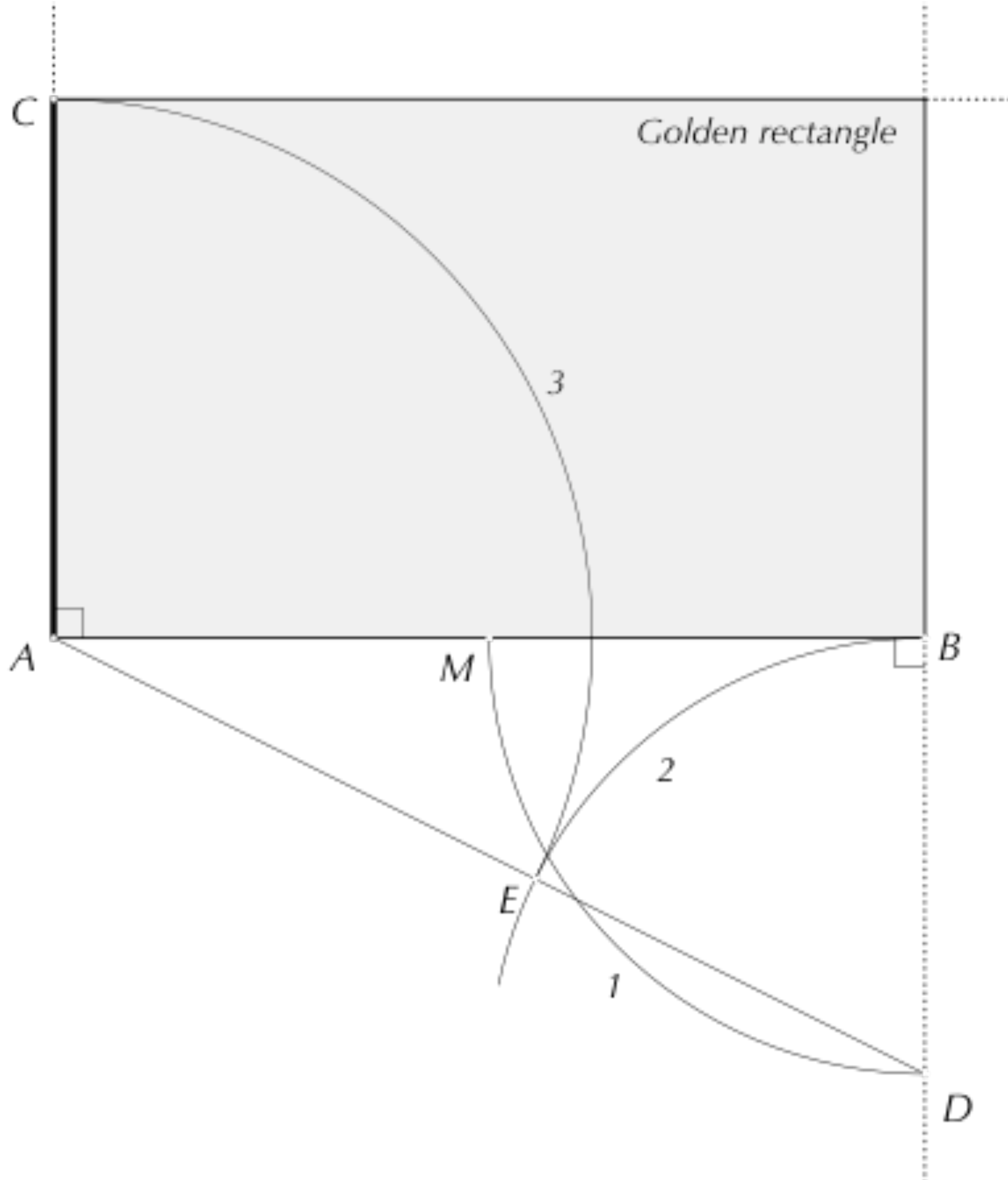




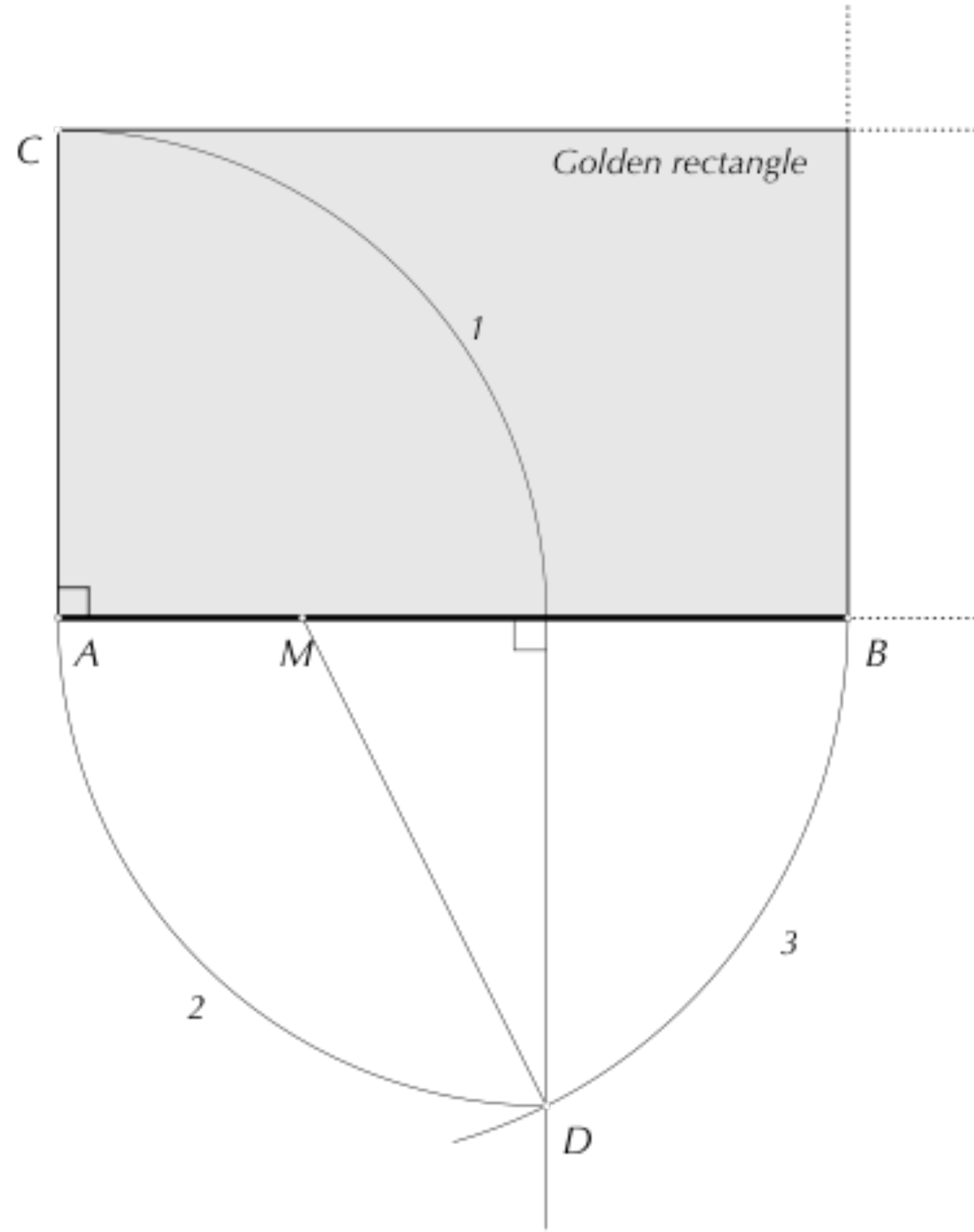
- golden rectangle



- golden spiral



- golden rectangle



golden rectangle