

8

Solids and Surfaces

Intuitively, solids are objects that have volume, that is, occupy a region of space, and are all of one piece. They have *insides* and *outsides* and generally represent real-world objects that can be physically constructed out of real materials. It is not entirely trivial to make this intuitive notion of what we mean by a solid mathematically precise.

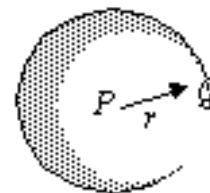
Surfaces do not have volume although they too occupy a region of space. The spatial boundary of a solid is a surface, and the geometry of surfaces can be studied in its own right. Again, a rigorous treatment demands more mathematical background than we can or are willing to provide and is certainly beyond the scope of this course. The present section treats solids informally but provides some useful terminology.

8.1 SOLIDS

We start informally by considering a solid as a set of points contained in a volume.

8.1.1 Neighborhoods

Points have *neighborhoods*, that is, for any point P , for some infinitesimal number r , there is a sphere of points that are at a distance less than r from P . Clearly, every point has infinitely many neighborhoods (for each possible value of r), and every neighborhood contains infinitely many points.

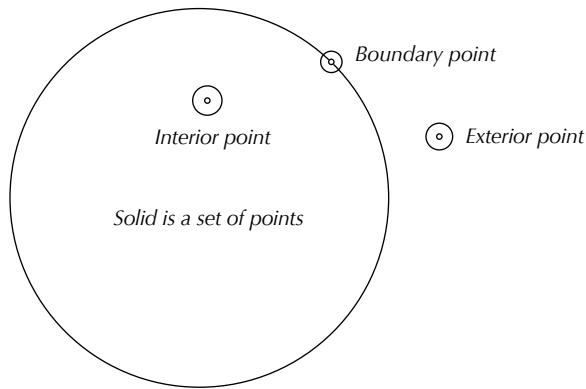


8-1

A neighborhood of a point

8.1.2 Interior and boundary points

Points can be inside a solid (indeed, a surface or any geometric figure), on the boundary of the figure or outside the figure. An *interior point* is one in which every neighborhood of the point is in the figure. A *boundary point* is one in which every neighborhood of the point contains a point that is in the figure and a point that is not in the figure. A boundary of a figure is made up of its boundary points. All other points are *exterior* to the figure.



8-2
Interior, exterior and boundary points

We say a point is *on* a solid if it is a boundary point. We say a point is *near* a solid if it is either an interior point or a boundary point.

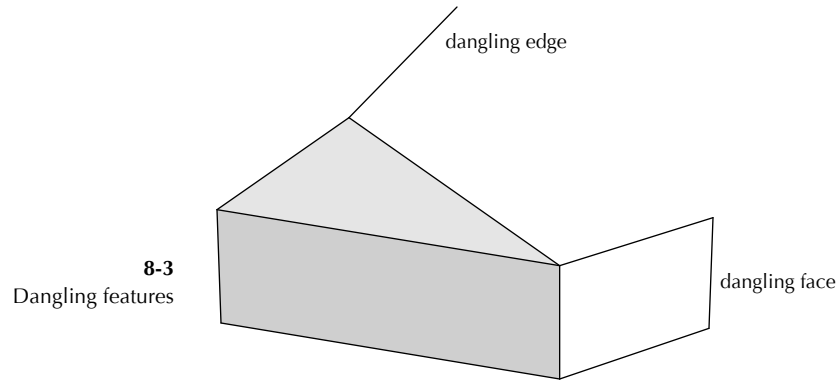
8.1.3 Connectivity

A solid is *connected* if it cannot be split into two solids (again, surfaces or geometric figures) such that each figure contains a point near the other. Connectivity expresses mathematically the intuitive notion that if a geometric figure is connected, it is all of one piece, and that it is not given as a collection of separate remote parts.

A spatial figure is a *solid* if it is connected, contains at least one interior point and all of its boundary points and its boundary equals the boundary of its interior.

This definition captures the following intuitive notions

- The fact that it contains at least one interior point (and consequently infinitely many interior points) assures that *a solid has volume* and thus it can be built with real materials
- The fact that it is connected captures our intuition that *a solid hangs together as a single piece*
- The fact that it contains its boundary is included for technical reason that we want to be sure that *whenever we are given a solid, we are sure that we are also given its boundary*.
- The fact its boundary is the boundary of its interior ensures that a solid has no dangling features; this, again, ensures that *it can be built with real materials*.



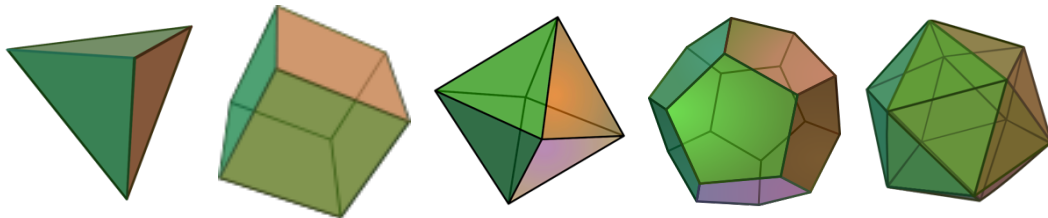
Although surfaces are much harder to define precisely we will rely on the notion that a connected subset of its boundary is a surface of the solid.

A word of caution must be added here. According to the above definition, we are not restricted to describing objects that are ‘solid’ in the normal sense, that is, constructed from real materials, and thus the opposite of a ‘void’, which signifies precisely the absence of physical material. Considered as sets of points, voids, like the space enclosed by walls, floors and ceilings, are solids as are the physical elements that enclose them.

Solids can be classified into the following types.

8.1.4 Polyhedral solids

A *polyhedron* is a solid whose boundary (or faces) consists of planar surfaces. Like polygons, a polyhedron is *convex* if it contains every segment whose endpoints are on the solid. A *regular* polyhedron has congruent faces. There are five such regular polyhedra, which are also known as the *Platonic solids*. These are shown in Figure 8-4: the tetrahedron bounded by four triangles; the cube bounded by six squares; the octahedron bounded by eight triangles; the dodecahedron bounded by twelve pentagons; and the icosahedron bounded by 20 triangles.



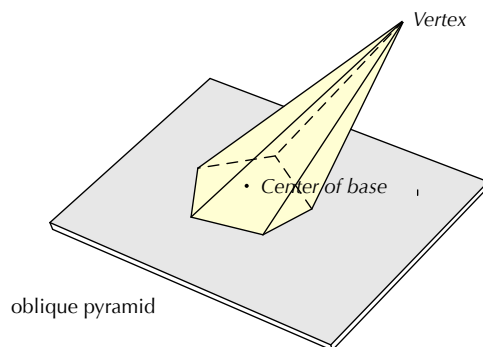
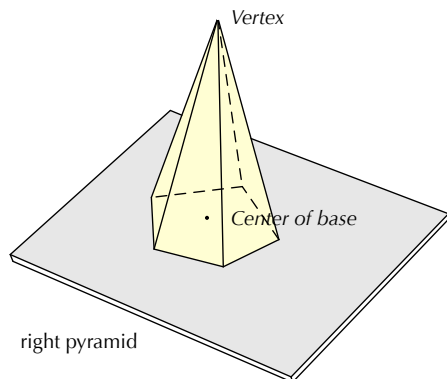
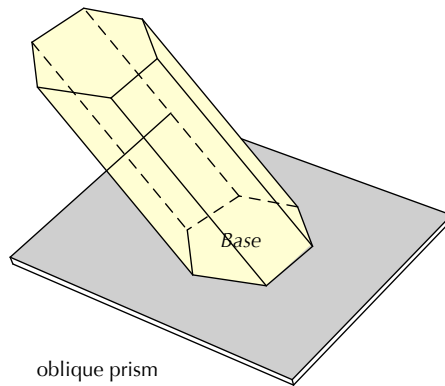
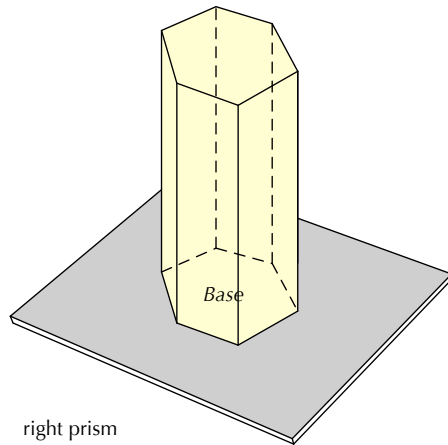
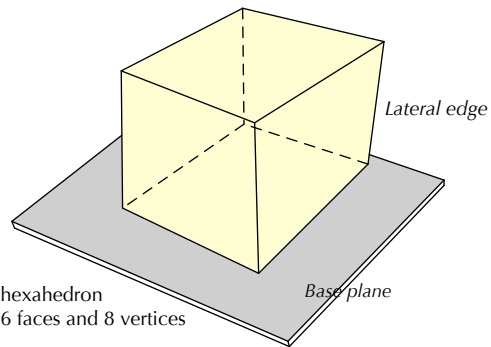
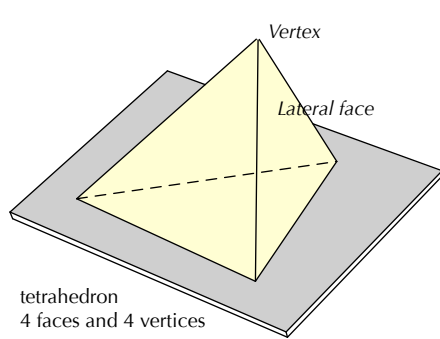
8-4
The Platonic Solids

Other or *irregular* polyhedra do not have this property and are described in other ways. Some are illustrated in Figure 8-5.

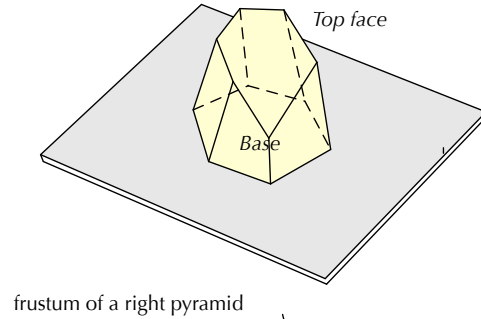
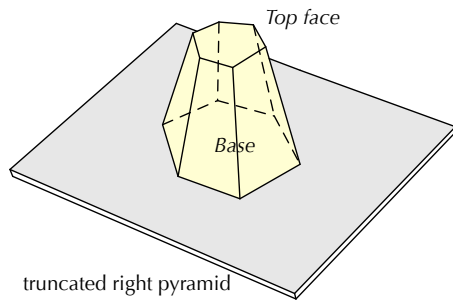
8.1.5 Euler's formula

All solid convex polyhedra satisfy a property known as Euler's formula: $V-E+F = 2$. That is, the number of vertices and faces is two more than the number of edges.

<i>Solid</i>	Tetrahedron	Hexahedron	Octahedron	Dodecahedron	Icosahedron
<i>Vertices</i>	4	8	6	20	12
<i>Edges</i>	6	12	12	30	30
<i>Faces</i>	4	6	8	12	20



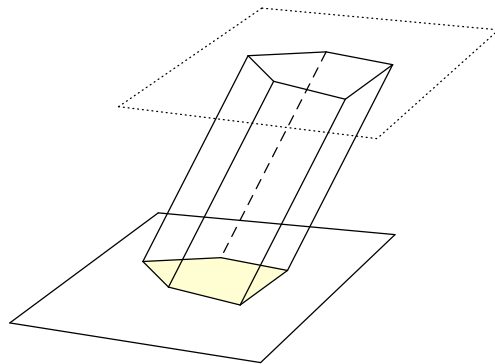
8-5
Example polyhedral solids



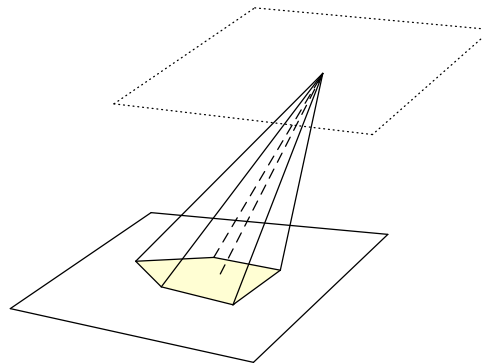
8-5(continued)
Example polyhedral solids

8.1.6 Extruded solids

Certain solids as shown in Figure 8-5 can be described by a process, termed *extrusion*, in which a simple polygon or a simply closed curve on a plane is swept along a line between the plane and another plane parallel to it in doing so gathering the points, which are contained in the extruded solid. These solids have a standard cross-section that does not vary over the length of the solid, and once a cross-section is known, only the line has to be specified to completely define the solid. An augmented form of extrusion adds a constant tapering of the cross-section over the length of the solid. Again, for a given cross-section, only the line has to be specified to completely define the solid. Both forms of extrusion are shown in Figure 8-6.



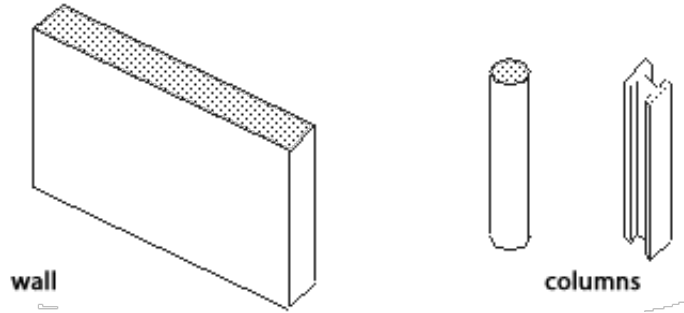
direction of the extrusion



extrusion with a constant taper

8-6
Extrusions

If a polygon with n sides is extruded, the extruded solid is an n -sided *prism*. A prism is a *parallelepiped* if the polygon is a parallelogram. If a polygon with n sides is extruded with taper, the extruded solid is an n -sided *pyramid*. Figure 8-7 illustrates extruded solids that are often encountered in buildings: walls and columns.



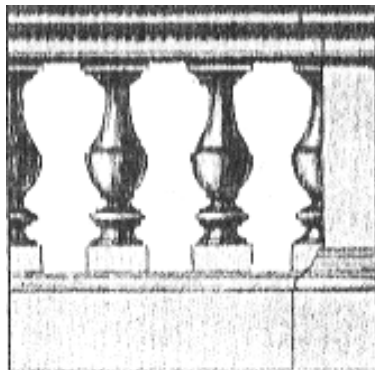
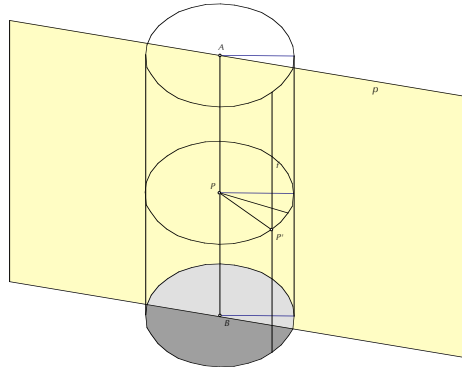
8-7
Extruded solids found in buildings

Extruded shapes often reflect a specific manufacturing process, one in which a malleable material is pressed through an opening or die whose form is the desired cross-section, very much like spaghetti are made by pressing amorphous pasta dough through a special die of a spaghetti machine.

8.1.7 Rotational solids

Here, solids are produced, by revolving a figure about a line. For each point on the line, we can specify a disc with radius equal to the perpendicular distance between the figure and line. The collection of points on the disc for all such discs constitutes a rotational solid. See Figure 8-8.

8-8
Creating a rotational solid



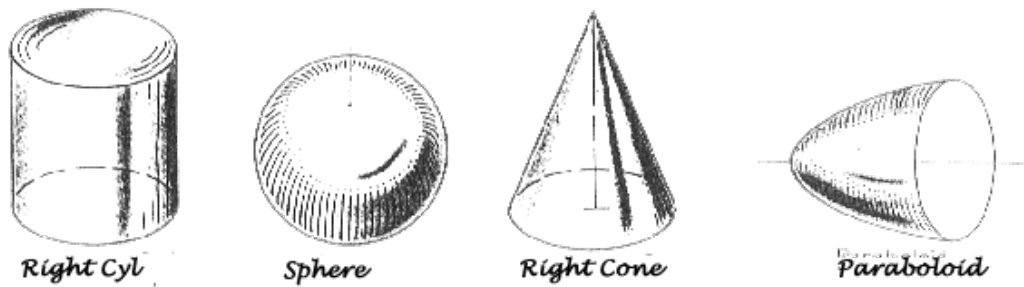
Column shafts with entasis or balusters of a balustrade are architectural examples of rotational solids.

8-9
Balustrade

When a line is revolved about a given line parallel to it, the solid generated by is a *rotational cylinder* or simply cylinder (which can also be considered as an extrusion solid.)

When the line revolves about a line also intersects it, the solid generated is a *right circular cone*. A cone may be considered as a rotational solid with constant taper.

When an ellipse is revolved about its major or minor axis, the solid generated is a *rotational ellipsoid*. If the ellipse is a circle, the solid is a sphere. Likewise, for a parabola, the solid generated is a *rotational paraboloid*, and for a hyperbola revolved about its major or minor axis, the solid generated is a *rotational hyperboloid*. See Figure 8-10 for examples of rotational solids.



8-10
Example rotational solids

The qualifier ‘rotational’ used in these examples distinguishes these solids from other solids in which a figure is revolved around a line in shapes other than disks.

Rotational solids may again reflect a special manufacturing process, for example one that uses a lathe that allows a piece of raw material to be rapidly rotated about a center line so that, when a cutting instrument is held at a constant distance from the center line, portions of material are removed at an equal distance from the center line. See Figure 8-11.



8-11
Solids of Rotation by Pedro Reyes (wood+paint)

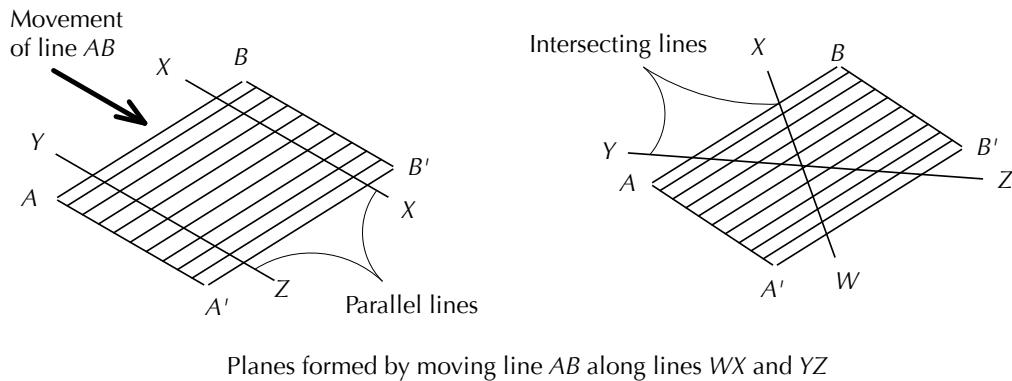
8.2 SURFACES

Planes, plane segments and boundaries of regular solids belong to a class of geometrical figures known as *surfaces*. While the precise definition of a general surface is somewhat complicated we will not attempt here; instead, we introduce some commonly encountered surfaces.

Surfaces fall into two major categories: ruled surfaces and double curved surfaces.

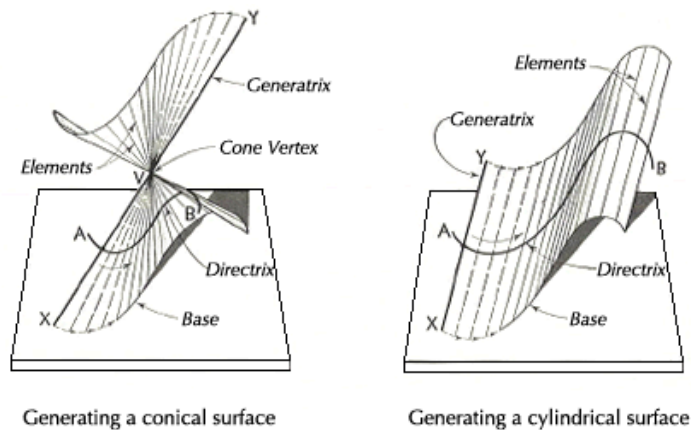
8.1.8 Ruled surfaces

A ruled surface is a surface that can be produced by moving a straight line. We have previously seen that parallel and intersecting lines can produce planes by moving a line parallel to itself while remaining in contact with the two given lines. See Figure 8-12.

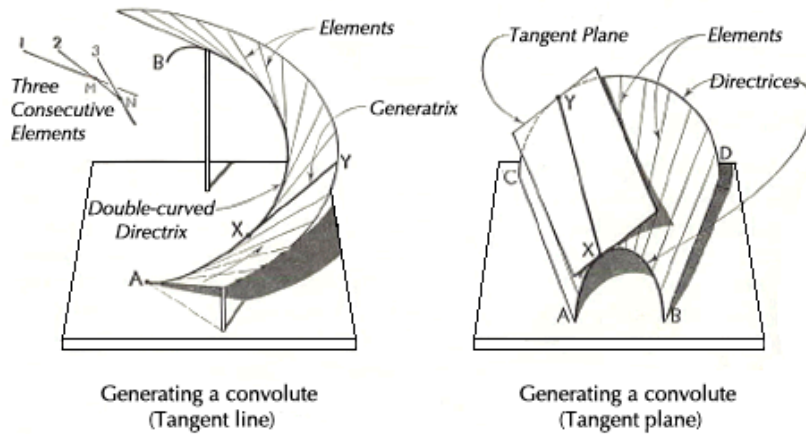


8-12
Producing planes by moving a line in continuous contact with two lines

Likewise, we can move a line whilst in continuous contact with a figure in a plane to produce a surface. See Figure 8-13.



8-13
Example ruled surfaces



8-13(continued)

Example ruled surfaces

When the figure is a curve, and the line is parallel to its initial position the surface is *cylindrical*. When the line is also in contact with a fixed point not on the curve, the surface generated is *conic*. Note that if the figure is a convex polygon, the respective surfaces are *prismatic* and *pyramidal*. When the line is a tangent to the curve, the surface generated is a *convolute*. Cylindrical, conic and convolute surfaces are examples of *single-curved surfaces*.

The lines on the surface are called its *elements* and are said to *generate* the surface. A representative element is called the *generatrix*. The curve is called the *directrix*. The surface is called *right* if the line is normal to the plane and *oblique*, otherwise. The surface is *circular* if the figure is a circle.



8-14

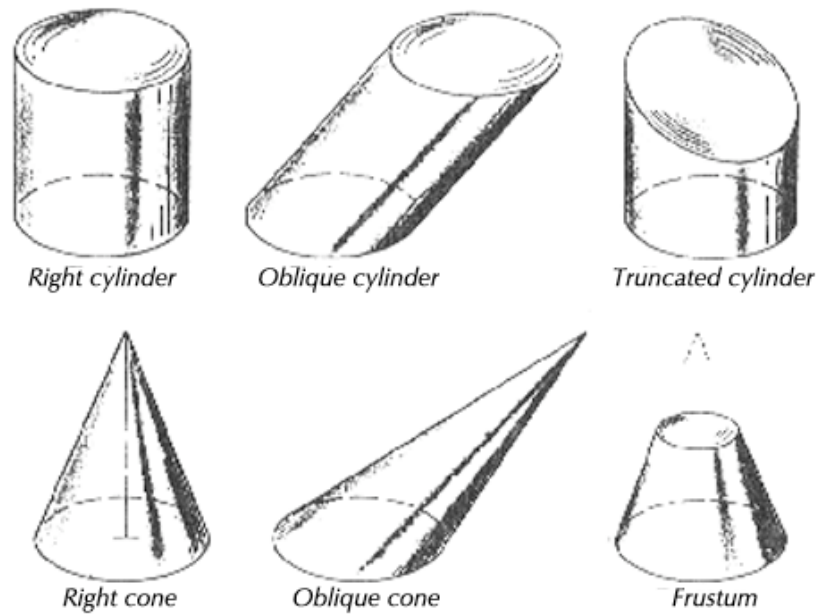
An example of a ruled surface

<http://www1.maths.leeds.ac.uk/~khouston/ruled.htm>

Ruled surfaces have the property that a straight line **on** the surface can be drawn through **any** point on the surface.

A ruled surface can become the boundary of a solid when its elements are restricted to the segments between two planes that intersect the elements. Some common ones are

shown in Figure 8-15. To complete the boundary, the segments on these planes that are between elements must be added. Terms like ‘cylinder’, ‘prism’, ‘pyramid’ and ‘cone’ are often used interchangeably to denote both a solid of that form or a surface.



8-15
Common ruled surfaces

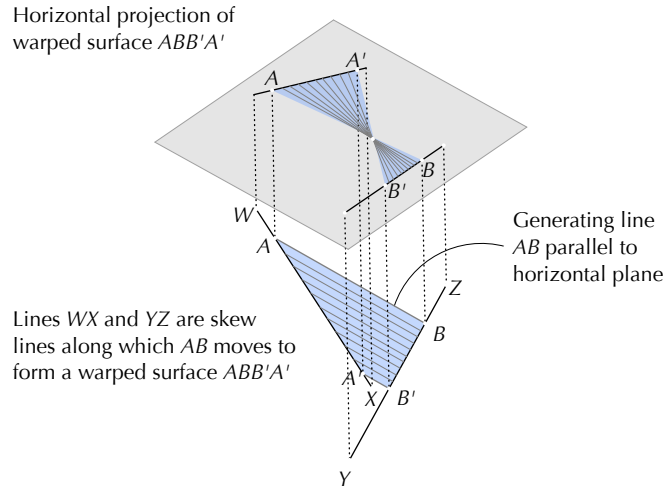
8.1.9 Warped Surface



8-16
The Philips Pavilion,
Brussels Expo 1958
Le Corbusier and Jannis Xenakis

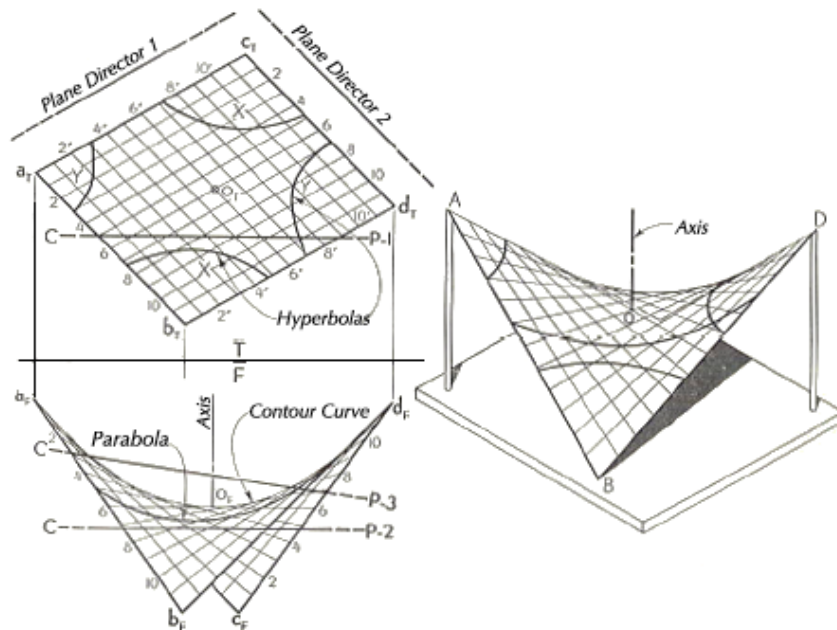
A ruled surface for which two successive elements are neither parallel nor pass through a common point is called *warped*. A well-known example is the *hyperbolic paraboloid* generated by line segments between two skew lines so that any two elements belong to

parallel planes. The surface is generated, by moving a line parallel to a given plane whilst maintaining continuous contact with two skew lines. See Figure 8-17.



8-17
Hyperbolic paraboloid

The standard position of a hyperbolic paraboloid is shown in Figure 8-18. In this position, sections with planes parallel to the two vertical planes are (congruent) parabolas, while sections with planes parallel to the horizontal plane are hyperbolas; thence the name of the surface.



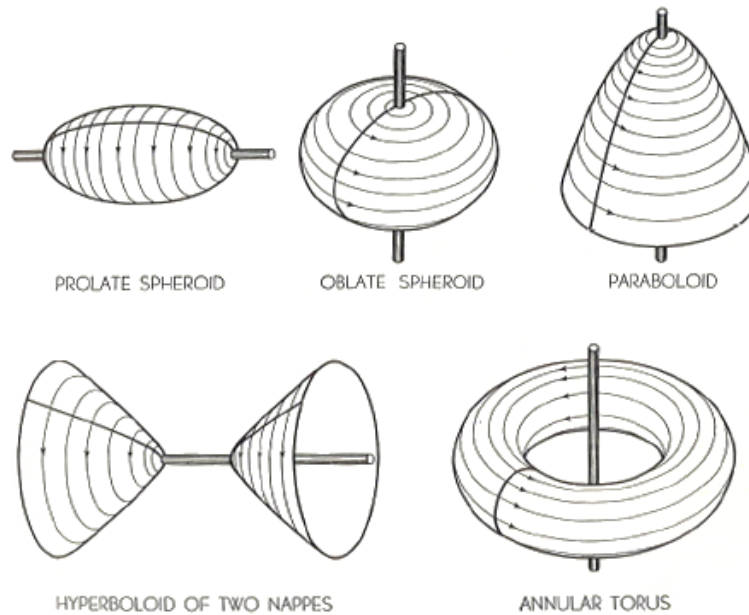
8-18
Parabolic hyperboloid – in the standard position

It is important to note that a warped surface cannot be **developed** accurately in that no two consecutive lines on the surface may lie in the same plane if the surface is laid onto a flat plane. The surface would be distorted.

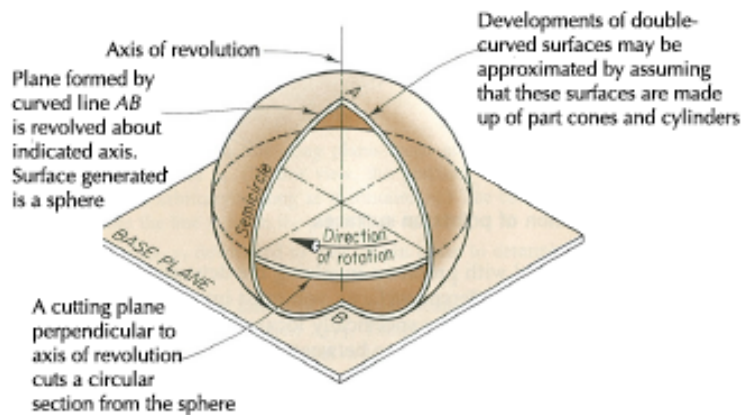
8.1.10 Double-curved surfaces

The boundaries of rotational solids generated by rotating a curve instead of a straight line are examples of double-curved surfaces. There are two kinds of double-curved surfaces: *surfaces of revolution* and *surfaces of evolution*. The former is produced by revolving a curved line about an axis; the latter by moving a curved line of constant or variable shape over a noncircular curved path. Figure 8-19 shows examples of double curved surfaces. The most common double curved surface is the sphere, which is obtained by rotating a semi-circle about an axis of revolution.

8-19
Double curved
surfaces



8-20
A double curved
surface



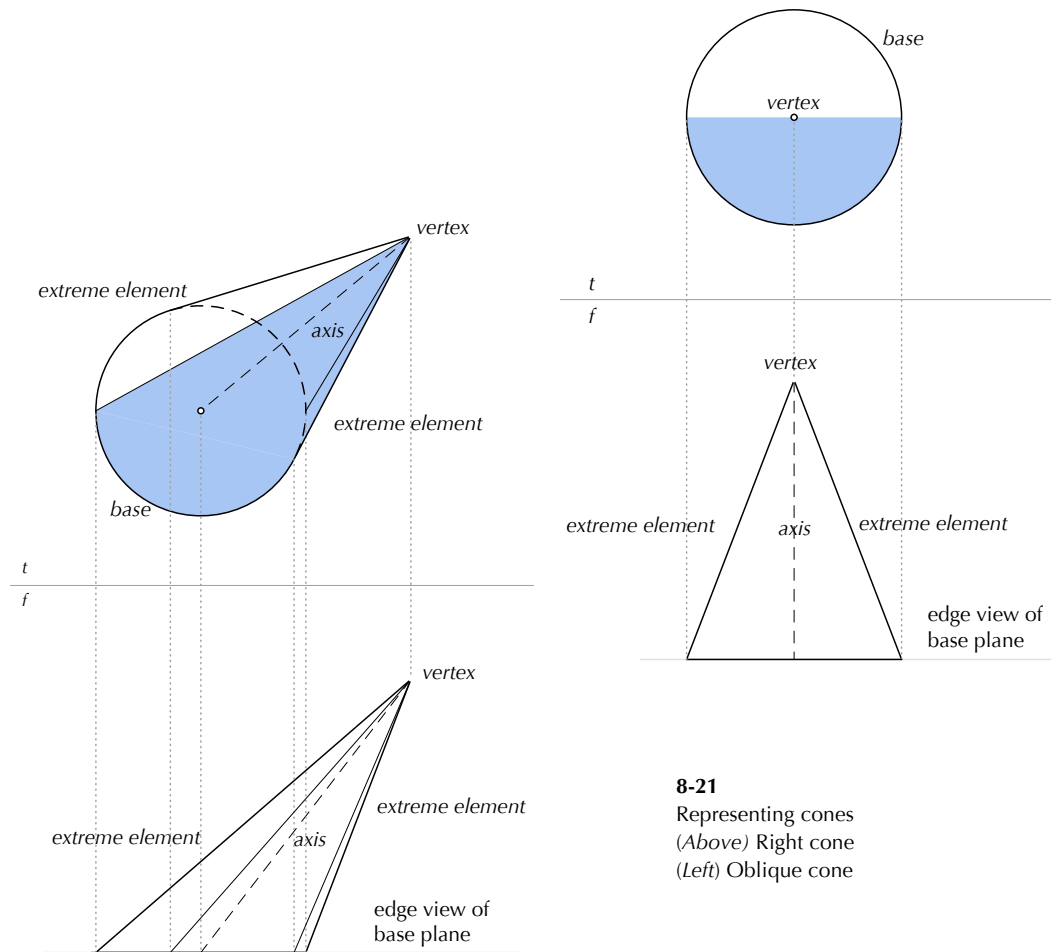
8.3 REPRESENTING SURFACES

Surfaces are typically represented in multi-view drawings by showing, in each view, the base curve(s) and/or directrices, the relevant elements, and the relevant vertices.

We consider two common surfaces: cones and cylinders. Figures 8-20, **Error! Reference source not found.** and 8-22 shows the top and front views of a right cone, an oblique cone, a right cylinder and oblique cylinder.

8.3.1 Cones

If the base curve is symmetric, then the axis may be shown as in Figure 8-21. Although a cone contains an infinite number of elements, it is sufficient to show in each view those elements that define the contour of the cone. In the top view for the oblique cone, these extreme elements are tangent to the base circle. The part of the cone that is visible in the front view is highlighted in the top view.

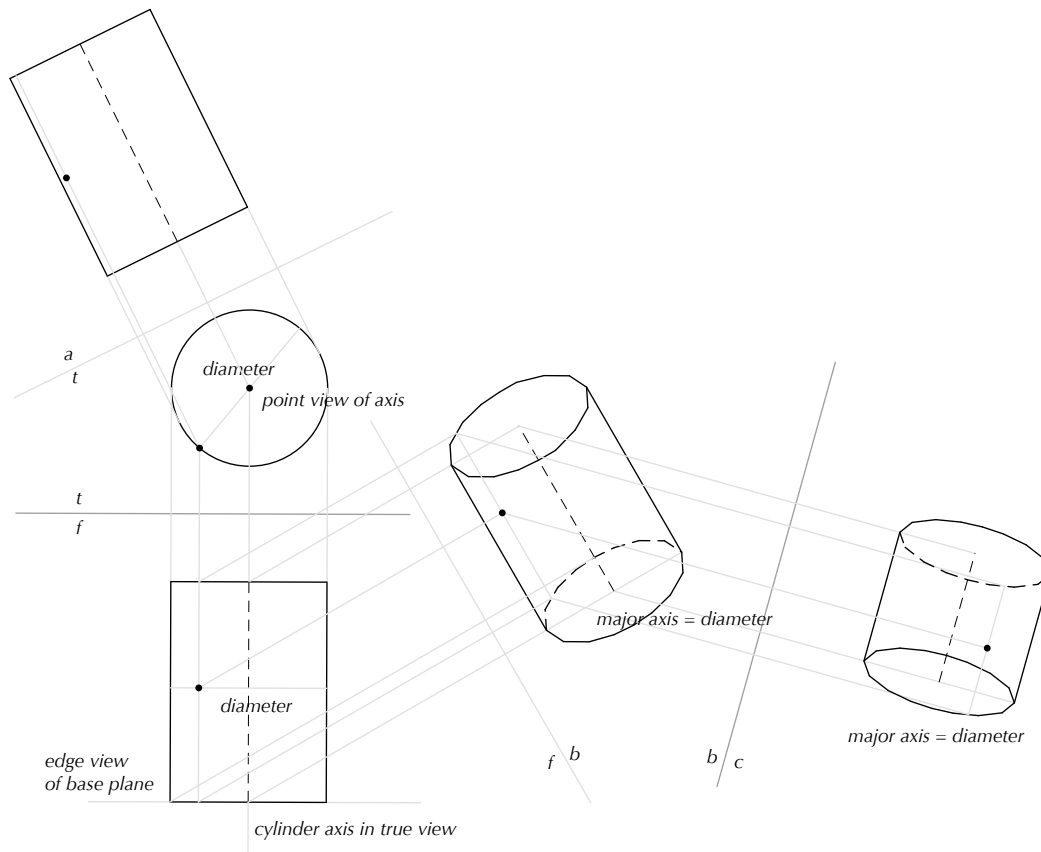


8-21
Representing cones
(Above) Right cone
(Left) Oblique cone

8.1.11 Cylinder

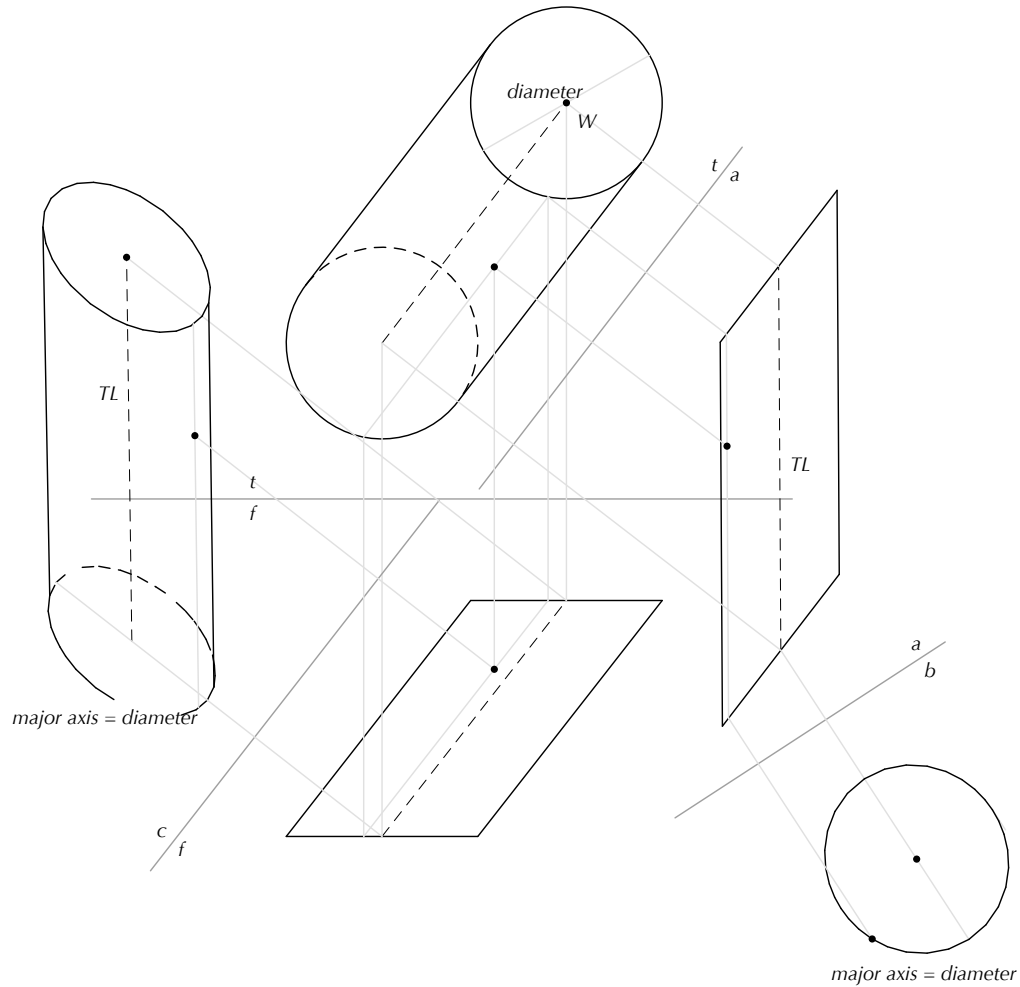
Cylindrical surfaces are represented in multi-view drawings by showing in each view the base curve(s) and the extreme elements. For regular cylinders the axis is also shown. See Figure 8-22.

Note that in the standard position, the base is seen in top view as a circle and in front view the axis of the cylinder is in true length. Auxiliary views *b* and *c* show the base as ellipses with the major axis equal to the diameter of the base. Figure 8-22 also shows how to locate a point on the surface of the cylinder.



8-22
Representation of a right cylinder

Figure 8-23 illustrates the representation of an oblique cylinder. Although the top and front views do not show the axis in true length, it shows the base circles as edges in front view. Views showing the axis in true length can be easily constructed. The base circle appears as ellipses in these views with their major axes equal to the diameter of the circular base.



8-23
Representation of an oblique cylinder

8.1.12 Locating points on surfaces

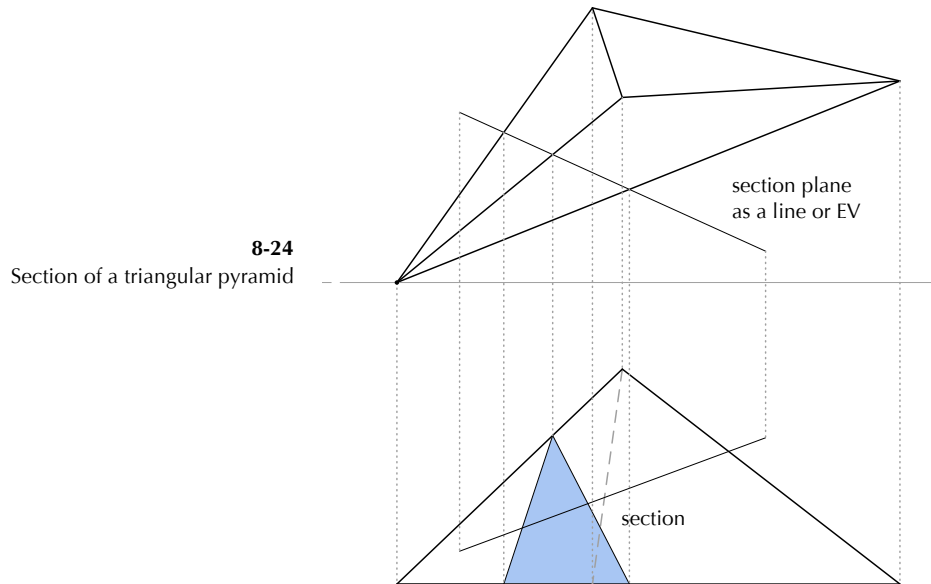
When one deals with problems in determining the lines of intersections between different surfaces, one must introduce methods by which points on the surface are specifically located. That is, determining the line of intersection between two intersecting surfaces actually means locating *points common to both surfaces*. One common way of looking at this is to consider section cuts.

8.4 SECTION OF A SOLID

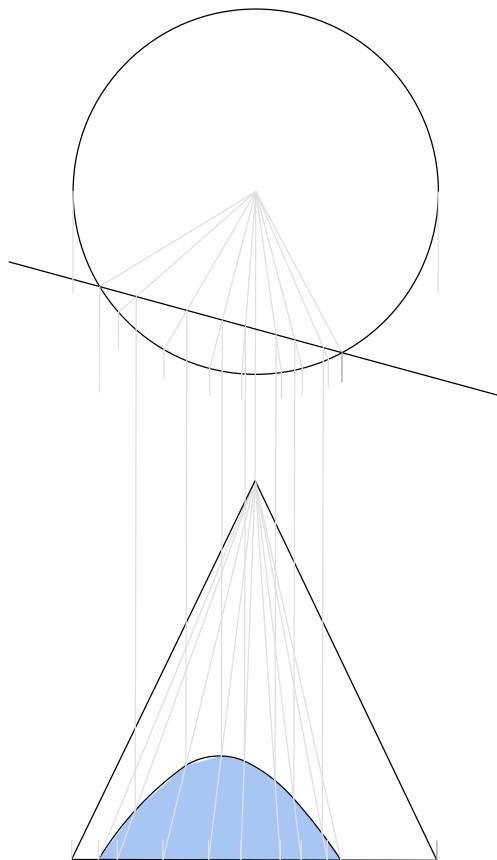
This idea is best illustrated by examples.

Figure 8-24 shows a triangular pyramid in adjacent views. In the top view, the 'section' plane that cuts through the solid is seen as a line, which may be considered as the edge view of the 'section' plane. If we project the points of intersection of the line with the

edges of the solid in top view into the front view, we obtain a section cut off from the solid, which is shown shaded.

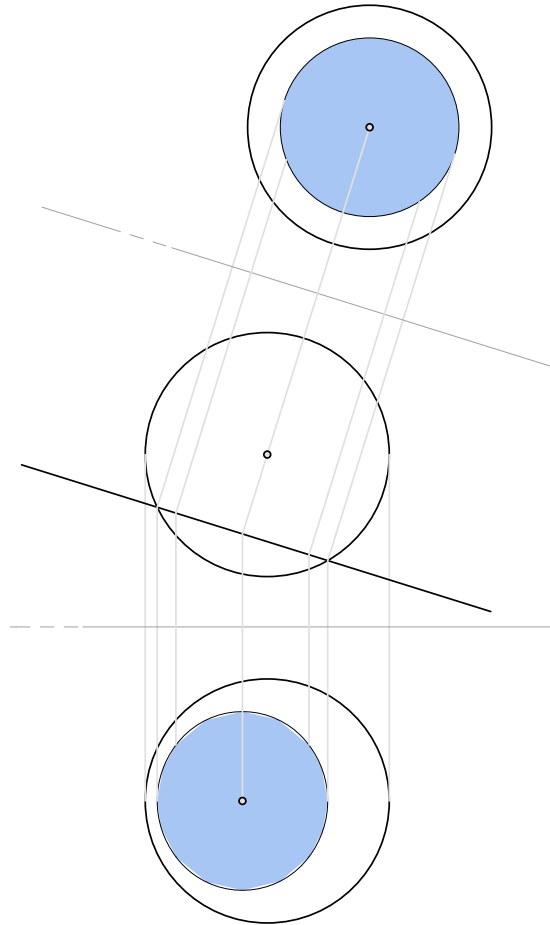


In the case of curved solids such as a sphere or a cone, a similar technique applies. Figures 8-25 and 8-27 illustrate respectively obtaining views of section cuts from a cone and a sphere.



8-25
Sections of a cone

In the case of the sphere, we use an auxiliary view that shows the section in true size to obtain the transfer distances needed to construct a view of the section.



8-26
Sections of a sphere

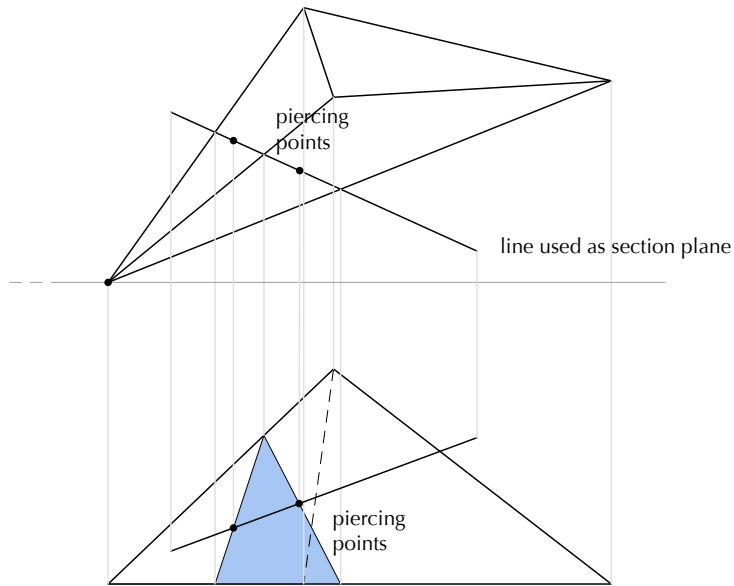
8.5 INTERSECTION OF A LINE AND A SOLID

In principle, we can use sectional views to determine the piercing points of a line and a solid. We treat the line in one view as the edge view of a section plane and determine the corresponding section in the other view. The points of intersection of the line (in the other view) and the section give the piercing points, which can then be projected back into the first view.

We will start with some simple examples.

8.1.13 Intersection of a line and pyramid

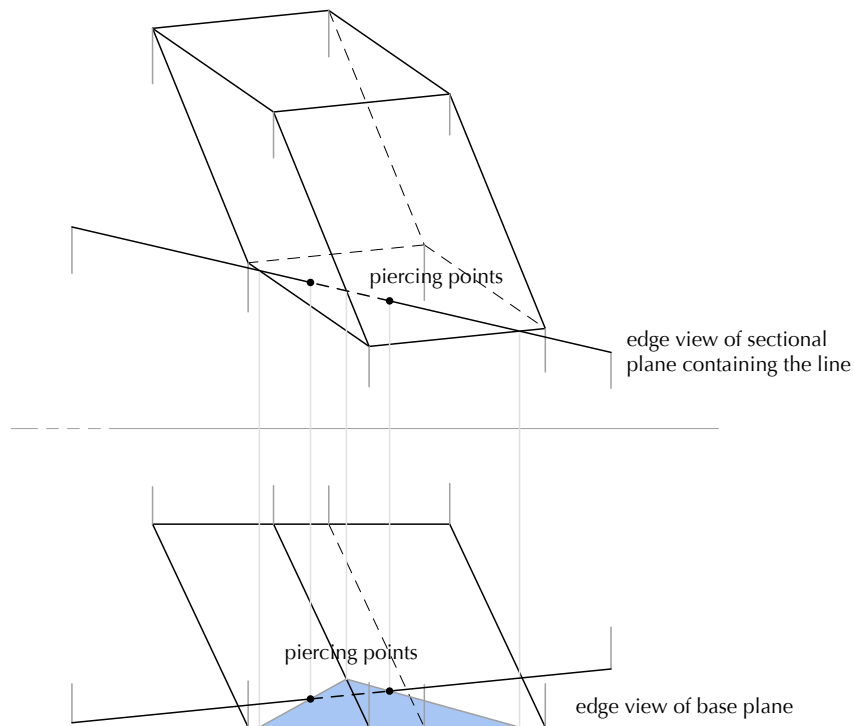
We consider the triangular pyramid in Figure 8-28. As suggested above, we first take a section cut using a view of the line as the edge view of a cutting plane and examine where the line meets the section in the other view.



8-27
Determining the piercing points of a line and a triangular pyramid

8.1.14 Intersection of a line and a prism

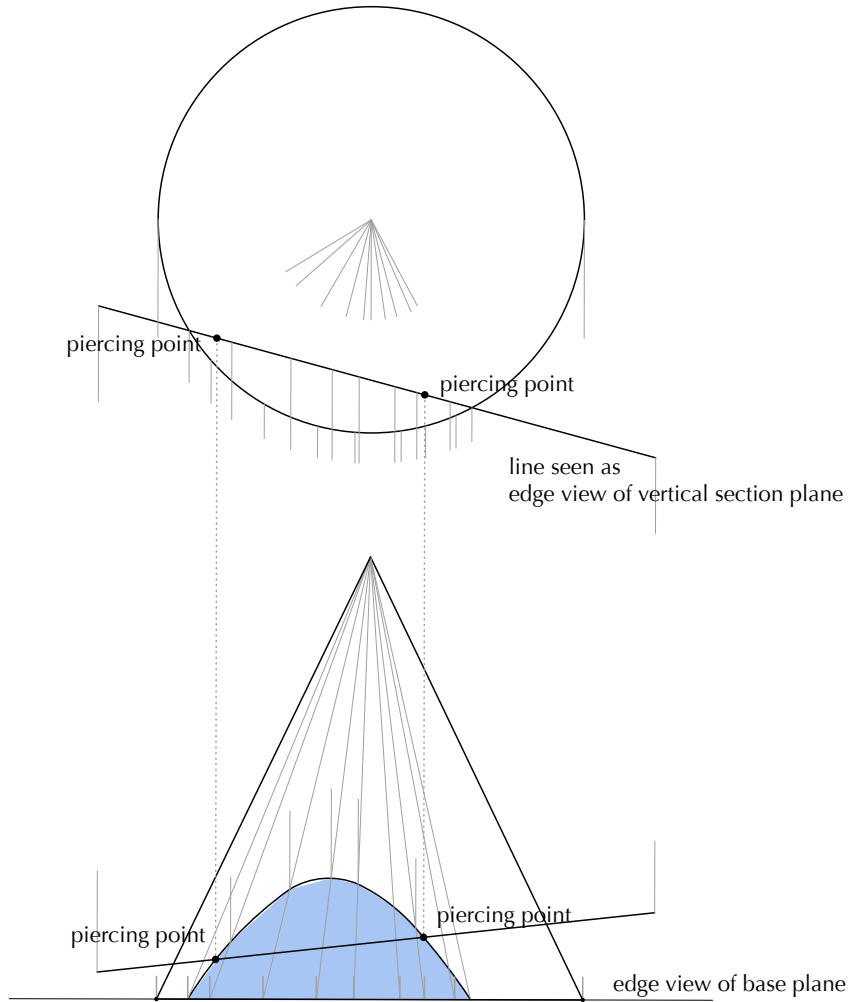
The same procedure can be applied to a prism to determine where a line meets it. Figure 8-28 illustrates the procedure for a simple inclined prism.



8-28
Intersection of a line and a prism

8.1.15 Intersection of a line with a right cone, a right cylinder and sphere

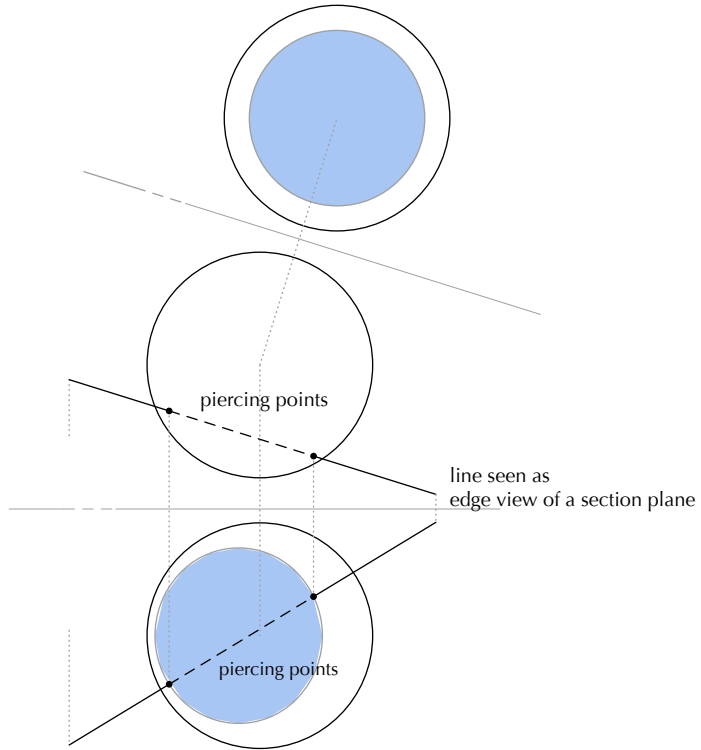
We employ a vertical section plane as illustrated in Figure 8-29 to demonstrate how to determine the intersection between a line and a right cone in a similar fashion to the intersection problems previously discussed.



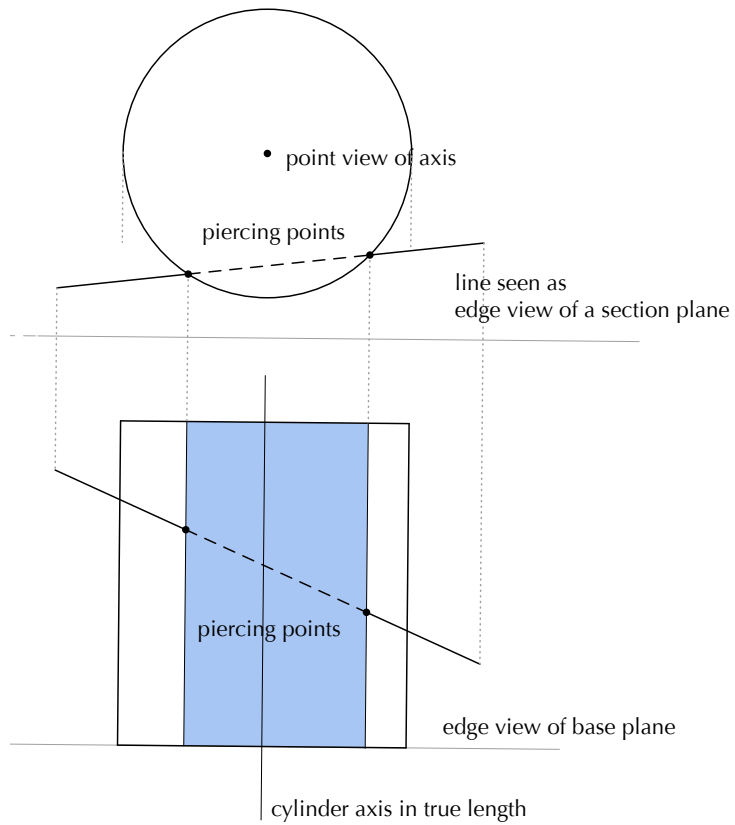
8-29
Intersection of a line and a right cone

The sphere and right cylinder can be treated in a similar fashion by also using a vertical section plane. See Figures 7.28 and 7.29.

8-30
Determining piercing points
of a line and sphere



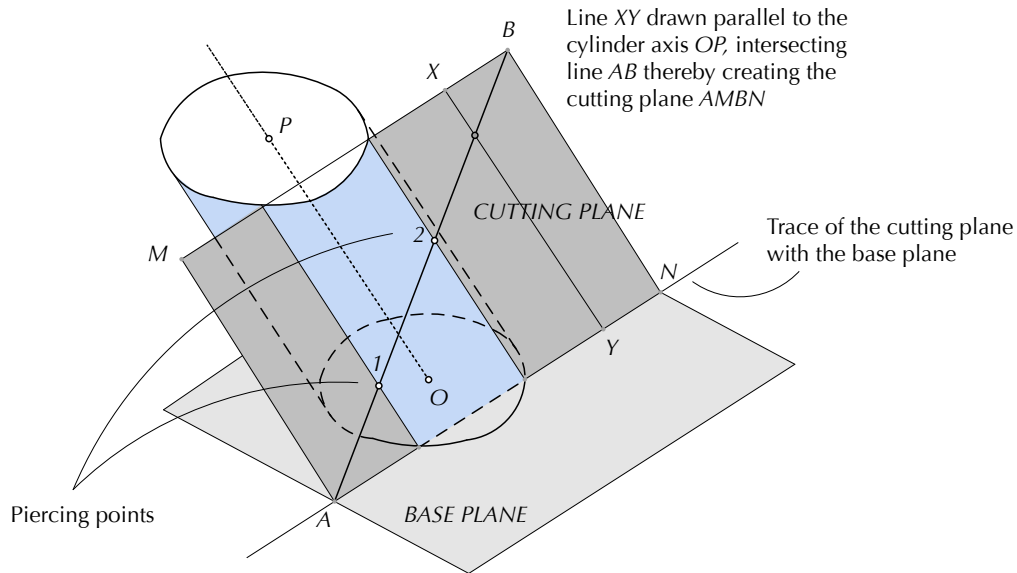
8-31
Determining piercing points
of a line and right cylinder



For other types of solids, special adaptations of the section or cutting plane method often lead to solutions.

8.1.16 Intersections of a line with an inclined cylinder

Figure 8-32 shows a line piercing a cylinder at two points and a cutting plane parallel to the axis of the cylinder containing the line.



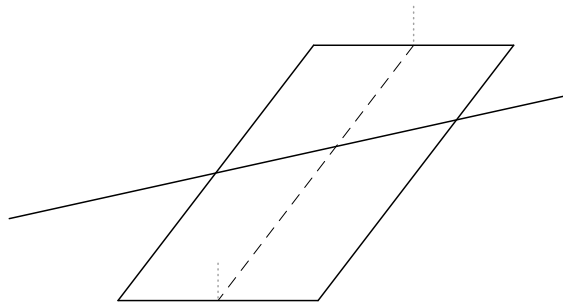
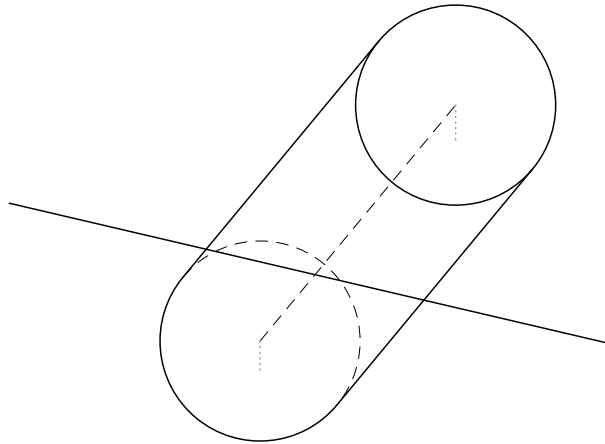
8-32
Cutting plane for a cylinder

Since the cutting plane is parallel to the cylinder axis, it cuts straight-line elements on the surface of the cylinder. When extended to the base plane, the cutting plane leaves a trace on the base plane that cuts the cylinder base at two points, which are the points through which the straight-line elements, determined by the cutting plane, pass. Since cutting plane contains the given line that intersects the cylinder, the points of intersection of the straight-line elements with this line are the piercing points for the given line and cylinder. If the cylinder and line are given in two views, it is easy to find the two line elements where the cutting plane and the cylinder intersect. The following construction shows this for an inclined cylinder, that is, a cylinder on an inclined axis.

Construction 8-1 *Intersection between an inclined cylinder and a line*

Given a line and an inclined cylinder in two adjacent views, t and f , where the axis of the cylinder is inclined but the base remains circular, find the intersection points between the line and the cylinder.

See Figure 7.31.

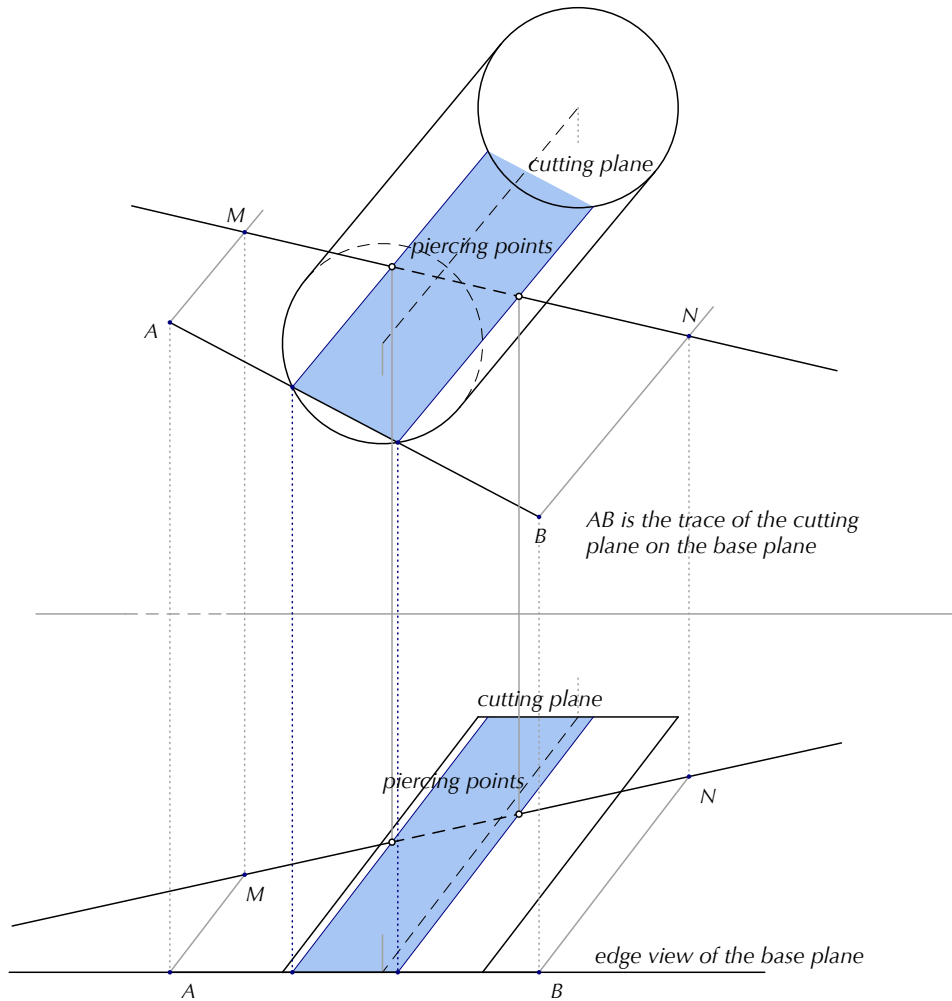


8-33

The problem – intersecting a line and an inclined cylinder

There are three steps:

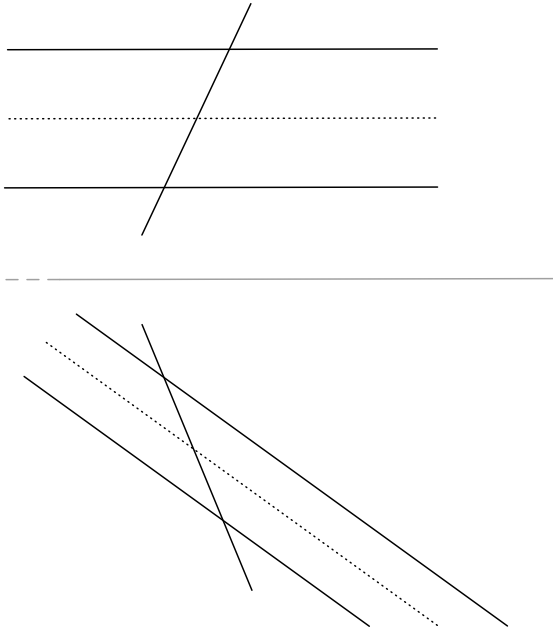
1. Select two points on the line, M and N , and draw two lines of the cutting plane through M and N , respectively, parallel to the axis of the cylinder in both views. These lines intersect with a plane parallel to a horizontal projection plane seen as an edge in the front view at points A and B . In Figure 7.32 the base plane of the cylinder in front view was selected for that purpose.
2. Project A and B into the top view. AB is the trace of the cutting plane in the top view. The cylinder intersects that plane in a circle. Its intersection with the trace gives two points of the cutting plane from which the intersection lines between the cutting plane and the cylinder can be drawn in both views.
3. The intersection between the intersection lines and the given line are the piercing points. We can then determine visibility between the line and cylinder, keeping in mind that different lines are on the outline of a cylinder in the different views.



8-34
Intersection of a line and an inclined cylinder

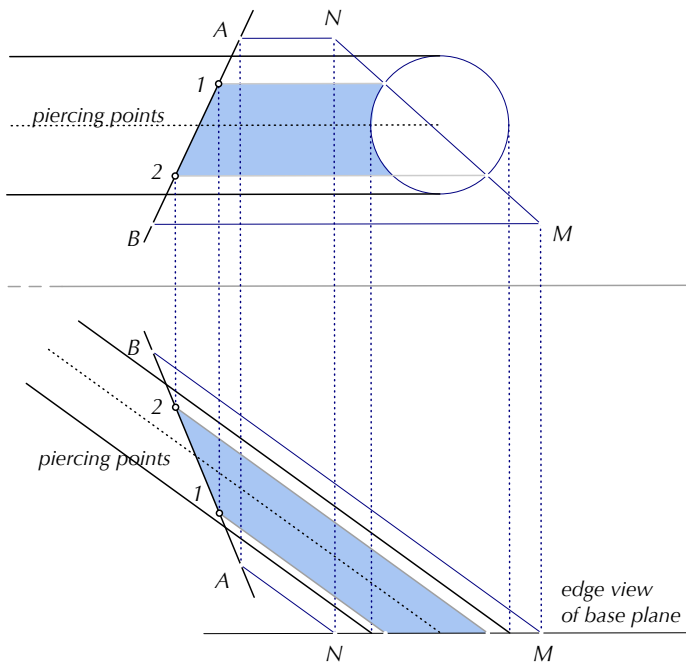
Construction 8-2
Intersection between a right cylinder seen obliquely and a line

A variation of this problem occurs when a right cylinder is oblique in both views. We consider a variation where the cylinder is parallel to the folding line in one view and oblique in the other. The construction is virtually similar in steps to the construction above and is shown in Figure 8-36.



8-35

A cylinder seen obliquely in front view and parallel to the folding line in the top view



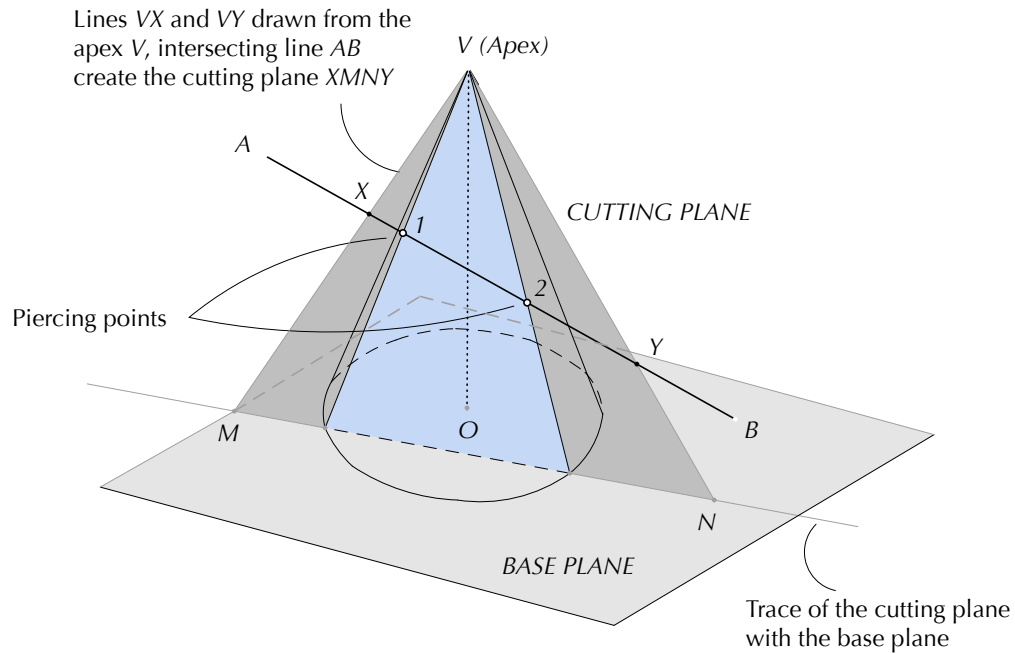
8-36

Intersection of a line and cylinder shown obliquely

The reader should note that when both views of a right cylinder are oblique, we first find an *auxiliary* view in which the axis of the cylinder is parallel to the folding line, and then apply this construction.

8.1.17 Intersections of a line with an inclined cone

Intersections between a line and an inclined cone can be constructed in a similar fashion using a cutting plane through the line and the vertex of the cone as illustrated in Figure 8-37.



8-37
Cutting plane for a cone

Construction 8-3

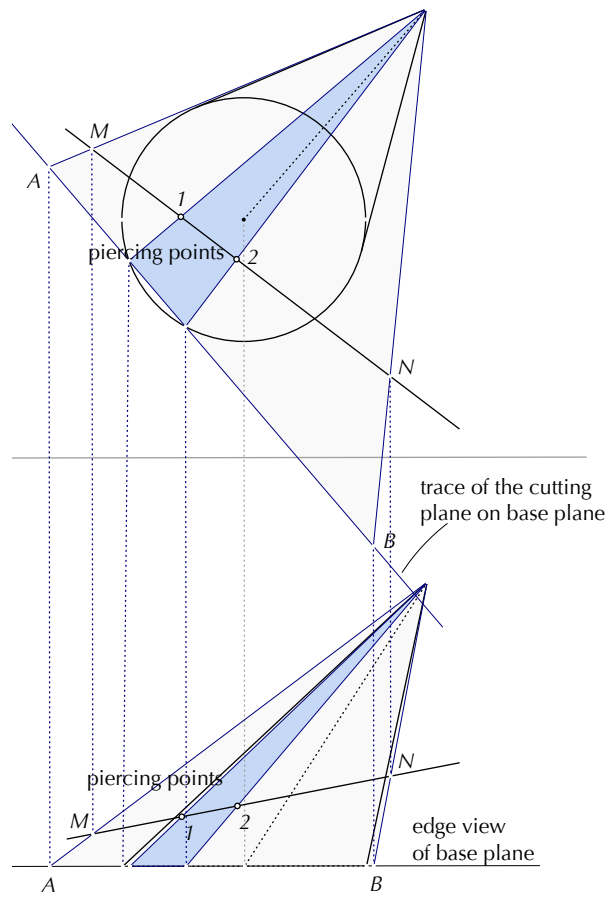
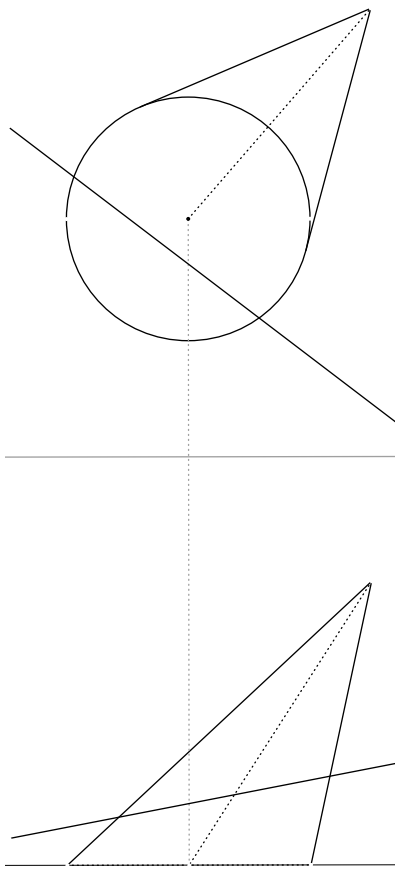
Intersection between an oblique cone and line

For a given line and a cone in two adjacent views, where the axis of the cone is inclined, find the intersection point(s) between the line and the cone.

There are three steps.

1. Select two points, M and N , on the line. Draw two lines through M and N , from the vertex V to intersect the base plane of the cone at points A and B . The lines VA and VB specify the cutting plane in both views.
2. Project A and B into the top view. AB is the trace of the cutting plane in the base plane. The cone intersects that plane in an ellipse. Its intersection with the trace gives two points of the cutting plane from which the intersection lines between the cutting plane and the cone can be drawn in both views.
3. The intersection between the intersection lines and the given line are the piercing points. Visibility between line and cone is determined in the usual way.

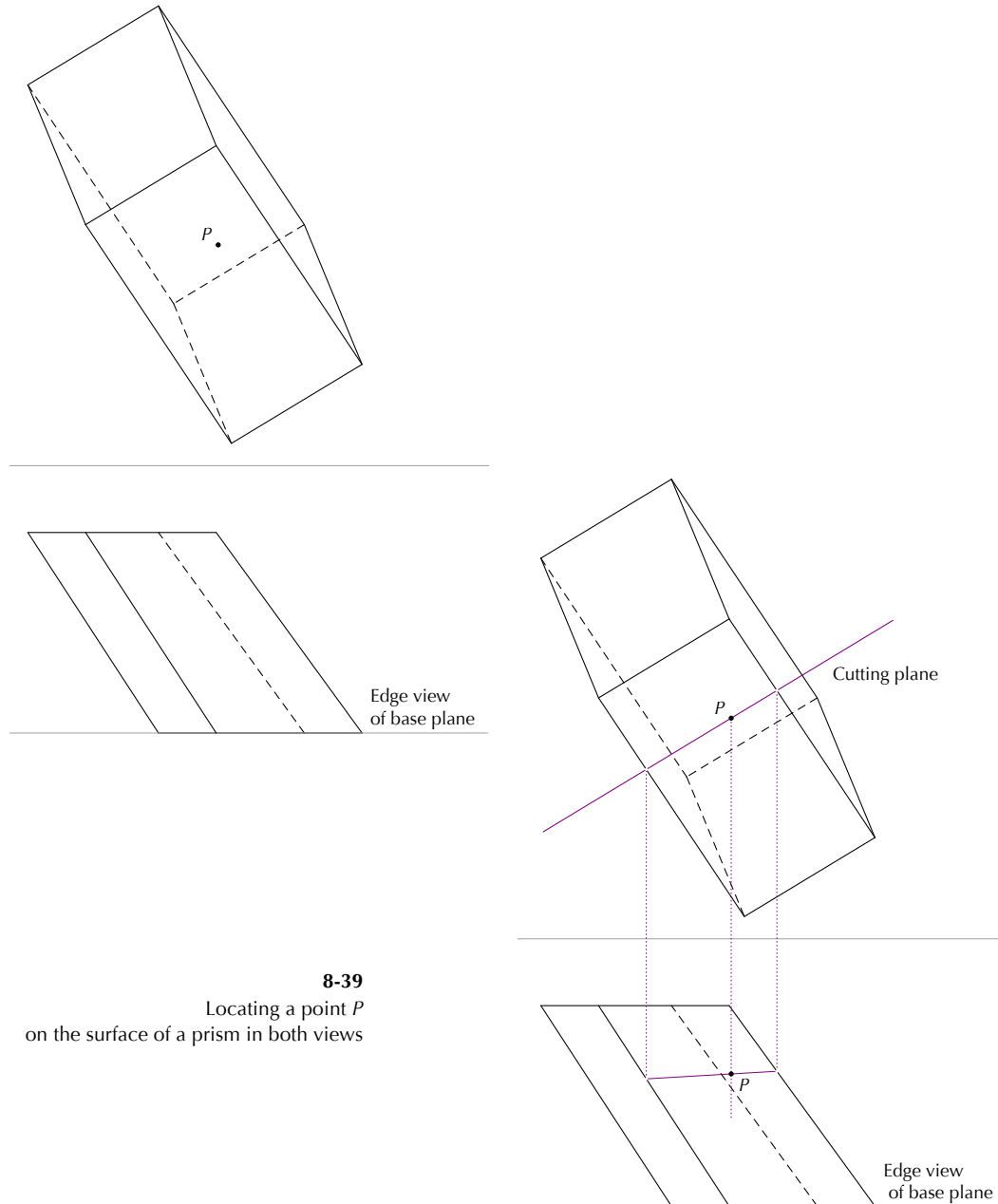
The construction is shown in Figure 8-38.



8-38
 Intersecting a line with an inclined cone
 (above) the problem
 (right) the solution

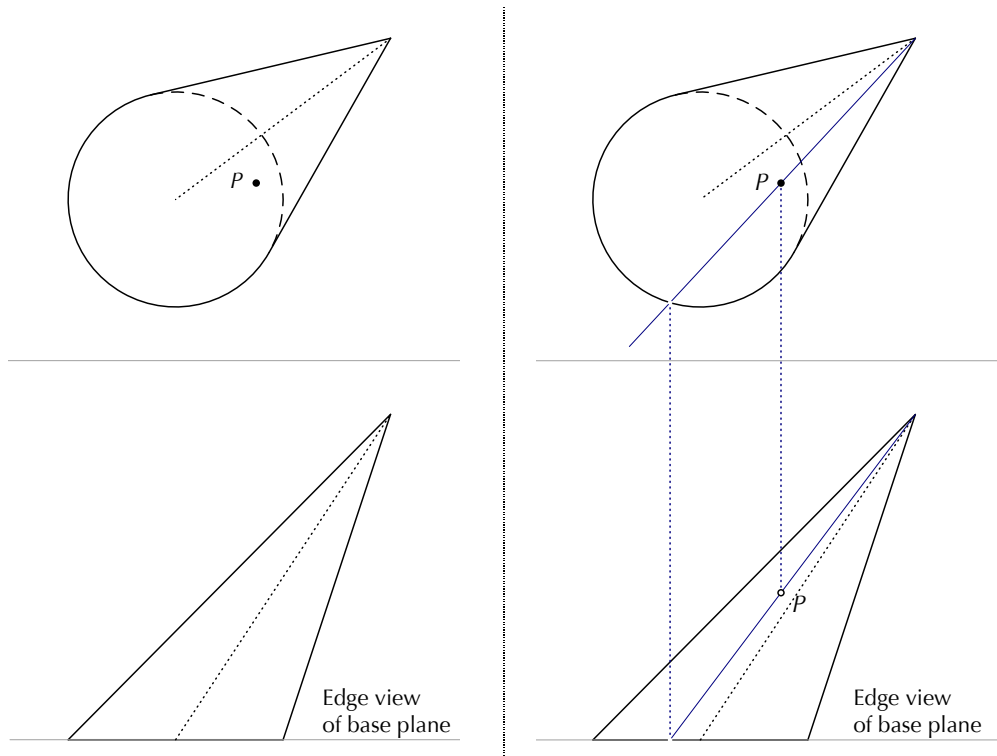
8.6 LOCATING POINTS ON COMMON GEOMETRICAL SURFACES

Implicit in the preceding constructions is the ability to locate points on surfaces of objects. For instance, locating a point on the surface of a prism or pyramid is easy by the cutting plane method. See Figure 8-39.



8-39
Locating a point P
on the surface of a prism in both views

To locate a point on the surface a cone is much simpler. Here one has to draw a line from the apex through the point to the base of the cone. See Figure 8.40. Cylinders were previously treated. See Figures 8-22 and 8-23.



8-40
Locating a point on the surface of a cone

8.7 PLANES TANGENT TO THE SURFACE OF A CONE AND CYLINDER

As we have seen we can use the cutting plane method for locating points and determining piercing points. When determining the intersection of certain geometrical objects such as cones and cylinders, we employ lines on the surfaces of these objects to construct points that lie on their line of intersection. Occasionally these cutting planes are tangential to the surface of the object. These plane tangents fall into three categories:

- Plane tangents to a specific point on the surface
- Plane tangents to the surface but contain points outside the surface
- Plane tangents to the surface but parallel to a line outside the surface

Construction 8-4

Plane tangents to a specific point on the surface

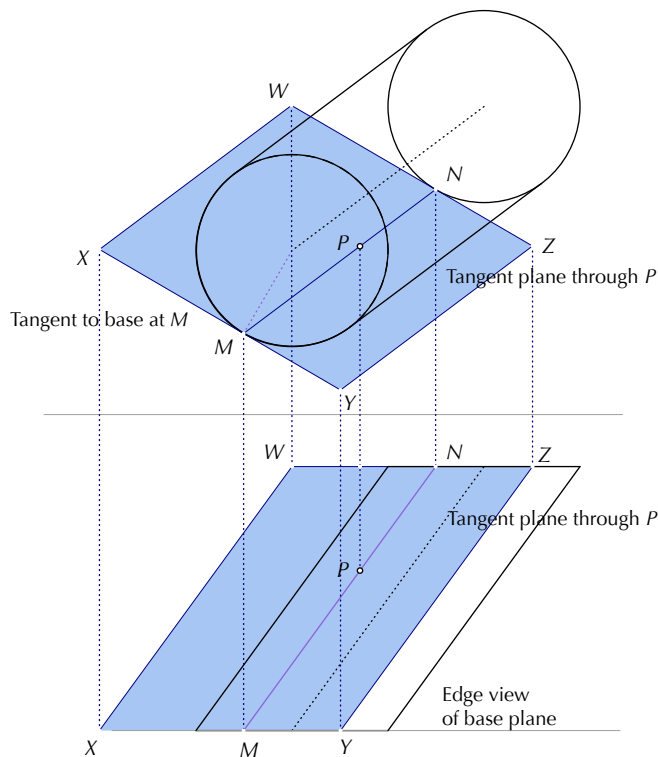
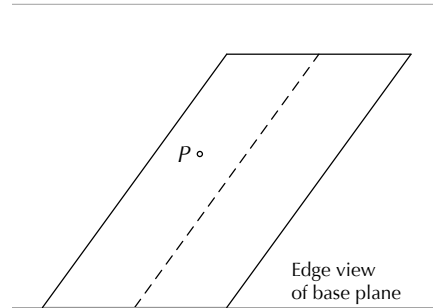
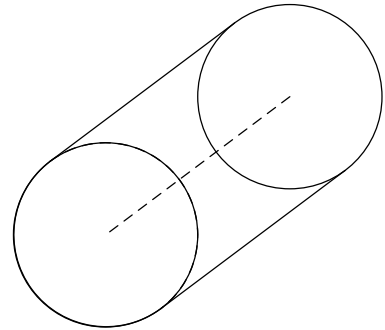
Cylinder

We are given a point P on the surface of a cylinder.

A tangent plane to a cylinder along any one of its line elements is parallel to the axis of the cylinder. Moreover, its trace is tangential to the base of the cylinder. We can use these facts.

The construction is shown in Figure 8-41. First, we locate point P in both views. For this, given P in either view, draw line parallel to the axis through P to meet the base at M . Project M to the other view and draw a line parallel to axis. Project P onto this line to locate P in the other view.

Next, in top view, construct a tangent at M and mark two points X and Y on it. Project these points to the front view. In top view, draw a line through N parallel to the tangent. From X and Y draw lines parallel to MN to meet the above line at W and Z . The points $WXYZ$ define the tangent plane. Project and locate W and Z in the front view.



8-41

Constructing a tangent plane through a point on the surface of a cylinder

Cone

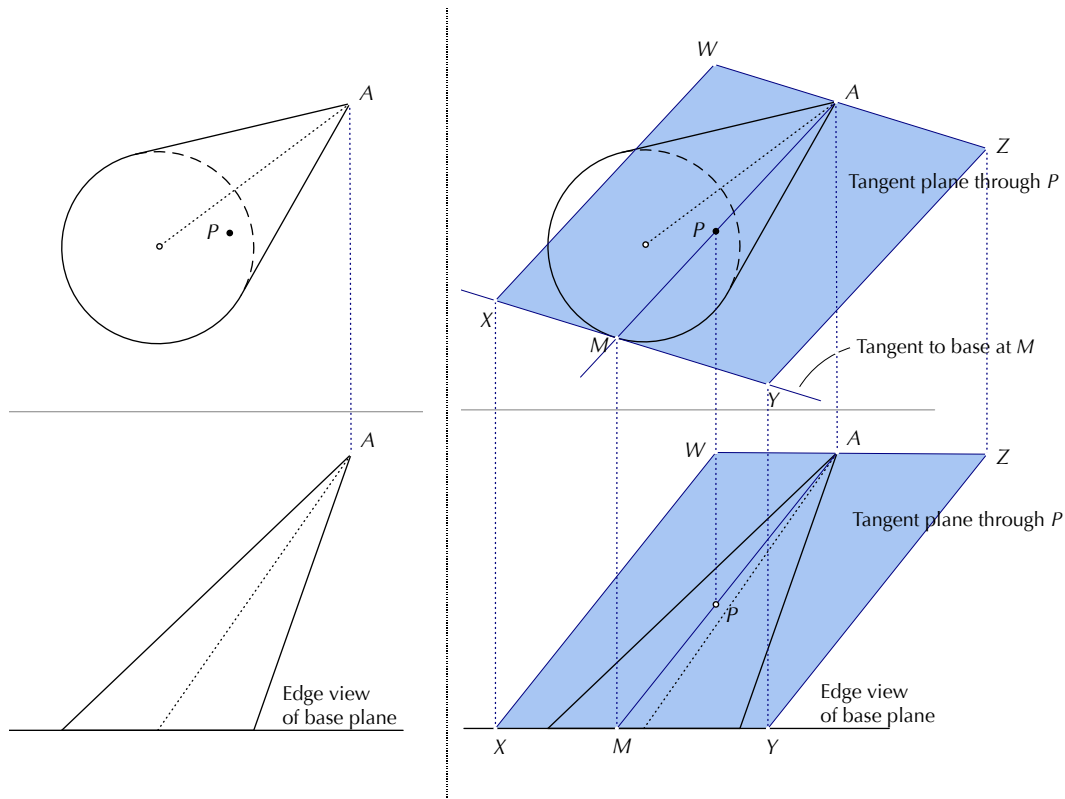
We are given a point P on the surface of a cone.

A plane tangent to the cone has a trace that is tangential to the base of the cone. This fact can be used to construct the plane tangent.

The construction is as follows.

As before, locate the point P in both views. In top view, construct a tangent at M and mark two points X and Y on it. Project these points to the front view. In top view, draw a line through apex A parallel to the tangent. From X and Y draw lines parallel to AM to meet the above line at W and Z . The points $WXYZ$ define the tangent plane. Project and locate W and Z in the front view.

See Figure 8-42.



8-42

Constructing a tangent plane through a point on the surface of a cone

Construction 8-5

Plane tangents through a point outside the surface

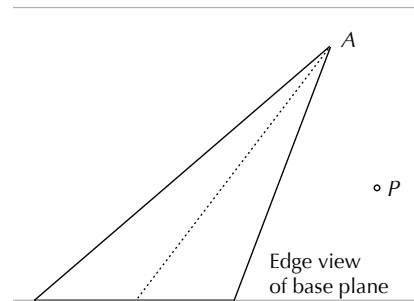
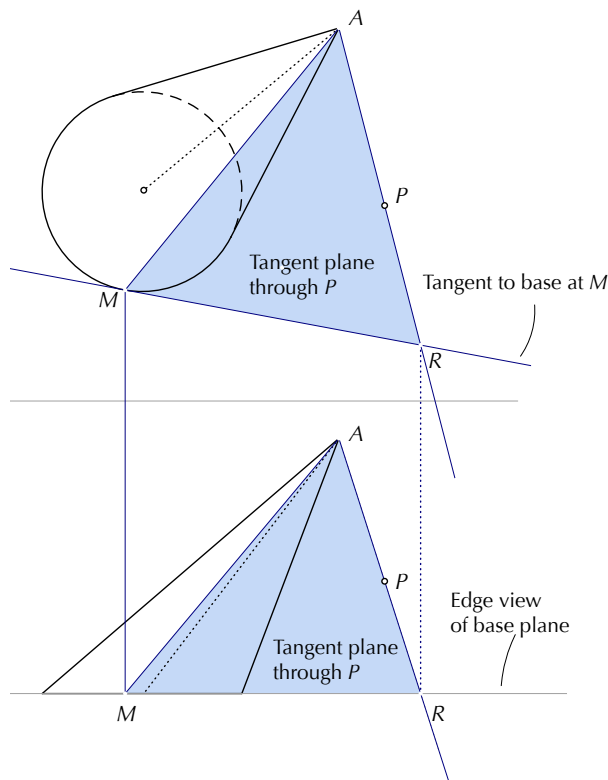
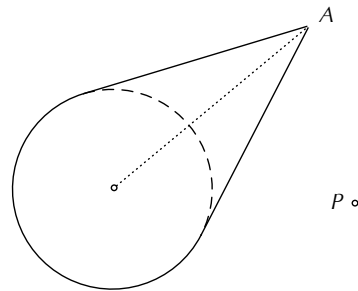
Cone

We are given a point P outside the surface of the cone.

We construct the tangent plane in a manner similar to the preceding construction.

In front view draw line from A through P to meet the base plane at R . Project R to the top view to meet the line through A and P . In the top view draw a tangent to M from R . AMR is the required tangent plane. Project M to the front view.

See Figure 8.43.



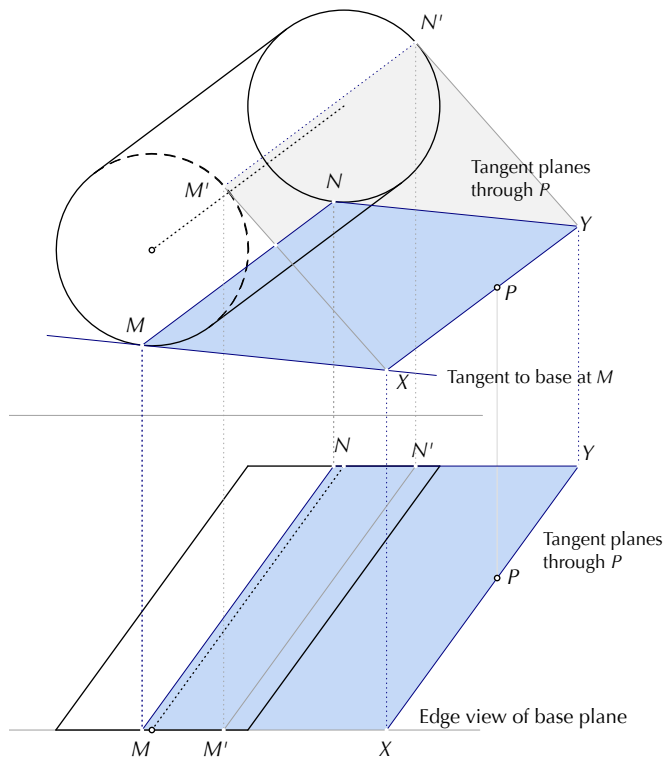
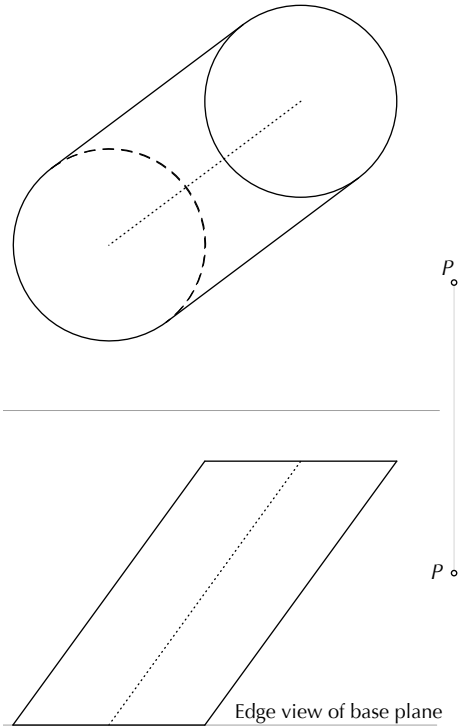
8-43

Constructing a tangent plane through a point outside the surface of a cone

Cylinder

We are given a point P outside the surface of the cylinder.

The construction is similar in to the above steps. A detailed description is omitted although the construction is shown in 8.42. Notice that there are two possible tangent planes $MNYX$ and $M'N'YX$ through point P outside the cylinder (Why?).



8-44
Constructing a tangent plane through a point outside a cylinder

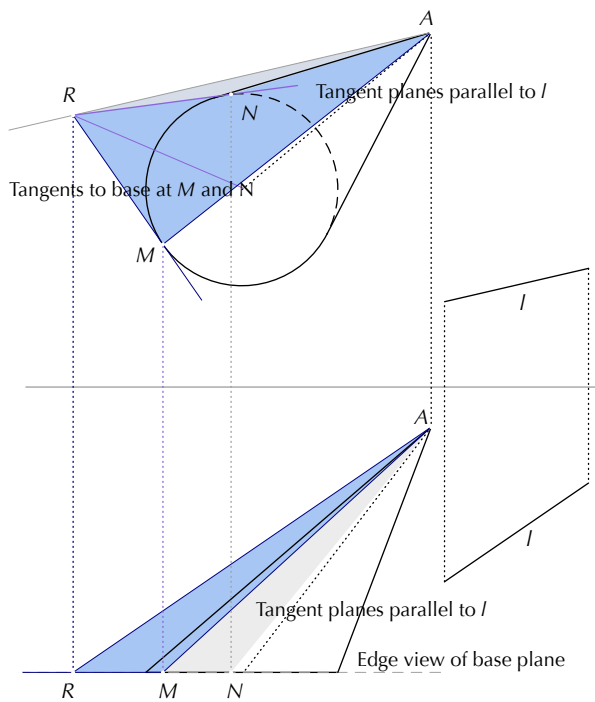
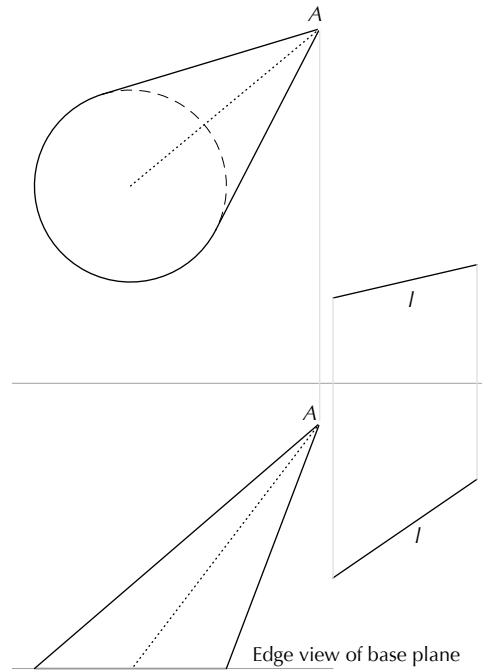
Construction 8-6

Plane tangents parallel to a line outside the surface

Cone

We are given a line l outside the surface of the cone

This construction is similar to that when given a point outside the surface of the cone. Here one determines the straight-line element along which the plane is tangent to the cone by constructing a plane through the apex and is parallel to the line outside. The construction is given in Figure 8-45.



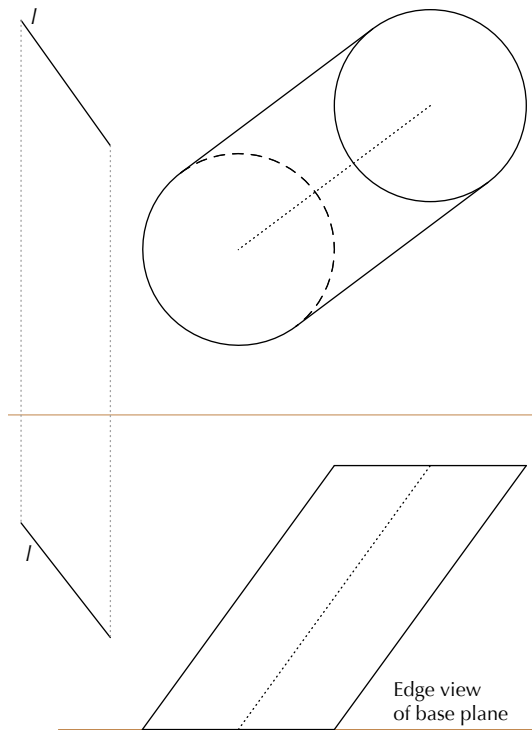
AR is parallel to l .

8-45
Plane tangents parallel to a line l
outside the surface of the cone

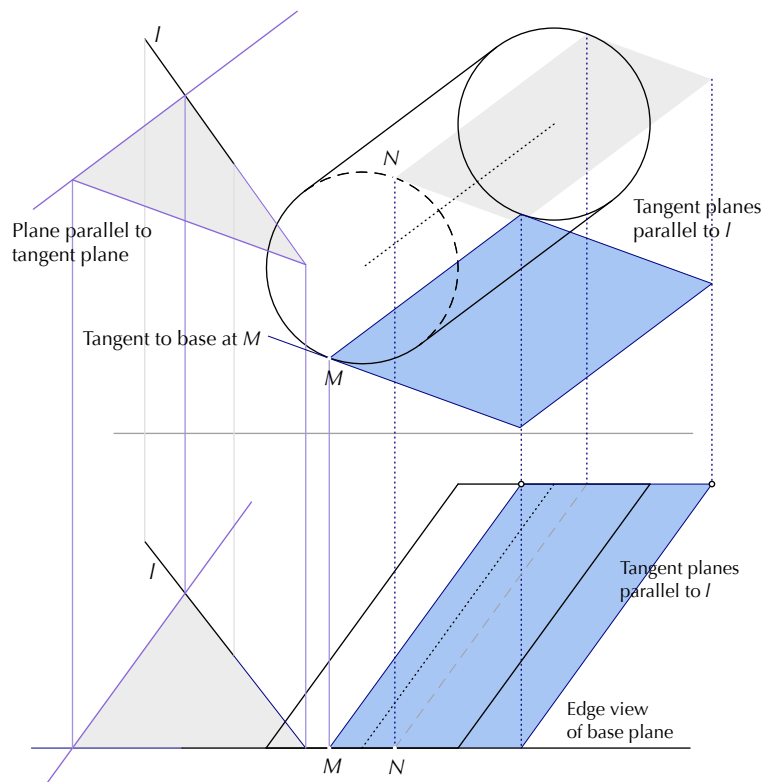
Cylinder

We are given a line l outside the surface of the cone.

(Right) The problem



8-46
Tangent plane
parallel to a line l
outside a cylinder



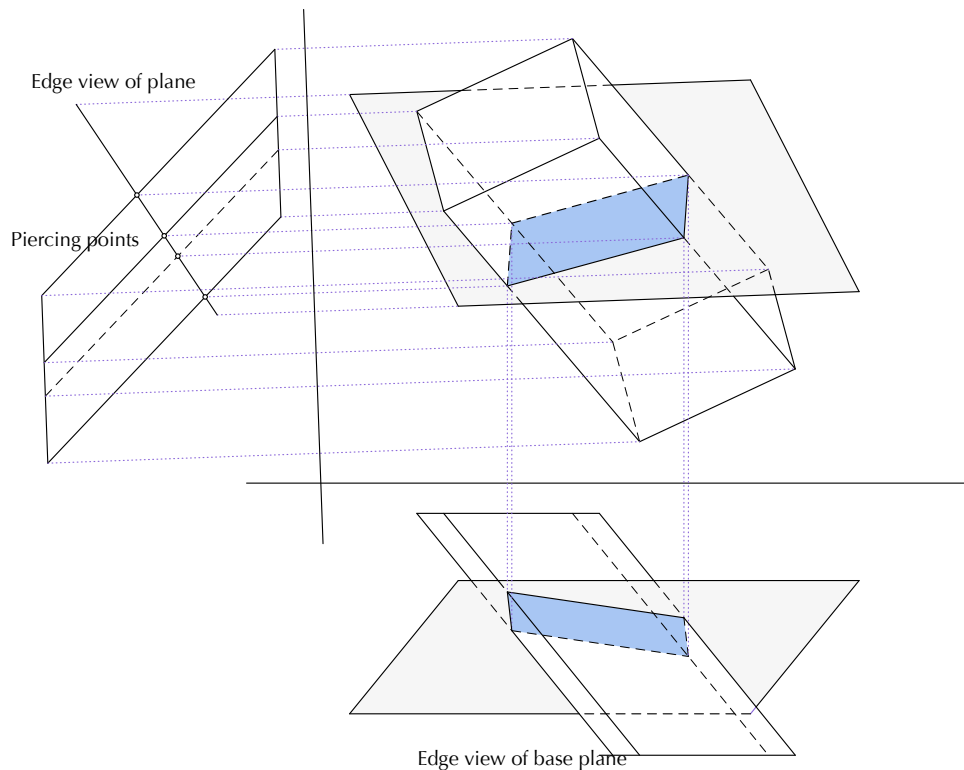
8.8 INTERSECTIONS OF COMMON GEOMETRICAL SOLIDS AND SURFACES

The study of the intersection of the solids and surfaces promotes visualization of three-dimensional space and the relationships, in this space, between various geometrical entities. To solve such problems one needs an understanding of i) planes and their properties; ii) relationship between lines and planes; and iii) basic characteristics of solids and surfaces.

In the case of the solids bounded by planar surfaces, the methods introduced below lead quickly to solutions to intersection problems. This is the case most often encountered in architectural applications. The roof example used to illustrate the applicability of Figures 6.14 and 6.15 can be taken as an illustration. For other solids, special adaptations of the cutting plane method can be used. We illustrate this with a few example constructions.

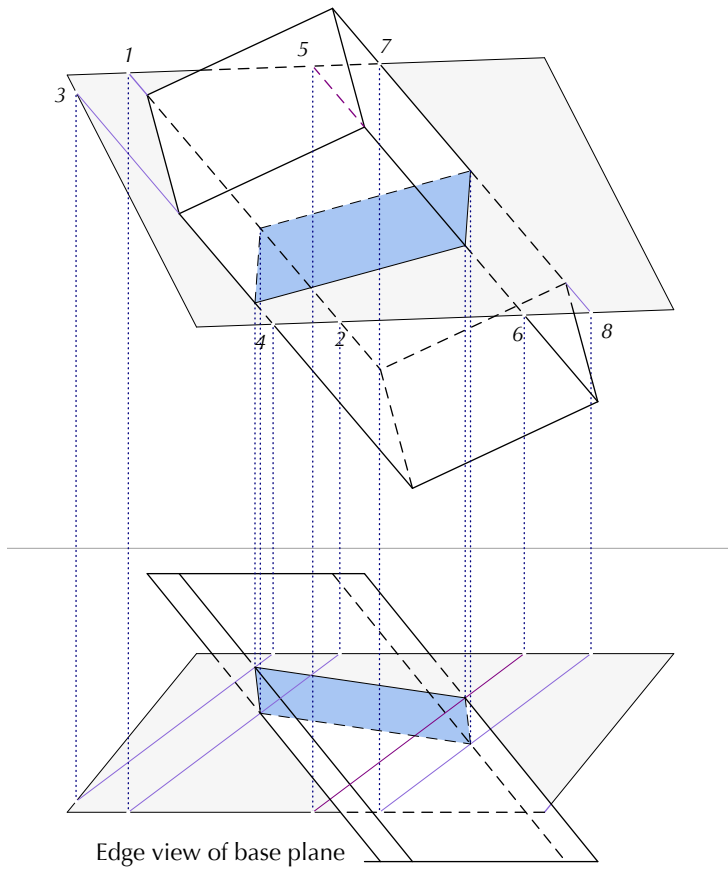
8.1.18 Lines of intersection of a plane surface and the faces of a prism

As we have seen in Chapter 6, the intersection of two flat surfaces is a line. Therefore, when a plane surface intersects the face of a prism it does so in a line. The individual lines of intersection between the plane and faces of the prism form the *complete* lines of intersection between the plane and the prism. We can determine the piercing points either by taking an edge view of the intersecting plane or by using the cutting plane method. Both methods were illustrated in Figures 6.11 and 6.12 respectively, which are reproduced again here.



8-47

Intersection of a plane surface with a prism (edge view method)



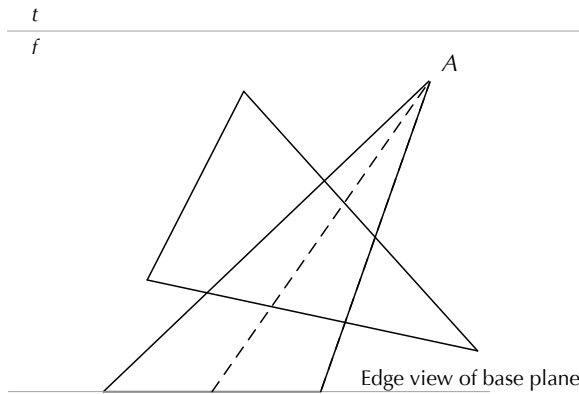
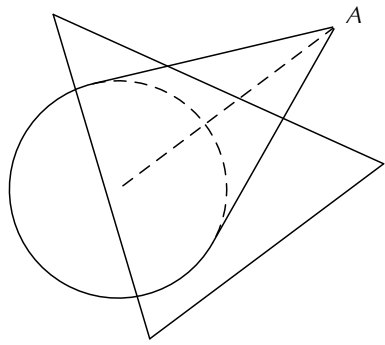
8-48
Intersection of a plane surface with a prism (cutting plane method)

Other the intersection of a plane surface with a polyhedron can be similarly constructed.

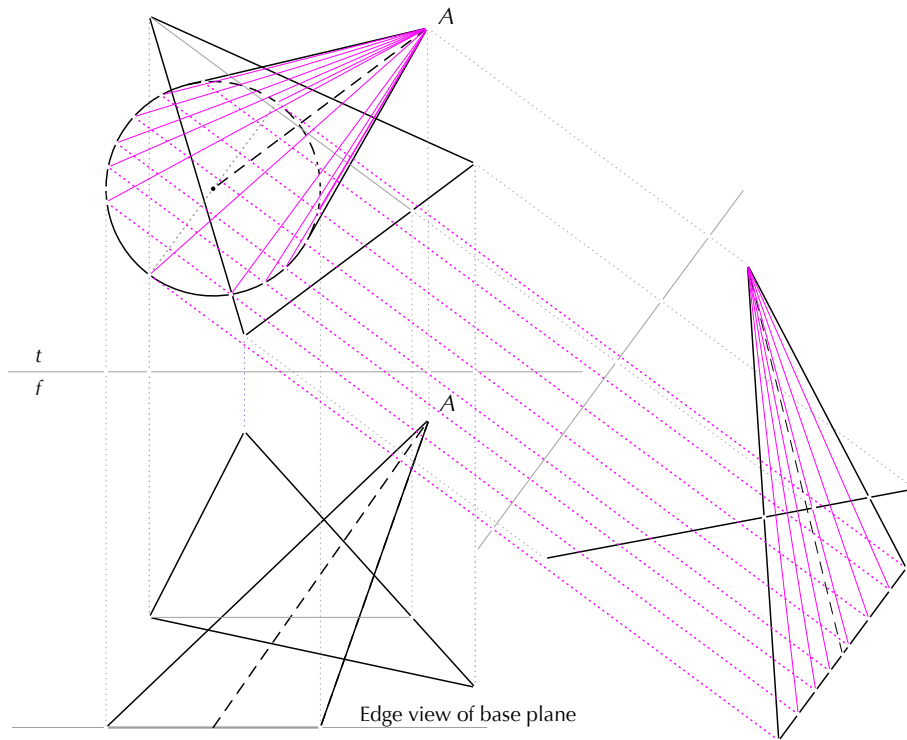
8.1.19 Intersection of a plane surface with an inclined cone

The intersection of a plane surface with a cone is a curved surface; in fact, it is one of the conic sections. See Figure 8-49. The construction is shown below as a series of steps.

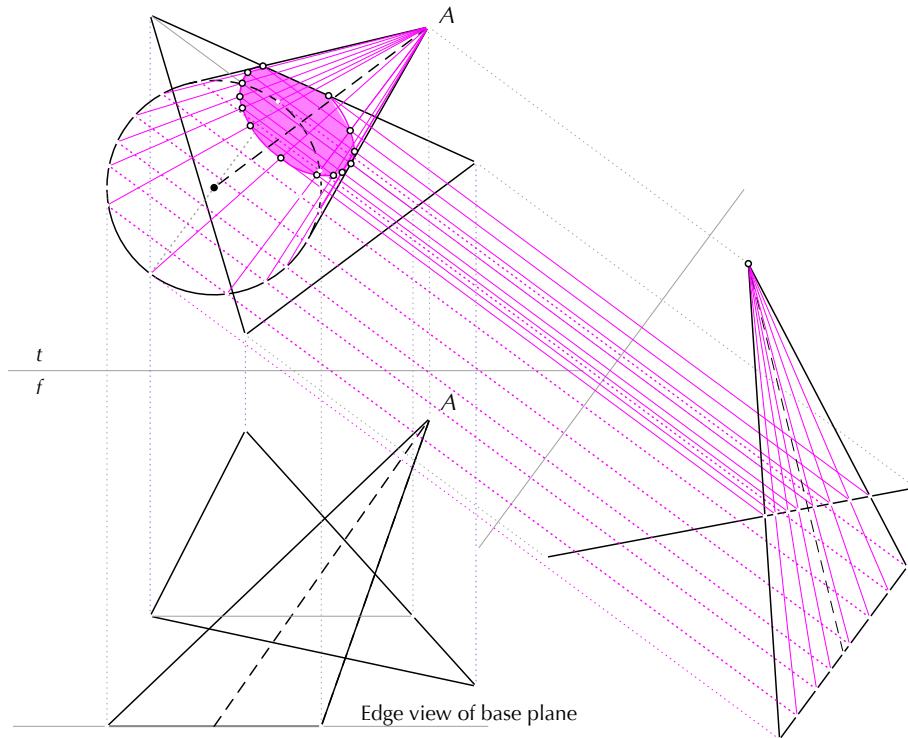
Firstly, it is convenient to take an edge view of the plane and using cutting planes that top correspond to ruled-line elements of the cone. See Figures 8-52 and 8-51. The last step projects the ruled lines and ellipse from the view to the front view. See Figure 8-52.



8-49
Intersection of a plane and a cone

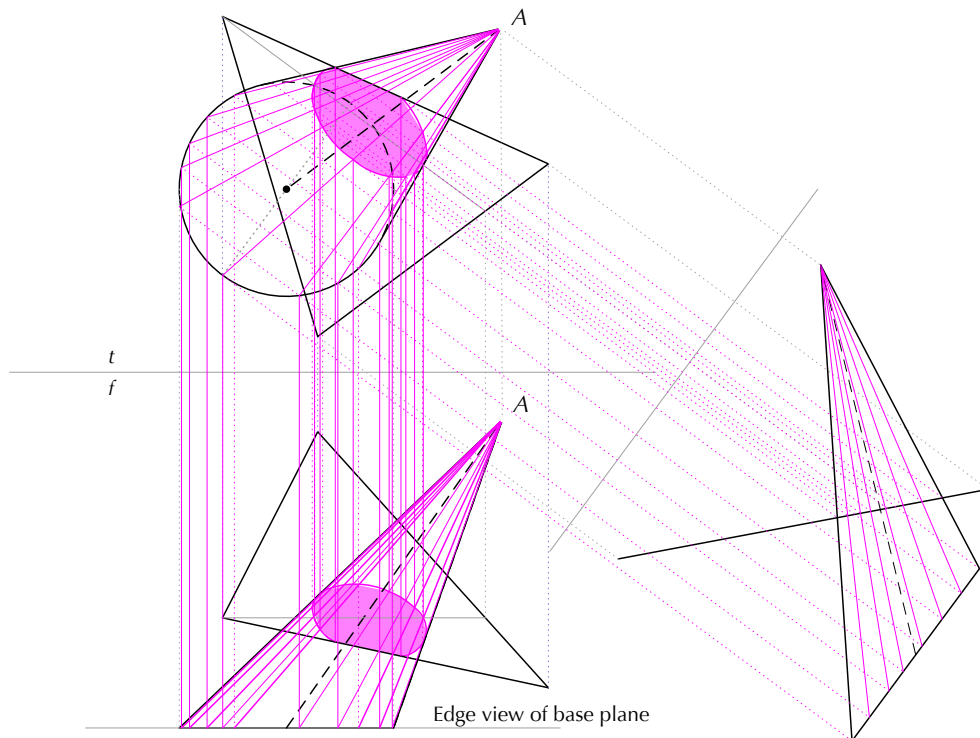


8-50
Step 1: Find edge view of plane and construct ruled lines for the cone



8-51

Step 2: Complete the ellipse in top view using the ruled lines as cutting planes



8-52

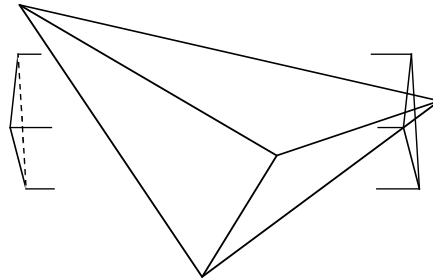
Step 3: Complete the intersection of a plane surface with an inclined cone

We can employ a similar approach with other single-curved surfaces such as a cylinder to determine the intersection with a plane surface.

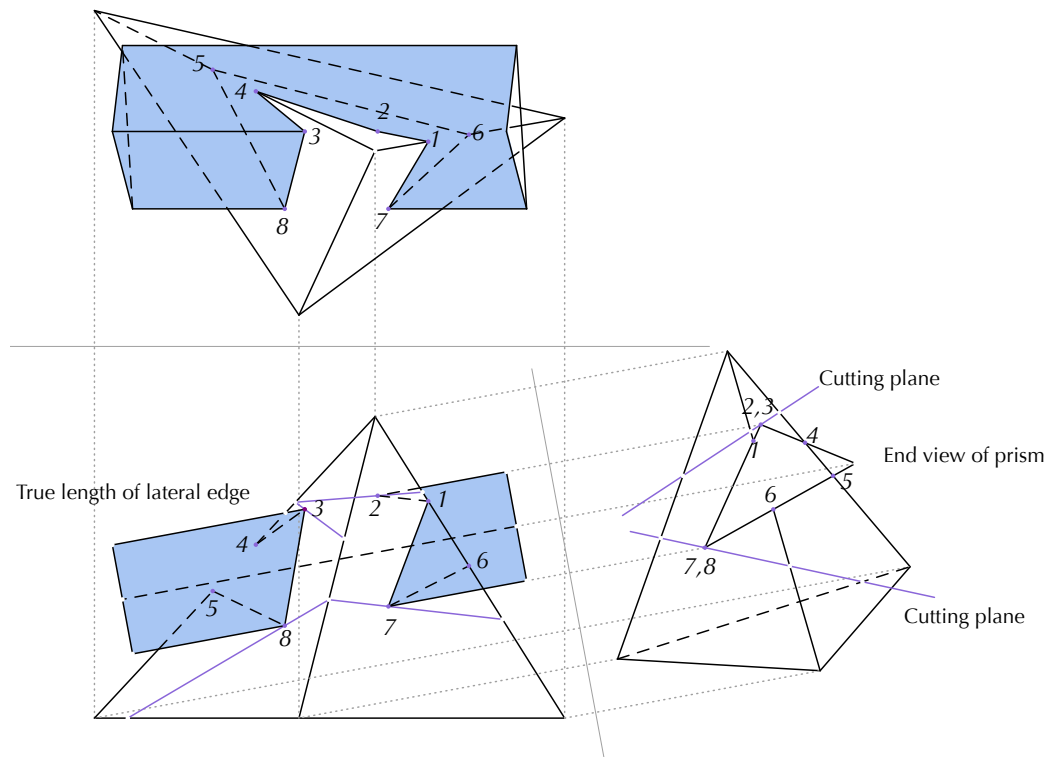
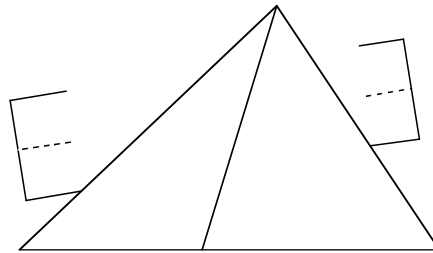
8.1.20 Lines of intersection of a prism and a pyramid

Figure 8-53 illustrates a prism and a pyramid, in which an auxiliary view (see Figure 8-54) is taken to show the end view of the prism in true shape.

In this case we employ a combination of the line method and the cutting plane method to determine the lines of intersection.



8-53
Intersecting prism and pyramid



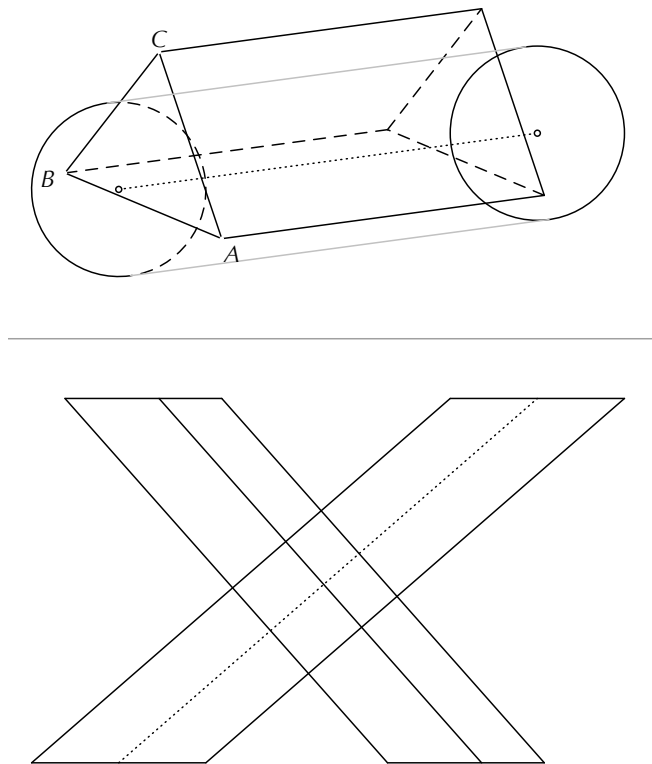
8-54
Solving the intersection of a prism and a pyramid

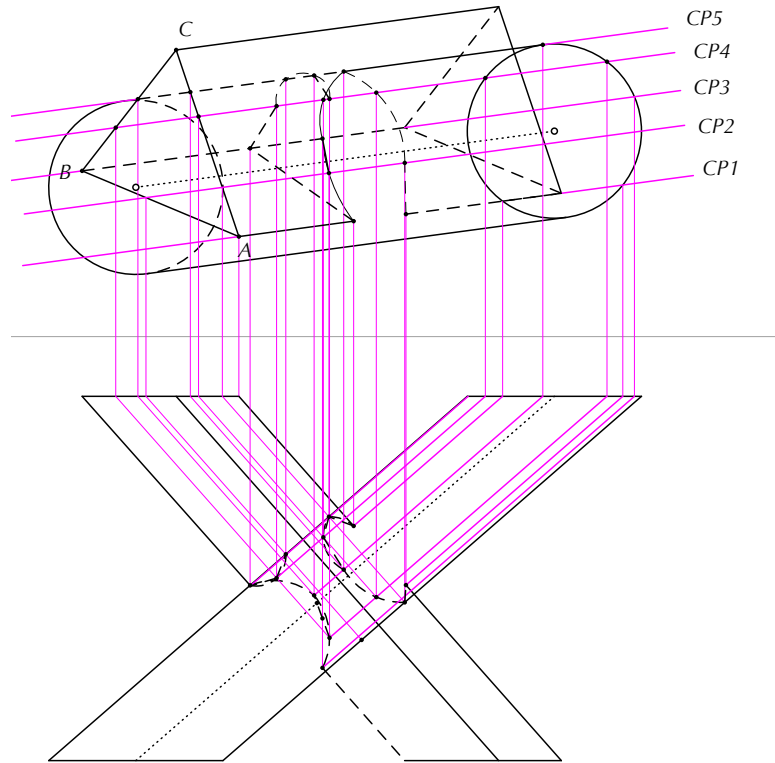
8.1.21 Intersection between a cylinder and a prism

The intersection of a cylinder and a prism (indeed, a cylinder and a cylinder) may be determined by cutting planes parallel to the axis of both surfaces which determine straight-line elements from each surface. In general, the intersection will be a curved line, continuous across intersected face of the prism, but broken at each ridge.

To illustrate the construction, we consider the case of an inclined cylinder and a triangular prism. To simplify the construction we consider the problem in a special position. In top view the axes of the prism and cylinder appear parallel in which case the cutting planes will appear in edge view. If the problem is specified in any other position, we may first reduce the problem to this position.

Figure 8-55 illustrates the construction. Five representative cutting planes are shown. *CP1* and *CP5* are the extreme planes. *CP1* passes through *A*; *CP5* is tangential to the cylinder. *CP3* passes through the ridge at *B*. *CP2* and *CP4* intersect the prism. Constructions for the piercing points are shown in detail.



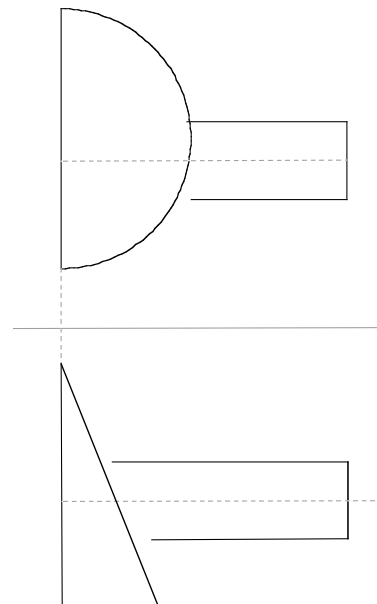


8-55
Intersection of an inclined cylinder with a prism

8.1.22 Intersection between a cylinder and cone

This is the classic example of the intersection of two single-curve surfaces, namely, a cylinder and a right cone. See Figure 8-56.

Other variations can be similarly dealt with though perhaps requiring much more detailed construction.



8-56
Intersection of a right cone and cylinder

