

# 4

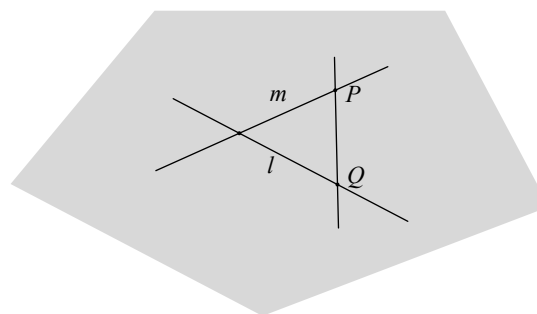
## Planes in 3-D Descriptive Geometry

---

### 4.1 SPECIFYING PLANES

Formally, for any two lines that intersect, the set of all points that lie on any line specified by two points one from each line specifies a *plane* defined by these two lines.

4-1  
Illustrating the definition of a plane



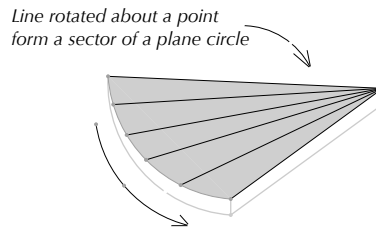
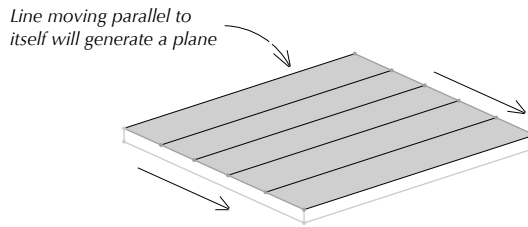
As a corollary:

#### **Definition 4-1: Plane**

*A spatial figure is a plane whenever, for any two points on the figure, the line specified by the points also lies on the figure.*

A plane surface has the characteristic being *flat*—a surface on which a line or straight edge may lie in any disposition. That is, every point on the line is in contact with the plane surface. Informally, the simplest way of specifying a plane can be specified by moving a line parallel to itself or by rotating a line about a point as shown in Figure 4-2.

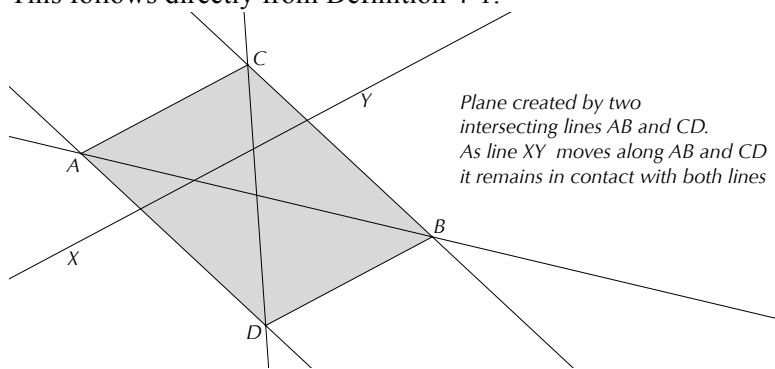
**4-2**  
 Generating a plane surface  
 by moving a line or rotating a line



Formally, a plane surfaces can be represented in four basic ways.

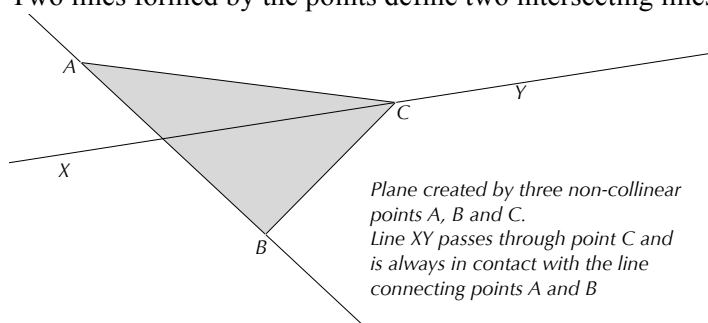
1. *By two intersecting lines*

This follows directly from Definition 4-1.



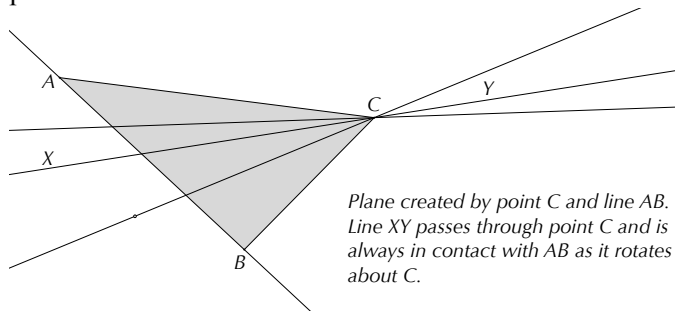
2. *By three distinct non-collinear points (points not in a straight line)*

Two lines formed by the points define two intersecting lines and thus define the plane.



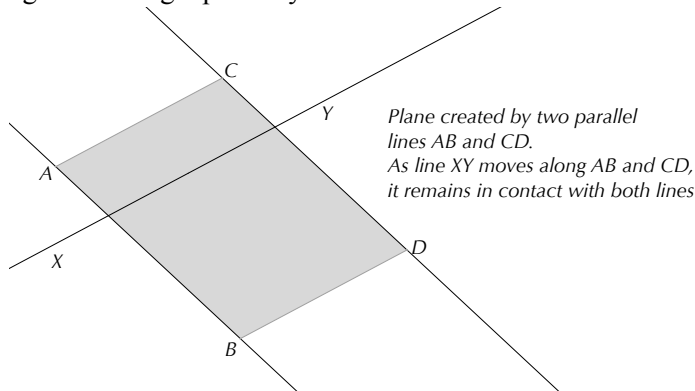
### 3. By a line and point not on the line

The point and any two points on the line define two intersecting lines and thus define a plane.



### 4. By two parallel lines

This reduces to the above case by selecting a point on one line and two on the other, again forming a plane by definition.

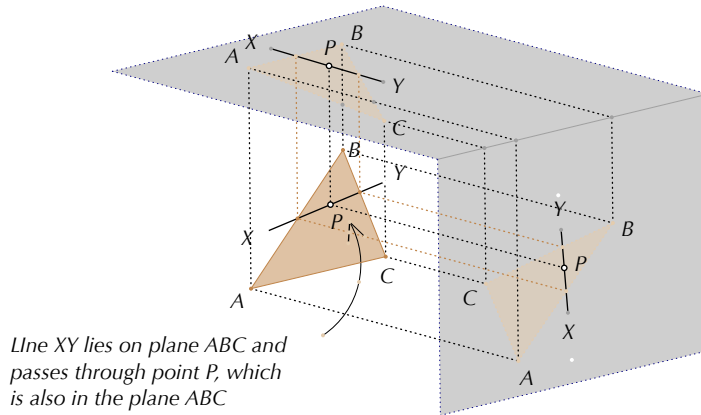


Note that planes are always drawn to have limited size. In principle, a plane has indefinite extent.

As a plane is completely and uniquely defined by three non-collinear points on the plane, we can use these points which form the corners of a triangle that belongs to the plane to delineate a bounded portion of the plane. Views of such triangles are generally used in descriptive geometry to define a plane. In practical applications, however, we often know more than three points on a plane. An example is a planar roof surface, which is defined by its four corners; any three of these points suffice to define the plane of the surface. The constructions that follow can be applied using any three convenient non-collinear points in these cases.

## 4.2 POINT ON A PLANE

Since a point lies in a plane surface whenever any line through the point lies on the surface, we can use this property to construct the view of any point on a plane surface.



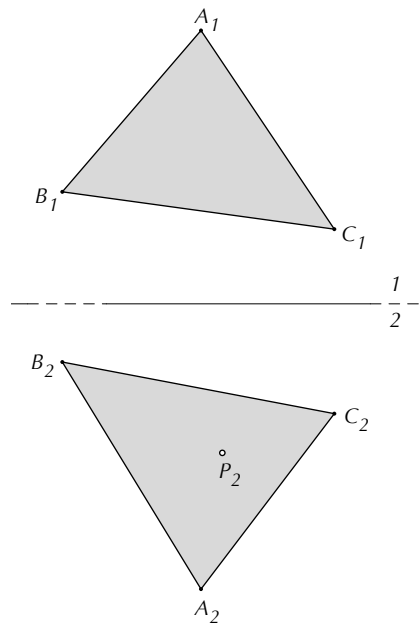
**4-3**  
Position of a point on a plane

The following construction determines the position of a point given a plane in adjacent views and a view of the point in a picture plane.

**Construction 4-1**  
*Locating the position of a point in a plane*

We are given a plane in two adjacent views and a view of a point. We are required to find the position of the point in the other view

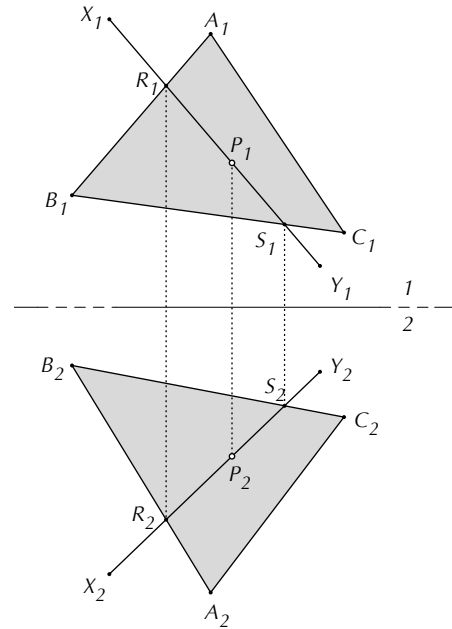
Suppose  $P_2$  is the given point in front view (view 2).



(Right) Problem configuration

There are three steps:

1. Draw any line  $X_2Y_2$  in view 2 passing through  $P_2$  and intersecting two sides of the plane represented by  $\triangle ABC$  at  $R_2$  and  $S_2$ .
2. Project the points into view 1 by constructing perpendicular projectors to locate points  $R_1$  and  $S_1$ , thereby, locating the projection of line  $X_2Y_2$  in view 1.
3. Pick  $P_2$  up into view 1. The point of intersection of the projector and the projection of  $X_1Y_1$  is the required point  $P_1$ .

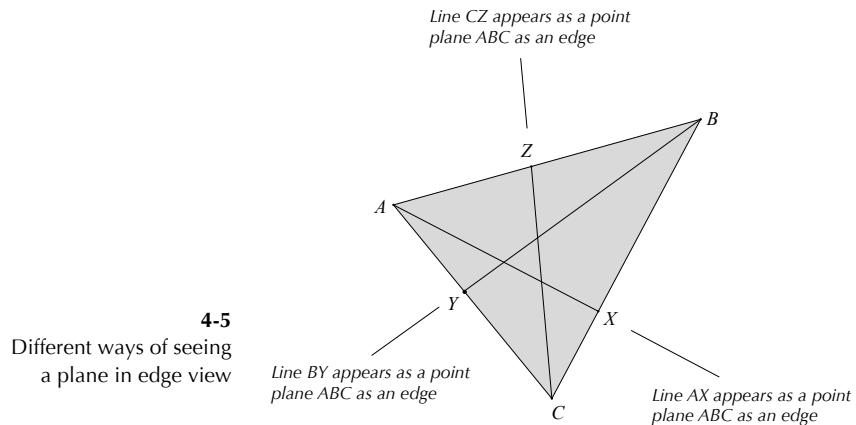


**4-4**  
Constructing a view of a point on a plane

### 4.2.1 Edge view of a plane

We know from Property 2-5 (on page 58; see Figure 2-10) that if a plane  $a$ , that is projected into a picture plane  $p$  by a line family  $f$ , contains at least one projection line in  $f$ , its image is the line where planes  $a$  and  $p$  meet; this line is called the *edge view* (EV) of plane  $a$ . The construction given below generates the edge view of a plane given by three non-collinear points or by the corners of a triangle.

In order to see a plane in edge view the viewer must assume a position in space where a line in the plane appears as a point.



**4-5**  
Different ways of seeing a plane in edge view

The edge view of a plane is needed for various investigations in descriptive geometry. It can, for example, be used to determine the distance between a point and a plane. All one has to do is generate an edge view of the plane and project the point into the same view; the distance between the point and the edge view, which is a line, is the required distance. We will explore such kinds of spatial relationships in the next chapter.

#### Construction 4-2

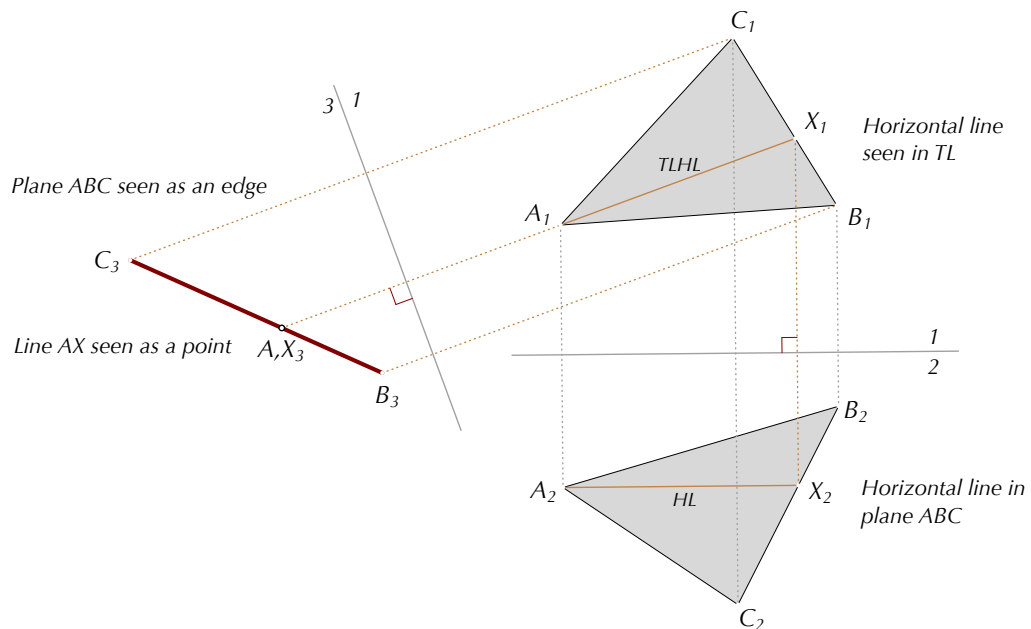
##### Edge view of a plane

Suppose we are given a plane surface represented by  $\triangle ABC$  in two adjacent views, 1 and 2. We are required to find an edge view of  $\triangle ABC$  (and hence, the plane).

There are four steps:

1. Select a view, say 2, and draw an interior segment of  $\triangle ABC$ , parallel to  $1 \parallel 2$ ; in many cases, it is convenient to select one corner of  $\triangle ABC$  as an endpoint of this segment; in Figure 4-6, corner  $A$  was selected and a segment  $A_2X_2$  drawn. This and the following step can be omitted if in one view, a side of  $\triangle ABC$  is parallel to  $1 \parallel 2$ . This side will play the same role as the segment in step 3
2. Project the endpoints of  $A_2X_2$  into view 1;  $A_1X_1$  now appears in TL. (Why?)
3. Select a folding line  $1 \parallel 3$  perpendicular to  $A_1X_1$  to define an auxiliary view 3.
4. Project  $\triangle ABC$  from 1 into 3. Points  $A, B$  and  $C$  will be collinear, and  $\triangle ABC$  (and the plane defined by it) appear in edge view in view 3.

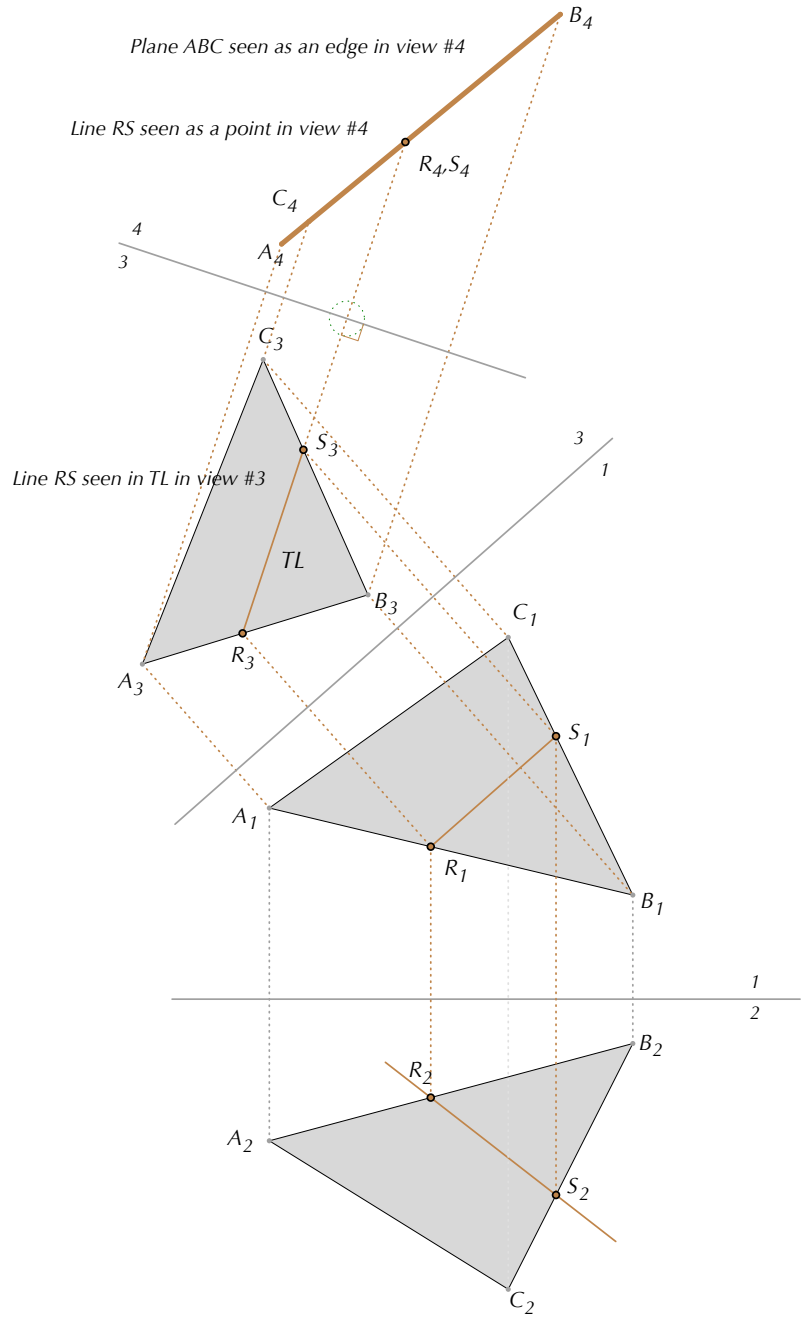
The construction is illustrated in Figure 4-6.



#### 4-6

Constructing the edge view (EV) of a plane surface

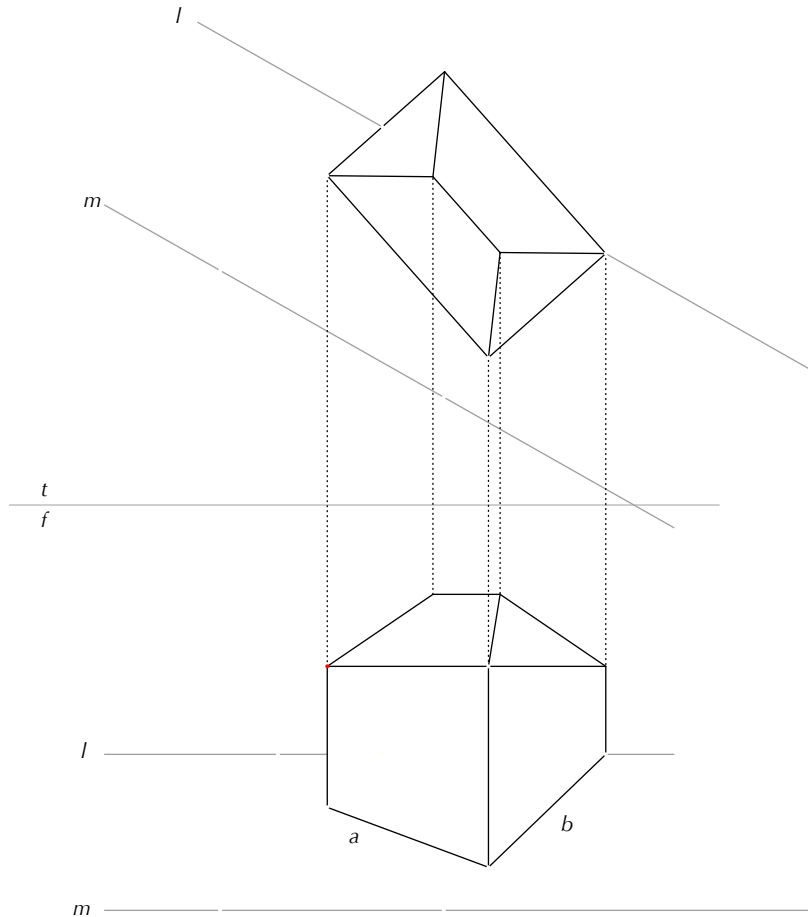
Instead of constructing a horizontal line as in step 1, we could have chosen an arbitrary line. In this case, we would need to create an additional auxiliary view that showed this line in TL before constructing an edge view of the plane. See Figure 4-7.



**4-7**  
Edge view of a plane

### 4.2.2 Worked example – House on sloping ground seen in edge view

We know that a plane can be described by a pair of parallel lines, which lie on the surface of the plane. This is a convenient way of describing sloping grounds such as hillsides. The example, shown in Figure 4-8, shows a house on a uniformly sloping ground. The problem is to find a view of the house showing the ground in edge view.



#### 4-8

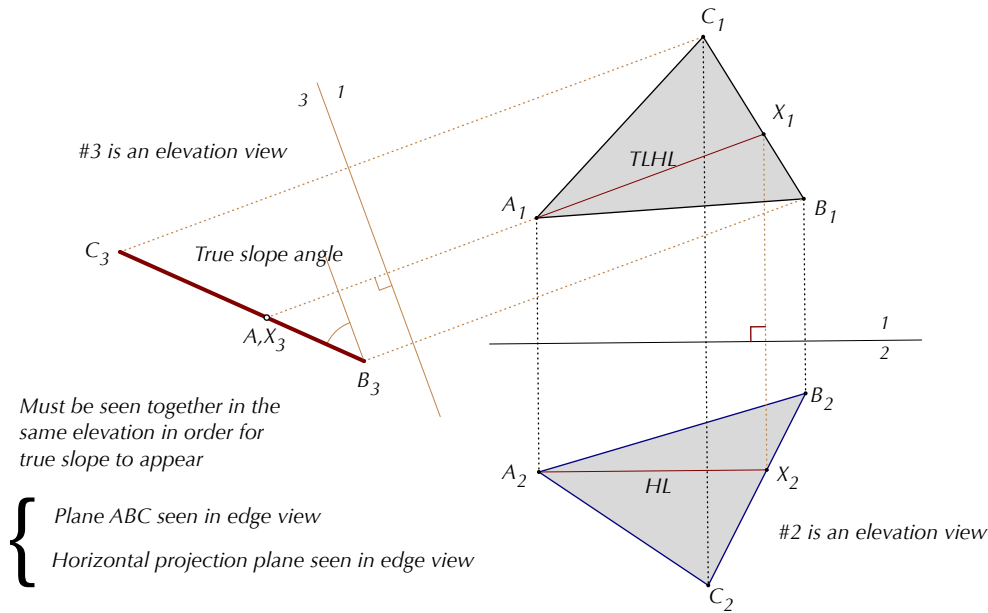
The problem: Finding a view of the house showing the sloping ground in edge view

In Figure 4-8 we are given the top (*t*) and front (*f*) views of the house and the sloping ground indicated by parallel lines *l* and *m*. Notice that the front view of the house shows where two exterior walls of the base of house meet the sloping ground. Lines, *a* and *b*, can, in fact, be constructed from the top view and an incomplete front view using techniques that will be discussed in subsequent chapters. However, for now we will assume that these lines are known.

The solution is shown in Figure 4-9. The first step is to find triangles that are coplanar with the sloping ground. The simplest way is to choose points *A*, *B*, *C* that lies on *l* and *m*. This guarantees that the triangle  $\Delta ABC$  is coplanar with the sloping ground. We







**4-10**  
True slope angle of a plane

#### 4.4 NORMAL (TRUE SHAPE) VIEW OF A PLANE

Another important view one might want to construct is a normal view of a plane, which is a view in a picture plane parallel to the plane. We know from Property 2-5 (on page 58: see Figure 2-10) that in such a projection, distances are preserved, and a normal view consequently shows any figure in the plane in its true shape and size. This view is therefore of general practical interest.

Before the true shape of the given plane can be determined, we must construct a view in which the plane appears as an edge. This EV of the plane can then be projected onto a parallel plane on which the true shape projection of the plane will appear. See the construction below illustrated in Figure 4-11.

##### **Construction 4-3** *Normal view of a plane*

Given  $\triangle ABC$  in two adjacent views, 1 and 2, find a normal view of  $\triangle ABC$  (that is, a view whose picture plane is parallel to the plane defined by  $\triangle ABC$ ).

There are three steps.

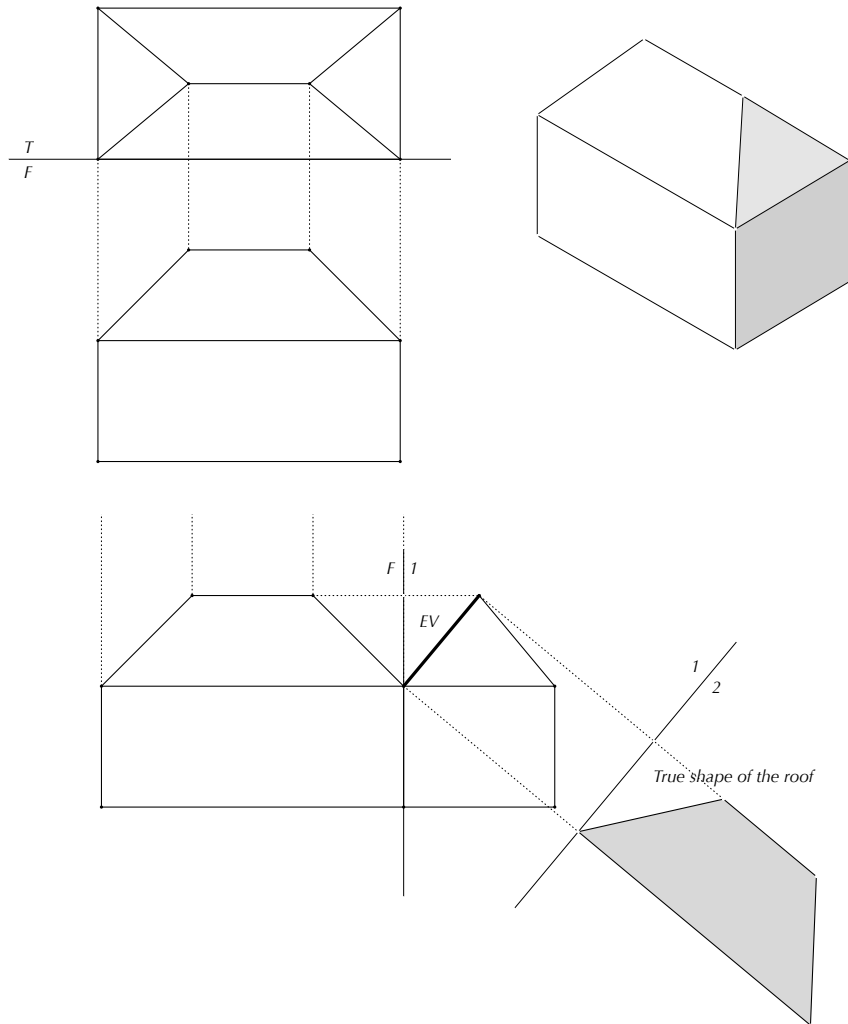
1. Construct an edge view of  $\triangle ABC$  in an auxiliary view 3.
2. Place folding line 3 | 4 in view 3 parallel to the edge view to define an auxiliary view 4.
3. Project  $\triangle ABC$  from view 3 into view 4. This is the desired normal view.



#### 4.4.1 Worked example – True size of a roof shape

Normal views of a plane are very important because they show every planar figure in the plane in its true size and shape. This is demonstrated by the following example.

Figure 4-12 shows a building with a hipped roof in top and front view, *T* and *F*, respectively. Neither view shows the major roof surface in true shape and size. The figure demonstrates how two successive auxiliary views result in a projection depicting the surface in normal view that shows its true shape and size. This view can be used, for example, to compute its surface area or to design a tiling pattern, which can then be projected back into the top and front views.



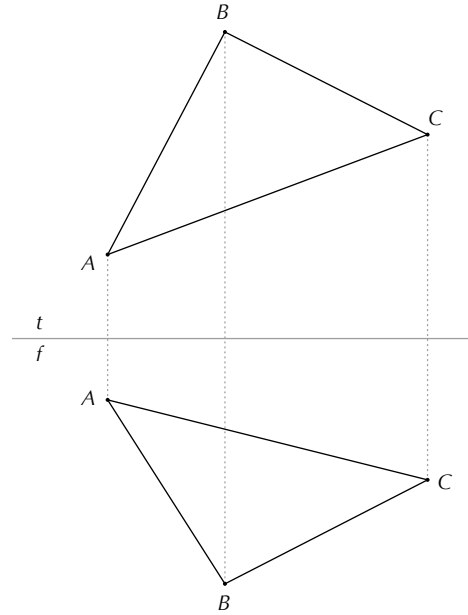
**4-12**  
True shape of a roof

The figure shows the complete auxiliary views; readers are encouraged to study these views in detail in order to deepen their understanding of the constructions under review and to develop their abilities to visualize shapes in three dimensions.

## 4.5 WORKED EXAMPLES

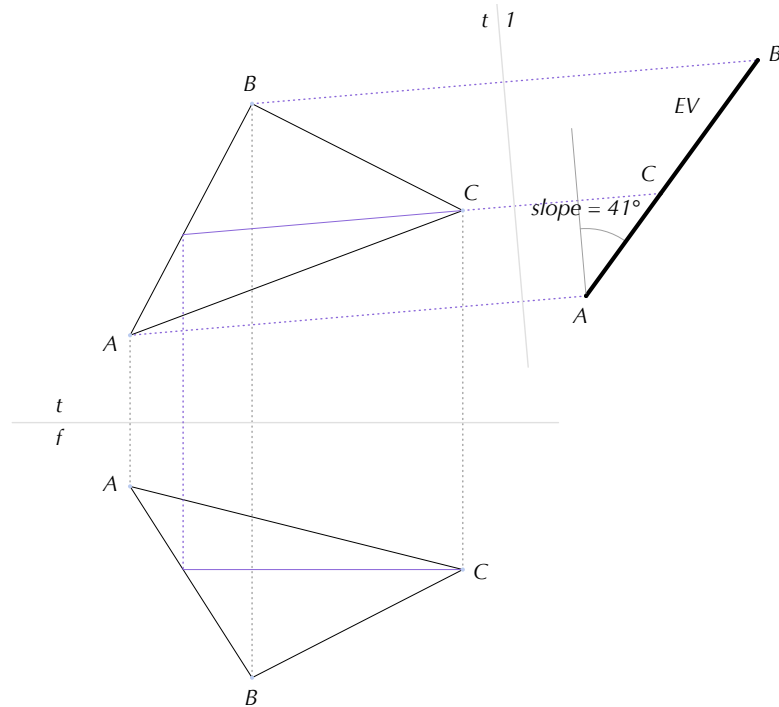
### 4.5.1 Largest inscribed circle

The figure on the right gives top and front views of a triangle. The problem is to determine its slope and the diameter of the largest circle that can be inscribed in this triangle showing the circle in both top and plan views.

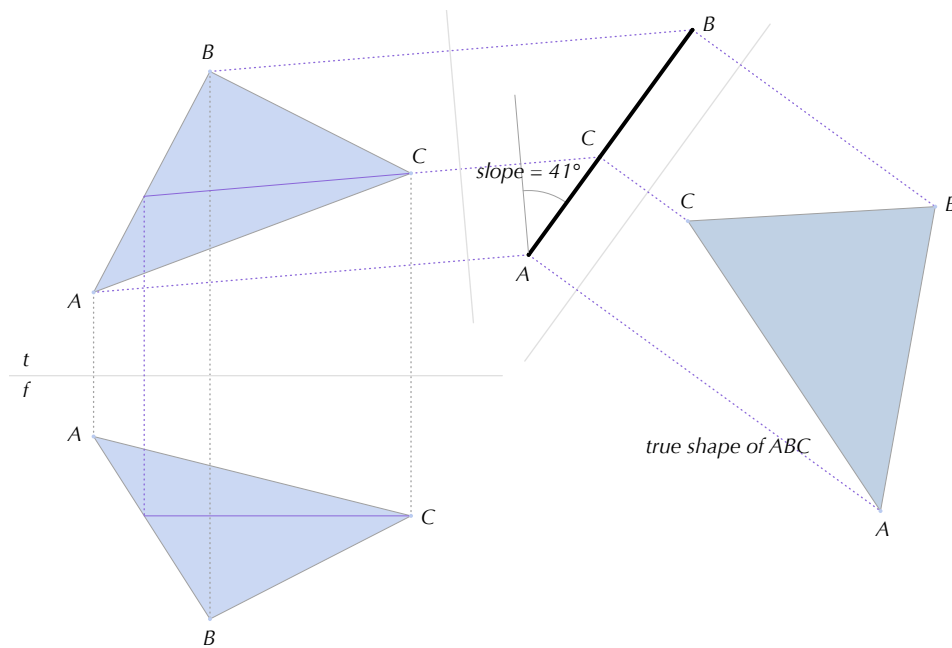


(Right)  
Problem configuration

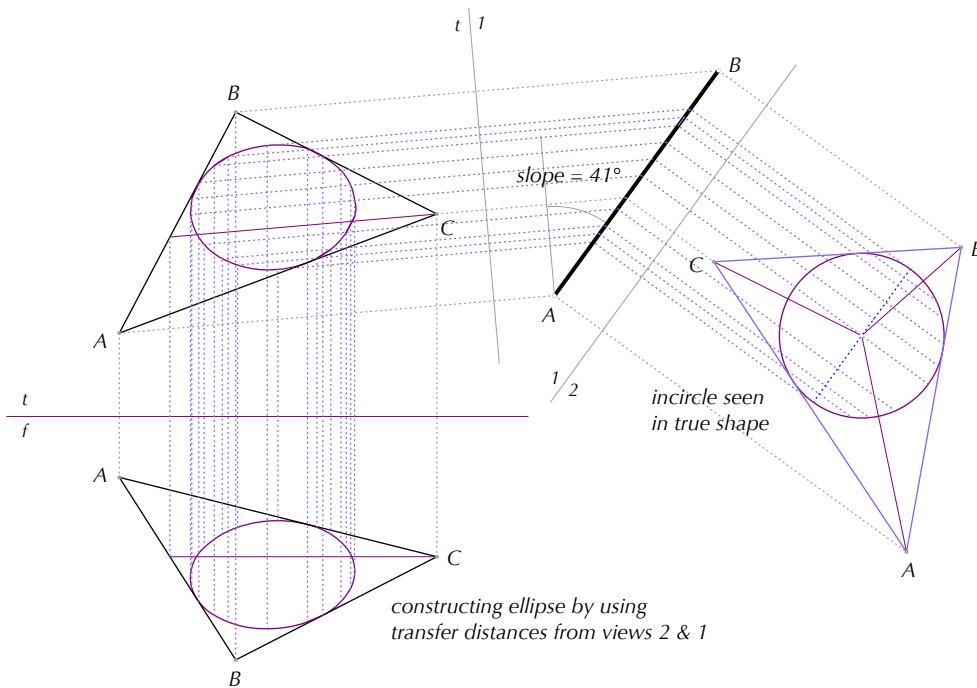
Let us first solve for the slope angle for which we will need to find the edge view of the triangle in elevation.



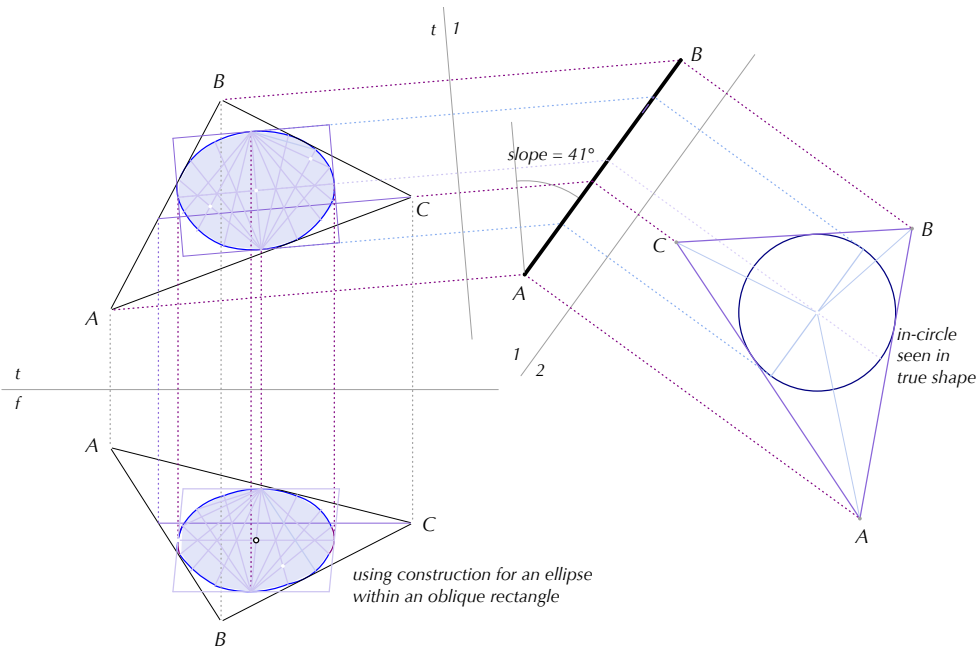
Next we find the true shape of the triangle in order to inscribe a circle.



Lastly, we construct the inscribed circle, and back project to the front and back views. The (in)-center of the circle is the intersection of the angle bisectors, its radius equaling the perpendicular distance from the in-center to any side. See Figures 4-13 and 4-14, the latter using the technique for constructing an ellipse within an oblique rectangle.



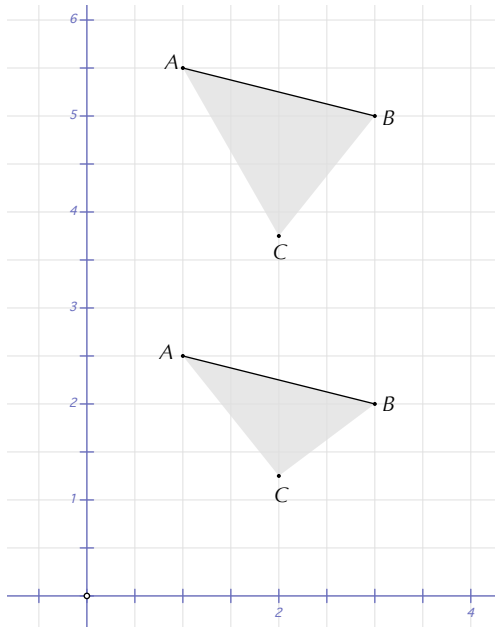
**4-13**  
Slope and in-circle of a triangle



**4-14**  
In-circle of a triangle using construction for an ellipse within an oblique rectangle

### 4.5.2 Completing the views of a plane

Suppose a plane is given by diagonal lines, say  $AB$  and  $CD$ . Suppose three of the points, say  $A$ ,  $B$  and  $C$  are given by their quad paper coordinates, for example,  $A(1, 2\frac{1}{2}, 5\frac{1}{2})$ ,  $B(3, 2, 5)$ , and  $C(2, 1\frac{1}{4}, 3\frac{3}{4})$ . See Figure 4-15.



**4-15**  
The problem statement

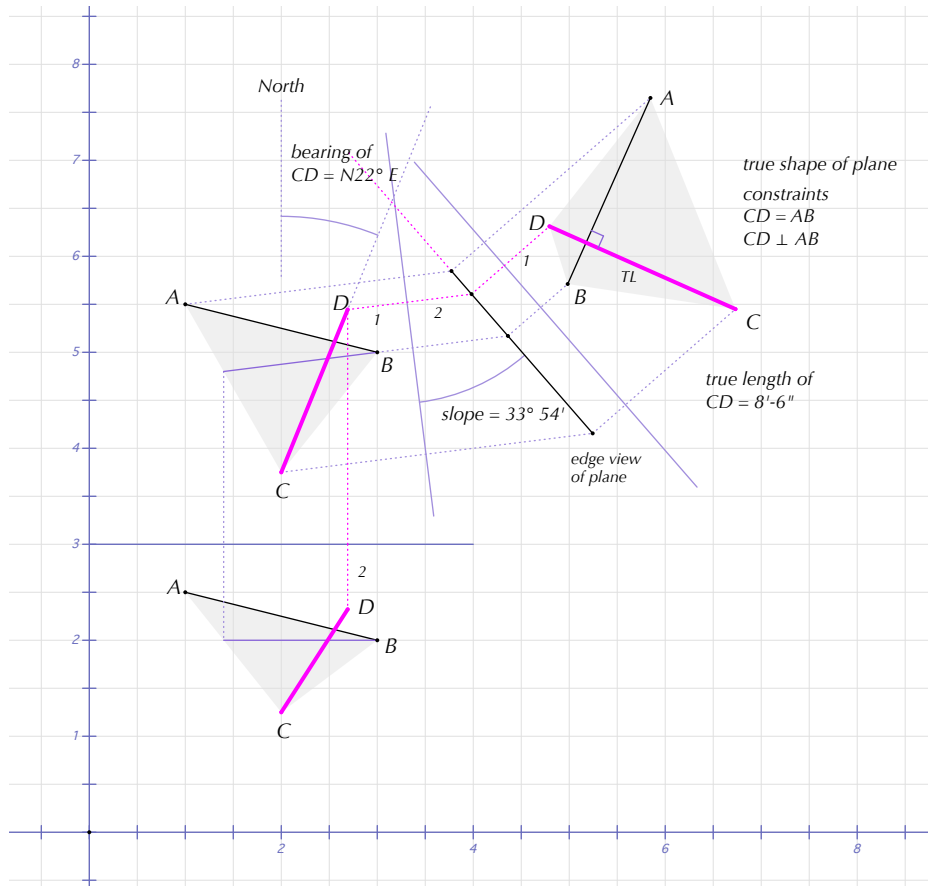
In order to determine  $D$  we will need further constraints. Suppose the diagonals are of equal length, that is,  $AB = CD$ ; suppose further that they intersect at right angles. Given that we are using quad paper, let us assume that the quad coordinates represent a scale, say  $1'' = 4'$ .

The problem is to determine the slope and true shape of the plane  $ABCD$ ; to find the true length and bearing of  $CD$ ; and to complete the top and front views of the plane.

We construct an auxiliary view from the top view in which plane  $ABC$  is seen in edge view. We can obtain the slope of the plane in this view. We construct a second view showing  $ABC$  in true shape. In this view  $AB$  is in true length. Construct a line from  $C$  perpendicular to  $AB$  and mark off  $D$  such that  $AB = CD$ .  $CD$  is in true length in this view. (Note that we will have to multiply the length measured off the quad paper by the scale for the drawing.)

Project  $D$  back to the first auxiliary view and then to top and then front views using transfer distances 1 and 2 in the second and first auxiliary views respectively. We obtain the bearing of  $CD$  from the top view.

The complete construction is given in Figure 4-16.



**4-16**  
Completing the views



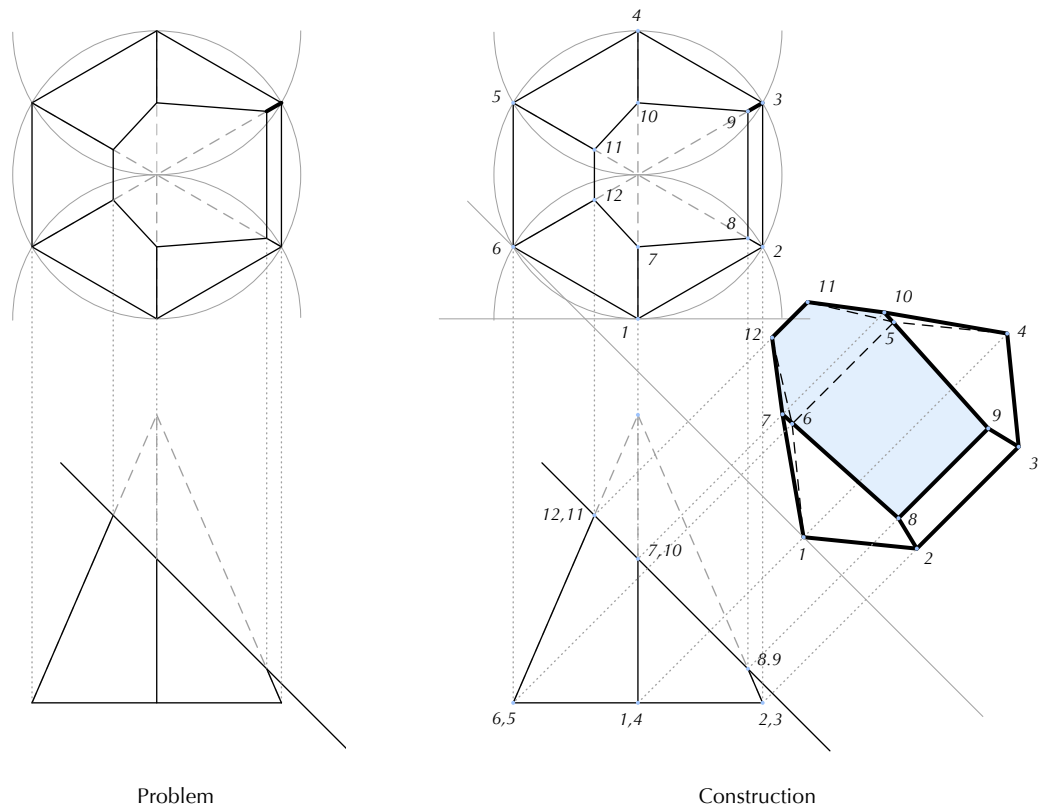
### 4.5.3 True shape of a truncated face

We revisit problem in Figure 3-35 (on page 118) of constructing the top and front views of a truncated pyramid shown on the left in Figure 4-17.

We are now interested in a view that shows *the true shape of the truncated face*.

Since the truncated face is the result of a sectional plane through the pyramid, it follows that the truncated face lies on the plane. Consequently, seeing the plane in true shape will give a view of the truncated face in true shape. Since the plane is seen in edge view in front elevation, all we need to do is take an auxiliary view normal to this edge view.

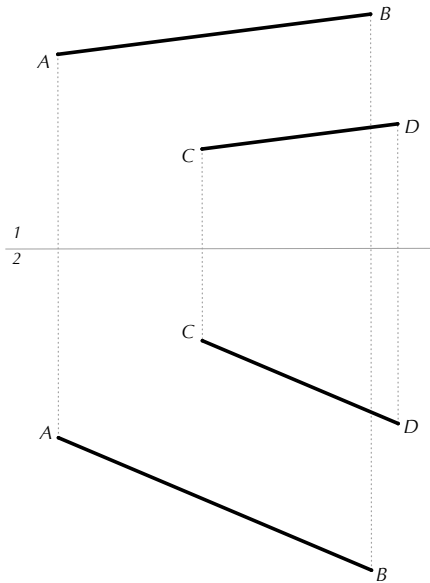
The construction is given in the right hand drawing in Figure 4-17. Notice that we have chosen conveniently located folding lines. We have also numbered the points to illustrate the construction. The true shape of the truncated face is shown shaded.



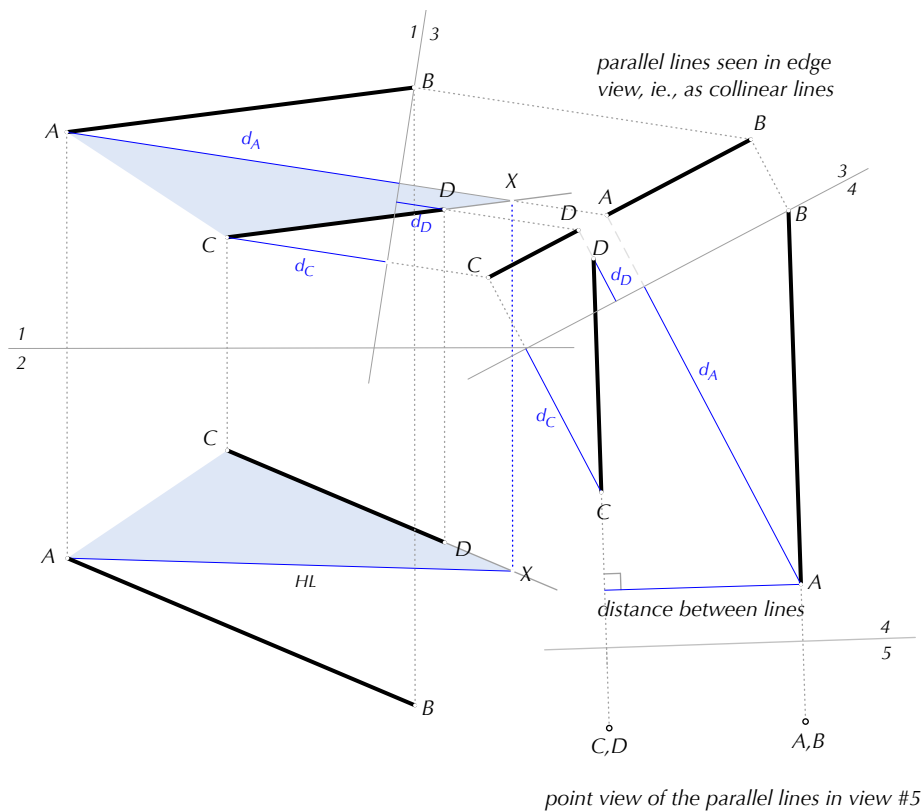
**4-17**  
Constructing the true shape of a truncated face

#### 4.5.4 Distance between parallel lines

We revisit construction in Figure 3-17 (on page 102) of determining the distance between two parallel lines, however, instead of a true length construction we employ the fact that parallel lines define a plane and use an edge view construction. The problem statement is given on the right in Figure 4-18, and its construction is given below in Figure 4-19.



**4-18**  
Distance between parallel lines – problem configuration



**4-19**  
Distance between parallel lines – the construction using planes

The first step in the construction is to determine a triangle that defines the plane specified by the two parallel lines. In front view, draw a horizontal line from  $A$  to meet



How would you determine the angle of intersection in this case?

