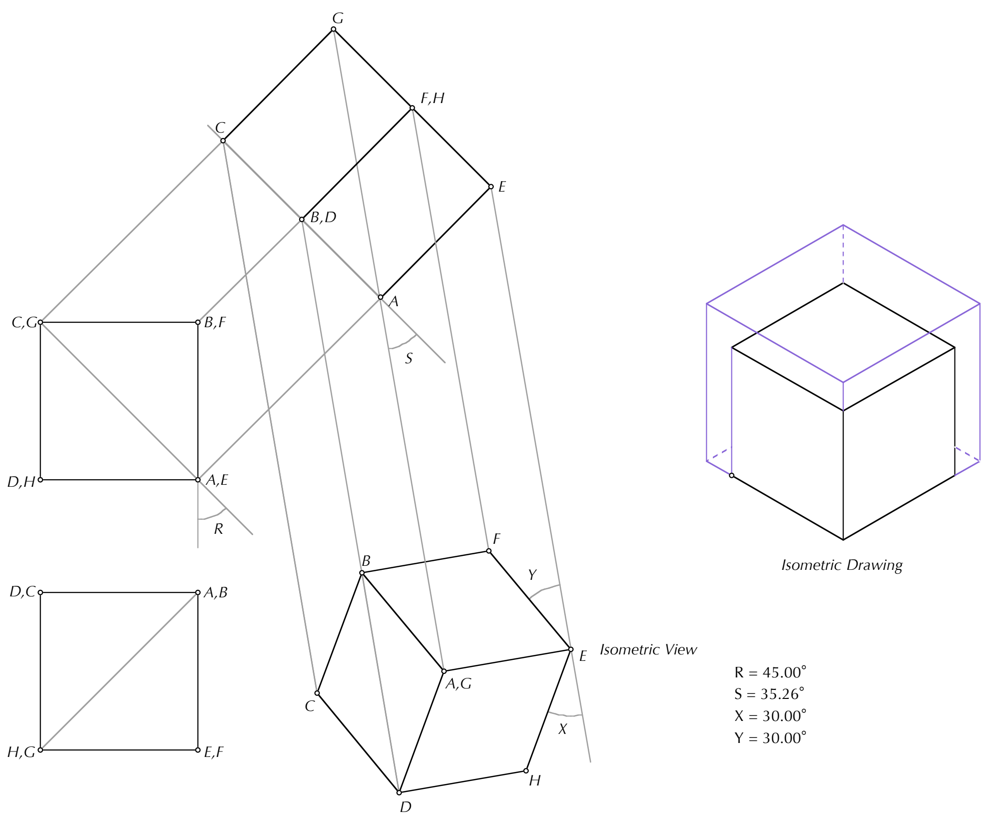
10
Axonometric Projections

10.1 AXONOMETRIC VIEWS

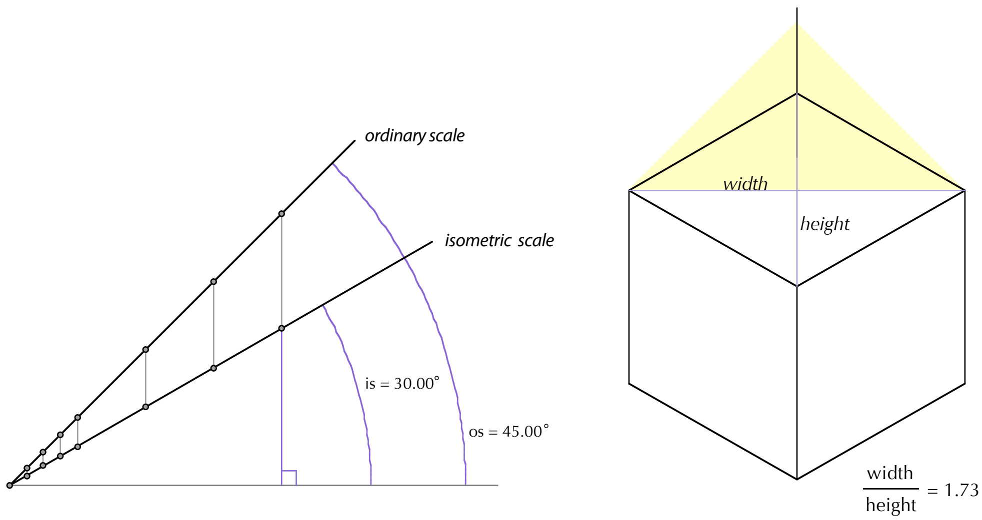


10-1
An isometric (and axonometric view) of a cube

Axonometric projections are parallel projections onto an oblique plane. Axonometric projections have the advantage that they give a pictorial view of the object, yet dimensions are measurable.

Manually, axonometric views can be constructed from orthographic views. This is best illustrated by an example. The construction in Figure 10-1 shows a cube in plan and elevation, from which an axonometric view of the cube is constructed in a direction parallel to one of its diagonals.

Notice that in this view each of the sides of the cube has been foreshortened equally (to 0.8165 of the actual length, or more precisely, $^{\sqrt{2}}/_{\sqrt{3}}$) and that the indicated angles X and Y are each 30°. Such a projection is also called an *isometric projection*, meaning equal measure. Isometric views can be drawn directly, as shown in Figure 10-2 where the view has been rotated until the vertical edge of the cube appears vertical.

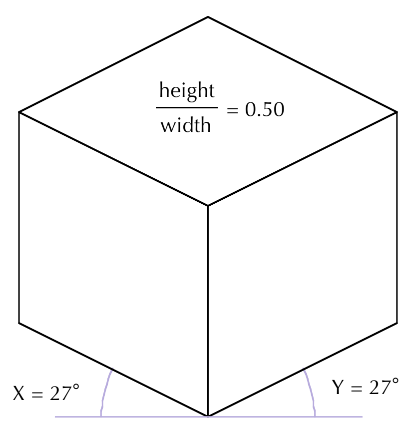


10-2
Isometric scale for the cube in **10-1**

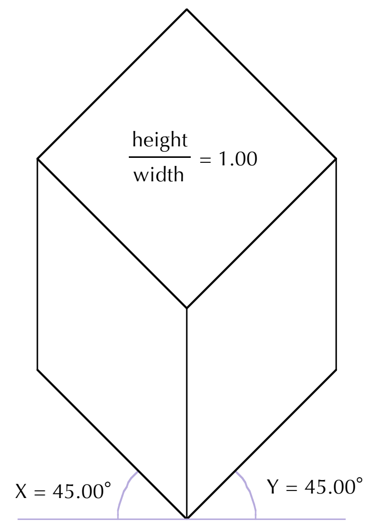
The 30° isometric projection has a height to width ratio of 1:√3. Two other common isometric views are shown in Figures 10-3 and 10-4. There are popular projections, which, however, are not true axonometric projections.

The projection shown on the right is a 27° isometric projection (actually, 26°34'12") also known as a 1:2 projection as this is the height to width ratio of the top rhombic face.

10-3
1:2 projection



The one on the left based is a 45° isometric view, also known as a *military projection*. It has a unit height to width ratio.



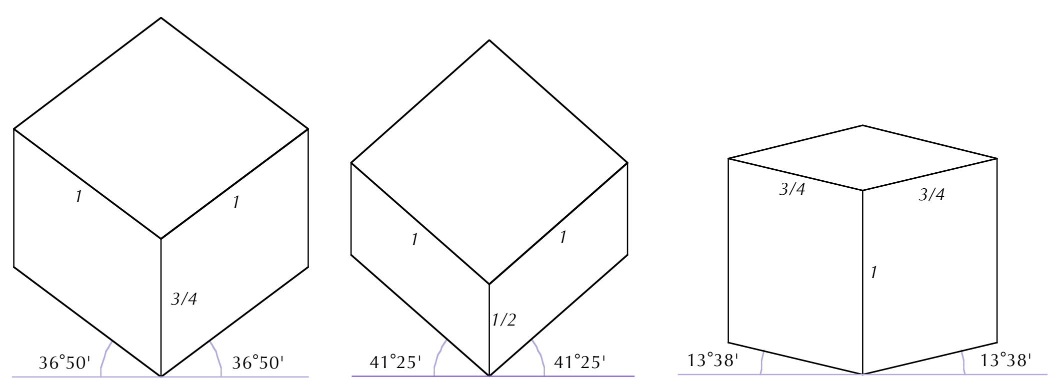
10-4
Military projection

10.1.1 Axonometric scales

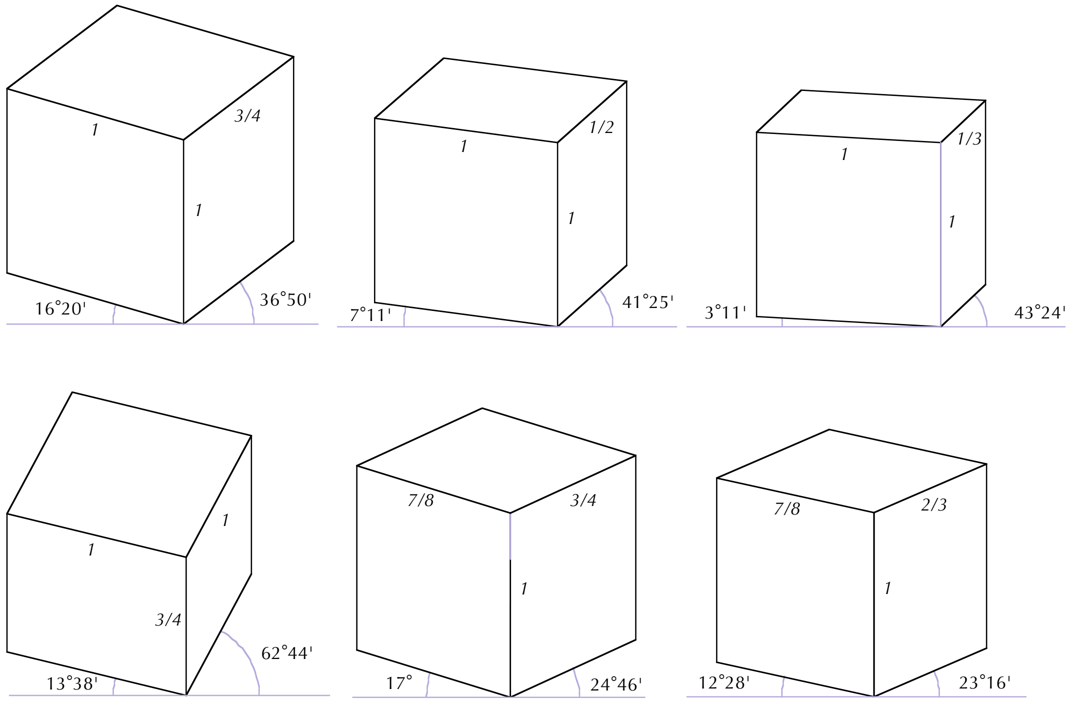
By adjusting the angles *X* and *Y*, views of the cube can be created according to a variety of axial scales. Notice that in some drawings two directions are equally scaled and one differently (called a *diametric projection*) and in other drawings all three directions are scaled differently (called a *trimetric projection*). These non-isometric axonometric projections tend to be more realistic in their depiction. In fact, Chinese scroll paintings tend to use diametric projections. See Figure 10-6 for an example.

Equally as is shown in Figure 10-1, every axonometric projection corresponds to a line of sight whose bearing is indicated by angle *R* and altitude (true angle of inclination) by angle *S*. Correspondingly, we can specify the axonometric scale by specifying the angles for the line of sight.

Table 10-1 gives the angles for the line of sight for the axonometric scales shown in Figure 10-5.



10-5
Various axonometric scales
Sides along the same axial direction have the same scale. Unmarked sides have unit value



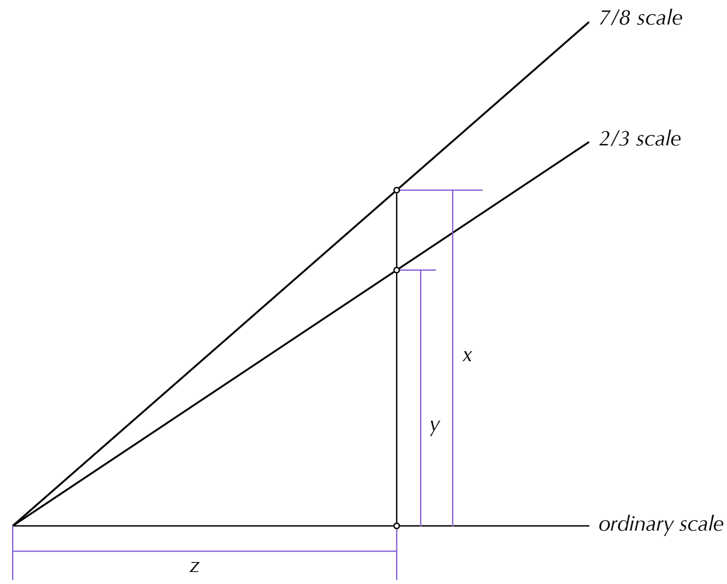
10-5 (continued)
Various axonometric scales

Table 10-1 Line of sight for the axonometric scales shown in Figure 10-5

Type of drawing	Scale ratios			Direction of the sight angles		Angle of drawing axes		True fore-shortening ratio
				R	S	X	Y	
Isometric	$X = 1$	$Y = 1$	$Z = 1$	45º	35º16'	30º	30º	0.8165
Dimetric	$X = 1$	$Y = 1$	$Z = {}^{3}/_{4}$	45º	48º30'	36º50'	36º50'	0.8835
Dimetric	$X = 1$	$Y = 1$	$Z = {}^{1}/_{2}$	45º	61º52'	41º25'	41º25'	0.9428
Dimetric	$X = {}^{3}/_{4}$	$Y = {}^{3}/_{4}$	$Z = 1$	45º	14º2'	13º38'	13º38'	0.9701
Dimetric	$X = 1$	$Y = {}^{3}/_{4}$	$Z = 1$	32º2'	27º56'	16º20'	36º50'	0.8835
Dimetric	$X = 1$	$Y = {}^{1}/_{2}$	$Z = 1$	20º42'	19º28'	7º11'	41º25'	0.9428
Dimetric	$X = 1$	$Y = {}^{1}/_{3}$	$Z = 1$	13º38'	13º16'	3º11'	43º24'	0.9733
Dimetric	$X = 1$	$Y = {}^{3}/_{4}$	$Z = {}^{3}/_{4}$	19º28'	43º19'	13º38'	62º44'	0.9701
Trimetric	$X = {}^{7}/_{8}$	$Y = {}^{3}/_{4}$	$Z = 1$	39º8'	22º3'	17º0'	24º46'	0.9269
Trimetric	$X = {}^{7}/_{8}$	$Y = {}^{2}/_{3}$	$Z = 1$	35º38'	17º57'	12º28'	23º16'	0.9513

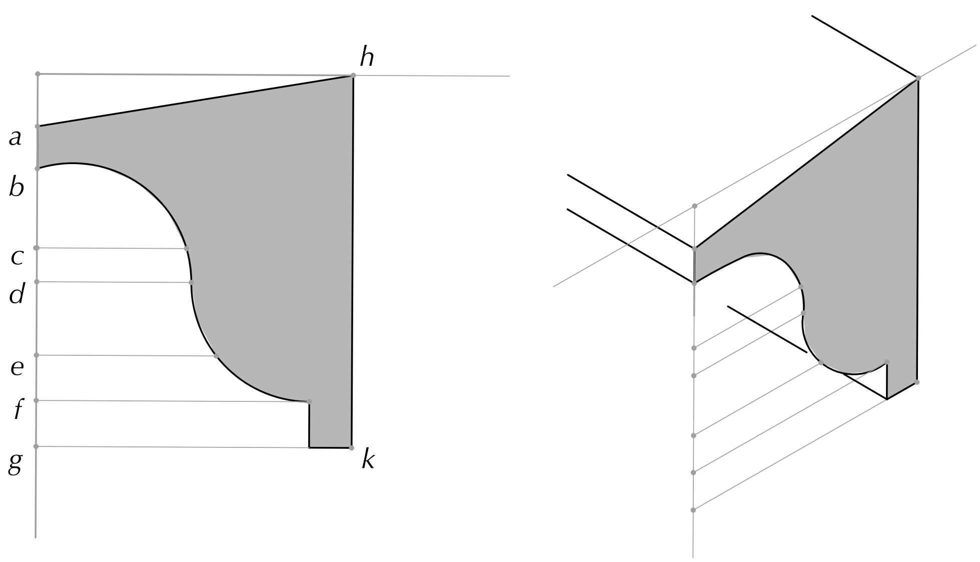
10-6
Imageries from the City of Cathay scroll painting illustrating diametric scale

A scale to measure off dimensions can be easily constructed as shown in Figure 10-7, which illustrates the trimetric scale $X = {}^{7}/_{8}$ $Y = {}^{2}/_{3}$ and $Z = 1$.



10-7
Measuring off dimensions for the trimetric scale $X = 7/8$ $Y = 2/3$ and $Z = 1$

Figure 10-8 illustrates constructing a 30º isometric drawing of a section of a molding. The section is outlined by two orthogonal axes, which are mapped to the isometric axes as shown on the right. Distances of points are then marked off the scale in Figure 10-2 along the isometric axial directions and the drawing completed as indicated.

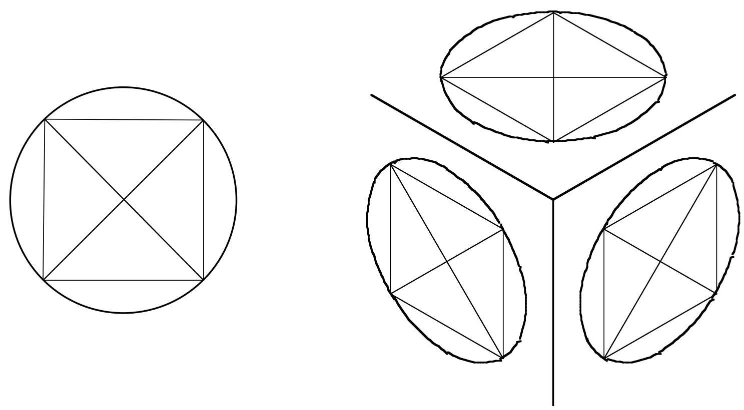


10-8
Constructing an isometric drawing of a section of a molding

10.2 CONSTRUCTING AXONOMETRIC VIEWS

10.2.1 Constructing an isometric view of a circle

The above technique can be used for circles and arcs of circles, but there simpler techniques. *The isometric projection of a circle is always an ellipse.*



10-9
Isometric views of a circle in the different axial planes

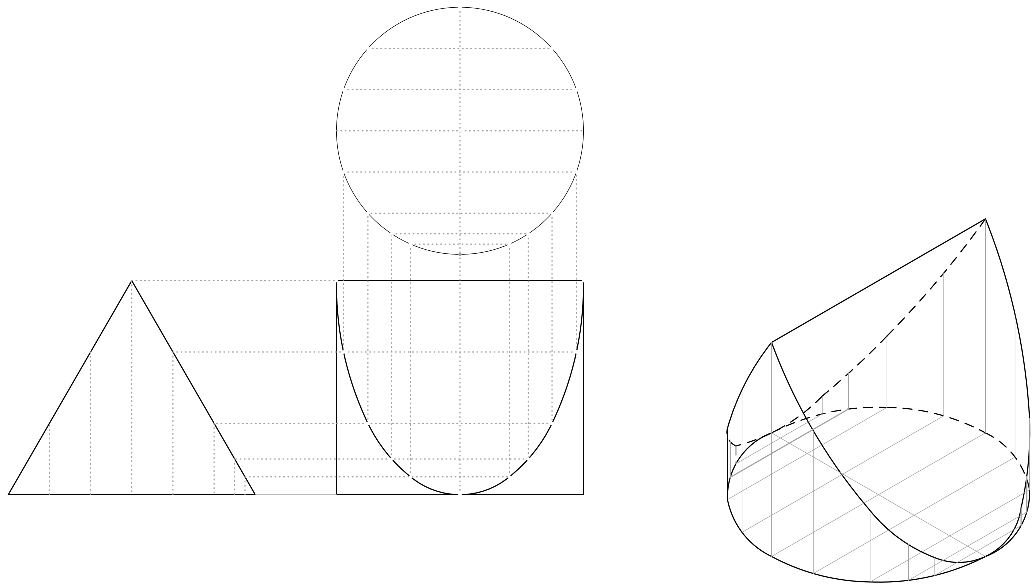
If the circles lie in an isometric plane, then the principal axes of the ellipse are oriented as shown in Figure 10-9.

If the circle does not lie in an isometric plane, first establish the axes of the isometric projection, then transfer distances along the original axes to the isometric axes for selected points on the circle. If the original drawings were drawn to isometric scales, then the distances can be read off directly from the original plan and elevation.

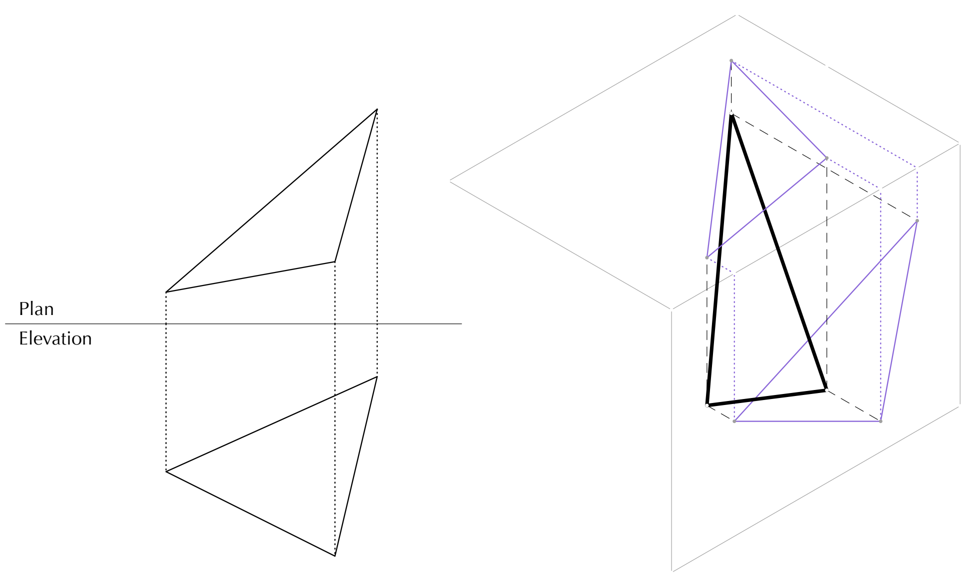
10.2.2 Constructing an isometric view from plan and elevation

Typically, the plan and elevation are drawn to the isometric scale and the distances transferred to the isometric view. In Figure 10-10, a cylindrical wedge is produced from the plan and side elevation although the elliptical base in the isometric view is first drawn and then the distances projected.

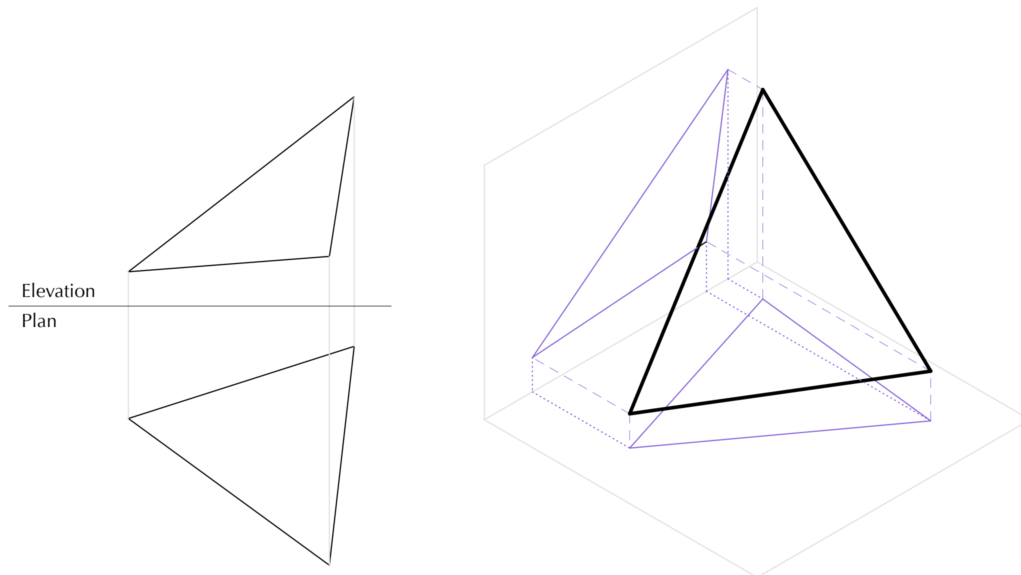
Another example shown in Figure 10-11 illustrates two ways of transferring the plan and elevation to the isometric planes and then projecting the view as shown. Each view is an axonometric from different viewing points. Alternatively, the plan and bottom can be seen in two different ways, the top figure being a standard third-angle projection with plan above and elevation below. The bottom figure shows a first-angle projection with the positions of plan and elevation reversed. This is more common view in engineering drawings. Again, in both figures, we assume that the plan and elevation were first redrawn according to the isometric scale.



10-10
Isometric views of a cylindrical wedge



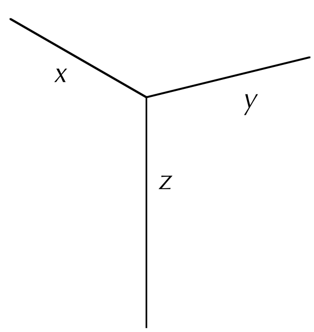
10-11
Isometric view from plan and elevation
(Third angle projection)



10-11
Isometric view from plan and elevation
(First angle projection)

10.2.3 Constructing the axonometric scale

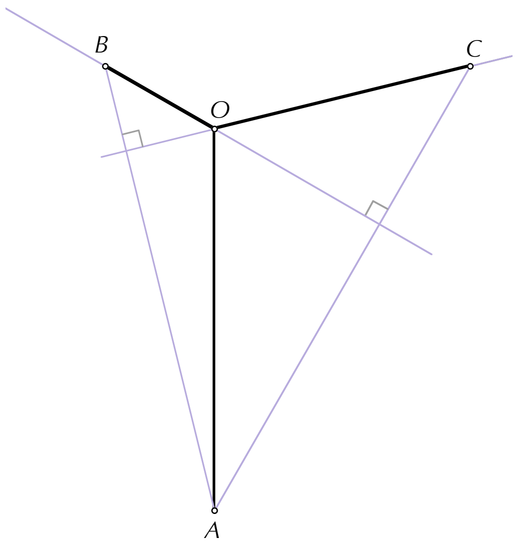
The axes of an axonometric view can be oriented arbitrarily. So if three axial directions are given, it is possible to determine the axial scales for each axis in order to directly create an axonometric drawing. See Figure 10-12, which shows three axial directions corresponding to the horizontal traces of three mutually orthogonal directions.



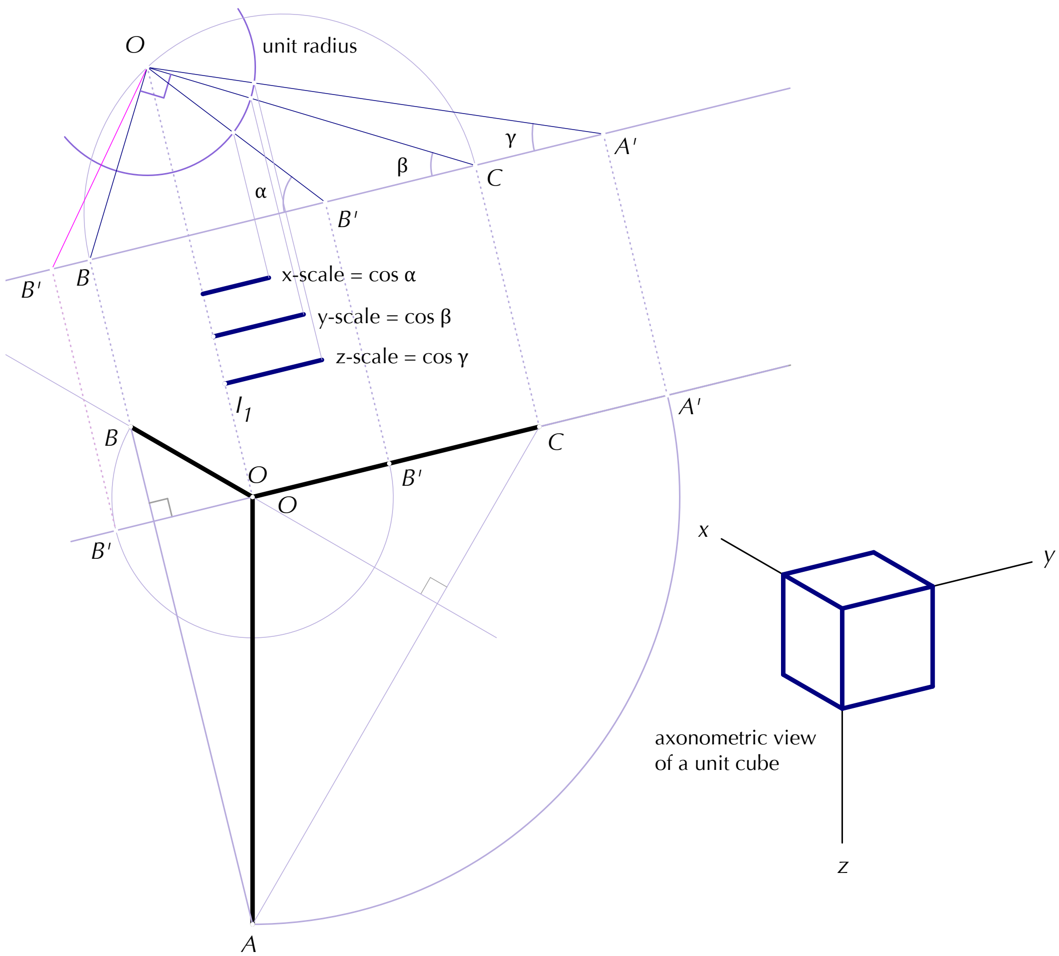
10-12
Arbitrary axial directions for an axonometric projection

There two main steps to the construction, which is shown in Figure 10-14.

The first step is to determine the inclination to the plane of projection, which can be achieved by drawing perpendiculars to two axial directions from the third axial direction to determine points on those axes. For this we construct a plane that cuts the axial directions. In Figure 10-13 we choose *OA* as the axial direction from which axial distances *OB* and *OC* are determined. *A*, *B* and C are the traces of the *OA, OB* and *OC* on this plane; that is, points *A, B* and *C* define the plane. As *OB* is perpendicular to *OA* and *OC*, it is also perpendicular to *AC*. Likewise *OC* is perpendicular to *AB*. Therefore,



10-13
Determining axial distances



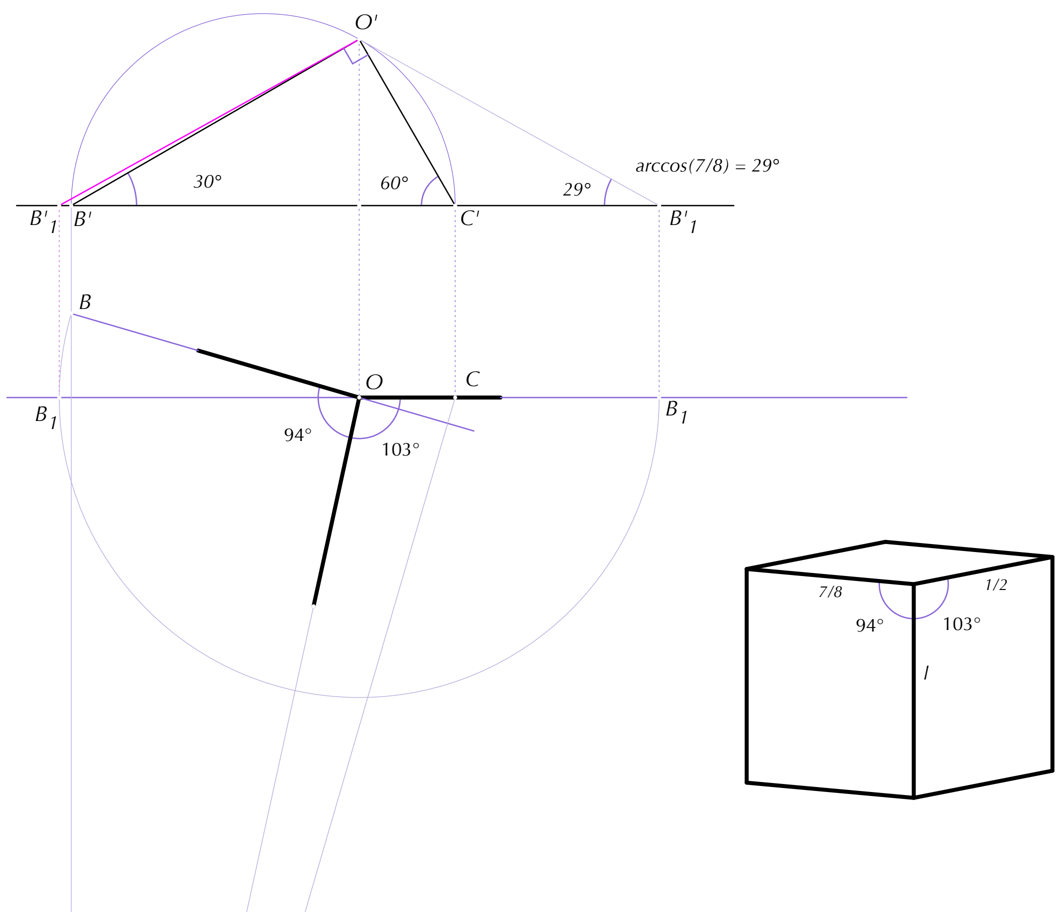
10-14
Constructing axonometric scales

The second step is to determine the axial scales. For this, we project *O, A*, *B* and *C* in true length onto an auxiliary view as shown. In this view, *OB* and *OC* are orthogonal.

In the auxiliary view, from *O*, draw a circle of unit radius to intersect *OA', OB'* and *OC*. Project the intersected points onto lines parallel to the folding line. The lengths on each of these lines give the axonometric scale for the specific axis as shown in the figure. Note that axonometric scales correspond to the cosines of the viewing direction angles α, β and γ to the projection planes as illustrated. These lengths can be used to construct an axonometric view of a unit cube.

10.2.4 Constructing the axonometric directions

Conversely if we have specific numbers for the scales, it is possible to (very nearly) determine the axial directions in order to obtain the axonometric axial directions. The construction is virtually the reverse of Figure 10-14 and is shown in Figure 10-15.



10-15
Constructing the axonometric axes for given axial scales
(Point *A* not shown in construction)

We will assume that for one of the axes, say, the vertical axis, the scale is unit. Clearly the scales lie between 0 and 1. For any scale, we can look up in a reference book, or from a website[3], an angle based on the cosine value. Suppose we want to construct the axial directions for a 1: $^7/_8$: $^1/_2$. That is, for scale = $^1/_2$, arccos($^1/_2$) = 60º. For a scale= $^7/_8$, arccos($^7/_8$)= 29º approximately. Note that accuracy relies more on the scale than on precise angle measurements. However, scales cannot be arbitrarily chosen and the two angles must sum to less than 90º. The steps are as follows.

- First, on a horizontal line *B′C′* draw any arbitrary semicircle. In this semicircle set off a line *C′O′* at an angle of arccos($^1/_2$) = 60º angle as shown.
- Next mark a point B'_1 on the line so that $B'_1O'C'$ makes an angle equal to arccos($^7/_8$) = approximately 29º.
- Draw any line parallel to *B′C′* and project B'_1, *O′* and *C′* to meet this line at B_1, *O* and *C* respectively. With *O* as center draw a circular arc OB_1 to meet the projector from *B′* at *B*. Join *OB*.
- Extend *OB* and draw a line from *C* perpendicular to it and let it intersect the vertical line from *B* at a point *A*. Join *OA*.
- *OA, OB* and *OC* specify the required axial directions relative to one another.
- On *OA, OB* and *OC* mark off points in the ratio 1: $^7/_8$: ½.

10.3 SHADES AND SHADOWS IN AXONOMETRIC VIEWS

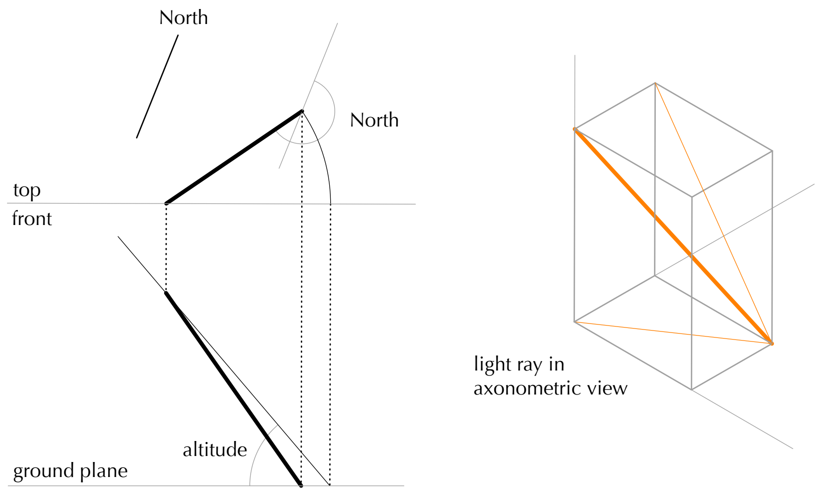
There is not much in the literature on this topic although it seems that if axonometric views are used properly, then shades and shadows would enhance any realism that they might offer. Clearly, one could create shades and shadows in orthographic projections, and then transfer them to the axonometric view. However, it is preferable to directly create shades and shadows in the axonometric view. The question is whether this is possible. The material here is speculative, based on educated reasoning.

Firstly, for any of light in an orthographic projection, we can create its axonometric view as shown in Figure 10-16. If the axonometric scales are different, then light rays would have to appropriately scaled.

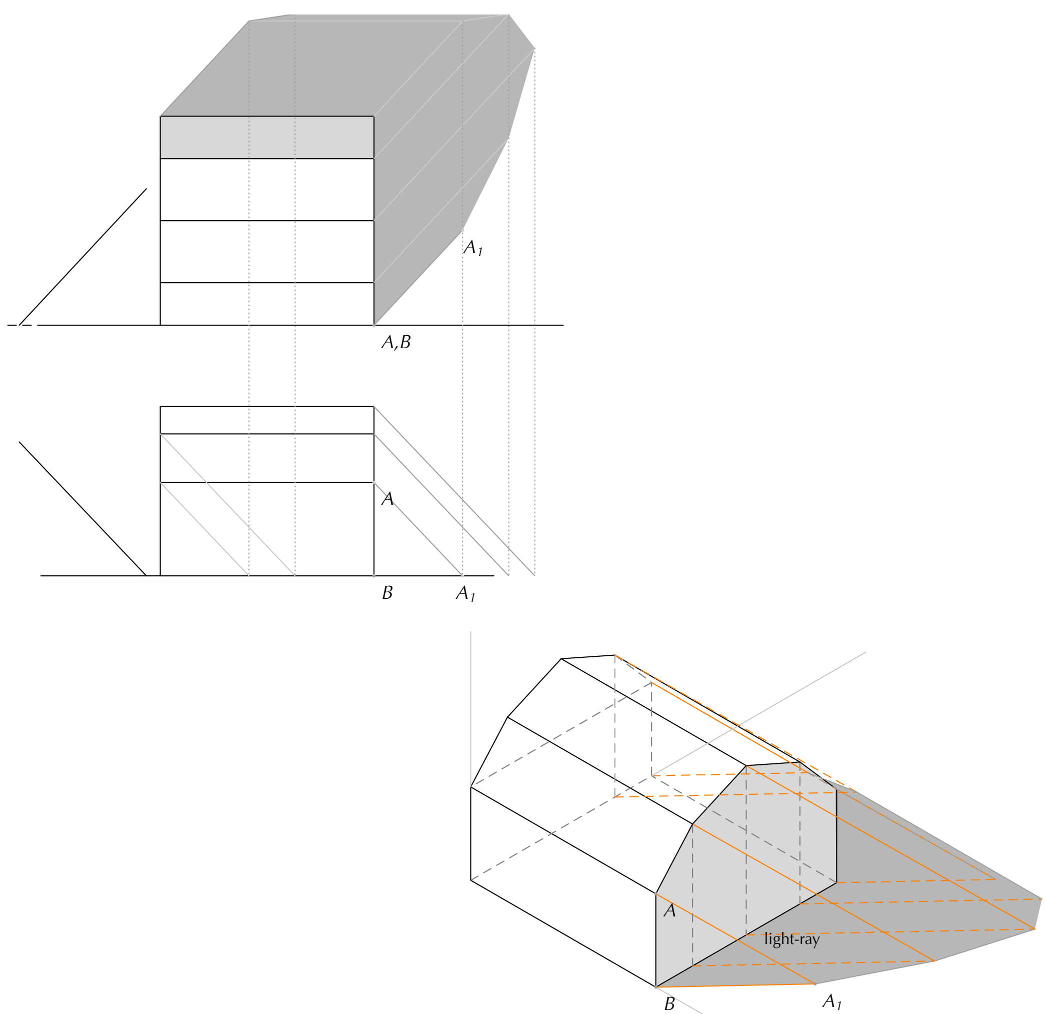
For the mansard roof example, shown originally in Figure 9-9, we either directly employ the light-ray, or employ a combination of determining a shadow point in orthographic view and then transferring the construction to the axonometric view. For shade line *AB*, we determine the shadow point A_1 corresponding to *A*. BA_1 is a shadow line and lies on the ground plane in the axonometric view. Note that BA_1 also indicates the bearing of the light ray. Moreover, AA_1 is the light-ray seen in elevation in the axonometric view. By drawing lines parallel to AA_1 and BA_1, passing through the shade points, we can directly construct the shade and shadow in the axonometric view. Shade and shadow are shown in the orthographic view for comparison. The construction is shown in 10-16.

---

[3] http://www.analyzemath.com/Calculators_2/arccos_calculator.html

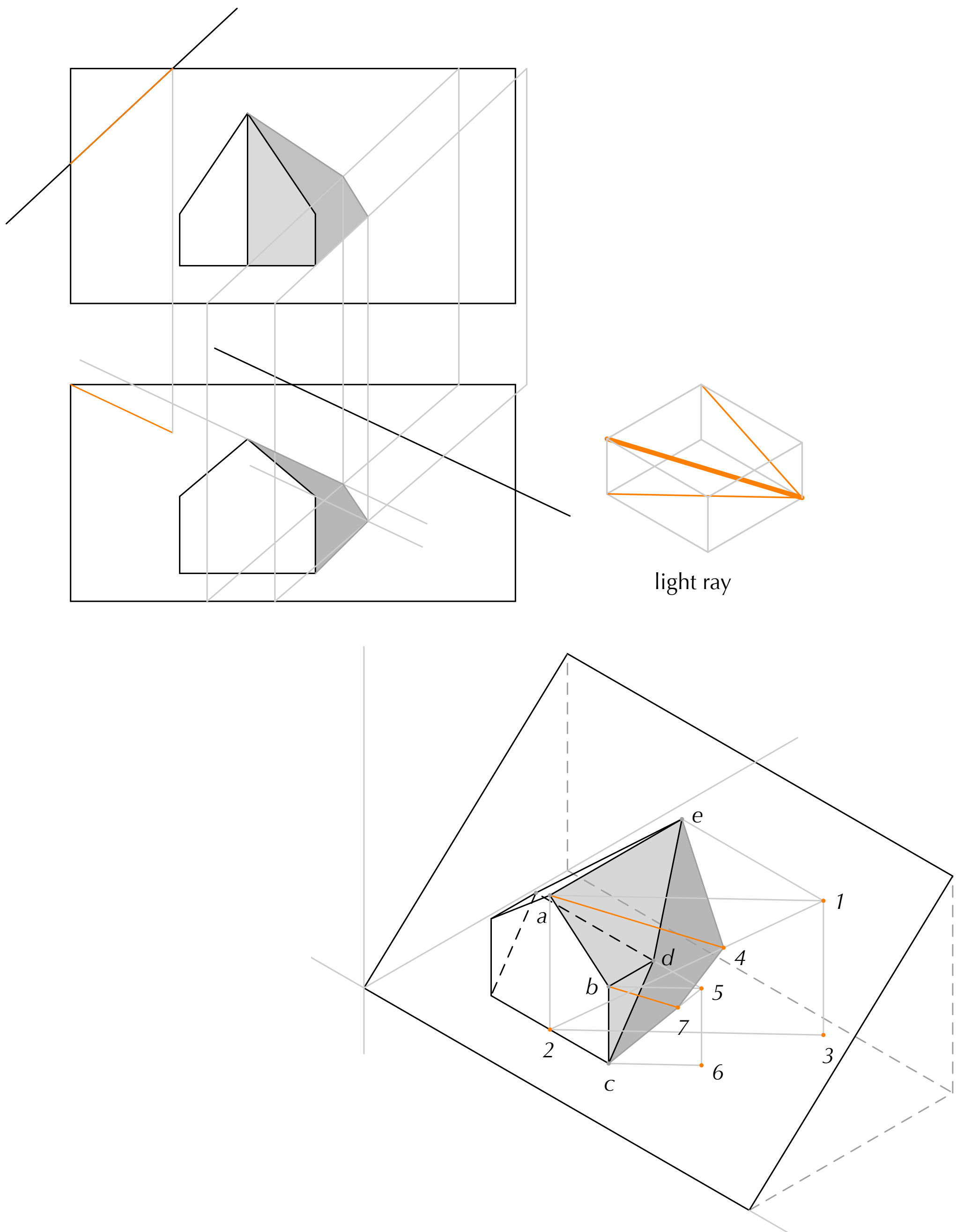


10-16
Light ray in axonometric view



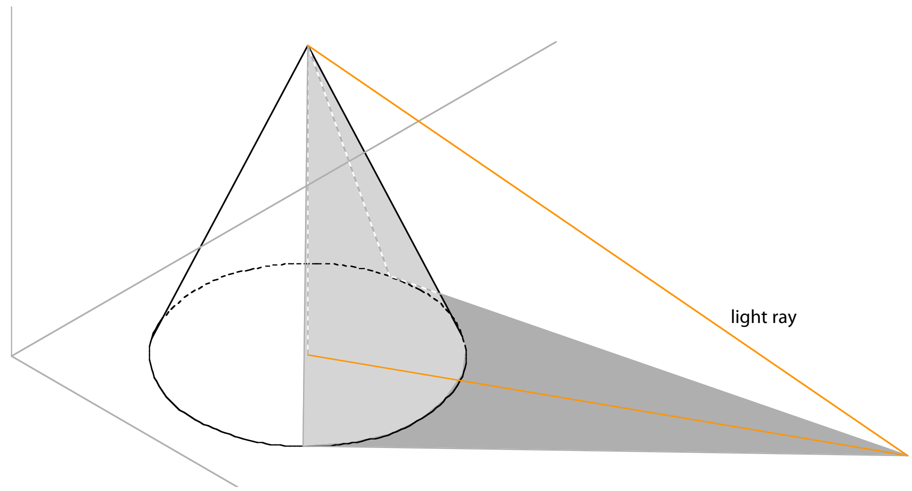
10-17
Shade and shadow in axonometric view

In the case of the dormer on a sloping roof, considered in Figure 9-12, except for the front of the dormer, none of planes are coincident with the axonometric axial planes. In this case, we determine the trace of the light ray on the roof plane. The construction is shown in Figure 10-18. *a132* is the cutting plane that contains the light ray from *a*. The line *12* lies on the roof plane and intersects the light ray from *a* at the piercing point *4*. Likewise, *b36*c is the cutting plane that contains the light ray from *b*. The line *c4* lies on the roof plane and intersects the light ray from *b* at the piercing point *7*.



10-18
Shades and shadows for a dormer in axonometric views

One final example is the cone. Here too, we should be able to apply shortcuts. Given the axial directions for the light we can map the vertex of the cone onto the ground plane to determine the shadow point for the vertex. Then, tangents to the base from the shadow point provide all the information needed to complete the shading and shadowing. See Figure 10-19.



10-19
Shades and shadows for a cone in axonometric views