SPATIAL RELATIONS, KINEMATICS AND ASSEMBLY

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Abstract

The description of designs for assemblies is given in terms of spatial relations between their components. For a given set of components and possible spatial relations many designs are possible. These may be constructed using shape rules based on the components shapes and the spatial relations. The ways in which the application of these rules generate designs is investigated and their description by means of a network is examined. It is suggested that this description of assemblies will provide the basis of a common language for the generation of designs and the operations for their manufacture.

Keywords: Spatial relation, assembly, shape rule.

1. INTRODUCTION

This paper will examine the description of assemblies, gripping and manipulation with robot arms in terms of spatial relations. The description of assemblies has two parts, namely the goal assembly and the intermediate stages in the assemblies. The latter includes the description of variable spatial relations between parts as they are brought to their desired final positions. The description of gripping is given in terms of the spatial relations between the fingers and the components. These spatial relations must allow the goal assemblies and intermediate assemblies to be realised. The description of manipulation is given in terms of variable spatial relations between the links of the manipulator.

It has often been noted that the procedures for assembly are dependent upon the design of the assembly. The design of the individual components and the relations between components in the finished assembly, are crucial determinants of the assembly operations required.

This paper will report research which is aimed at understanding the design of assemblies and attempt to draw together common threads from the descriptions of the assembly tasks, gripping and manipulation. In this way a common basis for design and assembly is possible which should facilitate the development of communication between the generation of designs and their manufacture.

This work deals mainly with the description and design of assemblies and attempts to bring together the description of assemblies for intelligent assembly [1,2] and the generation of languages of design using shape rules based on spatial relations [3,4]. In a broader context the description of assemblies is considered as a first step towards providing critical language component of a design machine (Fig. 1) [5] for assemblies. This language informs the design process and allows communication between the generation of designs and the processes for realizing the designs with effectors. In this case manipulators effect the assembly operations.

2. ASSEMBLIES

Assemblies may be considered as composed of desired spatial relations between components. At the elementary level their description requires the description of a spatial relation between two shapes, representing components.

Suppose, for the moment that we are not concerned with how the individual shapes are described, but want to describe their relationships. In constructing the design of an assembly, perhaps using a geometric solid modeller, there are three kinds of shapes. 'Primitive' shapes are used to construct the 'composite' shapes of the components. The components are then put together to form 'assembly' shapes.

The complexity of the component shapes is limited by many factors including the number of processes required and the number and type of the pre-assembly operations required for their manufacture. Thus in the assembly of electric motors individual components such as windings on cores are prepared before final assembly. Complex components may be assemblies in their own right.

In passing from primitive to composite component shapes and from components to assemblies, shapes are brought into desired relationships with one another. The description of these relations and the means of formulating rules for the generating designs in a 'language' of designs [3,4] is now given.

3. SPATIAL RELATIONS

Given two shapes S_1 and S_2 which are described in some way by their geometric features, perhaps by their surfaces, their boundary edges or their point description, using some geometric modelling technique.

Each shape is described relative to its own coordinate frame, and any description is given in terms of that coordinate frame. Let shape S_1 be described in terms of a frame F, fixed in the shape, and for the sake of convenience let S_1 denote the description. Similarly let shape S_2 be described in frame F_2.

A spatial relation between shapes S_1 and S_2 is given explicitly by the pair of shape descriptions $< S_1, S_2 >$ together with the description of the transformation τ_{12} between the coordinate frame F_1 and F_2. The transformation expresses the relation of coordinate frame F_2 to F_1 in terms of the coordinate frame F_1. At this stage the component shapes and the transformation are not expressed in a reference coordinate frame. The local coordinate frames in the individual shapes are used. A spatial relation between S_1 and S_2 is expressed by $< S_1, S_2, \tau_{12} >$. It is sometimes expedient to express the spatial relation as a pair of shapes, and the description $< S_1, \tau_{12}(S_2) >$ can be used. This provides a description of the spatial relation as a pair of shapes in the coordinate frame F_1.

Note that the description of a spatial relation is made with respect to 'local' coordinate frames in the shapes. Thus if the shapes are embedded in a reference coordinate system, the spatial relation between two shapes is irrespective of their position in the reference coordinate frame.

4. EQUIVALENT SPATIAL RELATIONS

If the shapes S_1 and S_2 are symmetrical there will be many representations of the spatial relation. Suppose that there is a transformation θ_1 such that $\theta_1(S_1) = S_1$: the transformation θ_1 is specified with respect to the coordinate frame F_1. The spatial relation $< S_1, S_2, \theta_1 * \tau_{12} >$ is equivalent to $< S_1, S_2, \tau_{12} >$ (Fig. 2) where τ_{12} is specified in the frame F_1 after it has been transformed by θ_1 and gives a new position of F_2. Similarly if there is a transformation θ_2 such that $\theta_2(S_2) = S_2$ where transformation θ_2 is specified in Frame F_2, then the spatial relation $< S_1, S_2, \tau_{12} * \theta_2 >$ is equivalent to the spatial relation $< S_1, S_2, \tau_{12} >$. In general the spatial relations $< S_1, S_2, \theta_1 * \tau_{12} * \theta_2 >$ where $\theta_1(S_1) = S_1$ and $\theta_2(S_2) = S_2$ are all equivalent to $< S_1, S_2, \tau_{12} >$.

The equivalent spatial relations are all pictorially the same, but the relation between the coordinate frames fixed in the shapes S_1 and S_2 is different in each case. The difference in the use of the alternative formulations of the spatial relations makes itself manifest when considering the 'composition' of two spatial relations.

5. COMPOSITIONS OF SPATIAL RELATIONS

Consider another spatial relation $< S_2, S_3, \tau_{23} >$. The spatial relation between S_1 and S_3 when this relation is satisfied between S_2 and S_3 and the relation $< S_1, S_2, \tau_{12} >$ is satisfied between S_1 and S_2 is given by the composition $< S_1, S_3, \tau_{12} * \tau_{23} >$. If the spatial relation $< S_1, S_2, \theta_1 * \tau_{12} >$ is used then the composite spatial relation is $< S_1, S_3, \theta_1 * \tau_{12} * \tau_{23} >$ which is equivalent to $< S_1, S_3, \tau_{12} * \tau_{23} >$. However, if the spatial relation $< S_1, S_2, \tau_{12} * \theta_2 >$ is used then the composite spatial relation is $< S_1, S_3, \tau_{12} * \theta_2 * \tau_{23} >$ (Fig. 3). This is not in general equivalent to $< S_1, S_3, \tau_{12} * \tau_{23} >$ unless one of the following conditions is satisfied.

(1) a symmetry transformation ϕ_1 of S_1 such that $\phi_1 * \tau_{12} = \tau_{12} * \theta_2$

(2) a symmetry transformation ϕ_3 of S_3 such that $\tau_{23} * \phi_3 = \theta_2 * \tau_{23}$

Thus condition (1) is equivalent to having $\tau_{12} * \theta_2 * \tau_{21}(S_1) = S_1$ and condition (2) is equivalent to $\tau_{32} * \theta_2 * \tau_{23}(S_3) = S_3$ where τ_{21} and τ_{32} are used to denote the inverses of τ_{12} and τ_{23} respectively. Further if there is a spatial relation $< S_2, S_3, \tau_{23} * \theta_3 >$ between S_2 and S_3 then the composite spatial relation $< S_1, S_3, \tau_{12} * \tau_{23} * \theta_3 >$ is equivalent to $< S_1, S_3, \tau_{12} * \tau_{23} >$. Finally, if a spatial relation $< S_2, S_3, \theta_2 * \tau_{23} >$ between S_2 and S_3 is used then the composite spatial relation is $< S_1, S_3, \tau_{12} * \theta_2 * \tau_{23} >$. This is not in general equivalent to $< S_1, S_3, \tau_{12} * \tau_{23} >$, except under the conditions given above. Symmetry of the shape S_2 thus gives many possible spatial relations for the composition.

The description of assemblies may be given by sets of spatial relations between components. However, if components possess summetry then the set of spatial relations is not sufficient to define the assembly. Particular cases of the spatial relationships must be selected.

6. DESCRIPTION OF ASSEMBLIES

The description of the assembly requires the specification of the spatial relations between components. As we have seen the specifications of a sequence of spatial relations does not describe the assembly uniquely, because of the possible symmetry transformations of the component shapes. The result of this non-uniqueness is that two given spatial relations $< S_1, S_2, \tau_{12} >$ and $< S_2, S_3, \tau_{23} >$ do not imply a third unique spatial relation $< S_1, S_3, \tau_{13} >$, between the shapes S_1 and S_3 but rather they imply a class of possible spatial relations. This arises because each spatial relation $< S_1, S_2, \tau_{12} >$ and $< S_2, S_3, \tau_{23} >$ corresponds to an equivalence class of spatial relations.

These difficulties may be overcome, but

first some advantages of using a spatial relation description of assemblies are indicated. The description may be given constructively in the sense that shapes may be brought together in desired spatial relations. This constructive description is thus close to the description of the kinds of physical assembly operations which will be required.

Another advantage arises from the fact that many designs may be generated for a given sequence of spatial relations. Further for a given set of spatial relations, the possible assemblies generated using these spatial relations, not necessarily in any specific sequence, represent classes of related designs. The designs share some desired spatial relations, although clearly other spatial relations will occur in the individual designs.

7. SHAPE RULES

The generation of assemblies using spatial relations can be formalized by 'translating' the spatial relations into rules [3]. For a spatial relations $< S_1, S_2, \tau_{12} >$ rules of the following types may be derived.

$$\begin{array}{ll} S_1 \longrightarrow S_1 + \tau_{12}(S_2) & \\ S_1 \longrightarrow S_2 + \tau_{21}(S_1) & (1) \end{array} \qquad \begin{array}{ll} S_1 + \tau_{12}(S_2) \longrightarrow S_1 & \\ \tau_{21}(S_1) + S_2 \longrightarrow S_2 & (2) \end{array}$$

The rules of type (1) take a shape S_1 or S_2 and add the other shape in the spatial relation $< S_1, S_2, \tau_{12} >$. The rules of the type (2) take two shapes S_1 and S_2 in the spatial relation $< S_1, S_2, \tau_{12} >$ and subtract or remove one of the shapes.

The rules apply in the following way. If a current subassembly possesses a subshape $\tau(S_1)$ for some transformation τ in a reference frame, then the application of a rule of type (1) takes the form $\tau(S_1) \longrightarrow \tau(S_1) + \tau * \tau_{12}(S_2)$. This method of rule application allows all the equivalent spatial relations to be realized by application of the rule.

More restrictive rule applications can be made by noting that each occurrence of a shape S_1, say, in the current shape representing a particular stage in the assembly will have its own coordinate frame and the transformations between this coordinate frame and the others in the current shape are known from the compositions of the transformations in individual spatial relations. If the coordinate system for S_1 is known then each rule application has a unique effect. The rule applies as it is written, by finding the shape S_1 in the subassembly, rather than by finding a transformation of S_1. This removes the explicit need for a reference frame in the generation of an assembly and places the onus of description upon the relations between components.

However, it might be argued that this is too restrictive and the freedom within the original spatial relations is not utilized in the rules. The rules may be made less restrictive by allowing each rule to belong to a class of rules.

Given the spatial relation $< S_1, S_2, \tau_{12} >$, then the general class of corresponding rules of type (1) will be $S_1 \longrightarrow S_1 + \theta_1 * \tau_{12} * \theta_2\ (S_2)$ where θ_1 and θ_2 are symmetry transformation of S_1 and S_2 respectively. Subclasses of this class of rules may be defined by restricting the transformations θ_1 and θ_2. Thus any choice for these restrictions will give a subclass of rules. If shapes S_1 and S_2 have n_1 and n_2 symmetry transformations respectively, then there are $2^{n1}\ 2^{n2}$ possible subclasses of rules corresponding to the given spatial relation.

The restrictions in the class of rules arising from a spatial relation may also be improved in another way, when using the application of the rule under transformation in a reference frame. These restrictions are imposed by 'reducing' the symmetry of the component shapes by adding marks or labels [3]. Suitably chosen marks, regarded as part of the shape (and transformed with the shape) will restrict the equivalence class of spatial relations and thus the range of possible rule applications corresponding to a given spatial relation. In a sense the marks 'guide' the generation of the assembly.

8. ASSEMBLY NETWORKS

Let us now return to the description of assemblies in terms of spatial relations. Suppose there is a set of spatial relations $\{< S_i, S_j, \tau_{ij} >\} = \{R_{ij}\}$ to be used in the description of an assembly. Suppose these relations are all distinct, that is no two are equivalent. Note that more than one spatial relation may be allowed between shapes S_i and S_j. Thus each R_{ij}, in general corresponds to a set of spatial relations. The application of an additive rule which appends a shape S_j to the shape S_i in spatial relation R_{ij} can be represented by arrows between shapes.

$$S_i \xrightarrow{\tau_{ij}} S_j$$

The application of symmetry transformations θ_j and θ_j' to S_j is denoted by:

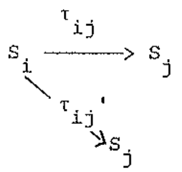
$$\begin{array}{l} S_j \xrightarrow{\quad \theta_j \quad} S_j \\ \ \searrow^{\theta_j'} \\ \qquad S_j \end{array}$$

and thus the general additive shape rule application based on the spatial relation R_{ij} is

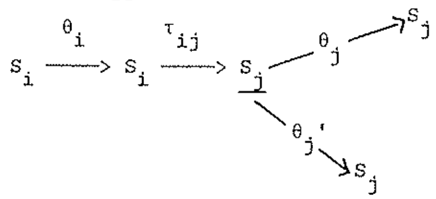
$$S_i \xrightarrow{\theta_i} S_i \xrightarrow{\tau_{ij}} S_j \xrightarrow{\theta_j} S_j$$

This representation gives a particular shape rule application from a class of shape rules.

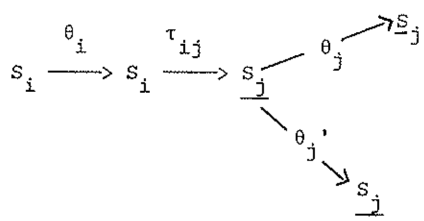
A description of an assembly is a network of shape rule applications in the form above. There may be more than one spatial relation R_{ij} between the shapes S_i and S_j. If this relation is τ_{ij}' then the application of the corresponding additive shape rules to S_i gives:



The subtractive rule application can be represented by underlining the shape instances which are removed. Thus if the rule $S_i + \tau_{ij}(S_j) \longrightarrow S_i$ is applied then the result is represented as the following for the particular instance of the spatial relation at which the subtractive rule is applied.



In some cases it is useful to use the subtractive rule to remove all instances of the shape S_j formed by symmetry transformations. This is effected by applying the appropriate class of subtractive rules to give the following effect:



The shape S_j in the particular relation to S_i has been used as a 'framework' for further spatial relations and may be removed when these have been generated. The assembly network has similarities to the 'assembly' graph in [1].

9. ANALYSIS OF ASSEMBLY NETWORKS

The network of spatial relations describing an assembly allows analysis of the interdependencies in the assembly. First, the relation between shapes connected by paths (not necessarily directed) can be given by tracing the transformations in the path. The networks are connected, thus the spatial relation between any pair of shapes in the assembly may be calculated.

Second, if a cycle is to be created in the network description of the assembly, then the transformations around the whole cycle must add to the identity. The possibility of making cycles in the network using the available spatial relations may thus be readily evaluated. Cycles may be created by requiring individual shapes to satisfy more than one spatial relation simultaneously.

How do we determine when two assemblies are the same from their networks of spatial relations? The assembly is really none other than a complex spatial relation among many shapes. The question posed is whether the network can be used to determine the equivalence of these composite spatial relations.

The isomorphism of two networks which preserves shapes and transformations (up to inverse for opposite directed edges) is a sufficient condition for two assemblies to be the same.

However, a given assembly does not necessarily possess a unique network. How can alternative networks for a single assembly be derived? A given network may have edges representing a rule application, which can be replaced by sequences of subtractive rule applications, based on the given spatial relations. These networks form the class of alternative networks for the given assembly. The class depends on the existence of sequences of spatial relations whose composition is equivalent to one of the given spatial relations.

10. ASSEMBLY OPERATIONS

The network of an assembly can be used to derive sequences of assembly operations, where each operation is based upon putting given relations into effect, including composite relations which occur when subtracted rule applications are used. The assembly network provides possible sequences for assembly operations, but choice must still be made between the sequences, based both upon manufacturing and manipulation considerations. There are usually particular components from which the assembly operations will start.

For example the assembly of components on a chassis framework or on a shaft will usually start with the shaft or framework and place the components one by one. The order in which the shapes are assembled will be determined by the manipulations which are required to effect the required relation, given the presence of previously assembled components. These are complex problems in the design of assembly tasks. The network representation of assembly will facilitate their resolution.

However, the spatial relation description of an assembly is really a goal configuration and intermediate assemblies are often required before given spatial relations are realized. In many cases a component is manipulated such that variable spatial relations are maintained, until the goal spatial relation is reached. For example, in placing a component in the corner of a framework for fixing, it may be slid down the corner until a stop is reached.

11. VARIABLE SPATIAL RELATIONS

The description of assemblies by spatial relations given above allows these variable spatial relations to be handled easily. A variable spatial relation $< S_1, S_2, \tau_{12} >$ has a variable transformation τ_{12}. The type of variation allowed is specified by the values of parameters in the description of the transformation. Further variable transformations can occur if the symmetry transforms θ_i of component shapes are allowed to take a continuous range of values. For example a cylindrical shaft has all rotations about its axis as symmetry transformations.

Variable spatial relations may be used to describe assemblies with moving parts or to find possible values of variable transformations in a cycle of spatial relations in an assembly network. The cycles provide dependencies among the variable transformations. The variable spatial relations can also describe the manipulator used in the assembly operations. The satisfaction of any given spatial relations is given by the composition of six variable 'independent' spatial relations between the links of the manipulator. Further, the relations between the fingers in a gripper and the component are variable relations. The different constraints by fingers, namely point, line and surface constraints, both with and without friction, represent variable spatial relations. For example, point constraint with friction allows motion around the point of contact and along the surface normal from the finger. The construction of finger constraints should imply fixed values for variable spatial relations between fingers and object implied by the simultaneous satisfaction of two or more variable spatial relations.

12. CONCLUSION

The description of the assemblies and the operations required for their construction in terms of spatial relations provides common descriptions for their design and manufacture. As such this type of description can form the basis for languages of design and manufacture in which assembly tasks may be described and executed. With a suitable language, the communications between design and manfacture are facilitated, making possible the automatic generation and intelligent construction of assemblies.

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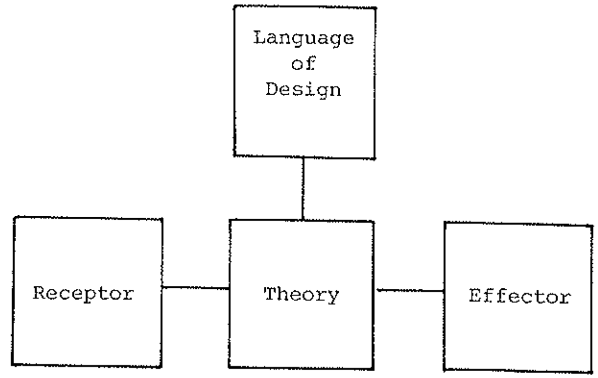

Figure 1: Schematic for Design Machine

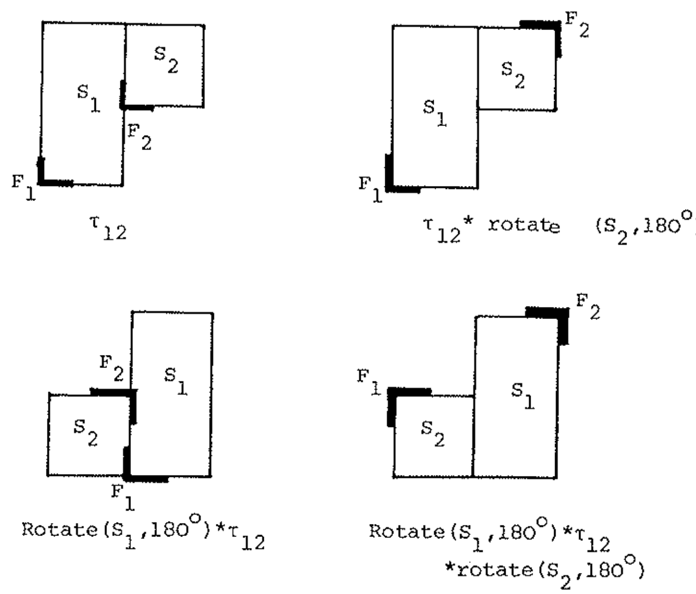

Figure 2: Equivalent spatial relations

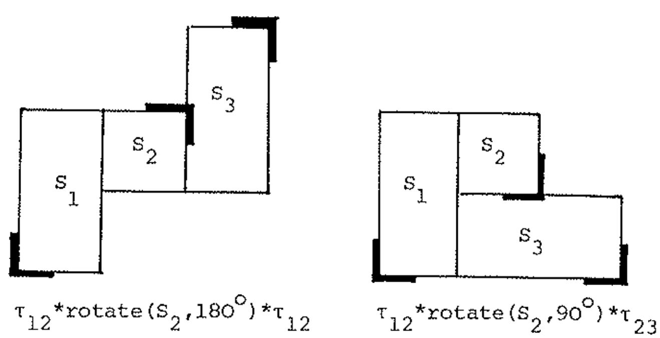

Figure 3: Compositions of spatial relations