Lists (15 points)

Name Key

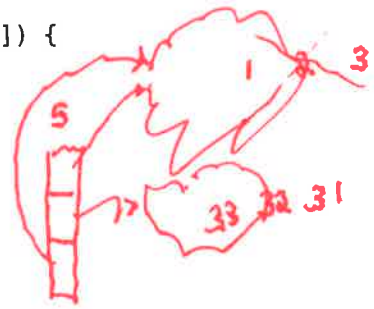
1. Consider the following Node and List classes.

```
class Node {

    private Object data;
    private Node next;

    public  Node(Object d, Node n) { data = d; next = n; }

    public  Node getNext() { return next; }

    public  Object getData() { return data; }

    public  void setNext(Node n) { next = n; }

    public  void setData(Object x) { data = x; }

    public  String toString() {

        String suffix = "";
        if(next == null)
            suffix = "--|";
        else   suffix = "-->";
        return data.toString() + suffix;
    }
}

public class List {

    private Node head;

    public List() { head = null;}

    public void add(Object t) { head = new Node(t,head); }

    public void addAlso(Object t) {
        if (head == null) {
            head = new Node(t,null);
        }
        else {
            Node p = head;
            Node q = p;
            while(p != null) {
                q = p;
                p = p.getNext();
            }
            Node m = new Node(t,null);
            q.setNext(m);
        }
    }


    public Object remove() {  Object t = head;
        head = head.getNext();
        return t;
    }
```


Key

Key

```
    public boolean isEmpty() { return (head == null); }

    public String toString() {

        if(head == null) return "--|";

        String result = "";

        Node p = head;

        while(p != null) {

            result = result + p;

            p = p.getNext();
        }

        return result;
    }

    public static void main(String a[]) {

        List s[] = new List[3];

        s[0] = new List();
        s[1] = new List();

        s[0].addAlso(1);
        s[0].addAlso(2);
        s[0].addAlso(3);

        s[1].add(31);
        s[1].add(32);
        s[1].add(33);

        // 1
        System.out.println(s[1]);

        // 2
        System.out.println(s[0]);

        //3
        while(!s[0].isEmpty())
            System.out.print(s[0].remove());

        //4
        s[2] = s[0];
        while(!s[2].isEmpty())
            System.out.print(s[2].remove());

    }
}
```



33→32→31 -||

1 → 2 → 3 -||

1 → 2 → 3 -||

NOTHING DISPLAYED

(a) The program compiles and executes without error. What is the exact output of the println statement marked in the code with a 1? (2 Points)

33 → 32 → 31 -||

Key

(b) The program compiles and executes without error. What is the exact output of the while loop marked in the code with a 2? (2 Points)

1 -->2 -->3 -||

(c) The program compiles and executes without error. What is the exact output of the println statement marked in the code with a 3? (2 Points)

1 --> 2 --> 3 --||

(d) The program compiles and executes without error. What is the exact output of the println statement marked in the code with a 4? (2 Points)

No output

(e) State a pre-condition that should be present in the remove method of the List class. (2 Point)

head != Null or List is not empty

(f) We need a public method for the List class that displays the list in reverse order. This method will be called "displayReversed" and is shown below. It will make use of a recursive method called "reverse" with the signature as shown. Complete the recursive method called "reverse". (5 Points)

```
private void reverse(Node p) {
    if (p != null) {
        reverse(p.getNext())
        System.out.println(p.getData())
    }

}

public void displayReversed() {
  reverse(head);
}
```

Key

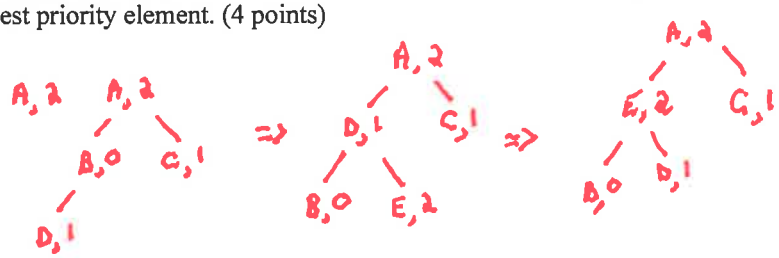
Priority Queues: (15 points)

2) (a) Sue has implemented a priority queue as an array of linked lists. Each list has an enQueue and deQueue method and these methods run in θ(1). The enQueue method adds data at the rear of the queue and the deQueue method removes data from the front of the queue. Each array cell holds a pointer to the front of the queue and a pointer to the rear of the queue. There are 3 priorities and so the array has 3 elements. The highest priority is 2 and the lowest is 0. Provide an algorithm that shows how the highest priority element will be removed from such a queue. Your algorithm will find the highest priority element and remove it by calling deQueue on the first non-empty linked list. The code that you write needs to be syntax error free Java. (5 Points)

```
object getHighPriority() {
    int i = 2
    while(i >= 0) {
        if(!a[i].isEmpty())
            return a[i].deQueue()
    }
    return null
}
```

(b) Bill has implemented a priority queue as a heap in an array. Draw a picture of the heap as a tree as it is being built. There are 3 priorities. The highest priority is 2 and the lowest is 0. Show each node being entered by redrawing the entire tree after each of the following insertions. Of course, Bill wants a deletion to always return the highest priority element. (4 points)

Ms. A has priority 2
Mr. B has priority 0
Ms. C has priority 1
Mr. D has priority 1
Ms. E has priority 2



(c) What person (A,B,C,D, or E) would be the first to be removed from the priority queue? (3 points)

A

(d) Suppose i is an array index into the heap in question 2b, complete the following method in error free Java. Be sure to express any pre-conditions that might be required.(3 Points)

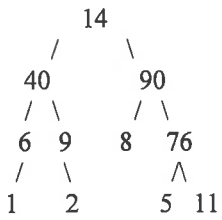
```
// i > 0 And i is a heap index
int parent(int i) {
    return (i-1) / 2;
}
```


Key

key

Trees (15 points)

3. Parts (a), (b), and (c) refer to the binary tree:



(a) List the data that would be accessed by a pre-order traversal on the given tree by writing out the values in the nodes as they would be accessed, separated by commas. (3 points)

14, 40, 6, 1, 9, 2, 90, 8, 76, 5, 11

(b) List the data that would be accessed by an in-order traversal on the given tree by writing out the values in the nodes as they would be accessed, separated by commas. (2 points)

1, 6, 40, 9, 2, 14, 8, 90, 5, 76, 11

(c) List the data that would be accessed by a post-order traversal on the given tree by writing out the values in the nodes as they would be accessed, separated by commas. (2 points)

1, 6, 2, 9, 40, 8, 5, 11, 76, 90, 14

(d) In general, if a binary tree is perfectly balanced (unlike the tree pictured here) and the tree is of height h then how many leaves will the tree have? (4 points) 2^h

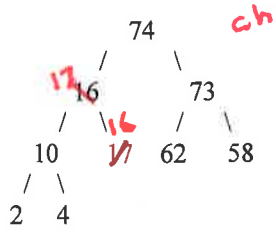
(e) Suppose we are working with a tree that has exactly 0 or 3 children per node. Suppose too that the tree is perfectly balanced and is of height h. how many leaves will the tree have? (4 points) 3^h

key

Key

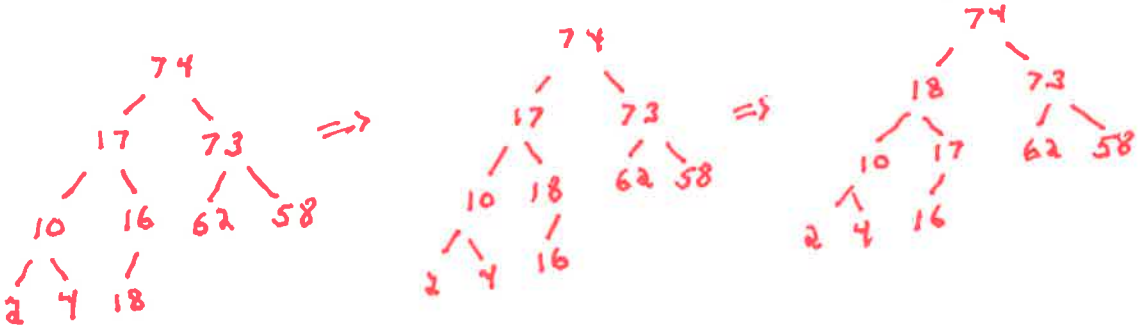
Heaps (10 points)

4. Consider the heap given below:



change on board

(a) Show the series of steps that would take place in adding a node containing 18 to the heap given above. In your answer, you should show what the heap looks like after each step in the addition process or reheapification process so that it is clear to the reader how the changes were made. (4 points)



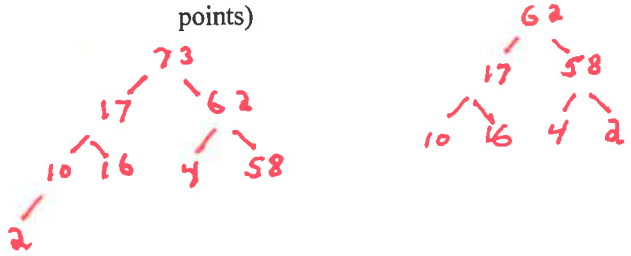
(b) Draw the heap in question 4 a. in an array. Be sure to show the array indexes. Draw the heap before the 18 was added to the heap in question 4 (a). (2 points)

(c) Consider again the heap in question 4 a. (without the 18). Describe a best case insertion into this heap. In other words, specify a value that you would add to this queue that would produce a best case insertion and specify the big θ performance of this insertion. (2 points)

$\{x \mid x \leq 16\}$ $\Theta(1)$

value _____ big theta _____

(d) Show the heap after deleting two values from the heap shown above. In your answer, for each deletion, you should show what the heap looks like after the deletion process or reheapification process. You should start with the original heap specified in the question, not the heap that resulted from the addition of 18 requested in part (a). Draw trees here. Do not draw arrays. (2 points)

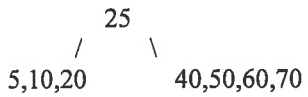


Key

B-Trees (20 points)

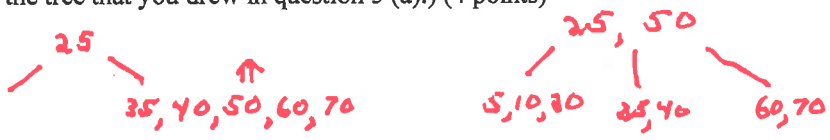
key

5. Consider the following B-Tree with a minimum of two and a maximum of four.



(a) Redraw the tree after inserting 22 and then inserting 4. (6 points)

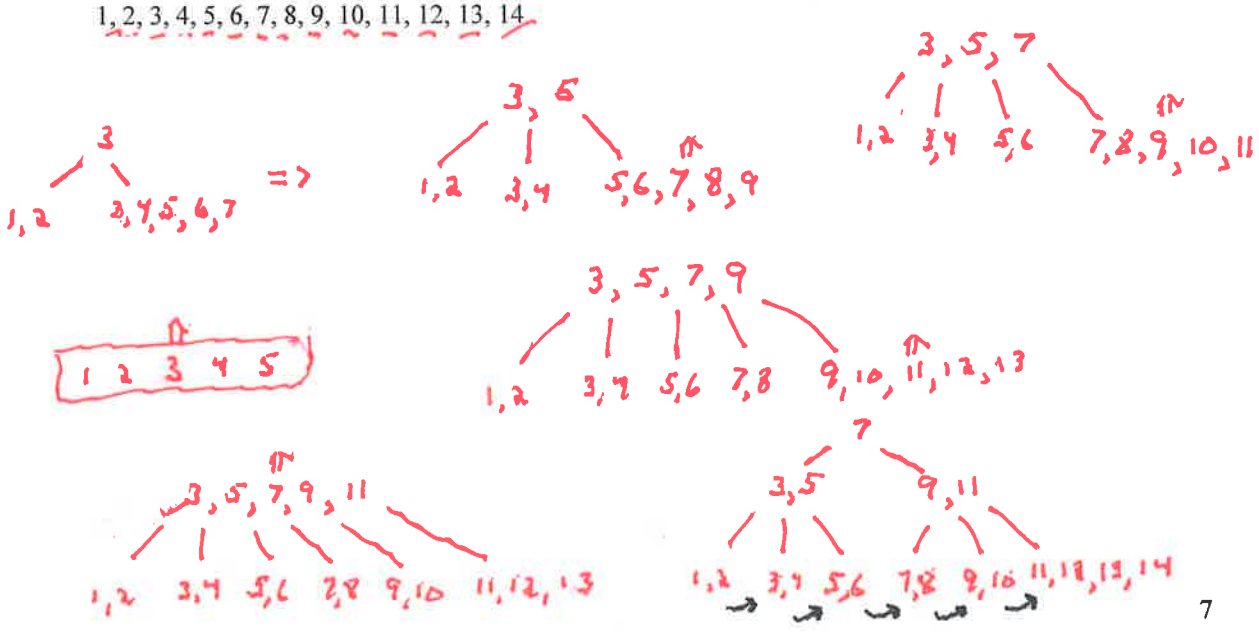
(b) Redraw the tree after inserting 35. (Begin work from the initial tree in question (5), do not work from the tree that you drew in question 5 (a).) (4 points)



(c) The original tree in question 5 has a height of 1. What is the maximum number of keys that this type of tree could hold with a height of 2? (4 points).

124 keys

(d) Insert the following numbers into a B+ Tree with minimum = 2. (6 Points)
1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14

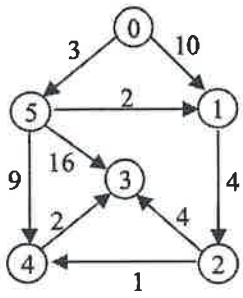


key

Graph Algorithms(15 points)

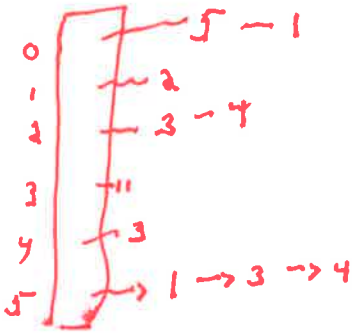
6. (a) What is the distance of the shortest path from node 0 to node 3 in the graph immediately below? (1 point) 12

(b) Draw the contents of the `distance` array for each iteration of Dijkstra's Algorithm as it works on this graph. Note how node 0 was selected and is shown to the left of the distance array. You need to show how the other nodes are selected in turn. Note the little blank "_" character next to each distance array. Fill that blank in with the next appropriate vertex number. Complete the first column first, and the next column second. (6 Points)



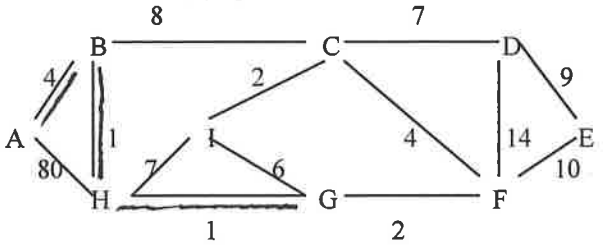
	0	1	2	3	4	5
0	0	?	?	?	?	?
5	0	10				3
1	0	5		19	12	3
2	0	5	9	19	12	3
4	0	5	9	13	10	3
3	0	5	9	12	10	3

(c) Draw an adjacency list representation of the graph shown immediately above (in part b). (3 Points)



Key

(d) Consider the following graph (the root vertex will be A):



Prim's algorithm first initializes the key and parent arrays in such a way that the root vertex has key 0 and a nil parent and all of the other vertices have an infinite key (?). It then continues to make selections and make updates to the key and parent arrays. Show the values that Prim's algorithm would compute for the second, third, and fourth pair of arrays below. (5 points)

A

	A	B	C	D	E	F	G	H	I
B	0	4	?	?	?	?	?	80	?
	nil	A						A	
H	0	?4	?8	?	?	?	?	?1	?
	nil	A	B					B	
G	0	?4	?8	?	?	?	?1	?	?7
	Nil	A	B				H		H
F	0	?4	?8	?	?	?2	?1	?1	?6
	Nil	A	B			G	H	B	G

Grade second now only

Key

7. Project Questions (10 Points)

(a) In Project 3 we wrote a Red Black tree to hold course names and integers (used to index an array). Suppose we wrote a method called traverse() that simply displayed each course name in the tree. Suppose too that traverse() was carefully written and did not waste time. Which of the following is true of the traverse() method? Circle all correct answers. (3 Points)

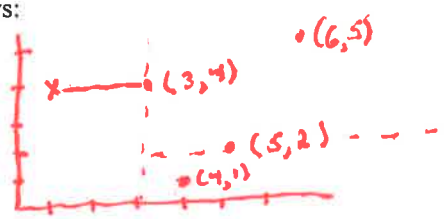
1. It runs in O(LogN)
2. It runs in O(1)
3. It runs in Ω(N^2)
4. It runs in Θ(N)
5. It runs in Θ(LogN)
6. It runs in Θ(1)
7. It runs in O(2^N)
8. It runs in O(3^N)
9. It runs in Ω(N)
10. It runs in Ω(1)

(b) Enter the following numbers into a Red Black Tree and draw the final tree. Red nodes must be circled and Black nodes must be left without circles. (3 points)

1, 2, 3, 4

(c) The following points, in a standard (x,y) coordinate plane, have been added to a 2-d tree. (3,4), (5,2), (4,1). The 2-d tree appears as follows:

(3,4)
\
(5,2)
/
(4,1)



Add the point (6,5) to this 2-d tree. Redraw the tree with this new point added. The first point, (3,4), breaks the plane vertically. (2 points)

(d) Consider the 2-d tree that you created, with the addition of (6,5), in (e). Suppose that we performed a nearest neighbor search for the point (1,4). Which point(s) in the tree need to be examined? (3,4)
(2 Points)

Key