Balanced Search Trees (15 points)

5. Consider the following B-Tree with a minimum of two and a maximum of four.

$$30$$
$$/ \qquad \backslash$$
$$10,20 \qquad 40,45,60,70$$
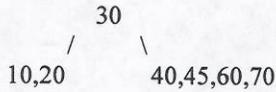
(a) Redraw the tree after inserting 35 and then 77. (3 points)

30
/ ⟍
10,20   35,40,45,60,70 ⬆

30,45
/ | ⟍
10,20   35,40   60,70,77
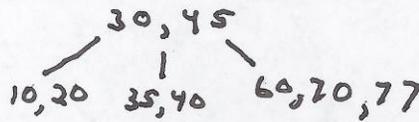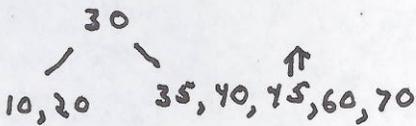
Key

(b) Redraw the tree after inserting 15 and then 16 and then 19. (Begin work from the initial tree in question (5), do not work from the tree that you drew in question 5 (a).) (3 points)

30
/ ⬆ ⟍
10,15,16,19,20   40,45,60,70

16,30
/ | ⟍
10,15   19,20   40,45,60,70
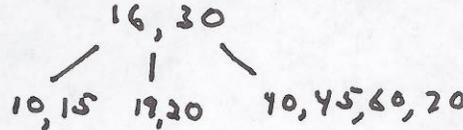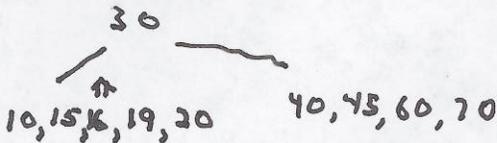
$$\frac{\begin{array}{r} 1\ 2\ 5 \\ 4 \end{array}}{5\ 0\ 0}$$

(c) The tree in question 5a has a height of 1. What is the maximum number of keys that this type of tree could hold with a height of 3? (3 points).

$$\begin{array}{r} 4 \\ 20 \\ 100 \\ 500 \\ \hline 624 \end{array}$$

4 — $4 \cdot 5^0$

$4 \cdot 5^1$

$4 \cdot 5^2$

$4 \cdot 5^3$
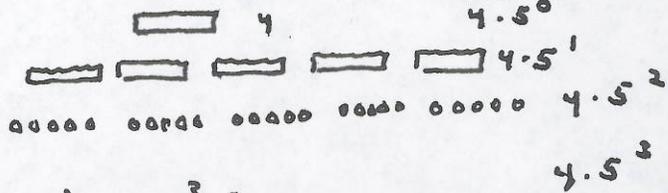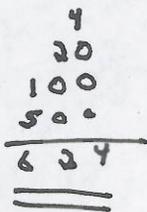
$$4 \cdot 5^0 + 4 \cdot 5^1 + 4 \cdot 5^2 + 4 \cdot 5^3 =$$
$$4 + 20 + 100 + 500 = 624$$

6. Insert the following numbers, one by one, into a Red-Black Tree. Show the tree after each insertion. Red vertices should be circled and Black vertices should appear without circles. (6 points)

1, 2, 3, 4, 5, 6

1

1 ⟍ ②

2 / ⟍ ① ③

2 / | ⟍ ① 3 ⟍ ④

2 / ⟍ ① ④

2 / | ⟍ 1 ④