21-260 Midterm 2 Review

Disclaimer: I don't write the exams or design any of the course material. But more practice is always good so I'm copying down (with modification) some exam problems my undergrad ODE teachers came up with to give you a fake exam experience. Have fun!

I'm abdicating all responsibility, so very likely there are errors and you're on your own. Still, kindly let me know via email (meihec@andrew.cmu.edu) and I'll fix them.

List of topics:

- ???

Carefully read the problem instructions.

1. Give a real general solution for each of the following equations.

   (a) $y'' - y = \dfrac{1}{1+e^t}$

   (b) $v'''' + 18v'' + 81v = 0$

2. The functions t and t^2 are two solutions of the homogeneous equation

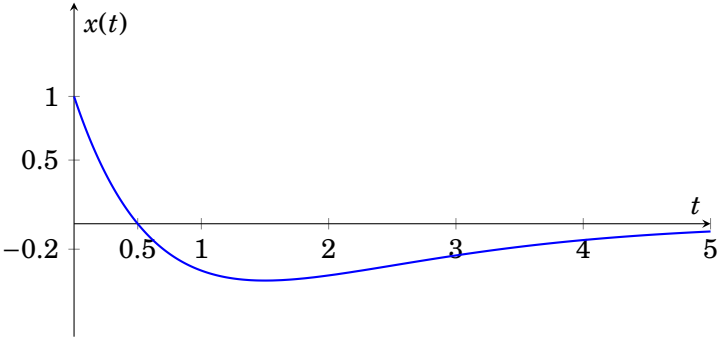
$$t^2y'' - 2ty' + 2y = 0 \quad \text{over } t > 0.$$

   (a) Compute their Wronskian.

   (b) Solve the IVP

$$t^2y'' - 2ty' + 2y = \frac{t^3}{t^2+4}, \quad y(1) = y'(1) = 0, \quad \text{over } t > 0.$$

3. Suppose the graph of the displacement of a damped harmonic oscillator as a function of time is given below.



   Based on the graph, can you determine whether the system is overdamped, critically damped, or underdamped? Explain.

4. Suppose $y_1(t) = t^2$ is a solution to the differential equation

$$ty'' - y' = 0, \qquad t > 0.$$

   (a) Find the Wronskian $W(y_1, y_2)(t)$ of a fundamental set of solutions.

   (b) Find a second linearly independent solution $y_2(t)$ of the differential equation.

5. Compute the Laplace transform of $f(t) = te^{-2t}u(t-5)$ from its definition. (Here u is the unit step function.) Give the domain of definition.

6. Use Laplace transform to solve the initial value problem

$$x'' + 4x = \delta(t-3), \qquad x(0) = 5, \quad x'(0) = 0.$$

You may refer to the table on the last page.

Function	**Laplace transform**
1	$\frac{1}{s}$
t^n	$\frac{n!}{s^{n+1}}$
e^{at}	$\frac{1}{s-a}$
$\sin(bt)$	$\frac{b}{s^2+b^2}$
$\cos(bt)$	$\frac{s}{s^2+b^2}$
$\sinh(bt)$	$\frac{b}{s^2-b^2}$
$\cosh(bt)$	$\frac{s}{s^2-b^2}$
te^{at}	$\frac{1}{(s-a)^2}$
$e^{at}\sin(bt)$	$\frac{b}{(s-a)^2+b^2}$
$e^{at}\cos(bt)$	$\frac{s-a}{(s-a)^2+b^2}$