FlowNotation: Uncovering Information Flow Policy Violations in C Programs

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Abstract

Programmers of cryptographic applications written in C need to avoid common mistakes such as sending private data over public channels, modifying trusted data with untrusted functions, or improperly ordering protocol steps. These secrecy, integrity, and sequencing policies can be cumbersome to check with existing general-purpose tools. We have developed a novel means of specifying and uncovering violations of these policies that allows for a much lighter-weight approach than previous tools. We embed the policy annotations in C's type system via a source-to-source translation and leverage existing C compilers to check for policy violations, achieving high performance and scalability. We show through case studies of recent cryptographic libraries and applications that our work is able to express detailed policies for large bodies of C code and can find subtle policy violations. To gain formal understanding of our policy annotations, we show formal connections between the policy annotations and an information flow type system and prove a noninterference guarantee.

1 Introduction

Programs often have complex data invariants and API usage policies written in their documentation or comments. The ability to detect violations of these invariants and policies is key to the correctness and security of programs. This is particularly important for cryptographic protocols and libraries as the security of a large system depends on its underlying secure protocols and primitives. As a result, there has been much interest in checking implementations of cryptographic protocols [3, 40, 14, 15, 13, 12, 27, 24]. These verification systems, while comprehensive in their scope, require expert knowledge of both the cryptographic protocols and the verification tool to be used effectively.

What remains missing is a lightweight and developer-friendly tool to help programmers identify programming errors at compile time that violate high-level policies on cryptographic libraries and protocols written in C. The policies that are particularly important are secrecy (e.g., sensitive data is not given to untrusted functions), integrity (e.g., trusted data is not modified by untrusted functions), and API call sequencing (e.g., the ordering of cryptographic protocol steps is maintained). These policies can be viewed as information flow policies.

In this paper, we present a framework called FlowNotation where C programmers can add lightweight annotations to their programs to express policy specifications. These policies are then automatically checked using a C compiler's type checker, potentially revealing policy violations in the implementation. Our annotations are in the same family as *type qualifiers* (e.g. CQual [57, 21, 36]), where qualifiers such as tainted and trusted are used to identify violations of integrity properties of C programs; supplying tainted inputs to a function that requires a trusted argument will cause a type error. Our work extends previous results to support more complex and refined sequencing properties. Consider the following policy: a data object is initially tainted, then it is sanitized using a encodeURI API, then serialized using a serialize API, and finally written to disk using a filewrite API. Such API sequencing patterns are quite common, but cannot be straightforwardly captured using previous type qualifier systems.

FlowNotation extends type qualifiers to include a sequence of labels for specifying policies similar to the above example. However, rather than implement a new type system, we develop a source-to-source transformation tool, which translates an annotated C program to another C program, through which a C compiler's type checker (indirectly) checks the annotated policies. The key insight is that qualified C types can be translated to C structures whose fields are the original C types. For instance, "trusted int" and "tainted int" can be translated to "typedef struct {int x;} int_trusted" and "typedef struct {int x;} int_tainted", respectively. Even though these two types are structurally equivalent, C's struct types are nominal types, and thus, attempts to use data of one type as the other will be reported as a compile-time error by a C type checker. Consequently, we can directly use C type checkers for policy checking. The benefit of this approach is that we can leverage performant C compilers to quickly type-check our policies over large codebases.

To gain a formal understanding of the type of errors that we can uncover with this system, we model the annotated types as *information flow types*, which augment ordinary types with security labels. We define a core language polC and prove that its information flow type system enforces noninterference. The novelty of polC's type system is that the security labels are sequences of secrecy and integrity labels, specifying the path under which data can be relabeled. Relabeling corresponds to *declassification* (marking secrets as public) and *endorsement* (marking data from untrusted source as trusted). The type system ensures that relabeling functions are called in the correct order.

We also define μ C, a core imperative language with nominal types but without information flow labels in order to model a fragment of C. We then formally define our translation algorithm based on *polC* and μ C. We prove the correctness of our translation algorithm: If the translated program is accepted by the type checker in μ C, then the original program is well-typed in *polC*. The formalism not only makes explicit assumptions made by our algorithm, but also provides a formal account of the properties being checked by the annotations.

To demonstrate the effectiveness of FlowNotation we implement a prototype for a subset of C and evaluate the prototype on several cryptographic libraries. Our evaluation shows that we are able to check useful information flow policies in a modular way and uncover subtle program flaws.

This paper makes the following technical contributions:

- We propose FlowNotation, a lightweight tool for finding errors that violate information flow policies in C programs.
- We connect annotations in FlowNotation to the information flow type system *polC*. We prove a noninterference theorem for *polC*'s type system, from which the property of correct API sequencing is a corollary.
- We define a translation algorithm from polC types to nominal types (modeled by μ C) and prove it correct.
- We implement a prototype and demonstrate the effectiveness of FlowNotation by evaluating it on several C cryptographic libraries and applications.

The rest of this paper is organized as follows: Section 2 presents a motivating example and describes the workflow of FlowNotation. Next, we define *polC* (Section 3) and μ C (Section 4) along with the algorithm for our translation process. In Section 5, we explain how the algorithms are implemented in C. Our case studies and evaluation results are presented in Section 6. Finally, we discuss related work in Section 7 and conclude in Section 8.

2 Overview and Motivating Examples

We illustrate how FlowNotation concretely works on the left side of Figure 1. First, to check an applicationspecific policy, a programmer writes the policy in C pragma annotations. Then our source-to-source translator takes the annotated program as input, and produces a translated C program. The resulting program is then type-checked using an off-the-shelf C compiler. If the compiler returns a type error, then this implies the policy is violated in the program.

Next we show example policies in the context of developing cryptographic applications.

2.1 Secrecy

Suppose a team of software developers is working on a large C project that uses customers' financial data. This project integrates a secure two-party computation component that allows Alice and Bob to find out which of the two is wealthier without revealing their wealth to the other or relying on a trusted third party. Let us assume that the program obtains Alice's balance using the function get_alice_balance, then calls function wealthierA to see whether Alice is wealthier than Bob. wealthierA's implementation uses a library that provides APIs for secure computation primitives.

```
int bankHandler() {
    int balA;
    balA = get_alice_balance();
    ...
    wealthierA(balA);
  }
```

The variable bala contains Alice's balance, and therefore should be handled with care. In particular, the programmer wants to check that the secrecy of bala is maintained. One method is to use information flow types (e.g. [50]), where the information flow type of bala is (int AlicePrivate), indicating that it is an integer containing an AlicePrivate type of secret.

In contrast, variables that do not contain secrets can be given the type (int Public). The information flow type system then makes sure that read and write operations involving balk are consistent with its secrecy label. For instance, if a function postBalance(int Public), which is meant to post the balance publicly, is called with balk as the argument, the type system will reject this program for violating the secrecy policy.

Our annotations are *information flow labels*, each of which has a *secrecy* component and an *integrity* component. Programmers can provide these annotations above the declaration of balk to specify the secrecy policy as follows:



In the annotation, **#requires** is a directive that allows our tool to parse this annotation (in practice, **#pragma** prefaces it). AlicePriv is a secrecy label. Finally, secrecy is a *projection*;

it specifies that we only care about the secrecy component of the label. bala's integrity component is automatically assigned bot, the lowest integrity. The information flow type of bala corresponding to this



Figure 1: Overview of FlowNotation and connections to the formal model.

annotation is **int** (AlicePrivate, bot). This annotation can be used to check this program for violations of the following policy P_1 .

 P_1 : balk should never be given as input to an untrusted function.

Here, trusted functions are those trusted by the programmer not to leak bala. Next, we discuss how a programmer can annotate trusted functions.

Our programmer trusts a secure computation library that provides secure computation primitives. Let us assume the API encodeA converts an integer argument into a bit representation similar to what is used in Obliv-C [53] for use with a garbled circuit. The API yao_execA takes a pointer to a function f and an argument for f, and runs f as a circuit with Yao's protocol [51]. Finally, at the end of the application's execution the API reveal is invoked to give the result of the function execution to both parties. The programmer constructs the following code for Alice (Bob's program is symmetric, which we omit):

```
int compare(int a, int b) { return a > b; }
int wealthierA(balA) {
   balA2 = encodeA(balA);
   int res = yao_execA(&compare, balA2);
   reveal(&res, ALICE);
   }
```

This program first encodes Alice's balance, and then calls yaolexecA with the comparison function and Alice's encoded balance bala2 as arguments, and finally calls reveal.

The code as it stands will not type-check after being translated, unless the programmer also appropriately annotates their trust in the secure computation APIs.

```
1 #param AlicePriv:secrecy
2 int encodeA(int balA);
3 #param(2) AlicePriv:secrecy
4 int yao_execA(void* compare, int balA);
```

These two annotations state that the functions must accept parameters with the label AlicePriv. In the second annotation, **#param(2)** specifies that the annotation should only apply to the second parameter. A violation of P_1 will be detected, when bala is given to a function that does not have this kind of annotation; e.g. that is not allowed (by the programmer) to accept AlicePriv-labeled data.

2.2 Integrity and Sequencing

A programmer can also use FlowNotation to check the program for violations of the following, more refined, policy P_2 .

 P_2 : balk should be used by the encoding function and then by the Yao protocol execution.

The annotation for bala is as follows.

```
1 #requires AlicePriv:secrecy then EncodedBal:integrity
2 int balA;
```

The keyword then allows for the sequencing of labels. Corresponding changes are made to the other annotations:

```
1 #param AlicePriv:secrecy
2 #return EncodedBal:integrity
3 int encodeA(int balA);
4 #param(2) EncodedBal:integrity
5 int yao_execA(void* compare, int balA);
```

labels	ℓ	::=	(s,ι)
policies	ρ	::=	$\perp \mid \top \mid \ell :: \rho$
1st order types	b	::=	$\operatorname{int} \operatorname{ptr}(s) T$
simple sec. types	t	::=	b ho unit
security types	s	::=	$t \mid [pc](t \to t)^{ ho}$
values	v	::=	$x n () f T \{v_1, \cdots, v_k\} loc$
expressions	e	::=	$v \mid e_1 \texttt{bop} \mid e_2 \mid v \mid e \mid let \mid x = e_1 in \mid e_2$
			$v.i { m if} v_1$ then e_2 else $e_3 v := e$
		Ì	$newe {\ast}v reLab(\ell'{::}\bot\leftarrow\ell{::}\top)v$
			Figure 2: Syntax of $polC$

The encodeA function, as before, requires the argument to have the AlicePriv secrecy label. In addition, the return value from encodeA will have the integrity label EncodedBal, stating that it is endorsed by the encodeA function to be properly encoded. The yao_execA function requires the argument to have the same integrity label. If only programmer-approved encoding functions are annotated with EncodedBal at their return value, the type system will check that an appropriate API call sequence (encodeA followed by yao_execA) is applied to the value stored in balA.

3 A Core Calculus for Staged Release

We formally define the syntax, operational semantics, and the type system of polC, which models annotated C programs that FlowNotation takes as input. We show that polC's type system can enforce not only secrecy and integrity policies, but also staged information release and data endorsement policies. We prove that our type system enforces noninterference, from which the property of staged information release is a corollary.

3.1 Syntax and Operational Semantics

The syntax of polC is summarized in Figure 2. We write ℓ to denote security labels, which consist of a secrecy tag s and an integrity tag ι . We assume there is a security lattice (S, \sqsubseteq_S) for secrecy tags and a security lattice (I, \sqsubseteq_I) for integrity tags. The security lattice $\mathcal{L} = (L, \sqsubseteq)$ is the product of the above two lattices. The top element of the lattice is (\top_S, \bot_I) (abbreviated \top), denoting data that do not contain any secret and come from the most trusted source; and the bottom element is (\bot_S, \top_I) (abbreviated \bot), denoting data that contain the most secretive information and come from the least trusted source. A policy, denoted ρ is a sequence of labels specifying the precise sequence of relabeling (declassification and endorsement) of the data. The example from Section 2.2 uses the following policy:

 $(AlicePrivate, \perp_I) :: (\perp_S, EncodedBal) :: \perp$

A policy always ends with either the top element, indicating no further relabeling is allowed, or the bottom element, indicating arbitrary relabeling is allowed. For our application domain, the labels provided by programmers are distinct points in the lattice that are not connected by any partial order relations except the \top and \perp elements.

A simple (first-order) security type, denoted t, is obtained by adding policies to ordinary types. Our core language supports integers (int), unit, pointers (ptr(s)), and record types (struct $T \{t_1, \dots, t_k\}$ to model C structs). Here T is the defined name for a record type. To simplify our formalism, we assume that defined type T is always a record type named T. Unlike ordinary information types, our information flow types use the policy ρ , rather than a single label ℓ . The meaning of an expression of type int ρ is that this expression is evaluated to an integer and it induces a sequence of declassification (endorsement) operations according to the sequence of labels specified by ρ . For instance, $e : \operatorname{int} H :: L :: \bot$ means that e initially is of int H, then it can be given to a declassification function to be downgraded to int L, the resulting expression can be further downgraded to bottom. $e : \operatorname{int} H :: L :: \top$ is similar except that the last expression cannot be declassified further; i.e., it stays at L security level. The annotated type for balk in Section 2.2 can be similarly interpreted.

We do not have a labeled unit type, because it is inhabited by one element () and thus does not contain sensitive information. A function type is of the form $[pc](t_1 \to t_2)^{\rho}$, where t_1 is the argument's type, t_2 is the return type, ρ is the security label of the function indicating who can receive this function, and pc, called the program counter, is the security label representing where this function can be called. For instance a function f of type $[L::\perp](t_1 \to t_2)^{H::\perp}$ cannot be called in an if branch that branches on secrets and the function itself cannot be given to an attacker whose label is $L::\perp$.

Our expressions are reminiscent of A normal forms: all elimination forms use only values (e.g., v.i, instead of e.i). This not only simplifies our proofs, but also the translation rules (presented in Section 4). The fragment of C that is checked in our case studies is quite similar to this form.

Values can be variables, integers, unit, functions, records, and store locations. Since we are modeling an imperative language, we do not have first-class functions. Instead, all functions are predefined, and stored in the context Ψ . Expressions include function calls, if statements, let bindings, and store operations. One special expression is the relabeling (declassification) operation, written $\mathsf{reLab}(\ell'::\perp \leftarrow \ell::\top) v$. This operation changes the label of v from $\ell::\top$ to $\ell'::\perp$. Such an expression should only appear in trusted declassification functions. For our applications, we further restrict the relabeling to be between two labels; from one ending with the top element to one ending with bottom element. We will explain this later when we explain the typing rules.

The judgement for small step semantics for polC is denoted $\Psi \vdash \sigma / e \longrightarrow \sigma' / e'$, where Ψ stores all the function code, σ is the store mapping locations to values and e is the expression to be evaluated. Appendix B contains a summary of all the operational semantic rules.

3.2 Typing Rules

The type system makes use of several typing contexts. We write D to denote the context for all the type definitions. We only consider type definitions of record (struct) types, written $T \mapsto \text{struct } T \{t_1, \dots, t_k\}$. The typing context for functions is denoted F. We distinguish two types of functions: ordinary functions, and declassification/endorsement functions whose bodies are allowed to contain relabeling operations, written $f:(d\&e)[pc]t_1 \rightarrow t_2$. F does not dictate the label of a function f. Instead, the context in which f is used decides f's label.

We write Σ to denote the typing context for pointers. It maps a pointer (heap location) to the type of its content. Γ is the typing context for variables, and *pc* is the security label representing the program counter.

Our type system has two typing judgments: $D; F; \Sigma; \Gamma \vdash v : t$ for value typing, and $D; F; \Sigma; \Gamma; pc \vdash e : t$ for expression typing. Selected typing rules are shown in Figure 3; full rules are in Appendix B.5.

We use a number of auxiliary definitions. First, we define the meaning of a policy ρ_1 being less strict than another, ρ_2 , written $\rho_1 \sqsubseteq \rho_2$, as the point-wise lifting of the label operation $\ell_1 \sqsubseteq \ell_2$. When one policy reaches its end, we use $\bot \sqsubseteq \rho$ or $\rho \sqsubseteq \top$. \bot represents a policy that can be arbitrarily reclassified and thus is a subtype of any policy ρ . On the other hand, \top is the strictest policy that forbids any reclassification; so any policy is less strict than \top .

Figure 3: Typing rules

The subtyping relation $s_1 \leq s_2$ is standard: most types are covariant except function argument types, which are contravariant, and pointer content types, which are invariant. $\rho \triangleright t$ denotes ρ guards t. It is defined as $\rho \sqsubseteq labOf(t)$. Here labOf(t) is the outermost label of type t; for instance, $\bot \triangleright$ int $(AlicePrivate, \bot_I)$, $(AlicePrivate, \bot_I) \triangleright$ int $(AlicePrivate, \bot_I)$. Finally $s \sqcup \rho$ is the type resulting from joining the policy of s with ρ .

Most of these typing rules are standard to information flow type systems. These rules carefully arrange the constraints on policies and the program counter so that noninterference theorem can be proven. Due to space constraints, we only explain the rule P-T-E-DE, which types the application of a declassification/endorsement function and is unique to our system. The first premise checks that v_f relabels data from ℓ_1 to ℓ_2 . The second premise checks that e_a 's type matches that of the argument of v_f ; further, e_a 's policy ρ has ℓ_1 and ℓ_2 as the first two labels, indicating that e_a is currently at security level ℓ_1 and the result of processing e_a has label ℓ_2 . Finally, the return type of the function application has the tail of the policy ρ . The policy of e_a does not change; instead, the policy of the result of the relabeling function inherits the tail of e_a 's policy. Therefore, our type system is not enforcing type states of variables as found in the Typestate system [47]. These declassification and endorsement functions only rewrite one label, not a sequence of labels. This allows us to have finer-grained control over the stages of relabeling.

3.3 Noninterference

We prove a noninterference theorem for polC's type system by adapting the proof technique used in FlowML [44]. We extend our language to include pairs of expressions and pairs of values to simulate two

executions that differ in "high" values. We only explain the key definitions for the theorem.

We first define equivalences of expressions in terms of an attacker's observation. We assume that the attacker knows the program and can observe expressions at the security level ℓ_A . To be consistent, when ℓ_A is not \top or \bot , the attacker's policy is written $\ell_A::\top$. Intuitively, an expression of type $b \rho$ should not be visible to the attacker if existing declassification functions cannot relabel data with label ρ down to $\ell_A::\top$. For instance, if $\rho = H::L::\bot$ and there is no declassification function from H to L, then an attacker at L cannot distinguish between two different integers v_1 and v_2 of type int ρ . On the other hand, if there is a function $f :_{d\&e}$ int $H::\top \to L::\bot$, then v_1 and v_2 are distinguishable by the attacker. We define when a policy ρ is in H with respect to the attacker's label, the function context, and the relabeling operations, in other words, when values of type $b \rho$ are not observable to the attacker, as follows. $\rho \in H$ if ρ cannot be *rewritten* to be a policy that is lower or equal to the attackers' policy.

$$\frac{\forall \rho', F; R \vdash \rho \rightsquigarrow \rho', \rho' \not\sqsubseteq \rho_A}{\rho_A; F; R \vdash \rho \in H}$$

Here $F; R \vdash \rho \rightsquigarrow \rho'$ holds when $\rho = \ell_1 :: \cdots :: \ell_i :: \rho'$ and there is a sequence of relabeling operations in F and R, using which ρ can be rewritten to ρ' . For instance, when $\ell_A = \bot$

$$\begin{array}{lll} F_1 &=& \texttt{encodeA}: (\texttt{d\&e})\texttt{int} \; (AlicePrivate, \bot_I) :: \top \\ & \to \texttt{int} \; (\bot_S, EncodedBal) :: \bot \\ F_2 &=& F_1, \texttt{yao_execA}: (\texttt{d\&e})\texttt{int} \; (\bot_S, EncodedBal) :: \top \to \texttt{int} \; \bot \\ & \quad \ell_A; \cdot; \cdot \vdash (AlicePrivate, \bot_I) \in H \\ & \quad \ell_A; F_1; \cdot \vdash (\bot_S, EncodedBal) \in H \\ & \quad \ell_A; F_2; \cdot \nvdash \; (\bot_S, EncodedBal) \in H \end{array}$$

Our noninterference theorem is formally defined below. The theorem states that given an expression e that is observable by the attacker, and two equivalent substitutions δ_1 and δ_2 for free variables in e, and both $e\delta_1$ and $e\delta_2$ terminate, then they must evaluate to the same value. In other words, the values of sub-expressions that are not observable by the attacker do not influence the value of observable expressions. The proof can be found in Appendix B.

Theorem 1 (Noninterference).

If $D; F; \Gamma; \perp \vdash e : s$, e does not contain any relabeling operations, given attacker's label ℓ , and substitution $\delta_1, \delta_2 \ s.t. \ F \vdash \delta_1 \approx_H \delta_2 : \Gamma$, and $\ell; F; \cdot \vdash labOf(s) \notin H$ and $\Psi \vdash \emptyset / e\delta_1 \longrightarrow^* \sigma_1 / v_1$ and $\Psi \vdash \emptyset / e\delta_2 \longrightarrow^* \sigma_2 / v_2$, then $v_1 = v_2$.

It follows from Noninterference that given $D; F; x: \ell_1::\cdots::\ell_n::\perp$ int $\vdash e:$ int $\ell_n::\top$ where the attacker's label is $\ell_n::\top$, the attacker can only gain knowledge about the value for x if there is a sequence of declassification/endorsement functions f_i s that remove label ℓ_i from the policy to reach $\ell_n::\top$. Further, if $\ell_i \not\subseteq \ell_{i+1}$, then the f_i s have to be applied in the correct order, as dictated by the typing rules.

4 Embedding in A Nominal Type System

The type system of polC can encode interesting security policies and help programmers identify subtle bugs during development. However, implementing a feature-rich language with polC's type system requires nontrivial effort. Moreover, only programmers who are willing to rewrite their codebase in this new language can benefit from it. Rather than create a new language, FlowNotation leverages C's type system to enforce policies specified by polC's types. The mapping between the concrete workflow of FlowNotation, polC and μ C, and the algorithms defined here is shown in Figure 1. We first define a simple imperative language μ C with nominal types and annotations, which models the fragment of C that FlowNotation works within. We show how the annotated types and expressions can be mapped to types and expressions in polC in Appendix C. Then in Section 4.2, we show how to translate polC programs back to μ C. These two algorithms combined describe the core algorithm of FlowNotation. We prove our translation correct in Section 4.3.

4.1 μ C and Annotated μ C

Expressions in μ C are the same as those in *polC*. The types in μ C do not have information flow policies, which are defined below. The names of the typing contexts remain the same.

Basic Types	π	::=	T int unit ptr(au)
Types	au	::=	$\pi \mid \pi_1 \to \pi_2$
Annotation	a	::=	πT at $ ho {\sf int}$ at $ ho {\sf ptr}(eta)$ at $ ho$
Typ. Annot.	β	::=	$a \mid a_1 \rightarrow a_2$
Expressions	e	::=	$\cdots \mid let x: eta = e_1 in e_2$
Annot. typedef	D_a	::=	$\cdot \mid D_a, T \mapsto struct \ T\{a_1, \cdots, a_k\}$
Annot. Func.	F_a	::=	$\cdot \mid F_a, f: a_1 \to a_2$
			$F_a, f: (d\&e)a_1 \to a_2$

We assume that programmers will provide policy annotations, denoted β . The annotated types β are very similar to labeled types s. We keep them separate, as programmers do not need to write out the fully labeled types. A programmer can annotate defined record types T at ρ , integers int at ρ , both the content and the pointer itself $ptr(\beta)$ at ρ , or the record type struct $T\{\beta_1, \dots, \beta_k\}$. The last case is used to annotate type declarations in the context D. We extend expressions with annotated expressions; let $x : a = e_1 \text{ in } e_2$. We assume that let bindings, type declarations, and function types are the only places where programmers provide annotations. A complete account of syntax and semantics can be found in Appendix A and C.

4.2 Translating Annotated Programs to μC

Instead of defining an algorithm to translate an annotated μC program e_a to another μC program, we first define an algorithm that maps e_a into a program e_l in polC; then an algorithm that translates e_l to a μC program.

Mapping from annotated μC to *polC*. This mapping helps make explicit all the assumptions and necessary declassification and endorsement operations needed to interpret those annotations as proper *polC* types and programs.

We write $\langle\!\langle \beta \rangle\!\rangle$ to denote the mapping of unannotated and annotated μC types to *polC* types. Unannotated types are given a special label U (unlabeled, defined as (\bot, \bot)); annotated types are translated as labeled types. All function types are given the pc label \bot , so the function body can be typed with few restrictions. The mapping from annotated types to *polC* types is summarized in Figure 4.

There are two sets of mapping rules for expressions:

 $D_a; F_a; \Gamma_a; s \vdash \langle\!\langle e \rangle\!\rangle \Rightarrow le \text{ and } D_a; F_a; \Gamma_a \vdash \langle\!\langle v \rangle\!\rangle \Rightarrow lv.$

The mapping rules use the annotated typing contexts: D_a , F_a , and Γ_a . The reading of the first judgement is that an annotated expression e is mapped to a labeled expression le given annotated typing contexts D_a , F_a , Γ_a , and *polC* type s, which e's type is supposed to be. The second judgment is similar, except that it only applies to values and the type of v is not given. Here le and lv are expressions with additional type annotations of form @s to ease the translation process from *polC* to μ C. For instance, n@int U means that n is an integer and it is supposed to have the type int U. This way, we can give the same integer different

$$\begin{split} &\frac{\pi \in \{\mathsf{int}, T\}}{\langle\!\langle \pi \rangle\!\rangle = \pi \ U} \quad \frac{\pi \in \{\mathsf{int}, T\}}{\langle\!\langle \pi \operatorname{at} \rho \rangle\!\rangle = \pi \ \rho} \quad \frac{\langle\!\langle \beta \rangle\!\rangle = s}{\langle\!\langle \mathsf{ptr}(\beta) \rangle\!\rangle = \mathsf{ptr}(s) \ U} \quad \frac{\langle\!\langle \beta \rangle\!\rangle = s}{\langle\!\langle \mathsf{ptr}(\beta) \operatorname{at} \rho \rangle\!\rangle = \mathsf{ptr}(s) \ \rho} \\ &\frac{\forall i \in [1, 2], \langle\!\langle a_i \rangle\!\rangle = t_i}{\langle\!\langle a_1 \to a_2 \rangle\!\rangle = [\bot](t_1 \to t_2)} \quad \frac{\forall i \in [1, 2], \langle\!\langle a_i \rangle\!\rangle = t_i}{\langle\!\langle (\mathsf{d\&e}) a_1 \to a_2 \rangle\!\rangle = (\mathsf{d\&e})[\bot](t_1 \to t_2)} \end{split}$$

Figure 4: Mapping annotations to types

$$\begin{split} \frac{D_a; F_a; \Gamma_a \vdash \langle\!\!\langle v \rangle\!\!\rangle \Rightarrow lv \quad tpOf(lv) = T \ \rho}{D_a(T) = (\mathsf{struct} \ T\{\beta_1, \cdots, \beta_n\}) \quad \forall i \in [1, n], \rho = labOf(\langle\!\langle \beta_i \rangle\!\rangle) \\ D_a; F_a; \Gamma_a; t \vdash \langle\!\langle v.i \rangle\!\rangle \Rightarrow lv.i} \ \mathsf{L}\text{-Field-U} \end{split}$$

$$\begin{array}{ll} D_a; F_a; \Gamma_a \vdash \langle\!\!\langle v \rangle\!\!\rangle \Rightarrow lv & tpOf(lv) = T \ \rho \\ D_a(T) = (\mathsf{struct} \ T\{\beta_1, \cdots, \beta_n\}) & \exists i \in [1, n], \rho \neq labOf(\langle\!\langle \beta_i \rangle\!\rangle) \\ \hline D_a; F_a; \Gamma_a; t \vdash \langle\!\langle v.i \rangle\!\rangle \Rightarrow \mathsf{let} \ y: T \perp = \mathsf{reLab}(\perp \Leftarrow \rho) \ lv \ \mathsf{in} \ (y@T \perp).i \end{array} \ \mathbf{L}\text{-FIELD}$$

$$\frac{D_a; F_a; \Gamma_a \vdash \langle\!\!\langle v \rangle\!\!\rangle \Rightarrow lv \qquad tpOf(lv) = b \ \rho}{D_a; F_a; \Gamma_a; t \vdash \langle\!\!\langle *v \rangle\!\!\rangle \Rightarrow \mathsf{let} \ y : b \ \bot = \mathsf{reLab}(\bot \Leftarrow \rho) \ lv \quad \mathsf{in} \ *(y@b \ \bot)} \ \mathsf{L-DereF}$$

 $\frac{D_a; F_a; \Gamma_a \vdash \langle\!\!\langle v \rangle\!\!\rangle \Rightarrow lv \qquad tpOf(lv) = \mathsf{ptr}(s) \ \rho \qquad D_a; F_a; \Gamma_a; s \vdash e \Rightarrow le}{D_a; F_a; \Gamma_a; t \vdash \langle\!\!\langle v := e \rangle\!\!\rangle \Rightarrow \mathsf{let} \ y : \mathsf{ptr}(s) \perp = \mathsf{reLab}(\perp \Leftarrow \rho) \ lv \ \mathsf{in} \ y @\mathsf{ptr}(s) \perp := le} \ \mathsf{L-Assign}(s) = \mathsf{L-Assign}(s)$

$$\begin{array}{l} D_a; F_a; \Gamma_a \vdash \langle\!\!\langle v_1 \rangle\!\!\rangle \Rightarrow lv_1 \\ \underline{tpOf}(lv_1) = \operatorname{int} \rho \qquad D_a; F_a; \Gamma_a; t \vdash \langle\!\!\langle e_2 \rangle\!\!\rangle \Rightarrow le_2 \qquad D_a; F_a; \Gamma_a; t \vdash \langle\!\!\langle e_3 \rangle\!\!\rangle \Rightarrow le_3 \\ \hline D_a; F_a; \Gamma_a; t \vdash \langle\!\!\langle \operatorname{if} v_1 \text{ then } e_2 \text{ else } e_3 \rangle\!\!\rangle \\ \Rightarrow \operatorname{let} x: \operatorname{int} \bot = (\operatorname{reLab}(\bot \Leftarrow \rho) \ lv_1) \text{ in if } x @\operatorname{int} \bot \text{ then } le_2 \text{ else } le_3 \end{array}$$

Figure 5: Mapping of expressions

types, depending on the context under which they are used: n@int U and $n@int \rho$ are translated into different terms.

A value is mapped to itself with its type annotated. For example, integers are given int U type, since they are unlabeled.

$$\overline{D_a; F_a; \Gamma_a \vdash \langle\!\langle n \rangle\!\rangle \Rightarrow n@int U}$$
 V-L-INT

Expression mapping rules are listed in Figure 5. The tricky part is mapping expressions whose typing rules in *polC* require label comparison and join operations. Obviously, the μ C type system cannot enforce such complex rules. Instead, we add explicit relabeling to certain parts of the expression to ensure that the types of the translated μ C program enforce the same property as types in the corresponding *polC* program.

There are two rules for record field access: one without explicit relabeling (L-FIELD) and one with (L-FIELD-U). Rule L-FIELD applies when all the elements in the record have the same label as the record itself. Rule L-FIELD-U explicitly relabels the record first, so the record type changes from $T \rho$ to $T \perp$, resulting in the field access having the same label as the element. This is because when the labels of the elements are not the same as the record, the typing rule P-T-E-FIELD will join the type of the field with the label of the record.

$$\frac{\rho \in \{U, \bot\}}{\llbracket \mathsf{int} \ \rho \rrbracket_D = (\mathsf{int}, \cdot)} \quad \frac{\rho \notin \{U, \bot\}}{\llbracket \mathsf{int} \ \rho \rrbracket_D = (T, T \mapsto \mathsf{struct} \ T \ \{\mathsf{int}\})} \quad \frac{\rho \notin \{U, \bot\}}{\llbracket \mathsf{int} \ \rho \rrbracket_D = (T, T \mapsto \mathsf{struct} \ T \ \{\mathsf{int}\})} \quad \frac{\rho \notin \{U, \bot\}}{\llbracket T \ \rho \rrbracket_D = (T', T' \mapsto \mathsf{struct} \ T' \ \{\tau_1, \cdots, \tau_n\})} = \frac{\rho \notin \{U, \bot\}}{\llbracket T \ \rho \rrbracket_D = (T', T' \mapsto \mathsf{struct} \ T' \ \{\tau_1, \cdots, \tau_n\})}$$

Figure 6: Type translation

However, this involves label operations, which μ C's type system cannot handle. L-DEREF and L-ASSIGN are similar. The mapping of if statements (L-IF) relabels the conditional v_1 to have int \perp type, so the branches are typed under the same program counter as the if expression. We write $\mathsf{reLab}(\perp \Leftarrow \rho)$ as a short hand for a sequence of relabeling operations $\mathsf{reLab}(\ell ::: \perp \Leftarrow \ell_n ::: \top) \cdots \mathsf{reLab}(\ell_i ::: \perp \Leftarrow \ell_{i-1} ::: \top) \cdots \mathsf{reLab}(\ell_2 ::: \perp \Leftarrow \ell_1 ::: \top)$ where $\rho = \ell_1 :: \cdots :: \ell_n :: \ell$ and ℓ is either \top or \bot . The implications of inserted relabeling operations are discussed at the end of this section.

Translation from *polC* to μ C. The translation of types is shown in Figure 6. It returns a μ C type and a set of new type definitions. We use a function $genName(t, \rho)$ to deterministically generate a string based on t and ρ as the identifier for a record type. It can simply be the concatenation of the string representation of t and ρ , which is indeed what we implemented for C (Section 5).

We distinguish between a type with a label that is U or \perp and a meaningful label. The translation of the type b U is simply b. This is because b U is mapped from an unannotated type b to begin with, so the translation merely returns it to its original type. Similarly $b \perp$ is generated by our relabeling operations during the mapping process, and should be translated to its original type b. On the other hand, a type annotated with a meaningful policy ρ is translated into a record type to take advantage of nominal typing. The translation also returns the new type definition. This would also prevent label subtyping based on the security lattice. However, this is acceptable given our application domain because the labels provided by programmers are distinct points in the lattice that are not connected by any partial order relations except the \top and \perp elements. Record types are translated to record types and types for the fields of the labeled record type $T \rho$ are the same as those for T, stored in the translated context D. This works because we assume that all labeled instances of the record type T (i.e., all $T \rho$) share the same definition.

Expression translation rules recursively translate the sub-expressions. We present a few interesting cases in Figure 7. The μ C type system is not asked to do complex label checking, so rule T-APP-DE has to insert label conversions. The label of the argument is cast from $\ell_1 :: \ell_2 :: \rho'$ to $\ell_1 :: \top$, as required by f, and the result of the function is cast from $\ell_2 :: \bot$ to $\ell_2 :: \rho'$. These operations are different from the ones inserted during the mapping process because they only exist to help μ C simulate the E-APP-DE typing rule in *polC*, but do not really have declassification or endorsement effects.

Next, we explain the translation of relabeling operations. Rule T-RELAB-N1 relabels a value whose type has a meaningful label to one with another meaningful label. The translated expression is a reassembled record using the fields of the original record. Rule T-RELAB-N2 relabels an expression with a U and \perp label to a meaningful label. In this case, the translated expression is a record. Rule T-RELAB-N3 translates an expression relabeled from a meaningful label to a U or \perp label to a projection of the record. The next rule, T-RELAB-SAME, does not change the value itself, because we are just relabeling between U and \perp labels. The final relabeling rule, T-RELAB-STRUCT, deals with records. In this case, we simply return the reassembled record because record types that only differ in labels have the same types for the fields, as shown in the last type translation rule in Figure 6.

Figure 7: Expression translation

4.3 Correctness

We prove a correctness theorem, which states that if our translated nominal type system declares an expression e well-typed, then the labeled expression e_l , where e is translated from, is well-typed under *polC*'s type system. Formally:

Theorem 2 (Translation Soundness (Typing)). If D_a ; F_a ; Γ_a ; $s \vdash \langle\!\langle e \rangle\!\rangle = le$, $\langle\!\langle D_a \rangle\!\rangle = D_l$, $\langle\!\langle F_a \rangle\!\rangle = F_l$, $\langle\!\langle \Gamma_a \rangle\!\rangle = \Gamma_l$, $[\![D_l]\!] = D$, $[\![\Gamma_l]\!]_D = (\Gamma, D_1)$, $[\![F_l]\!]_D = (F, D_2)$, $[\![le]\!]_D = (e', D_3)$, and $D \cup D_1 \cup D_2 \cup D_3$; F; \cdot ; $\Gamma \vdash e' : \tau$ implies D_l ; F_l ; \cdot ; $\Gamma_l \vdash tmOf(le) : s$ and $[\![s]\!] = (\tau, -)$

Here, tmOf(le) denotes an expression that is the same as le, with labels (e.g., @int U) removed. The proof is by induction over the derivation of D_a ; F_a ; Γ_a ; $s \vdash \langle \! \langle e \rangle \! \rangle \Rightarrow le$. The proof can be found in Appendix C.3.

It not hard to see that the translated program has the same behavior as the original program, because they have the same program structure except that the translated program has many indirect record constructions and field accesses.

4.4 Discussion

Relabeling Precision. It is clear from the mapping algorithm that a number of powerful relabeling operations are added. In all cases (except the if statement) we could do better by not relabeling all the way to bottom, but to the label of the sub-expressions. However, that would require a heavy-weight translation algorithm that essentially does full type-checking.

Implicit Flows. The security guarantees of programs that require relabeling operations to be inserted are weakened in the sense that in addition to the special declassification and endorsement functions, these inserted relabeling operations allow additional observation by the attacker. This means that the resulting program can implicitly leak information via branches, de-referencing, and record field access.

However, for our application domain we aim to check simple data usage and function call patterns which, as seen in our case studies, manifest errors with explicit flows. These policy violations are still detected if we don't have recursive types. The reason being those operations only cause relabeling of a *smaller* type. The API sequences keep the same basic type with changing labels. If we have recursive types, the above argument would be invalid. See the following example.

y: struct T {struct T (\perp_s , EncodedBal) :: \perp , int} ($AlicePrivate, \perp_I$) :: (\perp_s , EncodedBal) :: \perp

y.1 will have the same effect as encodeA, which violates the API sequence that we try to enforce using these types. Note that C doesn't allow this type, but we could use pointers to construct something quite similar. In our case studies, we do not have such interaction between policies and recursive types.

5 Implementation

We explain how the annotations and translation algorithms of FlowNotation are implemented for C.

Translation of annotations for simple types. Utilizing C's nominal typing via the typedef mechanism is key to realizing *polC* type system within the bounds of C's type system. The declaration of the *polC* type $t \rho$ in C will be: typedef struct { $t d_i$ } $\rho@t$; Here $\rho@t$ is a string representing the type $t \rho$ and it is simply a concatenation of the string representation of the policy ρ and the type t. Consider the annotated code snippet.

```
1 #requires l1:secrecy then l2:secrecy
2 int x;
```

In *polC*, the type of x is int $(l1, \perp_I) :: (l2, \perp_I) :: \perp$. The generated C typedef is: **typedef struct** {int d;} 115_12s_int;. This definition contains the original type, which allows access to the original data stored in x in the transformed program.

Structures and unions. We allow programmers to annotate structures in two ways: an instance of a structure can be annotated with a particular policy, or individual fields of an instance of a structure can be given annotations. The names of structures hold a particular significance within C since they are nominal types, and thus, they need to be properly handled. Unions are treated in a parallel manner, so we omit the details.

A policy on an instance of a structure is annotated and translated following the same formula as annotations on simple C types. Suppose we have the following annotation and code.

```
#requires 11:secrecy then 12:secrecy
```

```
2 struct foo x;
```

FlowNotation will produce the following generated type definition: typedef struct {struct foo d;} 115.125.foo;. This is different from the algorithm in Section 4, where structures are not nested and annotations are applied to structure definitions rather than instances. This is done in the implementation because the definition of foo might be external and therefore may not be known to the translation algorithm, so we simply nest the entire structure inside.

The second method allows annotations on particular fields of the structure as follows below.

```
#requires {f1:int, f2:int} l1:secrecy then l2:secrecy
struct foo x;
```

The following type definition will be generated.

Fields that have policy annotations are fields of the new struct. To allow access to other fields in the original struct, a copy of the original struct is nested inside this new struct. This is for the same reason as the structure nesting in the previous case.

Finally, we explain how member accesses are handled. Suppose a struct foo has members f1 and f2, and an annotation of policy p has been placed on member f1, but no annotation has been placed on member f2. The generated type definition for the structure is as follows: typedef struct { p_int f1; foo d; } p_foo;.

Assume x has type p_foo. Access to f1 is still x.f1, since there is a copy of it in x. Access to f2 is rewritten to x.d.f2. The field initialization is rewritten similarly: foo x={.f1=1,.f2=2}; is transformed to this: foo x={.f1=1,.d={.f2=2}};

Pointers. We provide limited support for pointers. Below is an example of how annotations on pointers are handled.

```
1 #requires AlicePriv:secrecy
2 int* x;
```

The translated code is below; a type definition of struct $AlivePrivS_int$ is generated: $AlicePrivS_int * x$; The following function can receive x as an argument because the annotation for its parameter matches that of x.

```
1 #param AlicePriv:secrecy
2 int f(int* x) {...}
```

The annotation for pointers only annotates the content of the pointer. Even though polC allows policies on the pointer themselves, we did not implement that feature. We also do not support pointer arithmetic, which is difficult to handle for many static analysis tools, especially lightweight ones like ours. However, our system will flag aliasing of pointers across mismatched annotated types. Our system will also flag pointer arithmetic operations on annotated types as errors. Programmers can encapsulate those operations in trusted functions and annotate them to avoid such errors.

Typecasts. The C type system permits typecasts, allowing one to redefine the type of a variable in unsound ways. Casting of non-pointer annotated types will be flagged as an error by **FlowNotation**. This is because our types are realized as C structures; type checkers do not allow arbitrary casting of structures. However, our tool cannot catch typecasts made on annotated pointers; a policy on a pointer will be lost if a typecast is performed.

Void. In this section we will discuss the handling of functions that have a **void** return type. We disallow the use of the **#return** annotation with such functions. The reasons for doing so will be explained below. Given that translation and the general purpose of the **void** type, it is clear that allowing an integrity annotation of a function with a **void** return type is not valid. Consider the following example:

```
1 #return trusted:integrity
2 void func() {...}
```

If we allowed this translation to proceed naively, the translated version of the code could look like this:

```
1 trustedI_void func() {...}
```

This is invalid for two reasons. First, as mentioned before this function is not returning anything and therefore an annotation on its return type is meaningless. Second, as this translation evidences, if we were to allow such an annotation, we would have created an invalid type, "trustedI_void". This type is invalid because, in order for it to be used in our annotation system, we need to generate functions that perform the relabeling operations to and from this type. However, no such operations can be generated, as they would effectively take nothing and endorse it to a trusted type.

Another case where **void** comes into play is in implicit void pointer conversion. In the case where a void pointer is being passed to a function for an annotated parameter, this will not be flagged as an error by our system.

Variadic Functions. We provide partial support for annotations on variadic functions. For example, with the following function:

1 **int** f(**int** a, **int** b, ...) { ... }

Only the first two arguments can have annotations.

Builtin Qualifiers Qualifiers are subsumed into the "original type" that our processing algorithm extracts from the source code. For instance, if we encounter the code:

```
1 #requires test:secrecy
2 volatile int x;
```

the qualifier **volatile** will be considered to be part of the base type "int". Thus, the translation of the code will be:

```
I __fln_testS_volatile_int x;
```

Rather than:

```
volatile ..fln..testS.int x;
```

This approach generalizes to multiple qualifiers on a type.

Builtin Operators. The labels we can add through our system are sometimes applied to variables with numeric types, e.g. **int**, **float**, **double**, etc. Binary and unary operations on these types are directly supported by C. After transformation arithmetic operations do not work out of the box on our transformed types. For instance, x+y will raise a type error if x and y are annotated because + is being applied to a struct, not an **int**. Programmers would need to define a plus function for the annotated type to circumvent this issue.

Code Generation. In addition to the above remarks on how specific C features are handled, we need to do some additional code generation and program reconstruction in order for our system to be straightforward for the end user to use. When processing a directory of annotated source files that includes one "root" file (typically the file containing the main function), our system does the following:

- 1. Recursively find and parse included files from the root
- 2. Gather annotations from each file

- 3. Generate header definitions for each file
- 4. Stitch together the original and generated files

Next, we explain two pieces of this process; header generation and program reconstruction.

Header Generation.

Header generation refers to the phase of the program transformation when all of the structure and function definitions for the annotated types in a particular file are generated. The generated structure and function definitions are collected into a single header file that is included where its definitions are needed during the program reconstruction phase.

To explain how the structure and function definitions are generated, let us consider the following code:

```
1 #requires AlicePriv:secrecy
2 int x;
```

Previously, we explained that for a variable definition of the form τx ; annotated with a policy ρ we need to generate a type $\tau @ \rho$. In our example, this generated type would be AlicePrivs_int. As we explained before, to give this type concrete meaning within the C type system, we instantiate it in the form of a typedef struct:

```
typedef struct {int d;} AlicePrivS_int;
```

This generated structure contains the original type as a member and interacts with the code as described in the subsection on structures (in section 5).

In order to be able to convert between the original type int, which we call the base type, and this new "type" AlicePrivS_int, which we call the policy type, two functions need to be generated:

```
privateS_int privateS_int_w(int x) {...};
int privateS_int_r(privateS_int x) {...};
```

The first function, given a regular integer will relabel the integer to the type AlicePrivS_int. The second function, will relabel AlicePrivS_int back to a regular integer.

Thus, we have the basis for what our header generation needs to accomplish. Each annotated type $\tau @ \rho$ can be viewed as a pair (base type, policy type). For each pair we must:

- 1. Generate a type of structure that has a base type member and is named $\rho_{-\tau}$
- 2. Generate a function from the base to the policy type
- 3. Generate a function from the policy to the base type

In order to prevent the duplication of generated structure or function definitions, we deduplicate the list so that it consists of only unique pairs.

Program Reconstruction. During the program reconstruction phase, header files that have been generated must be included at the right points in the program's dependency graph. If they are not included at the right points, then it is possible that a file containing transformed code that makes use of the generated structures and functions will be missing the definitions of those structures or functions and thus will not be compilable. In order to solve this issue, we recursively traverse the dependency graph starting from the root file. At each file that we visit in the graph, we include the generated header file containing the generated structures and definitions.

Pragmas. We have presented annotations without the pragma directive prefixing them for convenience of presentation. When using the actual implementation of FlowNotation we write, for instance, **#pragma requires AlicePriv:secrecy**. The use of the pragma directive allows C compilers to ignore our annotations, thus allowing developers to keep annotations in their codebases without the annotations interfering with normal compilation of the program.

Library	# Policies	Sec.	Int.	Seq.	LoA	$\sim m LoC$	Issues	Runtime (s)
Obliv-C Library	2	1	1	0	11	80	0	0.04
SCDtoObliv FP Circuits	4	4	0	0	10	43,000	1	5.55
ACK Oqueue	7	7	7	2	19	700	0	0.17
Secure Mux Application	4	3	4	0	11	150	0	0.06
Pool Framework	4	2	4	0	8	500	1	0.16
Pantaloons RSA	5	2	3	0	12	300	1	0.11
MiniAES	9	4	4	1	13	2000	0	0.08
Bellare-Micali OT	5	3	2	0	12	100	2	0.05
Kerberos ASN.1 Encoder	2	2	0	1	8	300	0	0.12
Gnuk OpenPGP-do	5	0	5	1	11	250	1	0.10
Tiny SHA3	3	3	0	1	6	200	0	0.10

Figure 8: Evaluation Results. Sec, Int, and Seq are the number of secrecy, integrity, and sequencing policies. LoA is lines of annotations, LoC is the lines of code.

6 Case Studies

We evaluate the effectiveness of FlowNotation at discovering violations of secrecy, integrity, and sequencing API usage policies on several open-source cryptographic libraries. Our results are summarized in Figure 8. We examine: Obliv-C, a compiler for dialect of C directed at secure computation [55, 53]; SCDtoObliv, a set of floating point circuits synthesized into C code [56]; the Absentminded Crypto Kit, a library of Secure Computation protocols and primitives [32, 33]; Secure Mux, a secure multiplexer application [60]; the Pool Framework, a secure computation memory management library [59, 60]; Pantaloons RSA, the top GitHub result for an RSA implementation in C [42]; MiniAES, an AES multiparty computation implementation [30, 29]; Bellare-Micali OT, an implementation of the Bellare-Micali oblivious transfer protocol [6]; Kerberos ASN.1 Encoder, the ASN.1 encoder module of Kerberos [1]; Gnuk OpenPGP-do, a portion of the OpenPGP module from gnuk [52]; Tiny SHA3, a reference implementation of SHA3 [45]. We determine application-specific policies and implement them with our annotations.

6.1 SCDtoObliv Floating Point Circuits

First, we show that FlowNotation can be used to discover flaws in large, automatically generated segments of code that would be very difficult for a programmer to manually analyze.

SCDtoObliv [56] synthesizes floating point circuit in C via calls to boolean gate primitives implemented in C. While this approach produces performant floating point circuits for secure computation applications, the resulting circuit files are hard to interpret and debug. The smallest of these generated circuit files is around 4000 lines of C code while the largest is over 14,000 lines. We annotate particular wires based on the circuit function to check that particular invariants such as which bits should be used in the output and which bits should be flipped are maintained.

FlowNotation uncovered a flaw in the subtraction circuit. The Obliv-C subtraction circuit actually uses an addition circuit to compute A + (-B). The function that does the sign bit flipping, __obliv_c__flipBit, is annotated so that it can only accept an input with the *needsFlipping* label as follows.

```
1 #param needsFlipping:secrecy
2 void __obliv_c__flipBit(OblivBit* src)
```

Our tool reports an error; rather than the sign bit of the second operand being given to __obliv_c__flipBit the sign bit of the *first* operand was given to __obliv_c__flipBit. Instead of computing A + (-B) the circuit computes (-A) + B; the result of evaluating the circuit is negated with respect to the correct answer.

6.2 A Potential Flaw in the Pool API

This case study is based on Pool, a Secure Computation tool [59, 60] and demonstrates that FlowNotation can help identify cross-module API constraints.

The Pool framework provides a set of APIs for users, some of which take function pointers as arguments. As a result, user-provided functions are called inside Pool APIs and interact with sensitive data from the framework. The following function pointer is used-accessible.

void (*Gate_Copy)(_, _, _, uint64_t indexs, _)

We have left most of the parameters opaque as they are unimportant to the flaw we discovered. According to the signature, the function pointed to by this pointer can accept *any* unsigned 64-bit integer as its fourth parameter (an index to a gate used by the Pool API).

We would like to check the property that only valid gates are being used in the protocol execution and that only trusted functions can use valid gates. We use the label *valid_gate* as both a secrecy and an integrity policy to prevent APIs from using invalid gates and untrusted functions from using valid gates. Here is an example of that annotation on a function that is said to produce a valid gate:

```
1 #return valid_gate: (secrecy, integrity)
2 uint64_t Next_Gate_in_Buffer(Pool *dst)
```

An error is reported for the following code.

Notice that the fourth argument of the Gate_Copy function is returned by the Next_Gate_in_Buffer function. The flaw is caused by the fact Gate_Copy is not trusted to take a valid gate as input, as far as can be told by its type and the project's documentation [58]. This error is similar to bugs found in kernels that give user-supplied callback functions private kernel data. To allow the translated code to compile, we would have to explicitly add an annotation to the Gate_Copy function to allow it to take a valid gate as input. By doing so, we are knowingly endorsing potentially dangerous user-supplied callback functions.

6.3 Gnuk OpenPGP-DO

The last case study shows that FlowNotation can uncover a previously known and patched null-pointer dereferencing bug and another potential bug in the gnuk OpenPGP-DO file, which handles OpenPGP Smart Card Data Objects (DO). We explain the latter in the next subsection.

The function w_k df handles the reading or writing of DOs that support encryption via a Key Derivation Function (KDF) in the OpenPGP-DO file.

```
static int rw_kdf (uint16_t tag, int with_tag,
const uint8_t *data, int len, int is_write)
```

If the data is being read, it is copied out to a buffer via the function copy_do_1:

```
static void copy_do_1(uint16_t tag, const uint8_t *do_data, int with_tag)
```

One invariant is that the do_data pointer must point to a valid segment of data; it must not be null. We provide the following annotation:

```
1 #param(2) check-valid-ptr:integrity
2 static void copy_do_1(uint16_t tag, const uint8_t *do_data, int with_tag)
```

This annotation states that the second parameter will only be accepted if it has been endorsed by a function that returns data annotated with the *check_valid_ptr* label. We provide such a function and rewrite all nullity checks to use it.

```
1 #return check_valid_ptr:integrity
2 const uint8_t *check_do_ptr(const uint8_t *do_ptr)
```

Returning back to the rw.kdf function, when data is being read, the following call of copy_do_1 occurs:

```
1 copy_do_1(tag, do_ptr[NR_DO_KDF], with_tag);
```

Compilation of the transformed code results in this error:

```
1 error: passing argument 2 of 'copy_do_1' from incompatible pointer type [-Werror=incompatible-
pointer-types]
2 copy_do_1(tag, do_ptr[1], with_tag);
3 ^~~~~~~
```

The issue is copy_do_1 is annotated to require a null-pointer check for parameter two, but that check was not performed.

6.4 Length Check in Gnuk OpenPGP-DO

We now demonstrate the discovery of a potential issue with the gnuk copy_do_1 function.

This utility function is responsible for performing a properly sized memcpy given a data array, in the format of a Tag-Length-Value data structure, that contains the data to by copied as well as metadata such as the size of the data to be copied. We focus our analysis on the size metadata, which is captured by the variable int len. We provide the following annotation:

```
1 #return check_len:integrity
2 int len;
```

The purpose of this annotation is to ensure that this length variable will be checked before it is given to memcpy to prevent a buffer overflow.

The copy_do_1 function does two slightly different things depending on the value of a conditional. In the first case, the array element do_data[0] is checked to not exceed its maximum size before it is assigned to len. In the second case, however, no check is made.

Thus, a potential faulting path exists: if the conditional is false and do_data[0] was previously assigned a negative value causing an overflow, when len is used as the size argument to memcpy, it could read past the end of the do_data array as it may not be null-terminated.

Our system alerts us to this issue:

```
1 evaluations/gnuk/openpgp-do_snip__fln.c:301:9: error: incompatible types when assigning to
    type `__fln__check_lenI_int {aka struct <anonymous>}` from type `uint8_t {aka int}`
2 len = do_data__fln_p[0];
3 ^
```

We contacted the maintainer of the library who assured us that every instantiation of the do_data array has the correct length and thus the potential issue we describe cannot come up in practice. However, we believe that addition of a check that would fulfill the policy we have described could be useful should a mistake be made with a do_data array.

6.5 Secure Multiplexer Application

Pool is a secure computation framework that was released by Zhu et al. [59, 60]. The authors provide an example application, a secure multiplexer, that makes use of the framework. We evaluate this application to check that the Pool API usage does not violate the secrecy or integrity properties of the garbler's or evaluator's data. We check first that the secrecy and integrity of each party's private data is maintained.

```
1 #requires AlivePriv: (secrecy, integrity)
2 bool* inputA;
```

At the next step of the protocol, Alice's input is assigned her private value by way of a helper function

```
inputA = int2bitsA(0x01AA);
```

Given that the int2bitsA function is Alice's way of assigning a value to her input, we accordingly annotate that it is trusted to provide integrity for the AlicePriv label:

```
1 #return AlicePriv:integrity
2 bool* int2bitsA(int x) {...}
```

On the side of the other party, Bob, parallel annotations can be made. Since only the functions int2bitsA and int2bitsB can provide an integrity endorsement to the AlicePriv and BobPriv respectively, our system can check that no other code will modify Alice and Bob's private input.

The next annotation we provide is a check on the data structure entities representing Alice and Bob. Alice is an instance of a Garbler structure and Bob is an instance of an Evaluator. Thus we provide a label GarblerProtected and apply both its secrecy and integrity projections to the Alice instantiation of Garbler:

```
1 #requires GarblerProtected:(secrecy, integrity)
2 Garbler alice;
```

All Pool framework functions that need to access the Garbler's (and respectively, the Evaluator's) data thus need to be trusted to maintain the secrecy of the Garbler's data. Thus, the following annotations are applied:

```
1 #param(1) GarblerProtected:secrecy
2 #param(2) PreparedFunction:secrecy
3 wire** execA(Garbler alice, wire** func,
4 wire** inpt) {...}
```

The annotation GarblerProtected makes it clear that this function is trusted to read the Garbler structure. The annotation PreparedFunction has not been explained before. Its role is specify that the function pointer wire** func must point to a function that fulfills the policy PreparedFunction. No policy violations were found.

6.6 Checking Initializations in Pool

Another annotation we provide adds checks to prevent users of the Pool framework from omitting initializations. It is an integrity endorsement:

```
1 #return initialized_pool:integrity
2 Pool* SetupPool(Pool *dst ...);
```

The reason for adding this annotation is that as the original framework code stands, there are no checks in functions that use the Pool structure that it is actually properly initialized. If a function uses an uninitialized Pool structure, the protocol evaluation could fail through an exception or could have some other undesirable behavior that may leak information to an attacker. By adding the above annotation as well as annotations of the form **#param(i)initialized_pool:integrity** to each of the functions that uses the Pool, we are able to statically check for cases where an uninitialized Pool structure is used.

A similar annotation checks for initialization of the ServiceConfig structure.

```
1 #return initialized_service:integrity
2 ServiceConfig* SetupService(Pool *dst ...);
```

We add corresponding annotations to each function that uses the **ServiceConfig** to only accept an initialized configuration.

6.7 Obliv-C Library

We demonstrate checking a secrecy property. The annotation we provide is a oblivious label. This policy is added to the oblivBit structure in the Obliv-C library [55]. The secrecy label is used to check that oblivious data is only being handled by functions that are trusted not to leak information about the oblivious data within the Obliv-C library. The integrity label is needed to check that only trusted APIs are allowed to generate oblivious data and update oblivious data structures.

We add annotations the OblivBit data structure as follows:

```
1 #requires oblivious:secrecy
2 OblivBit* data;
```

Functions that are trusted to process oblivious data are given an annotation that it is allowed to accept the oblivious data as an argument. See the example below.

```
1 #param oblivious:secrecy
2 void __obliv_c__copyBit(OblivBit* dest,
3 const OblivBit* src)
```

The use of this secrecy label also enforces the integrity of oblivious data structures. This is because unannotated data is assumed to have the special label U, so it cannot be used to update structures storing data labeled with *oblivious*. We did not find any policy violations in the Obliv-C library.

6.8 Kerberos ASN.1 Encoder

This case study concerns enforcing an API sequencing policy in a widely-used open-source program, Kerberos. More concretely, we consider the Kerberos ASN.1 Encoder which makes use of two functions free_atype and free_atype_ptr that work in tandem to free memory allocated to Kerberos C objects. Objects must first be freed by the free_atype function before they are freed by the free_atype_ptr function. We provide annotations for these functions to check for violations of this sequenced behavior.

The free_atype function takes as an argument a pointer to an object along with the struct atype_info containing a description of the object. We modify the function to return this atype_info struct.

const struct atype_info* free_atype(const struct atype_info *a, void *val)

In the function body, the appropriate freeing routine is called based on atype_info's type member. The freeing routine can take the form of recursive calls to free_atype, calls to other specialized freeing functions, or calls to the second freeing function free_atype_ptr:

static void free_atype_ptr(const struct atype_info *a, void *val)

This function is constructed similarly to free_atype except that it works only over pointer-type objects and only recursively calls itself.

We add the following annotations to those functions:

```
1 #param(1) freebase:secrecy
2 #return freeptr:secrecy
3 const struct atype_info* free_atype(const struct atype_info *a, void *val)
4 ...
5 #param(1) freeptr:secrecy
6 static void free_atype_ptr(const struct atype_info *a, void *val)
```

We add the following annotation to atype_info structs:

```
1 #requires freebase:secrecy then freeptr:secrecy
2 const struct atype_info* x;
```

The annotations above will check that the calling sequence invariant is maintained; no violations were found.

6.9 Oblivious Queue Data Structure

FlowNotation can be used to check granular invariants of data structures. This case study emphasizes the modularity of our approach. The case study is on an oblivious queue (oqueue) library [32]. The data structure is hierarchical and operations on this data structure should maintain the following invariants [54]: (1) The buffer at level i has 5×2^i data blocks. (2) The number of non-empty blocks at buffer level i is a multiple of 2^i . (3) Each level maintains a counter storing the next available empty block. (4) When the buffer at level i is full the last block is shifted down to level i + 1.

Invariants (1) and (3) can be violated through incorrect modification to the counter or the oqueue, so we should check that modifications are only done by trusted functions. Therefore, we use the labels *push_protect*, *pop_protect*, and *oqueue_tail*. To modify where the next element is placed in the oqueue, only functions that are trusted to modify data labeled with *oqueue_tail* can do so. Likewise, the binary counters <code>push_time</code> and <code>pop_time</code> should only be modified within the context of the push and pop operations.

Each field is given an integrity label to protect its access. One example annotation on the oqueue data structure is:

```
1 #requires {.push_time:int} push_protect:integrity
2 oqueue* this_layer;
```

Invariants (2) and (4) are checked at run time by conditional statement in the API code. We add two sets of annotations (symmetric for the push and pop functions) to model the checks for ensuring that data is shifted to a lower level or raised to a higher level in the queue when the current oqueue level is full or empty. We use the following label sequence policy: $oqueue_has_child \rightarrow oqueue_push_ready$. Considering just the conditional push case below, these labels form an endorsement sequence on the oqueue data structure. First, we endorse that the oqueue has a child via the has_child helper function that can check for the existence of a child, then we endorse that the oqueue is ready to be pushed to via the is_push_time} helper function that is trusted to access the oqueue's *push_protect*-labeled variable.

```
1 // oqueue → oqueue_has_child

2 layer1 = has_child(layer);

3 if (layer1) {

4 // oqueue_has_child → oqueue_push_ready

5 layer2 = is_push_time(layer1);

6 if (layer2) {

7 // oqueue_push_ready → oqueue

8 layer3 = tail_is_full(layer2);

9 ...
```

This illustrates an instance of a compositional check that our annotations are providing; not only are we checking for the existence of a particular endorsement sequence, but we also check that along the way, the



Figure 9: Processing Time vs Number of Annotations and Time Per Processing Stage

functions that act on our oqueue to provide those endorsements are only the functions that we trust. Finally, if all of these conditions are met, data is allowed to be shifted down to the lower level of the oqueue.

For the case of the conditional pop, the sequence of endorsement operations is similar, but we have another, higher-level, compositional guarantee. We add an annotation to the pop function itself labeling its layer parameter (which is an instance of the oqueue) as *oqueue_check_empty*. Thus, we enforce that only an oqueue that has been checked for emptiness can be used with the pop operation.

This also demonstrates the modularity of FlowNotation. We are able to provide annotations at many "levels" of the source code; in the above example there is a general check that the oqueue is non-empty before the oqueue_pop function is entered. Then within the body of the oqueue_pop function there are additional annotations that "refine" our knowledge about the state of the oqueue. These functions could come from the same library or across several libraries from different developers. FlowNotation allows policies to be collectively checked across different modules.

6.10 Performance Evaluation

We evaluate the performance of FlowNotation on synthetically generated C programs and annotations. The generation algorithm targets a specific number of lines of C code and annotations. The generated annotations include all three types of policies, **#requires**, **#param**, and **#return**, in combination with different primitive types, pointers, and structures. To elicit worse-case behavior, the generated annotations are predominantly sequencing annotations constructed from a set of templates representative of common API patterns from our case studies. The C programs are similarly generated from templates of our case studies. Experiments were run on a single-core Ubuntu 18.04 VM with 1GB of RAM, on a 2.7 GHz Intel Core i7 machine.

First, we evaluate how the runtime of FlowNotation is affected by the program size and the number of annotations. The results are summarized in Figure 9. We evaluate the runtime of four C programs, with 500, 1000, 2000, and 4000 lines of code respectively. For each program, we increase the number of annotations, up to 128 annotations. FlowNotation is efficient: all the the experiments finish within 4 seconds. FlowNotation is intended to be run on individual modules (libraries) that rarely exceed a couple thousand lines of code unless they are automatically generated, like the SCDtoObliv circuit file (14,000 LoC). Even then, FlowNotation finishes within 6 seconds.

To better understand how each component of FlowNotation contributes to the processing time, we profile execution time for each part. The results are summarized in Figure 9, which shows a cross-section of Figure 9 with only the samples with 128 annotations. The four stages of FlowNotation are: "Parse Files," where annotations are retreived; "Generate Header," where the header file containing type and structure definitions corresponding to the transformed types is generated; "Build AST," where the C parsing library, pycparser [7] builds an abstract syntax tree from the source code; "Transform," where the implementation of the translation algorithm of FlowNotation runs. Most of the stages take a negligible amount of time

compared to the stages Build AST and Transform. The majority of the overhead is due to the C parsing library we use.

We do not present the overhead added to compilation of transformed programs because developers do not need to compile the transformed programs. Once the transformed programs have been checked they can be discarded.

7 Related Work

Related work for FlowNotation spans four research areas: C program analysis tools, information flow types, linear types (type states), and cryptographic protocol verification.

Tools for Analyzing C Programs. Many vulnerabilities stem from poorly written C programs. As a result, many C program analysis tools have been built. Several C model checkers (e.g. [4, 10, 23, 37, 11]) and program analysis tools [26, 28, 20, 41] are open source and readily downloadable. Our policies can be encoded as state machines and checked by some of the tools mentioned above, which are general purpose and more powerful than ours but are not tuned for analyzing API usage patterns like ours. Further, our tool is backed by a sophisticated information flow type system.

Closest to our work is CQual [35]. Both theoretical foundations and practical applications of type qualifiers have been investigated [34, 17, 57, 21, 36]. Our annotations are type qualifiers and our work and prior work on type qualifiers share the same goal of producing a lightweight tool to check simple secrecy and integrity properties. We additionally support sequencing of atomic qualifiers, which is a novel contribution. Further, we prove noninterference of our core calculus, which other systems did not. Another difference is that CQual relies on a custom type checker, while our policies are translated and checked using C's type system. Finally, CQual supports qualifier inference, which can reduce the annotation burden on programmers. We do not have general qualifier inference because to do so would be tantamount to constructing a type checker for our system, which would defeat our goal of relying on a C compiler's type checker.

Information Flow Type Systems. Information flow type systems is a well-studied field. Several projects have extended existing languages to include information flow types (e.g., [44, 43]). Sabelfeld et al. provided a comprehensive summary in their survey paper [46]. Most information flow type systems do not deal with declassification. At most, they will include a "declassify" primitive to allow information downgrade, similar to our relabel operations. However, we have not seen work where the sequence of labels is part of the information flow type like ours, except for JRIF [38]. As a result, we are able to prove a noninterference theorem that implies API sequencing. JRIF uses finite state automata to enforce sequencing policies, which can entail a large runtime overhead.

Other projects that target enforcement of sequencing policies similar to those we have presented rely on runtime monitoring, not types [22, 49, 9, 5, 18, 19].

Linear Types and Typestate. Our sequencing policies are tangentially related to other type systems that aim to enforce API contracts. This line of work includes typestate and linear types [47, 2, 31]. The idea is that by using typestate/linear types one can model and check behaviors such as files being opened and closed in a balanced manner [2]. However, unlike in typestate the types on variables don't change in our system; when a part of a policy is fulfilled there is a new variable that "takes on" the rest of the policy.

Cryptographic Protocol Verification. Several projects have proposed languages to make verification of cryptographic programs more feasible: Jasmine, Cryptol, Vale, Dafny, F*, and Idris [3, 40, 15, 39, 48, 16], to name a few. There are also general tools for verifying cryptographic protocols [13, 12, 14, 27, 8, 24, 25]. These languages and tools are general purpose and more powerful than ours. However, none of these tools directly support checking properties of C implementations of cryptographic libraries like we do. Bhargavan et al.'s work uses refinement types to achieve similar goals as ours [13]. The annotated types can be viewed

as refinement types: $\{x : \tau \mid \rho\}$, where the policy is encoded as a predicate. Their system is more powerful, however it only supports F# code.

8 Conclusion

We have described FlowNotation, a lightweight annotation system for C that allows programmers to specify secrecy, integrity, and sequencing policies for their applications. FlowNotation is particularly useful in identifying errors at compile time that violate high-level policies in cryptographic libraries and applications. We have modeled our system formally and proved a noninterference guarantee. Finally, we have shown through a set of detailed case studies that FlowNotation can express and check complex policies for large bodies of C code and finds subtle implementation bugs.

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A Summary of μ C: A Core Calculus with Nominal Typing

We summarize the syntax, operational semantics, and typing rules for μC in this section. μC represents the fragment of C that FlowNotation works with.

A.1 Syntax

Basic Types	π	::=	$T {\sf int} {\sf unit} {\sf ptr}(au)$
Types	au	::=	$\pi \mid \pi_1 \to \pi_2$
Values	v	::=	$x \mid n \mid () \mid (T) \{v_1, \cdots, v_k\} \mid loc \mid f$
Expressions	e	::=	$v \mid e_1 \text{ bop } e_2 \mid v \mid e \mid e_1 x = e_1 \text{ in } e_2 \mid v.i$
			if v then e_1 else $e_2 \mid new(e) \mid v := e \mid v$
Type def. ctx	D	::=	$ D,T \mapsto struct \ T \ \{\pi_1,\cdots,\pi_k\}$
Func typing ctx	F	::=	$\cdot \mid F, f: \pi_1 \to \pi_2$
$Code \ ctx$	Ψ	::=	$\cdot \mid \Psi, f(x) = e$
$Typing \ ctx$	Γ	::=	$\cdot \mid \Gamma, x: au$
Store	σ	::=	$\cdot \mid \sigma, loc \mapsto v$
Store Typing	Σ	::=	$\cdot \mid \Sigma, loc: au$
Eval Ctx	E	::=	let x = [] in e new([]) v [] v := [] [] bop e v bop []

A.2 Operational Semantics

$$\begin{split} \frac{\Psi \vdash \sigma \ / \ e \longrightarrow \sigma' \ / \ e'}{\Psi \vdash \sigma \ / \ E[e] \longrightarrow \sigma' \ / \ E[e']} \ \text{N-E-CONTEXT} & \frac{v_1 \text{ bop } v_2 = v}{\Psi \vdash \sigma \ / \ i \text{ bop } v_2 \longrightarrow \sigma \ / \ i \ v} \ \text{N-E-BOP} \\ \hline \\ \overline{\Psi \vdash \sigma \ / \ e[e] \longrightarrow \sigma' \ / \ E[e']} \ \text{N-E-DEREF} & \overline{\Psi \vdash \sigma \ / \ loc \ \mapsto v_1 \ / \ ()} \ \text{N-E-ASSIGN} \\ \hline \\ \frac{loc \ fresh}{\Psi \vdash \sigma \ / \ new(v) \longrightarrow \sigma[loc \mapsto v] \ / \ loc} \ \text{N-E-NEW}} & \overline{\Psi \vdash \sigma \ / \ (\{v_1, \cdots, v_n\}).i \longrightarrow \sigma \ / \ v_i} \ \text{N-E-FIELD} \\ \hline \\ \\ \frac{\Psi \vdash \varphi \ / \ f(x) = e}{\Psi \vdash \sigma \ / \ f \ v \longrightarrow \sigma \ / \ e[v/x]} \ \text{N-E-APP}} & \overline{\Psi \vdash \sigma \ / \ let \ x = v \ in \ e \longrightarrow \sigma \ / \ e[v/x]} \ \text{N-E-LET} \\ \hline \\ \\ \\ \frac{n > 0}{\Psi \vdash \sigma \ / \ if \ n \ then \ e_1 \ else \ e_2 \longrightarrow \sigma \ / \ e_1} \ \text{N-E-IF-TRUE} \end{split}$$

$$\overline{\Psi \vdash \sigma \ / \ \text{if} \ 0 \ \text{then} \ e_1 \ \text{else} \ e_2 \longrightarrow \sigma \ / \ e_2} \ \text{N-E-IF-FALSE}$$

A.3 Typing Rules

$$\begin{array}{c} \hline D;F;\Sigma;\Gamma\vdash e:\tau \\ \hline \hline D;F;\Sigma;\Gamma\vdash n:\mathsf{int} \end{array} \overset{\mathrm{N-T-INT}}{\longrightarrow} \quad \hline \overline{D;F;\Sigma;\Gamma\vdash loc:\Sigma(loc)} \overset{\mathrm{N-T-Loc}}{\longrightarrow} \quad \hline \overline{D;F;\Sigma;\Gamma\vdash x:\Gamma(x)} \overset{\mathrm{N-T-VAR}}{\longrightarrow} \\ \hline \hline \overline{D;F;\Sigma;\Gamma\vdash f:F(f)} \overset{\mathrm{N-T-Fun}}{\longrightarrow} \quad \frac{D;F;\Sigma;\Gamma\vdash e_1:\mathsf{int} \quad D;F;\Sigma;\Gamma\vdash e_2:\mathsf{int}}{D;F;\Sigma;\Gamma\vdash e_1\mathsf{ bop } e_2:\mathsf{int}} \overset{\mathrm{N-T-Bop}}{\longrightarrow} \end{array}$$

$$\begin{split} \frac{T \mapsto \mathsf{struct} \ T \ \{\tau_1, \cdots, \tau_n\} \in D \quad \forall i, D; F; \Sigma; \Gamma \vdash v_i : \tau_i}{D; F; \Sigma; \Gamma \vdash v : \tau_i} \ \operatorname{N-T-Struct} \\ \frac{D; F; \Sigma; \Gamma \vdash v : T \quad T \mapsto \mathsf{struct} \ T\{\tau_1, \cdots, \tau_n\} \in D}{D; F; \Sigma; \Gamma \vdash v : \tau_i} \ \operatorname{N-T-Field} \quad \frac{D; F; \Sigma; \Gamma \vdash e : \tau}{D; F; \Sigma; \Gamma \vdash \mathsf{new}(e) : \mathsf{ptr}(\tau)} \ \operatorname{N-T-New} \\ \frac{D; F; \Sigma; \Gamma \vdash v : \mathsf{ptr}(\tau)}{D; F; \Sigma; \Gamma \vdash *v : \tau} \ \operatorname{N-T-DereeF} \quad \frac{D; F; \Sigma; \Gamma \vdash v : \mathsf{ptr}(\tau) \quad D; F; \Sigma; \Gamma \vdash e : \tau}{D; F; \Sigma; \Gamma \vdash v : e : \mathsf{unit}} \ \operatorname{N-T-Assign} \\ \frac{D; F; \Sigma; \Gamma \vdash v : \pi_1 \to \pi_2 \quad D; F; \Sigma; \Gamma \vdash v : e : \pi_1}{D; F; \Sigma; \Gamma \vdash v : \pi_2} \ \operatorname{N-T-Assign} \\ \frac{D; F; \Sigma; \Gamma \vdash v : \mathsf{nt} \quad D; F; \Sigma; \Gamma \vdash v : \pi_1}{D; F; \Sigma; \Gamma \vdash v : \pi_2} \ \operatorname{N-T-Aepp} \\ \frac{D; F; \Sigma; \Gamma \vdash v : \mathsf{nt} \quad D; F; \Sigma; \Gamma \vdash e_1 : \tau_1 \quad D; F; \Sigma; \Gamma \vdash e_2 : \tau_2}{D; F; \Sigma; \Gamma \vdash \mathsf{let} \ x : \tau_1 = e_1 \ \mathsf{in} \ e_2 : \tau_2} \ \operatorname{N-T-Let} \\ \frac{D; F; \Sigma; \Gamma \vdash v : \mathsf{int} \quad D; F; \Sigma; \Gamma \vdash e_1 : \tau}{D; F; \Sigma; \Gamma \vdash \mathsf{if} \ v \ \mathsf{then} \ e_1 \ \mathsf{else} \ e_2 : \tau} \ \operatorname{N-T-IF} \end{split}$$

5

Definitions and Meta-theory for polCΒ

B.1 polC Operational Semantics via Pairs

The operational semantic rules for polC include all the rules for μC and the following rule for relabeling.

$$\overline{\Psi \vdash \sigma \; / \; \mathsf{reLab}(\rho' \Leftarrow \rho) v \longrightarrow \sigma \; / \; v} \stackrel{\text{P-E-Relab}}{\longrightarrow}$$

B.2 Extension of Syntax with Pairs

To prove noninterference, we define a set of operational semantic rules that allow expression pairs, which effectively represent two executions differing in secrets. The syntax for the extended values and expressions are summarized below.

We write v^+ to denote values that may include pairs and e^+ to denote expressions that may include pairs. The definitions disallow nested pairs. For the rest of this section, when convient and clear from the context, we will write v and e to denote values and expressions that may contain pairs respectively.

B.3 Paired Operational Semantics

The operational semantics is summarized in Figure 10. Below are auxiliary definitions used by those rules.

B.4 Soundness and Completeness of the Paired Semantics

We first define projection relations.

$$\frac{\lfloor v^s \rfloor_i = \bullet}{\lfloor \sigma, loc \mapsto v^s \rfloor_i = \lfloor \sigma \rfloor_i} \qquad \frac{\lfloor v^s \rfloor_i \neq \bullet}{\lfloor \sigma, loc \mapsto v^s \rfloor_i = \lfloor \sigma \rfloor_i} \\
\lfloor \sigma / e^+ \rfloor_i = \lfloor \sigma \rfloor_i / \lfloor e^+ \rfloor_i$$

Next we define a number of well-formedness invariants of the runtime configuration $\sigma / i e$.

Definition 3 (Defined Pointers). We say that loc is defined in σ for execution i if the following holds

- $i = \bullet$ implies $\forall j \in \{1, 2\}, \ \lfloor \sigma(loc) \rfloor_j = v$
- $i \in \{1, 2\}$ implies $\lfloor \sigma(loc) \rfloor_i = v$

Definition 4 (In Scope Pointers). We say that loc is scope of $\sigma / i e^+$ where $i \in \{1, 2\}$ if $loc \in ((\bigcup_{loc' \in dom(\sigma)} fl(\lfloor \sigma(loc') \rfloor_i)) \cup fl(\lfloor e^+ \rfloor_i))$

Definition 5 (Closed Configurations). We say that $\sigma /_i e^+$ is closed if all of the following holds

- $i = \bullet$ implies $\forall i \in \{1, 2\}$, for all loc s.t. loc is in sope of $\sigma / i e^+$, loc is defined in σ for execution i.
- $i \in \{1,2\}$ implies for all loc s.t. loc is in sope of $\sigma / i e^+$, loc is defined in σ for execution i.

Lemma 6 (Preservation of Well-formednness).

- 1. For $i \in \{1, 2, \bullet\}$ if $\sigma_1 /_i e_1^+$ is closed and $\Psi \vdash \sigma_1 /_i e_1^+ \longrightarrow x /_i y$ then exists σ_2 and e_2^+ s.t. $x = \sigma_2$ and $y = e_2^+$ and $\sigma_2 /_i e_2^+$ is closed.
- 2. For $i \in \{1,2\}$ if $\sigma_1 /_i e_1$ is closed and $\Psi \vdash \sigma_1 /_i e_1 \longrightarrow x /_i y$ then exists σ_2 and e_2 s.t. $x = \sigma_2$ and $y = e_2$ and $\sigma_2 /_i e_2$ is closed.

Proof (sketch): By induction over the structure of the operational semantic rules.

Lemma 7 (Distributivity of Projection for Expressions). $\lfloor e^+[x \leftarrow v^+] \rfloor_i = \lfloor e^+ \rfloor_i [x \leftarrow \lfloor v^+ \rfloor_i]$

Proof. By induction over the structure of e^+ . Most cases can be proven by straightforward application of I.H., which we omit.

Case: $e^+ = x$

(1)
$$\begin{array}{l} \lfloor x[x \leftarrow v^+] \rfloor_i = \lfloor v^+ \rfloor_i \\ (2) \quad \lfloor x \rfloor_i [x \leftarrow \lfloor v^+ \rfloor_i] = x[x \leftarrow \lfloor v^+ \rfloor_i] = \lfloor v^+ \rfloor_i \\ \text{By (1) and (2)} \\ (3) \quad \lfloor x[x \leftarrow v^+] \rfloor_i = \lfloor x \rfloor_i [x \leftarrow \lfloor v^+ \rfloor_i] \end{array}$$

Case: $e^+ = \langle e_1 | e_2 \rangle$

 $\begin{array}{ll} (1) & \lfloor \langle e_1 \, | \, e_2 \rangle [x \Leftarrow v^+] \rfloor_i = \lfloor \langle e_1 [x \Leftarrow \lfloor v^+ \rfloor_1] \, | \, e_2 [x \Leftarrow \lfloor v^+ \rfloor_2] \rangle \rfloor_i = e_i [x \Leftarrow \lfloor v^+ \rfloor_i] \\ (2) & \lfloor \langle e_1 \, | \, e_2 \rangle \rfloor_i [x \Leftarrow \lfloor v^+ \rfloor_i] = e_i [x \Leftarrow \lfloor v^+ \rfloor_i] \\ \text{By (1) and (2)} \\ (3) & \lfloor \langle e_1 \, | \, e_2 \rangle [x \Leftarrow v^+] \rfloor_i = \lfloor \langle e_1 \, | \, e_2 \rangle \rfloor_i [x \Leftarrow \lfloor v^+ \rfloor_i] \end{array}$

Lemma 8. If for all $i \in \{1, 2\}$, $\mathcal{E} :: \Psi \vdash \sigma_1 /_i e_1 \longrightarrow \sigma_2 /_i e_2$ where $\sigma_1 /_i e_1$ is closed then $\Psi \vdash \lfloor \sigma_1 \rfloor_i / e_1 \longrightarrow \lfloor \sigma_2 \rfloor_i / e_2$ and $\lfloor \sigma_1 \rfloor_j = \lfloor \sigma_2 \rfloor_j$, where $\{i, j\} = \{1, 2\}$

Proof. Proof by induction on the structure of \mathcal{E} . For most cases, the store will not be updated. The proof follows directly by applying the same rule. We will present cases of memory operations.

$$\Psi \vdash \sigma /_i e \longrightarrow \sigma' /_i e'$$

$$\begin{split} \frac{\Psi \vdash \sigma \ / i \ e' \longrightarrow \sigma' \ / i \ e'}{\Psi \vdash \sigma \ / i \ E[e] \longrightarrow \sigma' \ / i \ E[e]} \ P\text{-E-CONTEXT} \\ \frac{\Psi \vdash \sigma \ / i \ e_i \longrightarrow \sigma' \ / i \ E[e] \longrightarrow \sigma' \ / i \ E[e]}{\Psi \vdash \sigma \ / i \ E[e] \longrightarrow \sigma' \ / i \ E[e]} \ P\text{-E-CONTEXT} \\ \frac{\Psi \vdash \sigma \ / i \ e_i \longrightarrow \sigma' \ / i \ e_i \ = e_j' \qquad \{i,j\} = \{1,2\}}{\Psi \vdash \sigma \ / (e_1 \mid e_2) \longrightarrow \sigma' \ / (e_1'\mid e_2')} \ P\text{-E-LIFT-APP} \\ \frac{\Psi \vdash \sigma \ / (e_1 \mid e_2) \lor \cdots \to \sigma \ / (e_1 \mid e_1) \ Pe^{-1} \ Pe^{-1}$$

Figure 10: Operational Semantics of Extended polC

Case: \mathcal{E} ends in P-E-DEREF

By assumption

(1) $\Psi \vdash \sigma /_i *loc \longrightarrow \sigma /_i (\mathsf{rd}_i \sigma(loc))$ By DEREF (2) $\Psi \vdash [\sigma]_i / *loc \longrightarrow [\sigma]_i / \mathsf{rd} [\sigma]_i(loc)$ By $\sigma /_i *loc$ is closed (3) $[\sigma(loc)]_i = v$, where v is a polC value By definition of rd : (4) $\mathsf{rd}_i \sigma(loc) = [\sigma(loc)]_i = v$ (5) $\mathsf{rd} [\sigma]_i(loc) = ([\sigma]_i)(loc) = v$ By (4) and (5) (6) $[\sigma]_i(loc) = [\sigma(loc)]_i = v$

Case: \mathcal{E} ends in P-E-Assign

By assumption $\Psi \vdash \sigma /_i \ loc := v \longrightarrow \sigma_2 /_i \ () \ and \ \sigma_2 = \sigma[loc \mapsto \mathsf{upd}_i \ \sigma(loc) \ v]$ (1)By Assign: (2) $\Psi \vdash |\sigma|_i / loc := v \longrightarrow \sigma' / ()$ and $\sigma' = \lfloor \sigma \rfloor_i [loc \mapsto \mathsf{upd} \lfloor \sigma \rfloor_i (loc) v]$ We show the case for when i = 1, the case for i = 2 can be proven similarly By definition of upd (3) $\sigma' = |\sigma|_i [loc \mapsto v]$ (4) $\sigma_2 = \sigma[loc \mapsto \langle |v|_1 | |\sigma(loc)|_2 \rangle]$ By v is a valid extended *polC* expression (5) v does not contain • and $v = \lfloor v \rfloor_1$ By the definition of projection (6) $|\sigma_2|_1 = |\sigma|_1 [loc \mapsto |v|_1] = \sigma'$ There are two subcases Subcase a. loc is defined in σ for execution 2 (a) $|\sigma_2|_2 = |\sigma|_2[loc \mapsto |\sigma(loc)|_2] = |\sigma|_2$ Subcase b. loc is not defined in σ for execution 2 (b) $\lfloor \sigma_2 \rfloor_2 = \lfloor \sigma[loc \mapsto \langle \lfloor v \rfloor_1 \mid \lfloor \sigma(loc) \rfloor_2 \rangle] \rfloor_2 = \lfloor \sigma[loc \mapsto \langle \lfloor v \rfloor_1 \mid \bullet \rangle] \rfloor_2 = \lfloor \sigma \rfloor_2$

Case: \mathcal{E} ends in P-E-NEW

By assumption (1) $\Psi \vdash \sigma /_i \operatorname{new}(v) \longrightarrow \sigma_2 /_i \operatorname{loc} \operatorname{and} \sigma_2 = \sigma[\operatorname{loc} \mapsto \operatorname{new}_i v]$ By NEW (2) $\Psi \vdash \lfloor \sigma \rfloor_i / \operatorname{new}(v) \longrightarrow \sigma' / \operatorname{loc} \operatorname{and} \sigma' = \lfloor \sigma \rfloor_i [\operatorname{loc} \mapsto \operatorname{new} v]$ We show the case for when i = 1, the case for i = 2 can be proven similarly By definition of new (3) $\sigma_2 = \sigma[\operatorname{loc} \mapsto \operatorname{new}_1 v] = \sigma[\operatorname{loc} \mapsto \langle v \mid \bullet \rangle]$ (4) $\sigma' = \lfloor \sigma \rfloor_1 [\operatorname{loc} \mapsto v]$ By v is a valid extended polC expression and the definition of projection (6) $\lfloor \sigma_2 \rfloor_1 = \sigma' = \lfloor \sigma \rfloor_1 [\operatorname{loc} \mapsto v]$ (7) $\lfloor \sigma_2 \rfloor_2 = \lfloor \sigma[\operatorname{loc} \mapsto \langle v \mid \bullet \rangle] \rfloor_2 = \lfloor \sigma \rfloor_2$

Lemma 9. $\lfloor E \rfloor_i [\lfloor e \rfloor_i] = \lfloor E[e] \rfloor_i$

Proof (sketch): Proof by induction on the structure of E.

Theorem 10 (Soundness). If $\mathcal{E} :: \Psi \vdash \sigma_1 / e_1^+ \longrightarrow \sigma_2 / e_2^+$ where σ_1 / e_1^+ is closed then for all $i \in \{1, 2\}, \Psi \vdash \lfloor \sigma_1 / e_1^+ \rfloor_i \longrightarrow \lfloor \sigma_2 / e_2^+ \rfloor_i$; or $\lfloor \sigma_1 / e_1^+ \rfloor_i = \lfloor \sigma_2 / e_2^+ \rfloor_i$.

Proof. Proof by induction on the structure of \mathcal{E} . Most cases are straightforward. We show a few key cases below.

Case: \mathcal{E} ends in CONTEXT

By assumption: (1) $\Psi \vdash \sigma / E[e] \longrightarrow \sigma' / E[e']$ (2) $\mathcal{E}' :: \Psi \vdash \sigma / e \longrightarrow \sigma' / e'$ By I.H. on \mathcal{E}' (3) $\Psi \vdash \lfloor \sigma / e \rfloor_i \longrightarrow \lfloor \sigma' / e' \rfloor_i, i \in \{1, 2\}$ By definition of projection (4) $\Psi \vdash \lfloor \sigma \rfloor_i / \lfloor e \rfloor_i \longrightarrow \lfloor \sigma' \rfloor_i / \lfloor e' \rfloor_i$ By (Context) and (4) (5) $\Psi \vdash \lfloor \sigma \rfloor_i / \lfloor E \rfloor_i [\lfloor e \rfloor_i] \longrightarrow \lfloor \sigma' \rfloor_i / \lfloor E \rfloor_i [\lfloor e' \rfloor_i]$ By Lemma 9, (5): (6) $\Psi \vdash \lfloor \sigma \rfloor_i / \lfloor E [e] \rfloor_i \longrightarrow \lfloor \sigma' \rfloor_i / \lfloor E [e'] \rfloor_i$

Case: \mathcal{E} ends in PAIR

By assumption (1) $\Psi \vdash \sigma / \langle e_1 | e_2 \rangle \longrightarrow \sigma' / \langle e'_1 | e'_2 \rangle$ (2) $\mathcal{E}' :: \Psi \vdash \sigma /_k e_k \longrightarrow \sigma' /_k e'_k, e_j = e'_j, \{k, j\} = \{1, 2\}$ By Lemma 8 and (2) (3) $\Psi \vdash \lfloor \sigma \rfloor_k / e_k \longrightarrow \lfloor \sigma' \rfloor_k / e'_k$ and (4) $\lfloor \sigma \rfloor_j = \lfloor \sigma' \rfloor_j$ Subcase a. i = kBy (3), the conclusion holds Subcase b. i = jBy (2), (5) $\lfloor \langle e_1 | e_2 \rangle \rfloor_j = \lfloor \langle e'_1 | e'_2 \rangle \rfloor_j$ By (4) and (5), the conclusion holds

Case: \mathcal{E} ends in DEREF

By assumption (1) $\Psi \vdash \sigma / *loc \longrightarrow \sigma / (\mathsf{rd} \ \sigma(loc))$ T.S. $\Psi \vdash \lfloor \sigma / *loc \rfloor_i \longrightarrow \lfloor \sigma / \mathsf{rd} \ \sigma(loc) \rfloor_i$ By DEREF (2) $\Psi \vdash \lfloor \sigma \rfloor_i / *loc \longrightarrow \lfloor \sigma \rfloor_i / \mathsf{rd} \ (\lfloor \sigma \rfloor_i)(loc)$ T.S. $\lfloor \mathsf{rd} \ \sigma(loc) \rfloor_i = \mathsf{rd} \ (\lfloor \sigma \rfloor_i loc)$ By definition of rd: (3) $\lfloor \mathsf{rd} \ \sigma(loc) \rfloor_i = \lfloor \sigma(loc) \rfloor_i$ (4) $\mathsf{rd} \ (\lfloor \sigma \rfloor_i)(loc) = \lfloor \sigma \rfloor_i(loc)$ By $\sigma / *loc$ is closed: (5) $\lfloor \sigma(loc) \rfloor_i = v$, where v is a polC value By projection definitions (6) $\lfloor \sigma \rfloor_i(loc) = \lfloor \sigma(loc) \rfloor_i = v$ Case: \mathcal{E} ends in ASSIGN

By assumption (1) $\Psi \vdash \sigma / loc := v \longrightarrow \sigma[loc \mapsto \mathsf{upd} \sigma(loc) v] / ()$ T.S. $\Psi \vdash \lfloor \sigma / loc := v \rfloor_i \longrightarrow \lfloor \sigma[loc \mapsto \mathsf{upd} \sigma(loc) v] / () \rfloor_i$ By ASSIGN: (2) $\Psi \vdash \lfloor \sigma \rfloor_i / loc := \lfloor v \rfloor_i \mapsto \lfloor \sigma \rfloor_i [loc \mapsto \mathsf{upd} \lfloor \sigma \rfloor_i (loc) \lfloor v \rfloor_i] / ()$ T. S. $\lfloor \sigma[loc \mapsto \mathsf{upd} \sigma(loc) v] \rfloor_i = \lfloor \sigma \rfloor_i [loc \mapsto \mathsf{upd} \lfloor \sigma \rfloor_i (loc) \lfloor v \rfloor_i]$ By definition of upd (3) $\sigma[loc \mapsto \mathsf{upd} \sigma(loc) v] = \sigma[loc \mapsto v]$ (4) $\lfloor \sigma \rfloor_i [loc \mapsto \mathsf{upd} \lfloor \sigma \rfloor_i (loc) \lfloor v \rfloor_i] = \lfloor \sigma \rfloor_i [loc \mapsto \lfloor v \rfloor_i]$ By v is a valid extended polC expression (5) v does not contain • By the definition of projection (6) $\lfloor \sigma[loc \mapsto v] \rfloor_i = \lfloor \sigma \rfloor_i [loc \mapsto |v|_i]$

Case: \mathcal{E} ends in NEW

By assumption (1) $\Psi \vdash \sigma / \operatorname{new}(v) \longrightarrow \sigma[loc \mapsto \operatorname{new} v] / loc$ T.S. $\Psi \vdash \lfloor \sigma / \operatorname{new}(v) \rfloor_i \longrightarrow \lfloor \sigma [loc \mapsto \operatorname{new} v] / loc \rfloor_i$ By NEW (2) $\Psi \vdash \lfloor \sigma \rfloor_i / \lfloor \operatorname{new}(v) \rfloor_i \longrightarrow \lfloor \sigma \rfloor_i [loc \mapsto \operatorname{new} \lfloor v \rfloor_i] / loc$ T.S. $\lfloor \sigma [loc \mapsto \operatorname{new} v] \rfloor_i = \lfloor \sigma \rfloor_i [loc \mapsto \operatorname{new} \lfloor v \rfloor_i]$ By definition of new (3) $\sigma[loc \mapsto \operatorname{new} v] = \sigma[loc \mapsto v]$ (4) $\lfloor \sigma \rfloor_i [loc \mapsto \operatorname{new} \lfloor v \rfloor_i] = \lfloor \sigma \rfloor_i [loc \mapsto \lfloor v \rfloor_i]$ By v is a valid extended polC expression (5) v does not contain • By the definition of projection (6) $\lfloor \sigma [loc \mapsto v] \rfloor_i = \lfloor \sigma \rfloor_i [loc \mapsto \lfloor v \rfloor_i]$

Case: \mathcal{E} ends in Let

By assumption: (1) $\Psi \vdash \sigma / \text{let } x = v \text{ in } e \longrightarrow \sigma / e[x \Leftarrow v]$ By LET (2) $\Psi \vdash \lfloor \sigma \rfloor_i / \text{let } x = \lfloor v \rfloor_i \text{ in } \lfloor e \rfloor_i \longrightarrow \lfloor \sigma \rfloor_i / \lfloor e \rfloor_i [x \Leftarrow \lfloor v \rfloor_i]$ By Lemma 7, (3) $\lfloor e[x \Leftarrow v] \rfloor_i = \lfloor e \rfloor_i [x \Leftarrow \lfloor v \rfloor_i]$

Lemma 11 (Projected run). If $\mathcal{E} :: \Psi \vdash \lfloor \sigma \rfloor_i / e \longrightarrow \sigma' / e'$ where e is a core polC constructs, $i \in \{1, 2\}$, then $\Psi \vdash \sigma /_i e \longrightarrow \sigma'' /_i e'$ and $\lfloor \sigma'' \rfloor_i = \sigma'$.

Proof (sketch): By induction over the structure of \mathcal{E} . For all the cases, we can apply the same evaluation rule of \mathcal{E} .

Lemma 12 (Projected execution completeness). If $\Psi \vdash \lfloor \sigma / e \rfloor_i \longrightarrow \sigma' / e'$ where $i \in \{1, 2\}$, then exists σ_1 , e_1 , and $k \in \{1, 2\}$ s.t. $\Psi \vdash \sigma / e \longrightarrow^k \sigma_1 / e_1$ and $\lfloor \sigma_1 / e_1 \rfloor_i = \sigma' / e'$.

Proof. By induction over the structure of e. For most cases, we consider one of the following three cases: the CONTEXT rule applies, a reduction applies, or a lift rule applies. We show one example case below. We also show the special case when e is a pair.

Case: $e = v e_1$

By assumption

(1) $\lfloor e \rfloor_i = v_2 e_2$ and $v_2 = \lfloor v \rfloor_i, \lfloor e_1 \rfloor_i = e_2$

Subcase a: CONTEXT applies

By assumption

(a1) $\Psi \vdash [\sigma]_i / v_2 e_2 \longrightarrow [\sigma]_i / v_2 e'_2$ and (a2) $\Psi \vdash [\sigma]_i / e_2 \longrightarrow [\sigma]_i / e'_2$ By I.H. on e_1 (a3) exists k, σ_2, e'_1 , s.t. $\Psi \vdash \sigma / e_1 \longrightarrow^k \sigma_2 / e'_1$ and $[\sigma_2 / e'_1]_i = [\sigma]_i / e'_2$. By applying CONTEXT (a4) $\Psi \vdash \sigma / v e_1 \longrightarrow^k \sigma_2 / v e'_1$ By projection and (1), (a3) (a5) $[\sigma_2 / v e'_1]_i = [\sigma]_i / v_2 e'_2$.

Subcase b: APP applies

By assumption (b1) $v_2 = f, e_2 = v_3$, and exists $v_1, e_1 = v_1$ (b2) $\Psi \vdash \lfloor \sigma \rfloor_i / v_2 e_2 \longrightarrow \lfloor \sigma \rfloor_i / e_3[x \leftarrow v_3]$ and (b3) $\Psi = \Psi', f(x) = e_3$ There are two cases: (I) $v = \langle f_1 | f_2 \rangle$ and (II) v = fFor (II), we can apply the APP rule, and use Lemma 7. We show details of proof of (I) below. By LIFT-APP rule (b4) $\Psi \vdash \sigma / \langle f_1 \mid f_2 \rangle v_1 \longrightarrow \sigma / \langle f_1 \mid v_1 \mid f_2 \mid v_1 \mid g_2 \rangle$ We show the case i = 1 and the other case can be proven similarly. By (1) and (b1)(b5) $f_1 = f$ and $v_1 = v_3$ By APP (b6) $\Psi \vdash \sigma /_i f v_3 \longrightarrow \sigma /_i e_3[x \Leftarrow v_3]$ BV PAIR (b7) $\Psi \vdash \sigma / \langle f_1 \mid v_1 \mid_1 \mid f_2 \mid v_1 \mid_2 \rangle \longrightarrow \sigma / \langle e_3[x \leftarrow v_3] \mid f_2 \mid v_1 \mid_2 \rangle$ By (b4) and (b7)the conclusion holds

Case: $e = \langle e_1 | e_2 \rangle$

By assumption (1) $\Psi \vdash \lfloor \sigma \rfloor_i / e_i \longrightarrow \sigma' / e'$ We prove the case when i = 1, the other case is similar By e_1 is a core *polC* construct and Lemma 11 (2) $\Psi \vdash \sigma /_1 e_1 \longrightarrow \sigma'' /_1 e'$ and $\sigma' = \lfloor \sigma'' \rfloor_1$ By PAIR and (2) (3) $\Psi \vdash \sigma / \langle e_1 | e_2 \rangle \longrightarrow \sigma'' / \langle e' | e_2 \rangle$

Theorem 13 (Completeness). If for all $i \in \{1, 2\}$, $\Psi \vdash \lfloor \sigma / e^+ \rfloor_i \longrightarrow^{n_i} \sigma_i / v_i$, then exists σ' , v', s.t. $\Psi \vdash \sigma / e \longrightarrow^* \sigma' / v'$ and for all $i \in \{1, 2\}$, $\lfloor \sigma' / v' \rfloor_i = \sigma_i / v_i$.

Proof. By induction over $n_1 + n_2$.

Base case $n_1 + n_2 = 0$

By assumption (1) $\lfloor e^+ \rfloor_i = v_i$ for $i \in \{1, 2\}$ By (1) and the definition of projection (2) e^+ is a value.

Inductive case $n_1 + n_2 = k + 1$

By assumption, at least one of the projections takes a step. We show one case and the other can be proven similarly.

$$\begin{split} \Psi &\vdash \lfloor \sigma \ / \ e^+ \rfloor_1 \longrightarrow \sigma'_1 \ / \ e'_1 \longrightarrow \overset{n_1-1}{\longrightarrow} \sigma_1 \ / \ v_1 \\ \Psi &\vdash \lfloor \sigma \ / \ e^+ \rfloor_2 \longrightarrow \overset{n_2}{\longrightarrow} \sigma_2 \ / \ v_2 \end{split}$$
(1)(2)By Lemma 12, (3) exists $k \in \{1, 2\}$, σ' and e_1^+ s.t. $\Psi \vdash \sigma / e^+ \longrightarrow^k \sigma' / e_1^+$ Subcase I: k = 1By the evaluation of a core polC term is deterministic (I1) $\sigma'_1 / e'_1 = \lfloor \sigma' / e_1^+ \rfloor_1$ By Theorem 10 and (2), we have two cases Subcase a: (a1) $\Psi \vdash \lfloor \sigma / e^+ \rfloor_2 \longrightarrow \lfloor \sigma' / e_1^+ \rfloor_2$ By the evaluation of a core *polC* term is deterministic (a2) $\Psi \vdash \lfloor \sigma / e^+ \rfloor_2 \longrightarrow \sigma'_2 / e'_2 \longrightarrow^{n_2-1} \sigma_2 / v_2$ (a3) $\sigma'_2 / e'_2 = \lfloor \sigma' / e_1^+ \rfloor_2$ By I.H. (1), (I1), (a2), (a3) (a4) $\sigma'', v', \text{ s.t. } \Psi \vdash \sigma' / e_1^+ \longrightarrow^* \sigma'' / v'$ (a5) and for all $i \in \{1, 2\}$, $[\sigma'' / v']_i = \sigma_i / v_i$ By (a4) and (3), the conclusion holds Subcase b: (b1) and $\lfloor \sigma / e^+ \rfloor_2 = \lfloor \sigma' / e_1^+ \rfloor_2$ By I.H. (1), (I1), (2), (b1) (b2) $\sigma'', v', \text{ s.t. } \Psi \vdash \sigma' / e_1^+ \longrightarrow^* \sigma'' / v'$ (b3) and for all $i \in \{1, 2\}$, $|\sigma'' / v'|_i = \sigma_i / v_i$ By (b2) and (3), the conclusion holds

Subcase II: k = 2

The proof is similar to the previous case. We need to case on whether the projection of the configuration to the right execution makes a step or remains the same. Finally invoke I.H.

$D;F;\Sigma;\Gamma\vdash v:s$

$$\begin{array}{ll} \hline \hline D;F;\Sigma;\Gamma\vdash n: \operatorname{int}\rho & \operatorname{P-T-V-INT} & \hline \hline D;F;\Sigma;\Gamma\vdash loc:\Sigma(x)\rho & \operatorname{P-T-V-Loc} \\ \hline \hline D;F;\Sigma;\Gamma\vdash x:\Gamma(x) & \operatorname{P-T-V-VAR} & \hline \hline D;F;\Sigma;\Gamma\vdash f:F(f)\rho & \operatorname{P-T-V-Fun} \\ \hline \hline \hline T\mapsto \operatorname{struct} T \{s_1,\cdots,s_n\}\in D & \forall i,D;\Sigma;\Gamma\vdash v_i:s_i \\ \hline D;F;\Sigma;\Gamma\vdash (T) \{v_1,\cdots,v_n\}:T\rho & \\ \hline \hline D;F;\Sigma;\Gamma\vdash v:s' & s'\leq s \\ \hline D;F;\Sigma;\Gamma\vdash v:s & \\ \hline D;F;\Sigma;\Gamma\vdash v:s & \\ \hline D;F;\Sigma;\Gamma\vdash v:s & \\ \hline D;F;\Sigma;\Gamma\vdash v_2:s & \rho \rhd s & \rho \in H \\ \hline D;F;\Sigma;\Gamma\vdash \langle v_1 \mid v_2 \rangle:s & \\ \end{array}$$

Figure 11: Typing Rules for Values in Extended polC

B.5 Summary of Typing Rules for Paired *polC*

First we define subtyping relations and policy operations below.

Figure 11 and 12 summarize typing rules for extended polC.

B.6 Preservation

Next we present the lemmas and proofs for the Preservation Theorem. We define $\Sigma \leq \Sigma'$ as $\Sigma' = \Sigma, \Sigma''$.

 $D;F;\Sigma;\Gamma;pc\vdash e:s$

$$\frac{D; F; \Sigma; \Gamma \vdash v: s}{D; F; \Sigma; \Gamma; pc \vdash v: s \sqcup pc} P^{\text{-}\text{T}\text{-}\text{E}\text{-}\text{VAL}}$$

$$\frac{D; F; \Sigma; \Gamma \vdash v: T \rho \quad T \mapsto \text{struct } \{s_1, \cdots, s_n\} \in D \quad pc \sqsubseteq \rho}{D; F; \Sigma; \Gamma; pc \vdash v: s \sqcup \rho} P^{\text{-}\text{T}\text{-}\text{E}\text{-}\text{FIELD}}$$

$$\frac{D; F; \Sigma; \Gamma; pc \vdash e: s \quad pc \triangleright \rho}{D; F; \Sigma; \Gamma; pc \vdash new(e) : ptr(s) \rho} P^{\text{-}\text{T}\text{-}\text{E}\text{-}\text{New}} \qquad \frac{D; F; \Sigma; \Gamma; pc \vdash v: s \sqcup \rho}{D; F; \Sigma; \Gamma; pc \vdash v: s \sqcup \rho} P^{\text{-}\text{T}\text{-}\text{E}\text{-}\text{DereF}}$$

$$\frac{D; F; \Sigma; \Gamma \vdash v_1 : ptr(s) \rho \quad D; F; \Sigma; \Gamma; pc \vdash e_2 : s \quad \rho \triangleright s}{D; F; \Sigma; \Gamma; pc \vdash v_1 : = e_2 : unit} P^{\text{-}\text{T}\text{-}\text{E}\text{-}\text{Assign}}$$

$$\frac{D; F; \Sigma; \Gamma \vdash v_f : [pc'](t_1 \to t_2)^{\rho} \quad D; F; \Sigma; \Gamma; pc \vdash e_a : t_1 \quad \rho \sqcup pc \sqsubseteq pc'}{D; F; \Sigma; \Gamma; pc \vdash v_f e_a : t_2} P^{\text{-}\text{-}\text{E}\text{-}\text{App}}$$

$$\frac{D; F; \Sigma; \Gamma \vdash v_f : [pc'](t_1 \to t_2)^{\rho} \quad D; F; \Sigma; \Gamma; pc \vdash e_a : t_1 \quad \rho \sqcup pc \sqsubseteq pc'}{D; F; \Sigma; \Gamma; pc \vdash let x : s_1 = e_1 \text{ in } e_2 : s_2} P^{\text{-}\text{-}\text{E}\text{-}\text{LET}}$$

$$\frac{D; F; \Sigma; \Gamma; pc \vdash e_1 : s_1 \quad D; F; \Sigma; \Gamma; pc \vdash e_2 : s \quad D; F; \Sigma; \Gamma; pc \sqcup \rho \vdash e_3 : s}{D; F; \Sigma; \Gamma; pc \vdash let x : s_1 = e_1 \text{ in } e_2 \text{ esc } s_1 : s} P^{\text{-}\text{-}\text{E}\text{-}\text{LET}}$$

$$\frac{D; F; \Sigma; \Gamma \vdash v_f : (d\&e)[pc'](b \ell_1 :: \top \rightarrow b \ell_2 :: \bot)^{\rho_f}}{D; F; \Sigma; \Gamma; pc \vdash e_a : b \rho \quad \rho = \ell_1 : \ell_2 : \rho' \quad \rho_f \sqcup pc \sqsubseteq pc'}} P^{\text{-}\text{-}\text{E}\text{-}\text{LE}}$$

$$\frac{D; F; \Sigma; \Gamma \vdash v_f : d\& \rho \quad pc \sqsubseteq \rho'}{D; F; \Sigma; \Gamma; pc \vdash v_f e_a : b \ell_2 :: L)^{\rho_f}} P_{\text{-}\text{-}\text{E}\text{-}\text{DE}}{D; F; \Sigma; \Gamma; pc \vdash relab(\rho' \neq \rho) : v_f \rho'} P^{\text{-}\text{-}\text{E}\text{-}\text{RELABEL}}$$

$$\frac{D; F; \Sigma; \Gamma \vdash v : b \rho \quad pc \sqsubseteq \rho'}{D; F; \Sigma; \Gamma; pc \vdash v_f e_a : b \ell_2 :: \rho'}} P^{\text{-}\text{-}\text{E}\text{-}\text{S}}{D; F; \Sigma; \Gamma; pc \vdash relab(\rho' \neq \rho) : v_f \rho'} P^{\text{-}\text{-}\text{E}\text{-}\text{RELABEL}}$$

$$\frac{D; F; \Sigma; \Gamma; pc \sqcup e : s}{D; F; \Sigma; \Gamma; pc \sqcup p' \vdash e_1 : s} D; F; \Sigma; \Gamma; pc \sqcup \rho' \vdash e_2 : s} \rho \vdash \rho = s }{D; F; \Sigma; \Gamma; pc \vdash e : s}} P_{\text{-}\text{T}\text{-}\text{E}\text{-}\text{RELABEL}}$$

Figure 12: Typing Rules for Expressions in Extended polC

Lemma 14. If $\rho' \sqsubseteq \rho$, $\rho \rhd s$, $s \le s'$, then $\rho' \rhd s'$

Proof (sketch): By examining $\rho \triangleright s$ and $s \leq s'$.

Lemma 15. If $\mathcal{E} :: D; F; \Sigma; \Gamma; pc \vdash e : s and pc' \sqsubseteq pc$ then $D; F; \Sigma; \Gamma; pc' \vdash e : s$.

Proof (sketch): By induction over the structure of \mathcal{E} . We use Lemma 14 in cases where pc is used in the premises.

Lemma 16. 1. If $\mathcal{E} :: D; F; \Sigma; \Gamma \vdash v : s \text{ and } \Sigma \leq \Sigma' \text{ then } D; F; \Sigma'; \Gamma \vdash v : s.$

2. If $\mathcal{E} :: D; F; \Sigma; \Gamma; pc \vdash e : s \text{ and } \Sigma \leq \Sigma' \text{ then } D; F; \Sigma'; \Gamma; pc \vdash e : s.$

Proof (sketch): By induction over the structure of \mathcal{E} . We use Lemma 14 in cases where pc is used in the premises.

Lemma 17 (Projection well-typed). If $\mathcal{E} :: D; \Sigma; \Gamma \vdash v : s$ then $\forall i \in \{1, 2\}, D; \Sigma; \Gamma \vdash |v|_i : s$

Proof (sketch): By induction over the structure of \mathcal{E} .

Lemma 18 (Substitution).

1. If $\mathcal{E} :: D; F; \Sigma; \Gamma, x:s \vdash v': s' \text{ and } D; F; \Sigma; \Gamma \vdash v: s \text{ then } D; F; \Sigma; \Gamma \vdash v'[x \leftarrow v]: s'$

2. If
$$\mathcal{E} :: D; F; \Sigma; \Gamma, x:s; pc \vdash e: s' and D; F; \Sigma; \Gamma \vdash v: s then D; F; \Sigma; \Gamma; pc \vdash e[x \leftarrow v]: s'$$

Proof (sketch): By induction over the structure of \mathcal{E} .

Lemma 19.

- 1. If $s \leq T \rho$ then $s = T \rho'$
- 2. If $s \leq \mathsf{ptr}(s') \ \rho$ then $s = \mathsf{ptr}(s') \ \rho'$

3. If $s \leq [pc_f](t_1 \to t_2)^{\rho}$ then $s = [pc'_f](t'_1 \to t'_2)^{\rho'}$ and $pc_f \sqsubseteq pc'_f$, $t_1 \leq t'_1$ and $t'_2 \leq t_2$.

Proof (sketch): By induction over the derivation $s \leq s'$.

Lemma 20 (Inversion).

- 1. If $D; F; \vdash (T)\{v_1, \dots, v_n\} : T \rho$, then $D(T) = \{s_1, \dots, s_2\}$ and $\forall i \in [1, n], D; F; \vdash v_i : s_i$.
- 2. If $D; F; \cdot \vdash loc : ptr(s) \ \rho, \ loc \in dom(\sigma), \ and \ D; F \vdash \sigma : \Sigma, \ then \ D; F; \cdot; pc \vdash \sigma(loc) : s.$
- 3. If $D; F; \vdash f: [pc_f](t_1 \to t_2)^{\rho}$, $f(x) = e \in \operatorname{dom}(\Psi)$ and $D; F \vdash \Psi$, then $D; F; x: t_1; pc_f \vdash e: t_2$.
- 4. If $D; F; \vdash \langle v_1 | v_2 \rangle$: s then $\forall i \in [1, 2], D; F; \vdash v_i$: s and $\exists \rho, s.t. \rho \rhd s$ and $\rho \in H$.

Proof (sketch): By induction over the typing derivation.

Lemma 21 (Value is typed w/o PC). If $\mathcal{E} :: D; F; \Sigma; \Gamma; pc \vdash v : s$ then $D; F; \Sigma; \Gamma \vdash v : s$.

Proof (sketch): By induction over the structure of \mathcal{E} . In the cases of E-SUB and E-PAIR, we directly apply I.H. and then apply the rule with the same name in value typing. In the case of E-VAL, we apply V-SUB.

Lemma 22 (Store). For all $i \in \{1, 2, \bullet\}$, if $D; F; \Sigma; \Gamma \vdash v : s$ and $i \in \{1, 2\}$ implies exists $\rho \in H$ s.t. $\rho \triangleright s$; then $D; F; \Sigma; \Gamma \vdash \mathsf{new}_i v : s$ and for all v' s.t. $D; F; \Sigma; \Gamma \vdash v' : s, D; F; \Sigma; \Gamma \vdash \mathsf{upd}_i v v' : s$.

Proof (sketch): By examining the definitions of these operations.

Lemma 23 (Value Has Flexible Label). Given a set of high labels H, if $\mathcal{E} :: D; F; \Sigma; \cdot \vdash v : b \rho$ and $\rho \in H$ iff $\rho' \in H$ then $D; F; \Sigma; \cdot \vdash v : b \rho'$.

Proof (sketch): By induction on the structure of v. The value typing rules assign an arbitrary ρ to the type of core *polC* values. In the case of pairs, the assumption that $\rho \in H$ iff $\rho' \in H$ allows us to apply V-PAIR rule.

Lemma 24. If $\mathcal{E} :: D; F; \Sigma; \Gamma; pc \vdash e : s$ then $pc \triangleright s$.

Proof (sketch): By induction over the structure of \mathcal{E} .

Lemma 25 (Preservation). If $\Psi \vdash \sigma /_i e \longrightarrow \sigma' /_i e'$, $D; F \vdash \Psi$, $D; F \vdash \sigma : \Sigma$ and $D; F; \Sigma; \cdot; pc \vdash e : s$, and $i \in \{1, 2\}$ implies $pc \in H$ then exists $\Sigma' \ge \Sigma$ s.t. $D; F \vdash \sigma' : \Sigma'$ and $D; F; \Sigma'; \cdot; pc \vdash e' : s$.

Proof. By induction over the structure of \mathcal{E} . The proofs are mostly standard and use Lemma 18 and 20. We only show cases where information flow labels or pairs are involved.

Case: \mathcal{E} ends in E-RELABEL

By assumption $D; F; \Sigma; \Gamma; pc \vdash \mathsf{reLab}(\rho' \Leftarrow \rho) \ v : b \ \rho'$ (1)(2) $\mathcal{E}' :: D; F; \Sigma; \Gamma \vdash v : b \ \rho \text{ and } pc \sqsubseteq \rho'$ By examining the operational semantic rules, there are two subcases Subcase a: v is not a pair (a3) $\Psi \vdash \sigma /_i \operatorname{reLab}(\rho' \Leftarrow \rho) v \longrightarrow \sigma /_i v$ By the definition of H(a4) $\rho \in H$ iff $\rho' \in H$ By Lemma 23, \mathcal{E}' , and (a4) (a5) $D; F; \Sigma'; \Gamma \vdash v : b \rho'$ Subcase b: $v = \langle v_1 | v_2 \rangle$ $\Psi \vdash \sigma \ / \ \mathsf{reLab}(\rho' \Leftarrow \rho) \langle v_1 \mid v_2 \rangle \longrightarrow \sigma \ / \ \langle \mathsf{reLab}(\rho' \Leftarrow \rho) v_1 \mid \mathsf{reLab}(\rho' \Leftarrow \rho) v_2 \rangle$ (b3)By Lemma 20 and \mathcal{E}' $\forall i \in [1, 2], D; F; \cdot \vdash v_i : b \ \rho \text{ and}$ (b4) $\exists \rho'', \text{ s.t. } \rho'' \triangleright b \rho \text{ and } \rho'' \in H$ (b5)By (b4) and E-RELAB (b6) $\forall i \in [1,2], D; F; \cdot; pc \sqcup \rho' \vdash \mathsf{reLab}(\rho' \Leftarrow \rho)v_i : b \ \rho'$ By the definition of H and (b5) (b7) $\rho \in H$ and $\rho' \in H$ By E-PAIR, (b6), (b7) (b8) $D; F; \Sigma; \Gamma; pc \vdash \langle \mathsf{reLab}(\rho' \Leftarrow \rho)v_1 \mid \mathsf{reLab}(\rho' \Leftarrow \rho)v_2 \rangle : b \rho'$

Case: \mathcal{E} ends in E-IF

By assumption

- (1) $D; F; \Sigma; \Gamma; pc \vdash \text{if } v \text{ then } e_2 \text{ else } e_3 : s$
- (2) $\mathcal{E}' :: D; F; \Sigma; \Gamma \vdash v : \text{int } \rho$
- (3) and $\mathcal{E}_2 :: D; F; \Sigma; \Gamma; pc \sqcup \rho \vdash e_2 : s$
- (4) and $\mathcal{E}_3 :: D; F; \Sigma; \Gamma; pc \sqcup \rho \vdash e_3 : s$

By examining the operational semantic rules, there are two subcases: v is not a pair

and v is a pair. We only show the case when $v = \langle v_1 | v_2 \rangle$

(5) $\Psi \vdash \sigma / \text{ if } \langle v_1 \mid v_2 \rangle$ then e_2 else $e_3 \longrightarrow \sigma / \langle \text{if } v_1 \text{ then} \lfloor e_2 \rfloor_1$ else $\lfloor e_3 \rfloor_1 \mid \text{if } v_2$ then $\lfloor e_2 \rfloor_2$ else $\lfloor e_3 \rfloor_2 \rangle$ By Lemma 20 and \mathcal{E}'

- (6) $\forall i \in \{1, 2\}, D; F; \cdot \vdash v_i : \text{int } \rho \text{ and }$
- (7) $\exists \rho'', \text{ s.t. } \rho'' \rhd \text{ int } \rho \text{ and } \rho'' \in H$
- By (7) and the definition of H

 $(8) \quad \rho \in H$

By Lemma 24, 14 and \mathcal{E}_2

 $\begin{array}{ll} (9) & \rho \rhd s \\ \text{By E-VAL and (6)} \\ (10) & \forall i \in [1,2], \ D; \ F; \ ;; \ pc \sqcup \rho \vdash v_i : \ \text{int } \ pc \sqcup \rho \\ \text{By Lemma 17 and (3), (4)} \\ (11) & D; \ F; \ \Sigma; \ \Gamma; \ pc \sqcup \rho \vdash \lfloor e_k \rfloor_m : s \ \text{where } k \in \{2,3\} \ \text{and } m \in \{1,2\} \\ \text{By E-IF and (3), (4), and (11)} \\ (12) & D; \ F; \ \Sigma; \ \Gamma; \ pc \sqcup \rho \vdash \ \text{if } v_i \ \ \text{then} \ \lfloor e_2 \rfloor_i \ \text{else } \ \lfloor e_3 \rfloor_i : s \ \text{where } i \in \{1,2\} \\ \text{By E-PAIR, (8), (9), and (12)} \\ (13) & D; \ F; \ \Sigma; \ \Gamma; \ pc \vdash \ \langle \text{if } v_1 \ \ \text{then} \ \lfloor e_2 \rfloor_1 \ \ \text{else } \ \lfloor e_3 \rfloor_1 \ \ \text{if } v_2 \ \ \text{then} \ \lfloor e_2 \rfloor_2 \ \ \text{else } \ \lfloor e_3 \rfloor_2 \rangle : s \end{array}$

Case: \mathcal{E} ends in E-DE

By assumption $D; F; \Sigma; \Gamma; pc \vdash v_f e_a : b \ell_2 :: \rho'$ (1)(2) $\mathcal{E}' :: D; F; \Sigma; \Gamma \vdash v_f : (\mathsf{d\&e})[pc'](b \ \ell_1 :: \top \to b \ \ell_2 :: \bot)^{\rho_f}$ (3) $\mathcal{E}'' :: D; F; \Sigma; \Gamma; pc \vdash e_a : b \rho$ (4) and $\rho = \ell_1 :: \ell_2 :: \rho', \rho_f \sqcup pc \sqsubseteq pc'$ By examining the operational semantic rules, there are three subcases **Subcase a:** e_a is not a value. This is a standard case and we omit. **Subcase b:** $e_a = v_a$ and v_f is not a pair (b1) $\Psi \vdash \sigma /_i v_f v_a \longrightarrow \sigma /_i e[x \Leftarrow v_a][v_f \Leftarrow v_f(x) = e]$ By Lemma 20 and $D; F \vdash \Psi$ (b2) $D; F; \Sigma; x: b \ \ell_1::\top, v_f: (\mathsf{d\&e})[pc'](b \ \ell_1::\top \to b \ \ell_2::\bot)^{\rho_f}; pc' \vdash e: b \ \ell_2::\bot$ By \mathcal{E}'' , Lemma 21 and V-SUB (b3) $\mathcal{E}'' :: D; F; \Sigma; \Gamma \vdash v_a : b \ \ell_1 :: \top$ By Lemma 18 (b2) and (b3) (b4) $D; F; \Sigma; \cdot; pc' \vdash e[x \leftarrow v_a][v_f \leftarrow v_f(x) = e] : b \ \ell_2 :: \bot$ Lemma 15 and (b4)(b5) $D; F; \Sigma; \cdot; pc \vdash e[x \Leftarrow v_a][v_f \Leftarrow v_f(x) = e] : b \ \ell_2 :: \bot$ By (b5) and V-SUB (b6) $D; F; \Sigma; \cdot; pc \vdash e[x \Leftarrow v_a][v_f \Leftarrow v_f(x) = e] : b \ \ell_2 :: \rho'$ Subcase c: $e_a = v_a$ and $v_f = \langle v_1 | v_2 \rangle$ (c1) $\Psi \vdash \sigma / \langle v_1 | v_2 \rangle v_a \longrightarrow \sigma / \langle v_1 | v_a |_1 | v_2 | v_a |_2 \rangle$ By Lemma 20 and \mathcal{E}' (c2) $\forall i \in [1,2], D; F; \vdash v_i : (\mathsf{d\&e})[pc'](c \ \ell_1 :: \top \to b \ \ell_2 :: \bot)^{\rho_f}$ and (c3) $\exists \rho'', \text{ s.t. } \rho'' \sqsubseteq \rho_f \text{ and } \rho'' \in H$ By Lemma 21, Lemma 17 and \mathcal{E}'' (c4) $D; F; \Sigma; \Gamma \vdash \lfloor v_a \rfloor_i : b \ \rho \text{ where } i \in \{1, 2\}$ By (4) and (c3)(c5) $\rho_f \sqcup pc \sqcup \rho'' \sqsubseteq pc'$ By (c2), (c4), and (c5) and E-DE (c6) $\forall i \in [1, 2], D; F; \cdot; pc \sqcup \rho'' \vdash v_i \mid v_a \mid_i : b \ \ell_2 :: \rho'$ By the definition of H and (c3) (c7) $\rho_f \in H$ By E-PAIR, (c6), (c7)(c8) $D; F; \Sigma; \Gamma; pc \vdash \langle v_1 \mid v_a \mid_1 \mid v_2 \mid v_a \mid_2 \rangle : b \ \ell_2 ::: \rho'$

Case: \mathcal{E} ends in E-PAIR

By assumption (1) $D; F; \Sigma; \Gamma; pc \vdash \langle e_1 | e_2 \rangle : s$ (2) $\mathcal{E}' :: D; F; \Sigma; \Gamma; pc \sqcup \rho' \vdash e_i : s, i \in \{1, 2\}$ (3) $\rho \triangleright s, \rho \in H$, and $\rho' \in H$ By examining the operational semantic rules (4) $\Psi \vdash \sigma / \langle e_1 | e_2 \rangle \longrightarrow \sigma' / \langle e'_1 | e'_2 \rangle$ (5) $\Psi \vdash \sigma /_i e_i \longrightarrow \sigma' /_i e'_i, e_j = e'_j, \text{ and } \{i, j\} = \{1, 2\}$ By I.H. on \mathcal{E}' (6) exists $\Sigma' \ge \Sigma$ s.t. $D; F \vdash \sigma' : \Sigma'$ (7) and $D; F; \Sigma'; \cdot; pc \sqcup \rho' \vdash e'_i : s$ By Lemma 16(2) and (6)(8) $D; F; \Sigma'; \cdot; pc \sqcup \rho' \vdash e'_j : s$ By PAIR (3), (7), and (8)(9) $D; F; \Sigma'; \Gamma; pc \vdash \langle e'_1 \mid e'_2 \rangle : s$

Theorem 26 (Preservation). If $\Psi \vdash \sigma / e \longrightarrow \sigma' / e'$ and $\vdash \Psi; \sigma; e$ then $\vdash \Psi; \sigma'; e'$.

Proof (sketch): By the definitions of $\vdash \Psi; \sigma; e$ and Lemma 25.

B.7 Noninterference

Finally, we present proofs for the Noninterference Theorem for polC.

$$\frac{F; R \vdash \rho_1 \rightsquigarrow \rho_2 \qquad F; R \vdash \rho_2 \rightsquigarrow \rho_3}{F; R \vdash \rho_1 \rightsquigarrow \rho_3} \qquad \frac{\rho_1 \sqsubseteq \rho_2 \qquad F; R \vdash \rho_2 \rightsquigarrow \rho_3}{F; R \vdash \rho_1 \rightsquigarrow \rho_3} \qquad \frac{\ell_2 :: \bot \Leftarrow \ell_1 :: \top \in R}{F; R \vdash \ell_1 :: \ell_2 :: \rho \rightsquigarrow \ell_2 :: \rho} \\
\frac{F = F', f: (d\&e)[pc'](b \ \ell_1 :: \top \rightarrow b \ \ell_2 :: \bot)^{\rho_f}}{F; R \vdash \ell_1 :: \ell_2 :: \rho \rightsquigarrow \ell_2 :: \rho} \\
\frac{\forall \rho', F; R \vdash \rho \rightsquigarrow \rho', \rho' \nvDash \rho_A}{\rho_A; F; R \vdash \rho \in H}$$

- -

Lemma 27. If ρ_A ; F; $R \vdash \rho_1 \in H$ and $\rho_1 \sqsubseteq \rho_2$ then ρ_A ; F; $R \vdash \rho_2 \in H$.

1. If $D; F; \Sigma; \cdot \vdash v : \text{int } \rho, \text{ and } \rho \notin H, \text{ then } |v|_1 = |v|_2.$ Lemma 28.

2. If $D; F; \Sigma; \cdot; \bot \vdash v : \text{int } \rho, \text{ and } \rho \notin H, \text{ then } \|v\|_1 = \|v\|_2$.

Proof (sketch): By induction over the typing derivation of the value.

Definition 29 (Equivalent substitution). We define $D; F \vdash \delta_1 \approx_H \delta_2 : \Gamma$ iff for all $x \in \text{dom}(\Gamma), D; F; \cdot; \cdot; \vdash$ $\delta_i(x) : \Gamma(x) \ (i \in \{1, 2\}) \ and \ \delta_1(x) = \delta_2(x) \ if \ labOf(\Gamma(x)) \notin H.$

$$\Gamma \vdash \delta_1 \bowtie \delta_2 = \delta$$

$$\begin{array}{l} \overline{\Gamma \vdash \cdot \bowtie \cdot = \cdot} & \frac{\Gamma \vdash \delta_1 \Join \delta_2 = \delta \quad labOf(\Gamma(x)) \in H}{\overline{\Gamma \vdash \delta_1, x \mapsto v_1 \Join \delta_2, x \mapsto v_2 = \delta, x \mapsto \langle v_1 \mid v_2 \rangle} \\ \\ \frac{\Gamma \vdash \delta_1 \bowtie \delta_2 = \delta \quad labOf(\Gamma(x)) \notin H}{\overline{\Gamma \vdash \delta_1, x \mapsto v_1 \Join \delta_2, x \mapsto v_2 = \delta, x \mapsto v_1} \end{array}$$

Lemma 30. If $D; F \vdash \delta_1 \approx_H \delta_2 : \Gamma$ and $\Gamma \vdash \delta_1 \bowtie \delta_2 = \delta$, then $\forall x \in \text{dom}(\Gamma), D; F; \cdot; \cdot \vdash \delta(x) : \Gamma(x)$.

Proof (sketch): By induction over the structure of Γ .

Theorem 31 (Noninterference).

If $D; F; \cdot; \Gamma; \perp \vdash e$: int ρ , let H be the set of labels not-observable by an attacker with label ρ_A , given substitution δ_1 , δ_2 s.t. $\delta_1 \approx_H \delta_2 : \Gamma$, and $\rho \notin H$ and $\Psi \vdash \emptyset / e\delta_1 \longrightarrow^* \sigma_1 / v_1$ and $\Psi \vdash \emptyset / e\delta_2 \longrightarrow^* \sigma_2 / v_2$, then $v_1 = v_2$.

Proof.

Let δ be the substitution from $\Gamma \vdash \delta_1 \bowtie \delta_2 = \delta$. By Lemma 30 (1) $\forall x \in \operatorname{dom}(\Gamma), D; F; \cdot; \cdot \vdash \delta(x) : \Gamma(x)$. By Substitution Lemma (Lemma 18) (2) $D; F; \cdot; \bot \vdash e \circ \delta : \operatorname{int} \rho$ By $\Gamma \vdash \delta_1 \bowtie \delta_2 = \delta$ (3) $\lfloor e \circ \delta \rfloor_i = e\delta_i, i \in \{1, 2\}$ By Completeness (Theorem 13) (4) $\Psi \vdash \emptyset / e \circ \delta \longrightarrow^* \sigma / v$ and for all $i \in \{1, 2\}, \lfloor v \rfloor_i = v_i$ By Preservation (Theorem 26) (5) $D; F; \cdot; \bot \vdash v : \operatorname{int} \rho$ By Lemma 28 (6) $v_1 = v_2 = \lfloor v \rfloor_1 = \lfloor v \rfloor_2$

C Definitions and Proofs of Translations from annotated μ C to μ C via *polC*

C.1 Mapping Annotated μC to *polC*

We first list all the rules for mapping annotated μC types to *polC* types. $\boxed{\langle \langle a \rangle \rangle = t} \boxed{\langle \langle \beta \rangle \rangle = s}$

$$\begin{split} \overline{\langle\langle \mathsf{unit}\rangle\rangle} &= \mathsf{unit} & \overline{\langle\langle \mathsf{int}\rangle\rangle} = \mathsf{int} \ U & \overline{\langle\langle \mathsf{ptr}(\beta)\rangle\rangle} = \mathsf{ptr}(s) \ U & \overline{\langle\langle T\rangle\rangle} = T \ U & \overline{\langle\langle T \ \mathsf{at} \ \rho\rangle\rangle} = T \ \rho \\ \overline{\langle\langle \mathsf{int} \ \mathsf{at} \ \rho\rangle\rangle} &= \mathsf{int} \ \rho & \frac{\langle\langle\beta\rangle\rangle = s}{\langle\langle\mathsf{ptr}(\beta) \ \mathsf{at} \ \rho\rangle\rangle = \mathsf{ptr}(s) \ \rho} & \frac{\forall i \in [1, 2], \langle\langle a_i\rangle\rangle = t_i}{\langle\langle a_1 \rightarrow a_2\rangle\rangle = [\bot](t_1 \rightarrow t_2)} \\ \overline{\langle\langle \mathsf{d\&e}\rangle a_1 \rightarrow a_2\rangle\rangle} &= (\mathsf{d\&e})[\bot](t_1 \rightarrow t_2) \end{split}$$

$$\frac{\forall i \in [1,k], \langle\!\langle a_i \rangle\!\rangle = t_i}{\langle\!\langle D_a, T \mapsto \text{struct } T \ \{a_1, \cdots, a_k\}\rangle\!\rangle = \langle\!\langle D_a \rangle\!\rangle, T \mapsto \text{struct } T \ \{t_1 \cdots, t_k\}}$$

We write lv and le to denote labeled polC values and expressions respectively. Values and expressions are mapped to labeled values and expressions to facilitate the translation process later.

Rules for mapping annotated μC values to labeled *polC* values are as follows. $D_a; F_a; \Gamma_a \vdash \langle\!\langle v \rangle\!\rangle \Rightarrow lv$

$$\begin{split} \overline{D_a; F_a; \Gamma_a \vdash \langle\!\langle (1) \rangle\!\rangle \Rightarrow \ (1)} & \text{V-L-UNIT} & \overline{D_a; F_a; \Gamma_a \vdash \langle\!\langle n \rangle\!\rangle \Rightarrow \ n@\text{int } U} & \text{V-L-INT} \\ \\ \overline{D_a; F_a; \Gamma_a \vdash \langle\!\langle x \rangle\!\rangle \Rightarrow \ x@\langle\!\langle \beta \rangle\!\rangle} & \text{V-L-VAR} & \overline{D_a; F_a; \Gamma_a \vdash \langle\!\langle f \rangle\!\rangle \Rightarrow \ f@\langle\!\langle F(f) \rangle\!\rangle} & \text{V-L-Fun} \\ \\ \\ \frac{\forall i \in [1, n], D_a; F_a; \Gamma_a \vdash \langle\!\langle v_i \rangle\!\rangle \Rightarrow lv_i}{D_a; F_a; \Gamma_a \vdash \langle\!\langle (T) \ \{v_1, \cdots, v_n\} \rangle\!\rangle \Rightarrow \ (T)\{lv_1, \cdots, lv_n\}@T \ U} & \text{V-L-Struct} \end{split}$$

Next, we summarize rules for mapping annotated μC expressions to labeled *polC* expressions below. $D_a; F_a; \Gamma_a; s \vdash \langle\!\langle e \rangle\!\rangle \Rightarrow le$

$$\frac{\langle\!\langle \Gamma_a(x)\rangle\!\rangle = s}{D_a; F_a; \Gamma_a; t \vdash \langle\!\langle n \rangle\!\rangle \Rightarrow n@t} \text{ L-INT} \qquad \frac{\langle\!\langle \Gamma_a(x)\rangle\!\rangle = s}{D_a; F_a; \Gamma_a; s \vdash \langle\!\langle x \rangle\!\rangle \Rightarrow x@s} \text{ L-VAR} \\
\frac{D_a(T) = \text{struct } T \{\beta_1, \cdots, \beta_n\} \quad \forall i \in [1, n], D_a; F_a; \Gamma_a; \langle\!\langle \beta_i \rangle\!\rangle \vdash \langle\!\langle v_i \rangle\!\rangle \Rightarrow lv_i}{D_a; F_a; \Gamma_a; T \rho \vdash \langle\!\langle (T) \{v_1, \cdots, v_n\} \rangle\!\rangle \Rightarrow (T) \{lv_1, \cdots, lv_n\} @(T \rho)} \text{ L-STRUCT}$$

 $\frac{D_a; F_a; \Gamma_a \vdash \langle\!\!\langle v \rangle\!\!\rangle \Rightarrow lv}{D_a(T) = (\mathsf{struct} \ T\{\beta_1, \cdots, \beta_n\}) \qquad \forall i \in [1, n], \rho = labOf(\langle\!\!\langle \beta_i \rangle\!\!\rangle)}{D_a; F_a; \Gamma_a; t \vdash \langle\!\!\langle v.i \rangle\!\!\rangle \Rightarrow lv.i} \ \mathsf{L}\text{-Field-U}$

 $\begin{array}{c} D_a; F_a; \Gamma_a \vdash \langle\!\!\langle v \rangle\!\!\rangle \Rightarrow lv \\ \frac{tpOf(lv) = T \ \rho \quad D_a(T) = (\mathsf{struct} \ T\{\beta_1, \cdots, \beta_n\}) \quad \exists i \in [1, n], \rho \neq labOf(\langle\!\langle \beta_i \rangle\!\!\rangle) \\ D_a; F_a; \Gamma_a; t \vdash \langle\!\langle v.i \rangle\!\!\rangle \Rightarrow \mathsf{let} \ y: T \perp = \mathsf{reLab}(\perp \Leftarrow \rho) \ lv \ \mathsf{in} \ (y@T \perp).i \end{array}$ L-FIELD

$$\frac{D_a; F_a; \Gamma_a; s \vdash \langle\!\langle e \rangle\!\rangle \Rightarrow le}{D_a; F_a; (\mathsf{ptr}(s) \ \rho) \vdash \langle\!\langle \mathsf{new}(e) \rangle\!\rangle \Rightarrow \mathsf{new}(le)@(\mathsf{ptr}(s) \ \rho)} \text{ L-New}$$

$$\frac{D_a; F_a; \Gamma_a \vdash \langle\!\langle v \rangle\!\rangle \Rightarrow lv \qquad tpOf(lv) = b \ \rho}{D_a; F_a; \Gamma_a; t \vdash \langle\!\langle *v \rangle\!\rangle \Rightarrow (\mathsf{let} \ y : b \ \bot = \mathsf{reLab}(\bot \Leftarrow \rho) \ lv \ \mathsf{in} \ *(y@b \ \bot)} \ \mathsf{L-DereF}$$

$$\begin{split} \frac{D_a; F_a; \Gamma_a \vdash \langle\!\langle v \rangle\!\rangle \Rightarrow lv \quad tpOf(lv) = \mathsf{ptr}(s) \rho \quad D_a; F_a; \Gamma_a; s \vdash e \Rightarrow le}{D_a; F_a; \Gamma_a; t \vdash \langle\!\langle v := e \rangle\!\rangle \Rightarrow \mathsf{let} \; y : \mathsf{ptr}(s) \perp = \mathsf{reLab}(\perp \leftarrow \rho) \; lv \; \mathsf{in} \; y@\mathsf{ptr}(s) \perp := le} \; \mathsf{L}\text{-}\mathsf{ASSIGN} \\ \frac{D_a; F_a; \Gamma_a \vdash \langle\!\langle v \rangle\!\rangle \Rightarrow lv \quad tpOf(lv) = [\bot](t_1 \to t_2)^\perp \quad D_a; F_a; \Gamma_a; t_1 \vdash \langle\!\langle e \rangle\!\rangle \Rightarrow le}{D_a; F_a; \Gamma_a; t_2 \vdash \langle\!\langle v \; e \rangle\!\rangle \Rightarrow lv \; le} \; \mathsf{L}\text{-}\mathsf{APP} \\ \frac{D_a; F_a; \Gamma_a \vdash \langle\!\langle v_f \rangle\!\rangle \Rightarrow lv_f \quad tpOf(lv_f) = (\mathsf{d\&e})[\bot](t_1 \to t_2)^\perp \quad D_a; F_a; \Gamma_a \vdash \langle\!\langle v_a \rangle\!\rangle \Rightarrow lv_a}{D_a; F_a; \Gamma_a; t_2 \vdash \langle\!\langle v_f \; v_a \rangle\!\rangle \Rightarrow lv_f \; lv_a} \; \mathsf{L}\text{-}\mathsf{APP}\text{-}\mathsf{DE} \\ \frac{D_a; F_a; \Gamma_a; \langle\!\langle \beta_1 \rangle\!\rangle \vdash \langle\!\langle e_1 \rangle\!\rangle \Rightarrow le_1 \quad D_a; F_a; \Gamma_a, x : \beta_1; t_2 \vdash \langle\!\langle e_2 \rangle\!\rangle \Rightarrow le_2}{D_a; F_a; \Gamma_a; t_2 \vdash \langle\!\langle \mathsf{let} \; x : \beta_1 = e_1 \; \mathsf{in} \; e_2 \rangle\!\rangle \Rightarrow \mathsf{let} \; x : \langle\!\langle \beta_1 \rangle\!\rangle = le_1 \; \mathsf{in} \; le_2 \; \mathsf{L}\text{-}\mathsf{LET} \\ \frac{tpOf(lv_1) = \mathsf{int} \; \rho \quad D_a; F_a; \Gamma_a; t \vdash \langle\!\langle \mathsf{if} \; v_1 \; \mathsf{then} \; e_2 \; \mathsf{else} \; e_3 \rangle}{D_a; F_a; \Gamma_a; t \vdash \langle\!\langle \mathsf{if} \; v_1 \; \mathsf{then} \; e_2 \; \mathsf{else} \; e_3 \rangle} \; \mathsf{L}\text{-}\mathsf{IF} \\ \Rightarrow \mathsf{let} \; x : \mathsf{int} \; \bot \; (\mathsf{reLab}(\perp \leftarrow \rho) \; lv_1) \; \mathsf{in} \; f \; x@\mathsf{int} \; \bot \; \mathsf{then} \; le_2 \; \mathsf{else} \; le_3 \end{split}$$

The mapping of a function definition is as follows. To make sure that programmers do not have to drastically change their programs, the mapping takes care of relabeling so the parameter can be used at its original type inside the function body. Similarly, the function body is relabeled from the original type to the annotated type.

$$\begin{split} & \langle\!\langle a_1\rangle\!\rangle = b_1\;\rho_1 \qquad \langle\!\langle a_2\rangle\!\rangle = b_2\;\rho_2\\ & D_a;F_a;\Gamma_a;b_2\;\rho_2 \vdash \langle\!\langle e[y/x]\rangle\!\rangle \Rightarrow le\\ \hline & D_a;F_a;\Gamma_a\vdash \langle\!\langle f(x):a_1\to a_2=e\rangle\!\rangle = \qquad f(x) = \mathsf{let}\;y:t_1\;U = \mathsf{reLab}(U \Leftarrow \rho_1)\;x\\ & \mathsf{in}\;\mathsf{let}\;z:t_2\;U = le\;\mathsf{in}\;\mathsf{reLab}(\rho_2 \Leftarrow U)\;z \end{split}$$

C.2 Translation from *polC* to μ C

 ρ

We have two type translation functions, one that does not take a type definition context as input and the one that does. The reason is that when translating the annotated type definition context, we need to generate new type definitions that are unknown at the time of translation, which are mapped to ? as a result. $\llbracket t \rrbracket = (\tau, D_{\Delta}) \boxed{\llbracket s \rrbracket = (\tau, D_{\Delta})}$

$$\begin{array}{ll} \displaystyle \frac{\rho \in \{U, \bot\}}{\llbracket T \ \rho \rrbracket = (T, \cdot)} & \displaystyle \frac{\rho \notin \{U, \bot\}}{\llbracket T \ \rho \rrbracket = (T', T' \mapsto T?)} & \displaystyle \frac{\rho \in \{U, \bot\}}{\llbracket \operatorname{int} \ \rho \rrbracket = (\operatorname{int}, \cdot)} \\ \\ \displaystyle \frac{\rho \notin \{U, \bot\}}{\llbracket \operatorname{int} \ \rho \rrbracket = (T, T \mapsto \operatorname{struct} T \ \{\operatorname{int}\})} & \displaystyle \frac{\rho \in \{U, \bot\}}{\llbracket \operatorname{int} \ \rho \rrbracket = (\operatorname{int}, \cdot)} \\ \\ \displaystyle \frac{\varphi \notin \{U, \bot\}}{\llbracket \operatorname{int} \ \rho \rrbracket = (T, T \mapsto \operatorname{struct} T \ \{\operatorname{int}\})} & \displaystyle \frac{\rho \in \{U, \bot\}}{\llbracket \operatorname{int} \ \rho \rrbracket = (\operatorname{int}, \cdot)} \\ \\ \displaystyle \frac{\varphi \notin \{U, \bot\}}{\llbracket \operatorname{int} \ \rho \rrbracket = (\operatorname{int}, \cdot) + \operatorname{struct} T \ \operatorname{int}\})} & \displaystyle \frac{\varphi \in \{U, \bot\}}{\llbracket \operatorname{int} \ \rho \rrbracket = (\operatorname{int}, \cdot)} \\ \\ \displaystyle \frac{\varphi \notin \{U, \bot\}}{\llbracket \operatorname{int} \ \rho \rrbracket = (\operatorname{int}, \cdot) + \operatorname{struct} T \ \operatorname{int}\}} & \displaystyle \frac{\varphi \in \{U, \bot\}}{\llbracket \operatorname{int} \ \rho \rrbracket = (\operatorname{int}, \cdot)} \\ \\ \displaystyle \frac{\varphi \notin \{U, \bot\}}{\llbracket \operatorname{int} \ \rho \rrbracket = (\operatorname{int}, \cdot) + \operatorname{struct} T \ \operatorname{int}\}} & \displaystyle \frac{\varphi \in \{U, \bot\}}{\llbracket \operatorname{int} \ \rho \rrbracket = (\operatorname{int}, \cdot)} \\ \\ \displaystyle \frac{\varphi \notin \{U, \bot\}}{\llbracket \operatorname{int} \ \rho \rrbracket = (\operatorname{int}, \cdot) + \operatorname{struct} T \ \operatorname{int} \ \rho \rrbracket = (\tau, D')} \\ \\ \displaystyle \frac{\varphi \oplus \{U, \bot\}}{\llbracket \operatorname{int} \ \rho \rrbracket = (\operatorname{int}, \cdot) + \operatorname{struct} T \ \operatorname{int} \ \rho \rrbracket = (\tau, D')} \\ \\ \displaystyle \frac{\varphi \oplus \{U, \bot\}}{\llbracket \operatorname{int} \ \rho \rrbracket = (\operatorname{int}, \cdot) + \operatorname{struct} T \ \operatorname{int} \ \rho \rrbracket = (\tau, D')} \\ \\ \displaystyle \frac{\varphi \oplus \{U, \bot\}}{\llbracket \operatorname{int} \ \rho \rrbracket = (\tau, D_i) + \operatorname{int} \ \rho \rrbracket = (\tau, D_i)} \\ \\ \displaystyle \frac{\varphi \oplus \{U, \bot\}}{\llbracket \operatorname{int} \ \rho \rrbracket = (\tau, D_i) + \operatorname{int} \ \rho \rrbracket = (\tau, D_i) + \operatorname{int} \ \rho \rrbracket = (\tau, D_i) + \operatorname{int} \ \rho \amalg \to \tau_2, D_1 \cup D_2)} \\ \end{array}$$

Translating the annotated type definition context needs two steps. The first step generates new type definitions, which are not filled as they themselves are being translated. In the second step, we fill these undefined type definitions using the translated type definition context. $\boxed{[D] = (D'; D_{\Delta})}$

$$\begin{array}{l} \forall i \in [1,k], \llbracket t_i \rrbracket = (\tau_i, D_i) \qquad [D] = (D'; D_\Delta) \\ \hline \hline [0,T \mapsto \text{struct } T \ \{t_1, \cdots, t_k\}] = (D', T \mapsto \text{struct } T \ \{\tau_1 \cdots, \tau_k\}; \cup_{i=1}^k D_i \cup \Delta_\Delta) \\ \hline D \vdash \texttt{fill}(D_1) = D_2 \end{array}$$

 $\overline{D \vdash \texttt{fill}(\cdot) = \cdot}$

$$\overline{D} \vdash \texttt{fill}(D_1, T \mapsto \texttt{struct} \ T \ \{\pi_1, \cdots, \pi_n\}) = D \vdash \texttt{fill}(D_1), T \mapsto \texttt{struct} \ T \ \{\pi_1, \cdots, \pi_n\}$$
$$D(T) = \texttt{struct} \ T \ \{\pi_1, \cdots, \pi_n\}$$

$$D(T) = \text{struct } T \{\pi_1, \cdots, \pi_n\}$$
$$D \vdash \text{fill}(D_1, T' \mapsto \text{struct } T?) = D \vdash \text{fill}(D_1), T' \mapsto \text{struct } T' \{\pi_1, \cdots, \pi_n\}$$

$$\frac{[D] = (D'; D_{\Delta}) \qquad D' \vdash \texttt{fill}(D_{\Delta}) = D''}{\llbracket D \rrbracket = D', D''}$$

C.3 Correctness of the Translation

We present definitions, lemmas, and proofs for the correctness of our translation algorithm.

$$\begin{array}{c} \hline \hline \mathbf{noBot}(\mathsf{unit}) & \frac{\rho \neq \bot}{\mathsf{noBot}(\mathsf{int}\ \rho)} & \frac{\mathsf{noBot}(s) \quad \rho \neq \bot}{\mathsf{noBot}(\mathsf{ptr}(s)\ \rho)} & \frac{\rho \neq \bot}{\mathsf{noBot}(T\ \mathsf{at}\ \rho)} \\ \\ \hline \frac{\forall i \in [1,2], \mathsf{noBot}(t_i)}{\mathsf{noBot}([\bot](t_1 \to t_2)^{\bot})} & \frac{\forall i \in [1,2], \mathsf{noBot}(t_i)}{\mathsf{noBot}((\mathsf{d\&e})[\bot](t_1 \to t_2)^{\bot})} \end{array}$$

Lemma 32 (Translation Pre-image Unique). If noBot(s) and noBot(s') and fst([[s]]) = fst([[s']]) or $fst([[s]]_D) = fst([[s']]_D)$ then s = s'.

Proof (sketch): By induction over the structure of s.

$$[\![le]\!]=e$$

$$\begin{array}{ll} \frac{\rho \in \{U, \bot\}}{\|\mathbb{R}^{(0)}(\mathbb{R})\|_{D}^{-} = (n, \cdot)} \ \mathrm{T-INT} & \begin{array}{ll} \frac{\rho \notin \{U, \bot\} & [\![\mathrm{int} \rho]\!]_{D} = (T, D')}{\|\mathbb{R}^{(0)}(\mathbb{R})\|_{D}^{-} = (n, \cdot)} \ \mathrm{T-INT-Pol} \\ \frac{\forall i \in [1, n], [\![lv_{i}]\!]_{D} = (v_{i}, D_{i}) & [\![T \rho]\!]_{D} = (T', D')}{\|[T \rho]\!]_{D} = (T', D') \ \mathrm{T-STRUCT} & [\![lv]\!]_{D} = (v, D') \\ \hline \|[v, i]\!]_{D} = (v, D') \\ \hline \|[U^{(1)}\}_{U^{(1)}}^{-} \cdots , v_{n}\}_{U^{(1)}}^{-} D_{i}^{-} D_{i}^{-} D_{i}^{-} D_{i}^{-} \\ \hline \\ \frac{\|[u^{(1)}]_{D} = (v, D') \\ \hline \|[v, i]\!]_{D} = (v, D') \\ \hline \|[v, i]\!]_{D} = (v, D') \\ \hline \|[v, i]\!]_{D} = (v, D') \\ \hline \\ \frac{\|[u^{(1)}]_{D} = (e, D') \\ \hline \\ p \notin \{U, \bot\} \\ \hline \\ \|[v, i]\!]_{D} = (v, p) \\ \hline \\ \frac{\|[v^{(1)}]_{D} = (v, p) \\ \hline \\ \|[v, i]\!]_{D} = (v, p) \\ \hline \\ \frac{\|[v^{(1)}]_{D} = (v, p) \\ \hline \\ \|[v, i]\!]_{D} = (v, p) \\ \hline \\ \frac{\|[v^{(1)}]_{D} = (v, p) \\ \hline \\ \|[v^{(1)}]_{D} = (v, p) \\ \hline \\ \frac{\|[v^{(1)}]_{D} = (v, p) \\ \hline \\ \|[v^{(1)}]_{D} = (v, p) \\ \hline \\ \frac{\|[v^{(1)}]_{D} = (v, p) \\ \hline \\ \|[v^{(1)}]_{D} = (v, p) \\ \hline \\ \frac{\|[v^{(1)}]_{D} = (v, p) \\ \hline \\ \|[v^{(1)}]_{D} = (v, p) \\ \hline \\ \frac{\|[v^{(1)}]_{D} = (v, p) \\ \hline \\ \|[v^{(1)}]_{D} = (v, p) \\ \hline \\ \frac{\|[v^{(1)}]_{D} = (v, p) \\ \hline \\ \|[v^{(1)}]_{D} = (v, p) \\ \hline \\ \frac{\|[v^{(1)}]_{D} = (v, p) \\ \hline \\ \frac{\|[v^{(1)}]_{$$

Figure 13: Translation Rules

Lemma 33. If β does not include \perp , then $noBot(\langle\!\langle \beta \rangle\!\rangle)$.

Proof (sketch): By induction over the structure of β . The translation rules does not insert \perp except for functions.

Lemma 34 (Value Translation Soundness). If $\mathcal{E} :: D_a; F_a; \Gamma_a \vdash \langle \! \langle v \rangle \! \rangle = lv$, tpOf(lv) = s, $\langle \! \langle D_a \rangle \! \rangle = D_l$, $\langle \! \langle F_a \rangle \! \rangle = F_l$, $\langle \! \langle \Gamma_a \rangle \! \rangle = \Gamma_l$, $\llbracket D_l \rrbracket = D$, $\llbracket \Gamma_l \rrbracket_D = (\Gamma, D_1)$, $\llbracket F_l \rrbracket_D = (F, D_2)$, $\llbracket lv \rrbracket_D = (v', D_3)$, and $D \cup D_1 \cup D_2 \cup D_3; F; : \Gamma \vdash v' : \tau$ implies $D_l; F_l; : \Gamma_l \vdash tmOf(lv) : s$ and $\llbracket s \rrbracket = (\tau, _)$.

Proof. By induction over the structure of \mathcal{E} .

Case: \mathcal{E} ends in V-L-INT rule.

By assumption:

$$\begin{split} \|v\|_{D} &= (v, D_{1}) & tpOf(lv) = b \\ \frac{b \text{ is not a struct type } \rho' \notin \{\bot, U\} & \rho \notin \{\bot, U\} & \|b \rho'\|_{D} = (T, D_{2}) \\ \hline \|reLab(\rho' \Leftrightarrow \rho)lv\|_{D} &= (\text{let } x = v.1 \text{ in } (T)\{x\}, D_{1} \cup D_{2}) \\ \hline \|lv\|_{D} &= (v, D_{1}) & tpOf(lv) = b \\ \frac{b \text{ is not a struct type } \rho' \notin \{\bot, U\} & \rho \in \{\bot, U\} & \|b \rho'\|_{D} = (T, D_{2}) \\ \hline \|reLab(\rho' \Leftrightarrow \rho)lv\|_{D} &= (T, D_{2}) \\ \hline \|reLab(\rho' \between \rho)lv\|_{D} &= (T, D_{2})$$

$$\llbracket \mathsf{reLab}(\rho' \Leftarrow \rho) lv \rrbracket_D = ((T)\{v\}, D_1 \cup D_2)$$

$$\begin{split} \underline{\llbracket lv \rrbracket_D = (v, D_1) & tpOf(lv) = b \ \rho & b \ \text{is not a struct type} \quad \rho \notin \{\bot, U\} \quad \rho' \in \{\bot, U\} \\ \underline{\llbracket reLab(\rho' \Leftarrow \rho)lv \rrbracket_D = (v.1D_1)} & \\ \\ \underline{\llbracket lv \rrbracket_D = (v, D_1) & labOf(lv) = b \ \rho & \rho, \rho' \in \{U, \bot\} \\ \underline{\llbracket reLab(\rho' \Leftarrow \rho)lv \rrbracket_D = (v, D_1)} & \\ \\ T-ReLAB-SAME & \\ \\ \underline{\llbracket reLab(\rho' \Leftarrow \rho)lv \rrbracket_D = (v, D_1)} & \\ \\ \underline{reLab(\rho' \Leftarrow \rho)lv \rrbracket_D = (T', D_1) & [\llbracket lv \rrbracket_D = (v, D_2) \\ \underline{\llbracket reLab(\rho' \Leftarrow \rho)lv \rrbracket_D = (et \ x_1 = v.1 \ \text{in } \cdots \text{let} \ x_n = v.n \ \text{in } (T')\{x_1, \cdots, x_n\}, D_1 \cup D_2)} & \\ \\ \\ \end{array}$$

Figure 14: Translation Rules

(1) $D_a; F_a; \Gamma_a \vdash \langle \langle n \rangle \rangle \Rightarrow n @int U$ By examining the translation rules, only T-INT applies (2) $[\![n@int U]\!]_D = (n, \cdot),$ By typing rules (3) $D \cup D_1 \cup D_2; F; \cdot; \Gamma \vdash n : int$ By typing rule V-INT (4) $D_l; F_l; \cdot; \Gamma_l \vdash n : int U$ By type translation (5) $[\![int U]\!]_D = (int, .)$

Case: \mathcal{E} ends in V-L-VAR rule.

By assumption: (1) $D_a; F_a; \Gamma_a \vdash \langle\!\langle x \rangle\!\rangle \Rightarrow x @\langle\!\langle \beta \rangle\!\rangle$ and $\Gamma_a(x) = \beta$ By examining the translation rules, only T-VAR applies (2) $[\![x @\langle\!\langle \beta \rangle\!\rangle]\!]_D = (x, \cdot),$ By typing rules (3) $D \cup D_1 \cup D_2; F; \cdot; \Gamma \vdash x : \Gamma(x)$ By typing rule V-VAR (4) $D_l; F_l; \cdot; \Gamma_l \vdash x : \Gamma_l(x)$ By assumption that $\langle\!\langle \Gamma_a \rangle\!\rangle = \Gamma_l$ and $[\![\Gamma_l]\!]_D = (\Gamma, D_1)$ (5) $[\![\langle\!\langle \beta \rangle\!\rangle]\!]_D = [\![\langle\!\langle \Gamma_a(x) \rangle\!\rangle]\!]_D = [\![\Gamma_l(x)]\!]_D = (\Gamma(x), .)$

Case: \mathcal{E} ends in V-L-FUN rule.

This case can be proved similarly as the previous case.

Case: \mathcal{E} ends in V-L-STRUCT rule.

By assumption: $D_a; F_a; \Gamma_a \vdash \langle\!\langle (T) \ \{v_1, \cdots, v_n\} \rangle\!\rangle \Rightarrow (T) \{lv_1, \cdots, lv_n\} @T U$ (1)and $\forall i \in [1, n], \mathcal{E}_i :: D_a; F_a; \Gamma_a \vdash \langle\!\langle v_i \rangle\!\rangle \Rightarrow lv_i$ (2)By examining the translation rules, only T-STRUCT applies $[\![(T)\{lv_1,\cdots,lv_n\}] @ (T \ U)]\!]_D = ((T')\{v'_1,\cdots,v'_n\},D_3)$ (3)and $[\![lv_i]\!]_D = (v'_i, D_{3i})$ and $[\![T \ U]\!] = (T', D_{3ii})$, and $D_3 = D_{3i} \cup D_{3ii}$ (4)By type translation rules (5) T' = T and $D_{3ii} = \cdot$ By typing rules (6) $D \cup D_1 \cup D_2 \cup D_3; F; \cdot; \Gamma \vdash (T)\{v'_1, \cdots, v'_n\}: T$ and $(D \cup D_1 \cup D_2 \cup D_3)(T) =$ struct $T\{\tau_1, \cdots, \tau_n\},\$ (7)(8)and $D \cup D_1 \cup D_2 \cup D_3; F; \cdot; \Gamma \vdash v'_i : \tau_i$ By $T \in \operatorname{dom}(D_a)$ (9) $T \in \operatorname{dom}(D)$ By I.H. on \mathcal{E}_i (10) $D_l; F_l; \cdot; \Gamma_l \vdash tmOf(lv_i) : s_i \text{ and } [\![s_i]\!]_D = (\tau_i, _)$ By well-formedness constraints, (11) $D_l(T) =$ struct $T\{s'_1, \cdots, s'_n\}$ By $\llbracket D_l \rrbracket = D$ and $D(T) = \text{struct } T\{\tau_1, \cdots, \tau_n\}$ (12) $[\![s'_i]\!] = \tau_i$ By Lemma 32, (10) and (12) (13) $s_i = s'_i$ By V-STRUCT and (10) and (13)(14) $D_l; F_l; \cdot; \Gamma_l \vdash tmOf((T)\{lv_1, \cdots, lv_n\}@(T \ U)): T \ U$ By type translation rules (15) $[T \ U]_D = (T, .)$

Lemma 35 (Relabel translation is sound). If $tpOf(lv) = b \rho$, $e = \texttt{fst}(\llbracket \texttt{reLab}(\rho' \leftarrow \rho) lv \rrbracket_D)$, $v = \texttt{fst}(\llbracket lv \rrbracket_D)$, and $D', D; F; \Gamma \vdash v : \texttt{fst}(\llbracket b \rho \rrbracket_D)$ then $D; F; \Gamma \vdash e : \texttt{fst}(\llbracket b \rho' \rrbracket_D)$.

Proof (sketch): By examining the translation rules for $[[reLab(\rho' \leftarrow \rho)lv]]_D$.

Lemma 36. Given $e = \texttt{fst}(\llbracket \texttt{reLab}(\rho' \leftarrow \rho) lv \rrbracket_D)$, then $D, D'; F; \Gamma \vdash e : \tau$ implies exists τ' s.t. $D, D'; F; \Gamma \vdash \texttt{fst}(\llbracket lv \rrbracket_D) : \tau'$.

Proof (sketch): By examining the translation rules for $[[reLab(\rho' \leftarrow \rho) lv]]_D$.

Theorem 37 (Expression Translation Soundness). If $\mathcal{E} :: D_a; F_a; \Gamma_a; s \vdash \langle\!\langle e \rangle\!\rangle = le, \langle\!\langle D_a \rangle\!\rangle = D_l, \langle\!\langle F_a \rangle\!\rangle = F_l, \langle\!\langle \Gamma_a \rangle\!\rangle = \Gamma_l, [\![D_l]\!] = D, [\![\Gamma_l]\!]_D = (\Gamma, D_1), [\![F_l]\!]_D = (F, D_2), [\![le]\!]_D = (e', D_3), and D \cup D_1 \cup D_2 \cup D_3; F; \cdot; \Gamma \vdash e': \tau implies D_l; F_l; \cdot; \Gamma_l \vdash tmOf(le) : s and [\![s]\!] = (\tau, _)$

Proof. By induction over the structure of \mathcal{E} .

Case: \mathcal{E} ends in L-INT rule.

By assumption: $D_a; F_a; \Gamma_a; s \vdash \langle\!\langle n \rangle\!\rangle = n@s$ (1)By examining the translation rules, there are two subcases subcase i. T-INT applies (i2) $s = \text{int } U \text{ and } \llbracket n @ \text{int } U \rrbracket_D = (n, \cdot),$ By typing rules (i3) $D \cup D_1 \cup D_2; F; \cdot; \Gamma \vdash n : int$ By typing rule P-T-V-INT and P-T-E-VAL (i4) $D_l; F_l; \cdot; \Gamma_l; \perp \vdash n : int U$ By type translation (i5) $\llbracket \operatorname{int} U \rrbracket_D = (\operatorname{int}, _)$ subcase ii. T-INT-POL applies (ii2) $s = \text{int } \rho \text{ and } [\![n@int \ \rho]\!]_D = ((T)\{n\}, D_e), [\![int \ \rho]\!]_D = (T, D_3)$ By (ii2) and type translation rules (ii3) $D_3 = T \mapsto \text{struct } T\{\text{int}\}$ By typing rules (ii4) $D \cup D_1 \cup D_2 \cup D_3; F; \cdot; \Gamma \vdash (T)\{n\} : T$ By typing rule P-T-V-INT and P-T-E-VAL (ii5) $D_l; F_l; \cdot; \Gamma_l; \perp \vdash n : int \rho$ By type translation (ii6) $[[int \rho]]_D = (T, _)$

Case: \mathcal{E} ends in L-VAR rule.

By assumption: (1) $D_a; F_a; \Gamma_a; s \vdash \langle \! \langle x \rangle \! \rangle \Rightarrow x@s \text{ and } \langle \! \langle \Gamma_a(x) \rangle \! \rangle = s$ By examining the translation rules, only T-VAR applies (2) $[\! [x@s]\!]_D = (x, \cdot),$ By typing rules (3) $D \cup D_1 \cup D_2; F; \cdot; \Gamma \vdash x : \Gamma(x)$ By typing rule P-T-V-VAR and P-T-E-VAL (4) $D_l; F_l; \cdot; \Gamma_l; \bot \vdash x : \Gamma_l(x)$ By assumption that $\langle \! \langle \Gamma_a \rangle \! \rangle = \Gamma_l$ and $[\! [\Gamma_l]\!]_D = (\Gamma, D_1)$ (5) $[\! [s]\!]_D = [\! \langle \! \langle \Gamma_a(x) \rangle \! \rangle \!]_D = [\! [\Gamma_l(x)]\!]_D = (\Gamma(x), -)$

Case: \mathcal{E} ends in L-STRUCT rule.

By assumption:

(1) $D_a; F_a; \Gamma_a; T \ \rho \vdash \langle\!\langle (T) \ \{v_1, \cdots, v_n\} \rangle\!\rangle \Rightarrow (T) \{ lv_1, \cdots, lv_n \} @T \ \rho$

(2) $D_a(T) = \text{struct } T\{\beta_1, \cdots, \beta_n\}$

(3) and $\forall i \in [1, n], \mathcal{E}_i :: D_a; F_a; \Gamma_a; \langle\!\langle \beta_i \rangle\!\rangle \vdash \langle\!\langle v_i \rangle\!\rangle \Rightarrow lv_i$

By examining the translation rules, only T-STRUCT applies

(4) $\llbracket (T)\{lv_1, \cdots, lv_n\} @ (T \rho) \rrbracket_D = ((T')\{v'_1, \cdots, v'_n\}, D_3)$

- (5) and $[\![lv_i]\!]_D = (v'_i, D_{3i})$ and $[\![T \ \rho]\!]_D = (T', D')$
- By type translation rules

(6) $T' = genName(T, \rho)$ and $D' = T' \mapsto \text{struct } T'\{\tau_1, \cdots, \tau_n\}$ and $D(T) = \text{struct } T\{\tau_1, \cdots, \tau_n\}$ By typing rules

(7) $D \cup D_1 \cup D_2 \cup D_3; F; \cdot; \Gamma \vdash (T') \{v'_1, \cdots, v'_n\} : T'$

(8) and $(D \cup D_1 \cup D_2 \cup D_3)(T') =$ struct $T'\{\tau_1, \cdots, \tau_n\},$

(9) and $D \cup D_1 \cup D_2 \cup D_3$; $F; \cdot; \Gamma \vdash v'_i : \tau_i$ By I.H. on \mathcal{E}_i (10) $D_l; F_l; \cdot; \Gamma_l; \bot \vdash tmOf(lv_i) : \langle\!\langle \beta_i \rangle\!\rangle$ and $[\![\langle\!\langle \beta_i \rangle\!\rangle]\!]_D = (\tau_i, _)$ By P-T-V-STRUCT and P-T-E-VAL and (6) and (10) (11) $D_l; F_l; \cdot; \Gamma_l; \bot \vdash tmOf((T)\{lv_1, \cdots, lv_n\}@(T \ \rho)) : T \ \rho$ By (5) (12) $[\![T \ \rho]\!]_D = (T', _)$

Case: \mathcal{E} ends in L-FIELD-U rule.

By assumption:

(1) $D_a; F_a; \Gamma_a; s \vdash \langle\!\langle v.i \rangle\!\rangle \Rightarrow lv.i$ $\mathcal{E}' :: D_a; F_a; \Gamma_a \vdash \langle\!\!\langle v \rangle\!\!\rangle \Rightarrow lv \text{ and } tpOf(lv) = T \ \rho$ (2)(3)and $D_a(T) = ($ struct $T\{\beta_1, \cdots, \beta_n\})$ and $\forall i \in [1, n], \rho = labOf(\langle\!\langle \beta_i \rangle\!\rangle)$ (4)By examining the translation rules, only T-FIELD applies (5) $[[lv.i]]_D = (v'.i, D_3)$ and $[[lv]]_D = (v', D_3)$ By assumption and typing rules (6) $D \cup D_1 \cup D_2 \cup D_3; F; \cdot; \Gamma \vdash v'.i : \tau_i$ By inversion of (6)(7) $D \cup D_1 \cup D_2 \cup D_3; F; \cdot; \Gamma \vdash v': T'$ and $(D \cup D_1 \cup D_2 \cup D_3)(T') =$ struct $T'\{\tau_1, \cdots, \tau_n\}$ (8)By Lemma 34 on \mathcal{E}' , (5) and (8) (9) $D_l; F_l; \cdot; \Gamma_l \vdash tmOf(lv) : T \rho \text{ and } \llbracket T \rho \rrbracket_D = (T', _)$ By (9), P-T-E-VAL (10) $D_l; F_l; \cdot; \Gamma_l; \perp \vdash tmOf(lv) : T \rho$ By (3) and $\langle\!\langle D_a \rangle\!\rangle = D_l$ (11) $D_l(T) = \text{struct } T \{ \langle \langle \beta_1 \rangle \rangle, \cdots, \langle \langle \beta_n \rangle \rangle \}$ By (10), (11), and P-T-E-FIELD (12) $D_l; F_l; \cdot; \Gamma_l; \perp \vdash tmOf(lv.i) : \langle\!\langle \beta_i \rangle\!\rangle \sqcup \rho$ By (4) and (12)(13) $D_l; F_l; \cdot; \Gamma_l; \perp \vdash tmOf(lv.i) : \langle\!\langle \beta_i \rangle\!\rangle$ By (9) and $\llbracket D_l \rrbracket = D$ (14) $D(T) = \text{struct } T\{\tau_1, \cdots, \tau_n\} \text{ and } D(T') = \text{struct } T'\{\tau_1, \cdots, \tau_n\}$ By (11) and (14)(15) $\llbracket \langle \langle \beta_i \rangle \rangle \rrbracket_D = (\tau_i, \Box)$

Case: \mathcal{E} ends in L-FIELD rule.

By assumption:

(1) $D_a; F_a; \Gamma_a; s \vdash \langle\!\langle v.i \rangle\!\rangle \Rightarrow le \text{ and } le = \text{let } y: T \perp = \text{reLab}(\perp \Leftarrow \rho) lv \text{ in } (y@T \perp).i$ (2) $\mathcal{E}':: D_a; F_a; \Gamma_a \vdash \langle\!\langle v \rangle\!\rangle \Rightarrow lv \text{ and } tpOf(lv) = T \rho$ (3) and $D_a(T) = (\text{struct } T\{\beta_1, \cdots, \beta_n\})$ By examining the translation rules, only T-LET applies (4) $[\![le]\!]_D = (e', D'_3 \cup D''_3) \text{ and } e' = \text{let } y: T = e_1 \text{ in } y.i,$ (5) and $[\![\text{reLab}(\perp \Leftarrow \rho) lv]\!]_D = (e_1, D_3)$ By assumption and typing rules (6) $D \cup D_1 \cup D_2 \cup D_3; F; \cdot; \Gamma \vdash e': \tau_i$ By inversion of (6)

(7) $D \cup D_1 \cup D_2 \cup D_3; F; \cdot; \Gamma \vdash e_1 : T$ $D \cup D_1 \cup D_2 \cup D_3; F; \cdot; \Gamma, y: T \vdash y.i: \tau_i$ and (8)(9) $D_l(T) = \text{struct } T \{ \langle \! \langle \beta_1 \rangle \! \rangle, \cdots, \langle \! \langle \beta_n \rangle \! \rangle \}$ By (9) P-T-E-FIELD (10) $D_l; F_l; \cdot; \Gamma_l, y: T \perp; \perp \vdash y.i: \langle\!\langle \beta_i \rangle\!\rangle$ By (9) and $\llbracket D_l \rrbracket = D$ (11) $D(T) = \text{struct } T\{\tau_1, \cdots, \tau_n\}$ By (9) and (11)(12) $\llbracket \langle \langle \beta_i \rangle \rangle \rrbracket_D = (\tau_i, ...)$ By Lemma 36 and (7)(13) $D \cup D_1 \cup D_2 \cup D_3; F; \cdot; \Gamma \vdash \mathsf{fst}(\llbracket lv \rrbracket_D) : \tau'$ By Lemma 34 on \mathcal{E}' , (5), (7), (13) (14) $D_l; F_l; \cdot; \Gamma_l \vdash tmOf(lv) : T \rho$ By (14), P-T-E-RELAB (15) $D_l; F_l; \cdot; \Gamma_l; \bot \vdash tmOf(\mathsf{reLab}(\bot \Leftarrow \rho) \ lv) : T \perp$ By P-T-E-LET, (15), (10), (16) $D_l; F_l; \cdot; \Gamma_l; \bot \vdash le : \langle\!\langle \beta_i \rangle\!\rangle$

Case: \mathcal{E} ends in L-NEW rule.

By assumption:

 $D_a; F_a; \Gamma_a; (\mathsf{ptr}(s) \ \rho) \vdash \langle\!\langle \mathsf{new}(e) \rangle\!\rangle \Rightarrow \mathsf{new}(le)@(\mathsf{ptr}(s) \ \rho)$ (1)(2) $\mathcal{E}' :: D_a; F_a; \Gamma_a; s \vdash \langle\!\langle e \rangle\!\rangle \Rightarrow le$ By examining the translation rules, there are two subcases Subcase i: T-NEW applies (i3) $\rho \in \{U, \bot\}, [[new(le)@(ptr(s) \rho)]]_D = (new(e'), D_3) \text{ and } [[le]]_D = (e', D_3)$ By assumption and typing rules (i4) $D \cup D_1 \cup D_2 \cup D_3; F; \cdot; \Gamma \vdash \mathsf{new}(e') : \mathsf{ptr}(\tau)$ By inversion of (i4) (i5) $D \cup D_1 \cup D_2 \cup D_3; F; \cdot; \Gamma \vdash e': \tau$ By I.H. on \mathcal{E}' , (i5) and (i3) (i6) $D_l; F_l; \cdot; \Gamma_l; \perp \vdash tmOf(le) : s \text{ and } [s]_D = (\tau,]$ By (i6), P-T-E-NEW $D_l; F_l; \cdot; \Gamma_l; \perp \vdash tmOf(\mathsf{new}(le)) : \mathsf{ptr}(s) \ \rho$ (i7) (i8) $\llbracket \mathsf{ptr}(s) \ \rho \rrbracket_D = (\mathsf{ptr}(\tau), _)$ Subcase ii: T-NEW-POL applies (ii3) $\rho \notin \{U, \bot\}, [[new(le)@(ptr(s) \rho)]]_D = (T\{new(e')\}, D_3)$ and $\llbracket le \rrbracket_D = (e', D_3)$ and $\llbracket \mathsf{ptr}(s) \ \rho \rrbracket = T$ (ii4)By assumption and typing rules (ii5) $D \cup D_1 \cup D_2 \cup D_3; F; \cdot; \Gamma \vdash (T) \{\mathsf{new}(e')\} : T$ By inversion of (i5) $D \cup D_1 \cup D_2 \cup D_3; F; \cdot; \Gamma \vdash e' : \tau$ (ii6)(ii7) and $(D \cup D_1 \cup D_2 \cup D_3)(T) = \text{struct } T\{\text{ptr}(\tau)\} \text{ and } [s]_D = (\tau, _)$ By I.H. on \mathcal{E}' , (ii6) and (ii4) (ii8) $D_l; F_l; \cdot; \Gamma_l; \perp \vdash tmOf(le) : s \text{ and } [s]_D = (\tau, _)$ By (ii8), P-T-E-NEW (ii9) $D_l; F_l; \cdot; \Gamma_l; \perp \vdash tmOf(\mathsf{new}(le)) : \mathsf{ptr}(s) \rho$

Case: \mathcal{E} ends in L-DEREF rule.

By assumption:

(1) $D_a; F_a; \Gamma_a; s \vdash \langle\!\langle *v \rangle\!\rangle \Rightarrow le \text{ and } le = \mathsf{let } y : b \perp = \mathsf{reLab}(\perp \Leftarrow \rho) \ lv \text{ in } *(y@b \perp)$ (2) $\mathcal{E}' :: D_a; F_a; \Gamma_a \vdash \langle\!\langle v \rangle\!\rangle \Rightarrow lv \text{ and } tpOf(lv) = b \rho$ By examining the translation rules, only T-LET applies (3) $[\![le]\!]_D = (e', D'_3 \cup D''_3)$ and $e' = \text{let } y : b = e_1 \text{ in } * y$, (4) and $\llbracket \mathsf{reLab}(\bot \leftarrow \rho) \ lv \rrbracket_D = (e_1, D_3)$ By assumption and typing rules (5) $D \cup D_1 \cup D_2 \cup D_3; F; \cdot; \Gamma \vdash e': \tau$ By inversion of (5)(6) $D \cup D_1 \cup D_2 \cup D_3; F; \cdot; \Gamma \vdash e_1 : \tau_y$ (7) $\tau_y = \mathsf{ptr}(\tau), \text{ and } [\![b]\!]_D = (\tau_y, ...)$ (8) $D \cup D_1 \cup D_2 \cup D_3; F; \cdot; \Gamma, y : \tau_y \vdash *y : \tau$ By (7)(9) $b = ptr(s) \text{ and } [s]_D = (\tau, .)$ By P-T-E-Deref (10) $D_l; F_l; \cdot; \Gamma_l, y: b \perp; \perp \vdash *y: s$ By Lemma 36, (4) and (6)(13) $D \cup D_1 \cup D_2 \cup D_3; F; \cdot; \Gamma \vdash \mathsf{fst}(\llbracket lv \rrbracket_D) : \tau'$ By Lemma 34 on \mathcal{E}' , (5), (7), and (13) (14) $D_l; F_l; \cdot; \Gamma_l \vdash tmOf(lv) : b \rho$ By (14), P-T-E-RELAB (15) $D_l; F_l; \cdot; \Gamma_l; \bot \vdash tmOf(\mathsf{reLab}(\bot \Leftarrow \rho) \ lv) : b \perp$ By P-T-E-LET, (15),(9) (10), (16) $D_l; F_l; \cdot; \Gamma_l; \bot \vdash le : s$

Case: \mathcal{E} ends in L-Assign rule.

By assumption:

by assumption.
(1)
$$D_a; F_a; \Gamma_a; s \vdash \langle\!\langle v := e \rangle\!\rangle \Rightarrow le$$

and $le = \text{let } y : \text{ptr}(s) \perp = \text{reLab}(\perp \Leftarrow \rho) lv \text{ in } y@\text{ptr}(s)$.
(2) $\mathcal{E}' :: D_a; F_a; \Gamma_a \vdash \langle\!\langle v \rangle\!\rangle \Rightarrow lv \text{ and } tpOf(lv) = \text{ptr}(s) \rho$
(3) $\mathcal{E}'' :: D_a; F_a; \Gamma_a; s \vdash e \Rightarrow le_2$
By examining the translation rules, only T-LET applies
(4) $[\![le]\!]_D = (e', D'_3 \cup D''_3) \text{ and } e' = \text{let } y : \tau_y = e_1 \text{ in } y := e_2,$
(5) and $[\![reLab(\perp \Leftarrow \rho) lv]\!]_D = (e_1, D'_3)$
(6) and $[\![le_2]\!]_D = (e_2, D''_3),$
(7) and $[\![ptr(s) \perp]\!]_D = (\tau_y, .),$
By assumption and typing rules
(8) $D \cup D_1 \cup D_2 \cup D_3; F; \cdot; \Gamma \vdash e' : \tau'$
By inversion of (7)
(9) $D \cup D_1 \cup D_2 \cup D_3; F; \cdot; \Gamma \vdash e_1 : \tau_y$
(10) $\tau_y = \text{ptr}(\tau), \text{ and } \tau' = \text{unit},$
(11) $D \cup D_1 \cup D_2 \cup D_3; F; \cdot; \Gamma, y : \tau_y \vdash e_2 : \tau$
By (7) and (10)
(12) $[\![s]\!]_D = (\tau, .)$
By I.H. on $\mathcal{E}'', (3), (6), (11)$
(13) $D_l; F_l; \cdot; \Gamma_l; \bot \vdash tmOf(le_2) : s$
By T-ASSIGN, (13),
(14) $D_l; F_l; \cdot; \Gamma_l, y : \tau_y \perp; \bot \vdash y@\text{ptr}(s) \bot := le_2 : \text{unit}$

 $\perp := le_2$

By Lemma 36, (5) and (9) (15) $D \cup D_1 \cup D_2 \cup D_3; F; \cdot; \Gamma \vdash fst([\![lv]\!]_D) : \tau'$ By Lemma 34 on \mathcal{E}' , (2), (15), (16) $D_l; F_l; \cdot; \Gamma_l \vdash tmOf(lv) : ptr(s) \rho$ By (16), P-T-E-RELAB (17) $D_l; F_l; \cdot; \Gamma_l; \bot \vdash tmOf(reLab(\bot \Leftarrow \rho) \ lv) : ptr(s) \bot$ By P-T-E-LET, (17) (14), (18) $D_l; F_l; \cdot; \Gamma_l; \bot \vdash le : unit$

Case: \mathcal{E} ends in L-IF rule.

By assumption:

(1) $D_a; F_a; \Gamma_a; s \vdash \langle\!\langle \text{if } v_1 \text{ then } e_2 \text{ else } e_3 \rangle\!\rangle \Rightarrow le$ and $le = \text{let } x : \text{int } \perp = (\text{reLab}(\perp \leftarrow \rho) \ lv_1)$ in if x@int \perp then le_2 else le_3 $\mathcal{E}' :: D_a; F_a; \Gamma_a \vdash \langle\!\langle v \rangle\!\rangle \Rightarrow lv \text{ and } tpOf(lv) = \mathsf{int } \rho$ (2) $\mathcal{E}'' :: D_a; F_a; \Gamma_a; s \vdash \langle\!\!\langle e_2 \rangle\!\!\rangle \Rightarrow le_2$ (3) $\mathcal{E}''' :: D_a; F_a; \Gamma_a; s \vdash \langle\!\langle e_3 \rangle\!\rangle \Rightarrow le_3$ (4)By examining the translation rules, only T-LET applies $[[le]]_D = (e', D'_3 \cup D''_3)$ and $e' = \text{let } x : \text{int} = e_1 \text{ in if } x \text{ then } e'_2 \text{ else } e'_3$, (5)(6)and $\llbracket \mathsf{reLab}(\bot \Leftarrow \rho) \ lv \rrbracket_D = (e_1, D'_3)$ (7)and $[\![le_2]\!]_D = (e'_2, D''_3)$, and $[\![le_3]\!]_D = (e'_3, D''_3)$, By assumption and typing rules $D \cup D_1 \cup D_2 \cup D_3; F; \cdot; \Gamma \vdash e': \tau$ (8)By inversion of (8)(9) $D \cup D_1 \cup D_2 \cup D_3; F; \cdot; \Gamma \vdash e_1 : int$ (10) $D \cup D_1 \cup D_2 \cup D_3; F; \cdot; \Gamma, x: \mathsf{int} \vdash e'_2: \tau$ (11) $D \cup D_1 \cup D_2 \cup D_3; F; \cdot; \Gamma, x: \mathsf{int} \vdash e'_3: \tau$ By I.H. on \mathcal{E}'' , (3), (7), (10) (12) $D_l; F_l; \cdot; \Gamma_l; \perp \vdash tmOf(le_2) : s \text{ and } \llbracket s \rrbracket_D = (\tau, _)$ By I.H. on \mathcal{E}''' , (4), (7), (11) (13) $D_l; F_l; \cdot; \Gamma_l; \perp \vdash tmOf(le_3): s$ By T-IF, (12), (13) (14) $D_l; F_l; \cdot; \Gamma_l, x: \text{int } \perp; \perp \vdash tmOf(\text{if } x@\text{int } \perp \text{ then } le_2 \text{ else } le_3): s$ By (5) there are two subcases Subcase $\rho \in \{\perp, U\}$ **T-RELAB-SAME** applies (i1) $e_1 = v_1$ and $[[lv]]_D = (v_1, D_3)$ By Lemma 34 on \mathcal{E}' , (6), and (i11) (i2) $D_l; F_l; \cdot; \Gamma_l \vdash tmOf(lv) : int \rho$ By (i2), P-T-E-VAL (i3) $D_l; F_l; \cdot; \Gamma_l; \perp \vdash tmOf(lv) : int \rho$ (i4) $D_l; F_l; \cdot; \Gamma_l; \perp \vdash tmOf(\mathsf{reLab}(\perp \Leftarrow \rho) \ lv) : \mathsf{int} \perp$ By P-T-E-LET, (i4), (15), (i5) $D_l; F_l; \cdot; \Gamma_l; \bot \vdash le : s$ Subcase $\rho \notin \{\bot, U\}$ **T-RELAB-N3** applies (ii1) $e_1 = v'.1$ and $[\![v]\!]_D = (v', D_3)$ By inversion of (9)(ii2) $D \cup D_1 \cup D_2 \cup D_3; F; \cdot; \Gamma \vdash v': T',$

(ii3) $(D \cup D_1 \cup D_2 \cup D_3)(T') = \text{struct } T'\{\text{int}\}$ By Lemma 34 on \mathcal{E}' , (2), (ii3), and (ii1) (ii4) $D_l; F_l; \cdot; \Gamma_l \vdash tmOf(lv) : \text{int } \rho \text{ and } \llbracket \text{ptr}(s) \rho \rrbracket_D = (T', D'_3)$ By (ii4), P-T-E-VAL (ii5) $D_l; F_l; \cdot; \Gamma_l; \bot \vdash tmOf(lv) : \text{int } \rho$ (ii6) $D_l; F_l; \cdot; \Gamma_l; \bot \vdash tmOf(\text{reLab}(\bot \Leftarrow \rho) \ lv) : \text{int } \bot$ By P-T-E-LET, (ii6) (15), (ii7) $D_l; F_l; \cdot; \Gamma_l; \bot \vdash le : s$

Case: \mathcal{E} ends in L-LET rule.

By assumption:

 $D_a; F_a; \Gamma_a; s \vdash \langle\!\langle \mathsf{let} \ x : \beta_1 = e_1 \ \mathsf{in} \ e_2 \rangle\!\rangle \Rightarrow le$ (1)and $le = \text{let } x : \langle\!\langle \beta_1 \rangle\!\rangle = le_1 \text{ in } le_2$ $\mathcal{E}' :: D_a; F_a; \Gamma_a; \langle\!\langle \beta_1 \rangle\!\rangle \vdash \langle\!\langle e_1 \rangle\!\rangle \Rightarrow le_1$ (2) $\mathcal{E}'' :: D_a; F_a; \Gamma_a, x : \beta_1; t_2 \vdash \langle\!\langle e_2 \rangle\!\rangle \Rightarrow le_2$ (3)By examining the translation rules, only T-LET applies $\llbracket le \rrbracket_D = (e', D'_3 \cup D''_3)$ and $e' = \text{let } x : \tau = e'_1 \text{ in } e'_2$ (4)(5) $\llbracket \langle \langle \beta_1 \rangle \rangle \rrbracket_D = (\tau, _)$ (6) and $[\![le_1]\!]_D = (e'_1, D'_3)$, and $[\![le_2]\!]_D = (e'_2, D''_3)$, By assumption and typing rules (7) $D \cup D_1 \cup D_2 \cup D_3; F; \cdot; \Gamma \vdash e': \tau'$ By inversion of (7)(8) $D \cup D_1 \cup D_2 \cup D_3; F; \cdot; \Gamma \vdash e'_1 : \tau$ $(9) \quad D \cup D_1 \cup D_2 \cup D_3; F; \cdot; \Gamma, x : \tau \vdash e'_2 : \tau'$ By I.H. on \mathcal{E}' , (2), (6), (8) (10) $D_l; F_l; \cdot; \Gamma_l; \perp \vdash tmOf(le_1) : \langle\!\langle \beta_1 \rangle\!\rangle \text{ and } [\![\langle\!\langle \beta_1 \rangle\!\rangle]\!]_D = (\tau, _)$ By I.H. on \mathcal{E}''' , (3), (6), (9) (11) $D_l; F_l; \cdot; \Gamma_l, x : \langle\!\langle \beta_1 \rangle\!\rangle; \bot \vdash tmOf(le_2) : s$ By T-LET, (10), (11) (12) $D_l; F_l; \cdot; \Gamma_l; \bot \vdash le : s$

Case: \mathcal{E} ends in L-APP rule.

By assumption:

(1) $D_a; F_a; \Gamma_a; t_2 \vdash \langle\!\langle v \ e \rangle\!\rangle \Rightarrow lv \ le$ $\mathcal{E}' :: D_a; F_a; \Gamma_a \vdash \langle\!\langle v \rangle\!\rangle \Rightarrow lv \text{ and } tpOf(lv) = [\bot](t_1 \to t_2)^{\bot}$ (2) $\mathcal{E}'' :: D_a; F_a; \Gamma_a; t_1 \vdash e \Rightarrow le$ (3)By examining the translation rules, only T-APP applies $[v \ le]_D = (v' \ e', D'_3 \cup D''_3)$ and (4)and $[\![lv]\!]_D = (v', D'_3)$ (5)(6)and $[\![le]\!]_D = (e', D''_3),$ By assumption and typing rules (7) $D \cup D_1 \cup D_2 \cup D_3; F; \cdot; \Gamma \vdash v' e' : \tau_2$ By inversion of (7) $(8) \quad D \cup D_1 \cup D_2 \cup D_3; F; \cdot; \Gamma \vdash v' : \tau_1 \to \tau_2$ $(9) \quad D \cup D_1 \cup D_2 \cup D_3; F; \cdot; \Gamma \vdash e' : \tau_1$ By I.H. on \mathcal{E}'' , (6), (9) (10) $D_l; F_l; \cdot; \Gamma_l; \bot \vdash tmOf(le) : t_1 \text{ and } [[t_1]]_D = (\tau_1, _)$

By Lemma 34 on \mathcal{E}' , (5), (8) (11) $D_l; F_l; :; \Gamma_l \vdash tmOf(lv) : [\bot](t_1 \to t_2)^{\rho} \text{ and } [[[\bot](t_1 \to t_2)^{\rho}]]_D = \tau_1 \to \tau_2$ By (11), P-T-E-VAL (12) $D_l; F_l; :; \Gamma_l; \bot \vdash tmOf(lv) : [\bot](t_1 \to t_2)^{\bot}$ By P-T-E-APP, (10), (12), (13) $D_l; F_l; :; \Gamma_l; \bot \vdash lv \ le : t_2$ By (11) (14) $[[t_2]]_D = (\tau_{2, -})$

Case: \mathcal{E} ends in L-APP-DE rule.

By assumption:

 $D_a; F_a; \Gamma_a; t_2 \vdash \langle\!\langle v_f \ v_a \rangle\!\rangle \Rightarrow lv_f \ lv_a$ (1) $\mathcal{E}' :: D_a; F_a; \Gamma_a \vdash \langle\!\langle v_f \rangle\!\rangle \Rightarrow lv_f \text{ and } tpOf(lv_f) = (\mathsf{d\&e})[\bot](t_1 \to t_2)^{\bot}$ (2) $\mathcal{E}'' :: D_a; F_a; \Gamma_a \vdash v_a \Rightarrow lv_a$ (3)By examining the translation rules, only T-APP-DE applies (4) $\llbracket v_f \ v_a \rrbracket_D = (e', D'_3 \cup D''_3 \cup D'''_3 \cup D'''_3 \cup D''''_3)$ and $e' = \mathsf{let} \ y : \tau_1 = e_a$ in $\mathsf{let} \ z : \tau_2 = v'_f \ y$ in e_2 $tpOf(lv_a) = b \ \rho \text{ and } \rho = \ell_1 :: \ell_2 :: \rho'$ (5) $\llbracket \mathsf{reLab}(\ell_1 :: \top \Leftarrow \rho) lv_a \rrbracket = (e_a, D'_3)$ (6)(7) $\llbracket \mathsf{reLab}(\ell_2 :: \rho' \leftarrow \ell_2 :: \bot)(z@b \ \ell_2 :: \bot) \rrbracket = (e_2, D_3'')$ $\llbracket t_1 \rrbracket = (\tau_1, D_3'')$ and $\llbracket t_2 \rrbracket = (\tau_2, D_4''')$ (8)(9)and $[\![lv_f]\!]_D = (v'_f, D''''')$ By assumption, (10) $D \cup D_1 \cup D_2 \cup D_3; F; \cdot; \Gamma \vdash e': \tau'$ By inversion of (10)(11) $D \cup D_1 \cup D_2 \cup D_3; F; \cdot; \Gamma \vdash e_a : \tau_1$ (12) $D \cup D_1 \cup D_2 \cup D_3; F; \cdot; \Gamma, y : \tau_1 \vdash v'_f y : \tau_2$ (13) $D \cup D_1 \cup D_2 \cup D_3; F; \cdot; \Gamma, y : \tau_1, z : \tau_2 \vdash e_2 : \tau'$ By inversion of (12)(14) $D \cup D_1 \cup D_2 \cup D_3; F; \cdot; \Gamma, y : \tau_1 \vdash v'_f : \tau_1 \to \tau_2$ By Lemma 34 on \mathcal{E}' , and (14) (15) $D_l; F_l; \cdot; \Gamma_l; \perp \vdash tmOf(lv_f) : (\mathsf{d\&e})[\perp](t_1 \to t_2)^{\perp}$ By Lemma 36, (5), and (11) (16) $D \cup D_1 \cup D_2 \cup D_3; F; \cdot; \Gamma \vdash \mathsf{fst}(\llbracket v_a \rrbracket_D) : \tau' \text{ and } \tau' = \mathsf{fst}(\llbracket b \rho \rrbracket_D)$ By Lemma 34 (17) $D_l; F_l; \cdot; \Gamma_l \vdash tmOf(lv_a) : b \ \rho$ and By Lemma 35 on (6) (11) (16) (18) $\tau_1 = \texttt{fst}(\llbracket b \ \ell_1 :: \top \rrbracket_D)$ By Lemma 32 on (8) (18)(19) $t_1 = b \ \ell_1 :: \top$ By Lemma 36 and (7)(20) $D \cup D_1 \cup D_2 \cup D_3; F; : \Gamma \vdash \mathsf{fst}(\llbracket z @b \ \ell_2 :: \bot \rrbracket_D) : \tau'' \text{ and } \tau'' = \mathsf{fst}(\llbracket z @b \ \ell_2 :: \bot \rrbracket_D)$ By Lemma 32 on and z has type τ_2 (8) (20) (21) $t_2 = b \ \ell_2 :: \bot$ By P-T-E-APP-DE (22) $D_l; F_l; \cdot; \Gamma_l; \perp \vdash tmOf(lv_f \ lv_a) : b \ \ell_2 :: \rho'$