Mining Connection Pathways for Marked Nodes in Large Graphs

Leman Akoglu

Computer Science · Stony Brook University

Hanghang Tong

CUNY

Jilles Vreeken

Universiteit Antwerpen

Polo Chau

GT

Nikolaj Tatti

Katholieke Universiteit Leuven

Christos Faloutsos

Carnegie Mellon University

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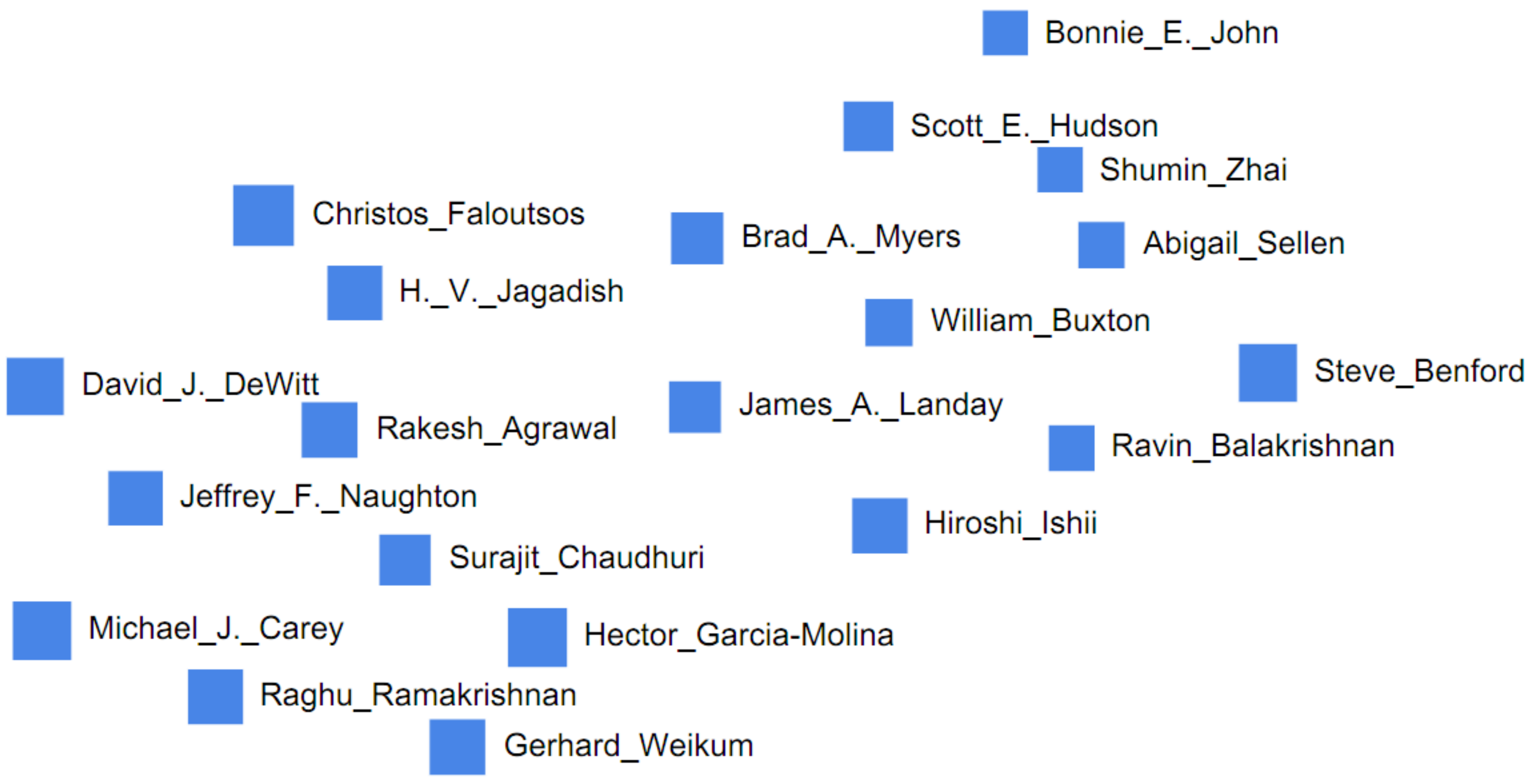
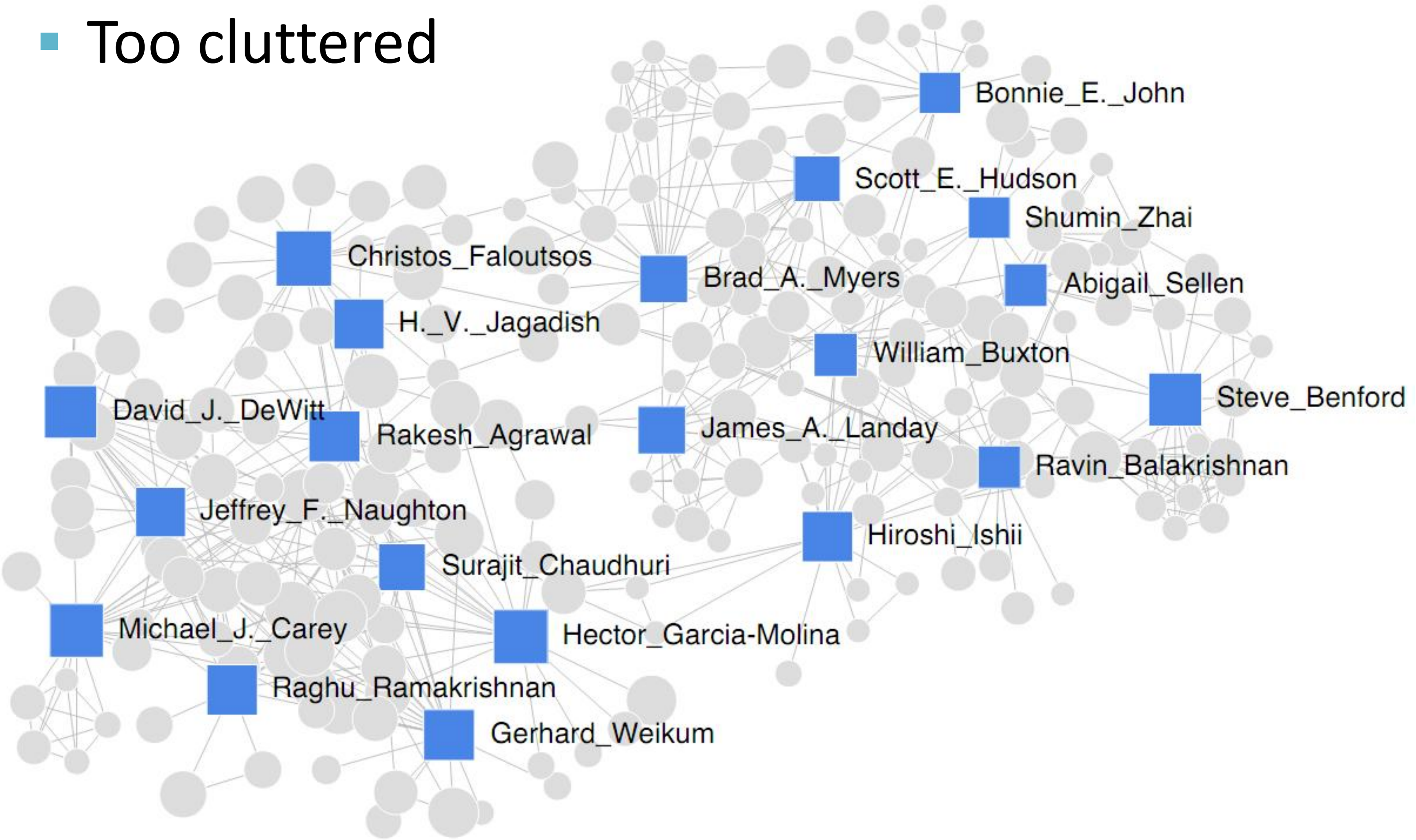
Example

- What can we say about this “list” of authors?
  - Use relational information



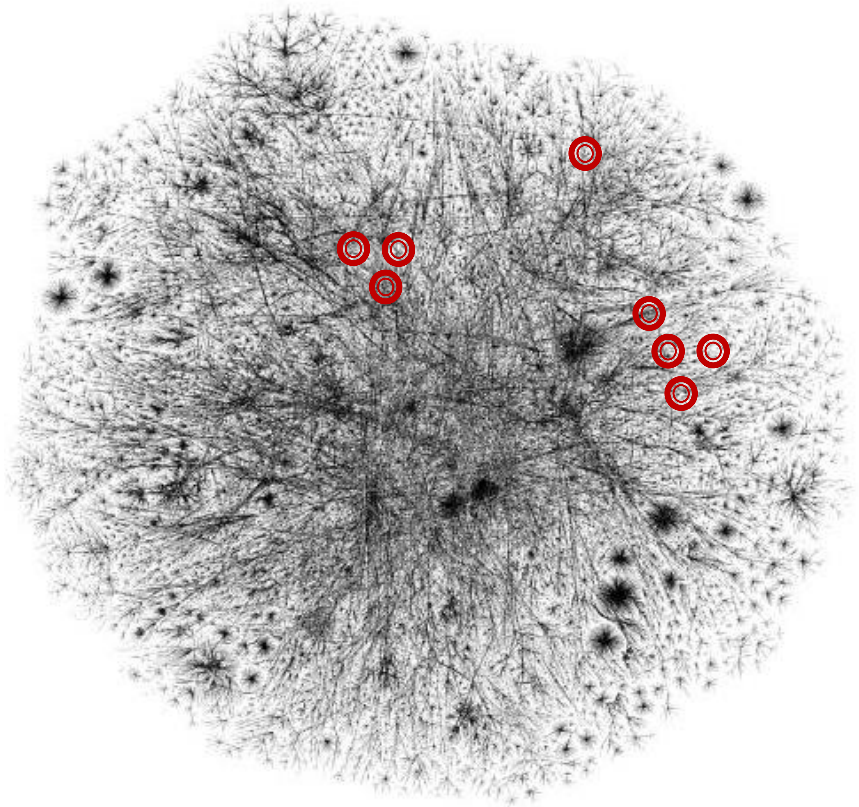
Example

- Any patterns in the co-authorship graph?
  - Too cluttered



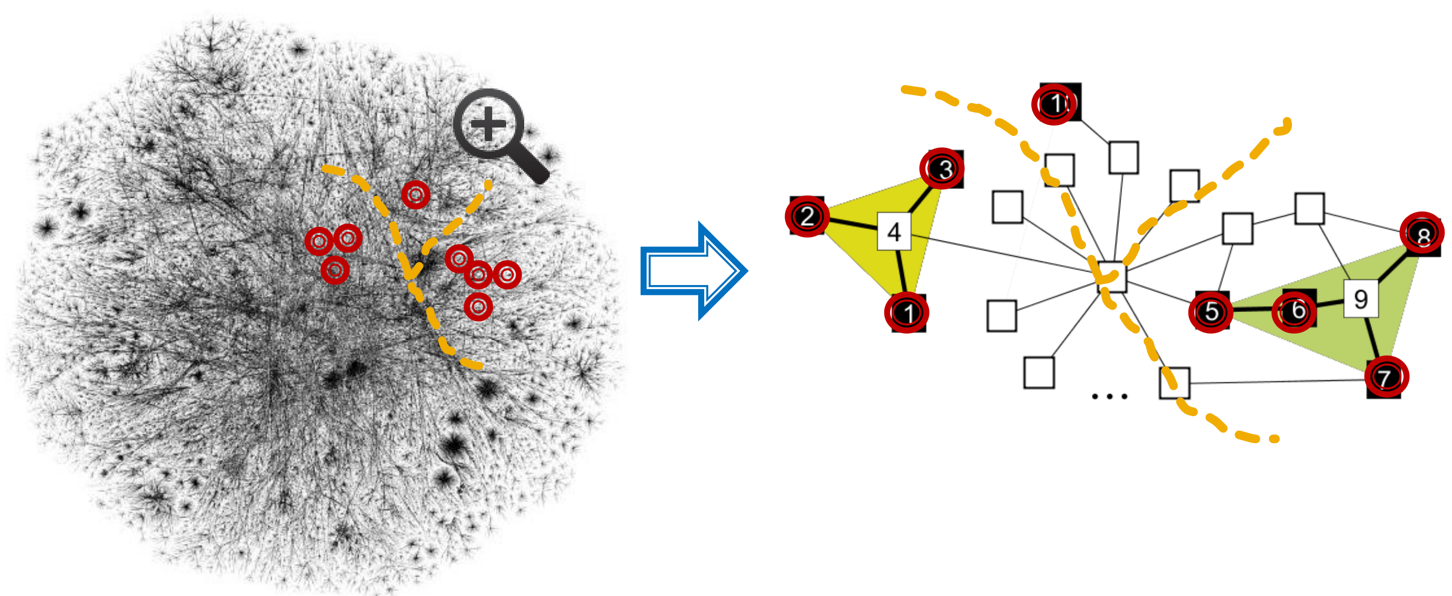
Problem

- Given
  - a **large graph G**
  - a **handful of nodes S** marked by an external process
- What can we say?
  - are **S** close by?
  - are **S** segregated?
  - how many groups do they form?
- How can we connect them?
  - with "simple" paths
  - who are "good" connectors?



Our approach

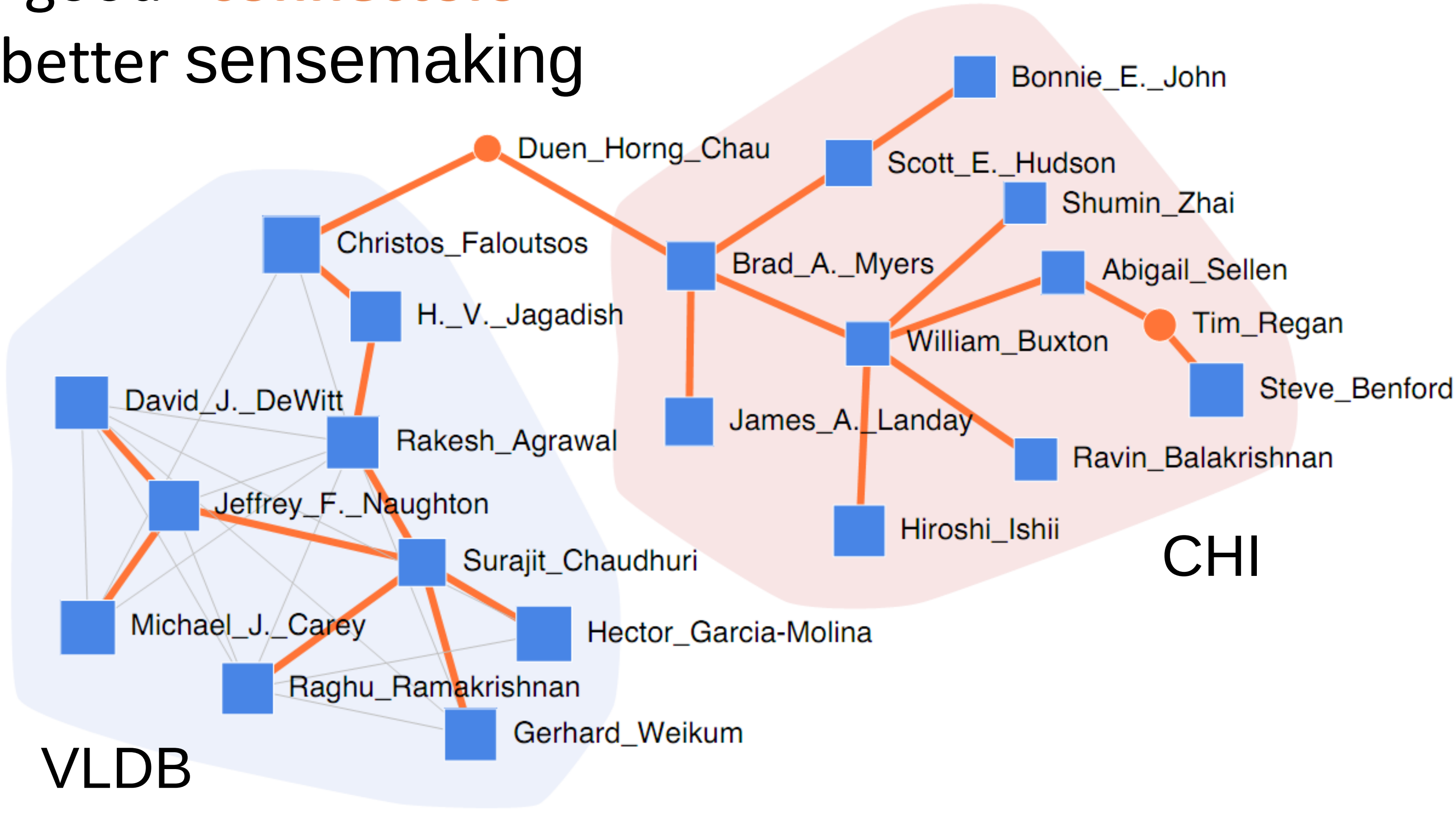
- Use the network structure to explain **S**
- Partition **S** into **groups** of nodes, such that:
  - "simple" **paths** connect the nodes in each **group**,
  - nodes in different groups are "not easily reachable"



- Use the Minimum Description Length principle
  - Best partitioning requires the "least number of bits"

Example

- “Simple” connection **pathways**
  - “good” **connectors**
  - better sensemaking

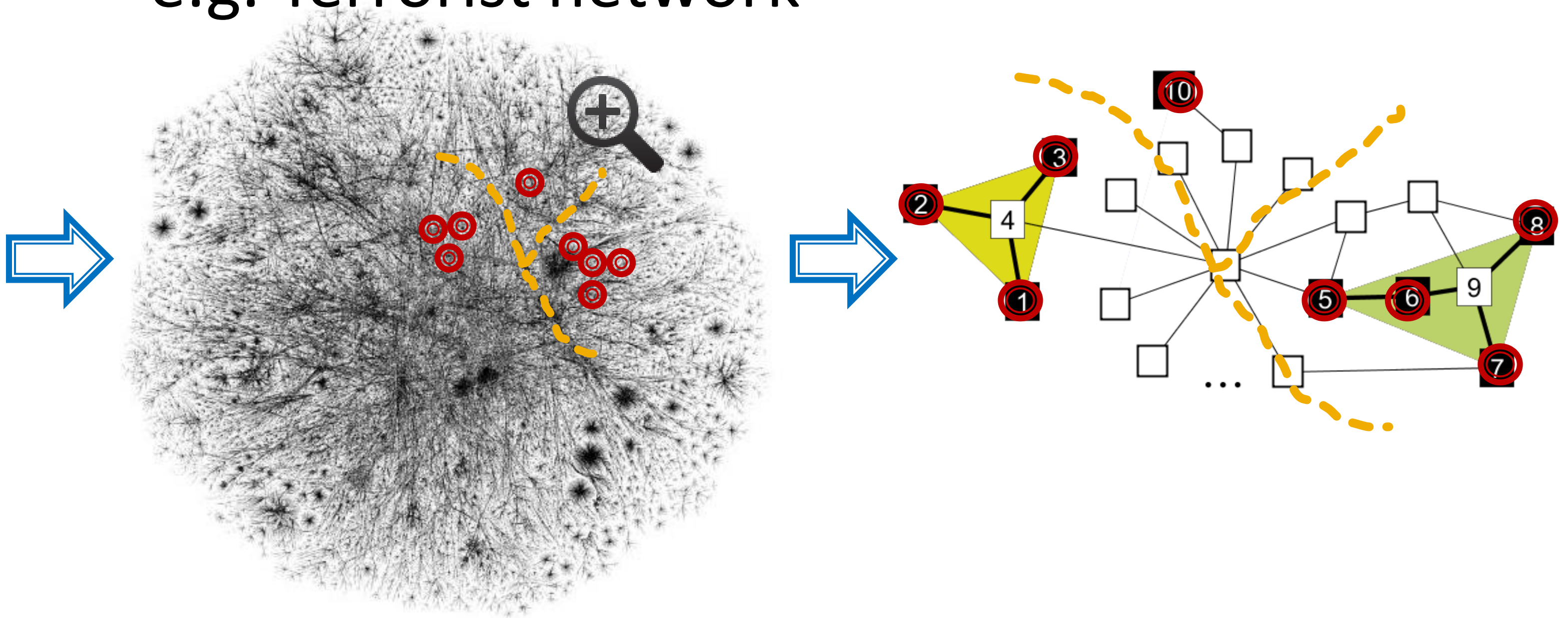


Applications

- 1. Graph anomaly description/summarization

e.g. Terrorist network

Top-k anomalies



✓ Summarize top-k node anomalies by groups

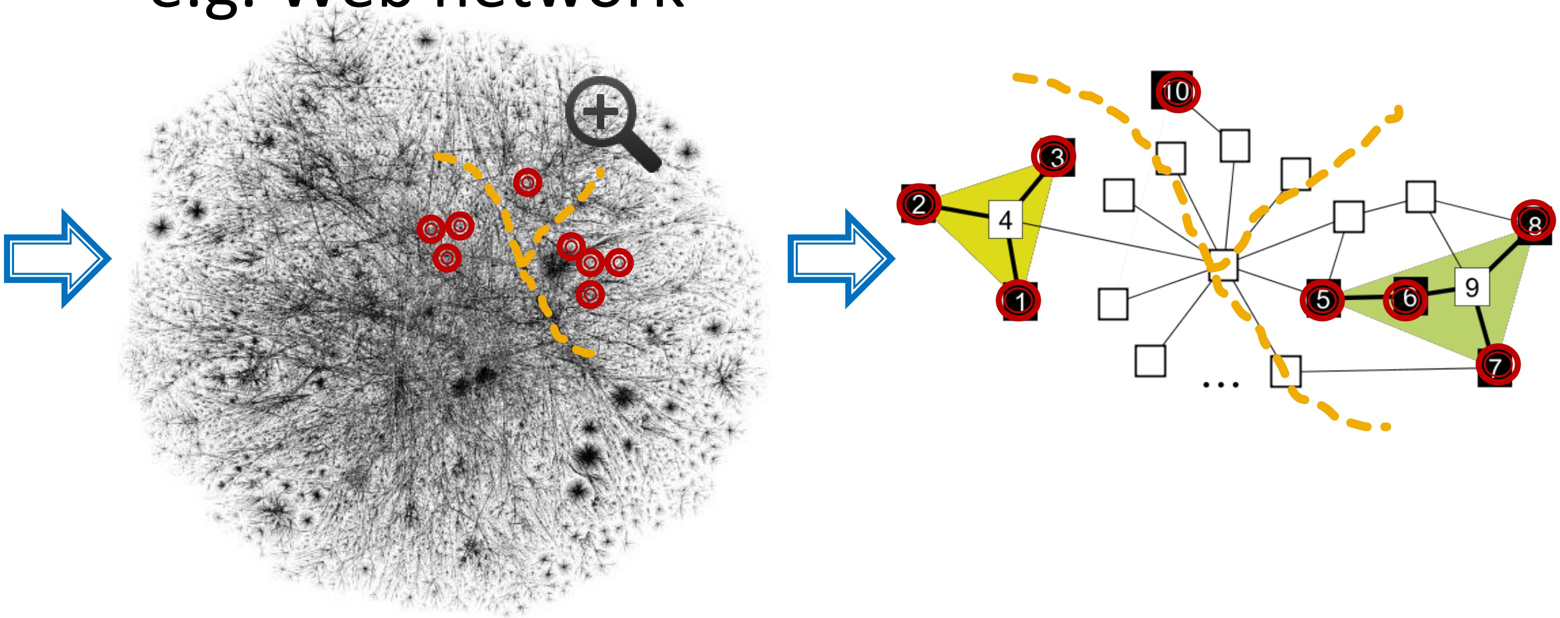
✓ Find connections/connectors among groups

Applications

- 2. Query summarization

e.g. Web network

Top-ranked Web pages



✓ Summarize top-k query pages by groups

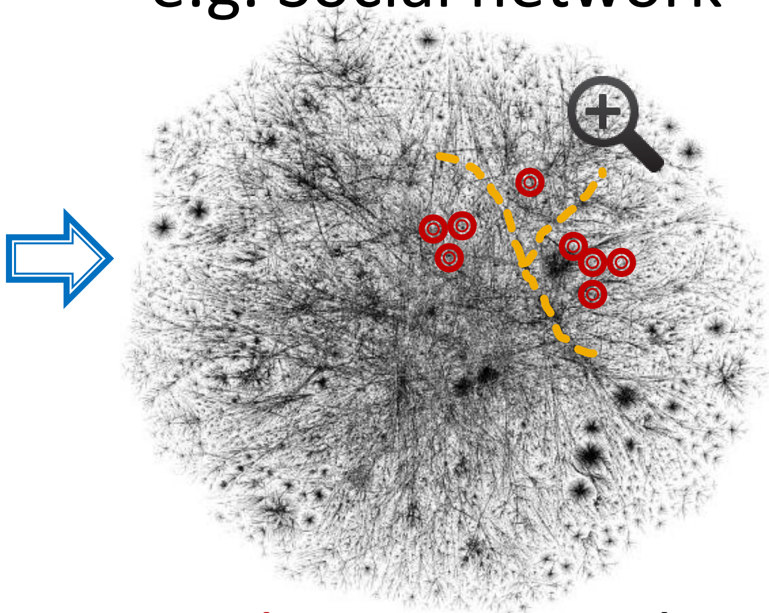
✓ Find connections/connectors among groups

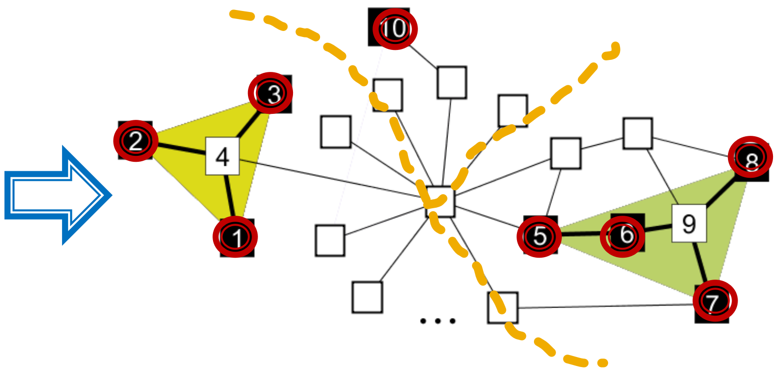
Applications

- 3. Understanding dynamic events on graphs

e.g. Social network

Affected people





Group people s.t. network structure can be associated with the spread of event

✓ within groups (number of points of infection)

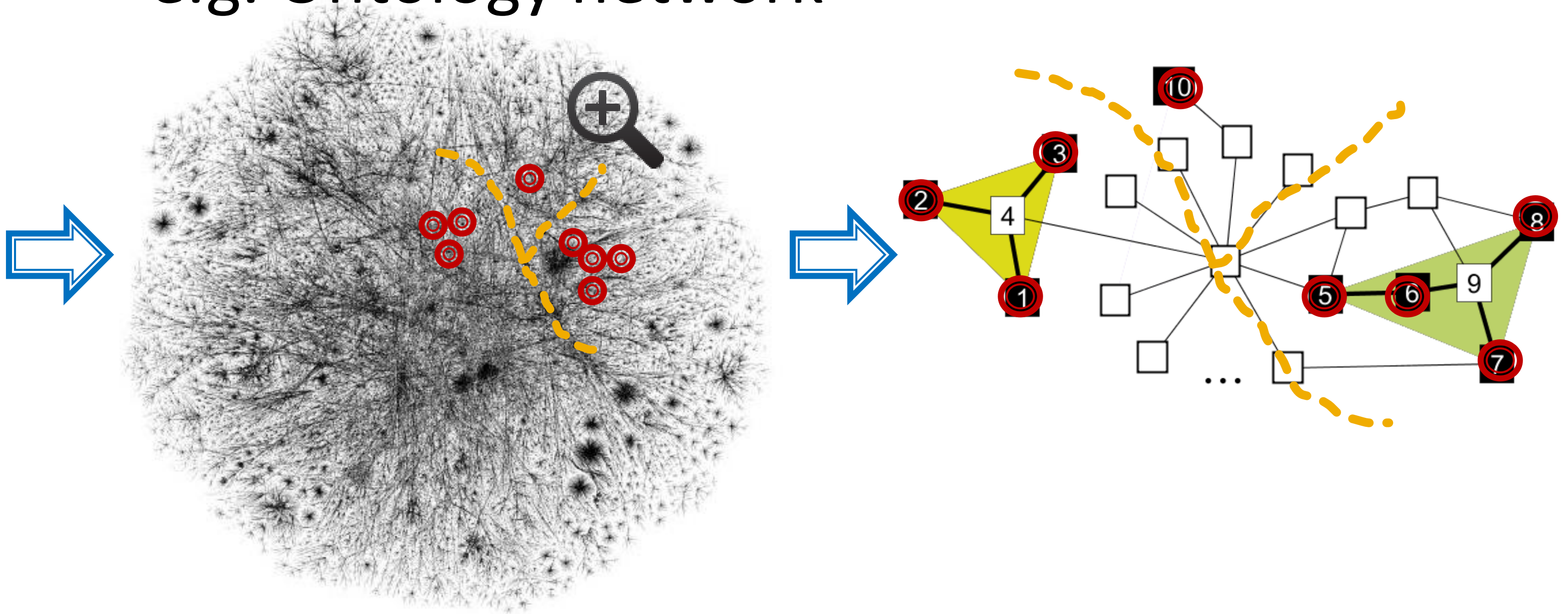
✓ but not quite across groups

Applications

- 4. Understanding semantic coherence

e.g. Ontology network

Set of words



✓ Summarize words by semantically coherent groups

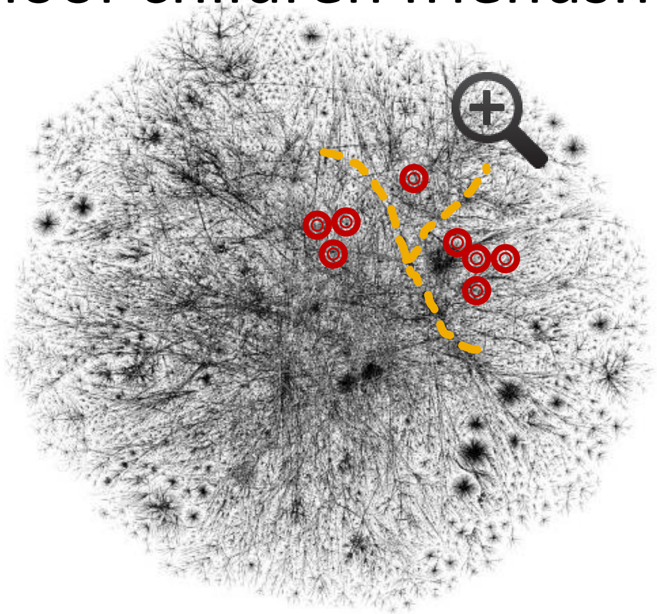
✓ Find connectors (other relevant words) among groups

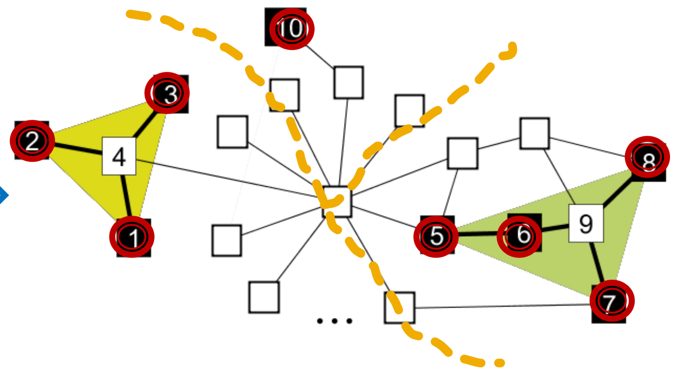
Applications

- 5. Understanding segregation (social science)

e.g. School-children friendship network

Students with attributes of interest





✓ Summarize students by their social "circles"

✓ Study groups (and groups within groups)

Roadmap

- Problem description
- Approach
- Applications
- Problem formulation
  - Problem definition
  - MDL intuition
  - Objective formulation
- Algorithms
- Experiments

Problem (formally)

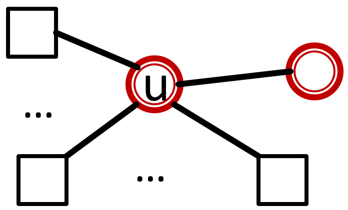
Problem Definition Given a graph $G = (V, E)$ and a set of marked nodes $M \subseteq V$

Problem 1. Optimal partitioning Find a coherent partitioning P of M. Find the optimal number of partitions $|P|$.

Problem 2. Optimal connection subgraphs Find the minimum cost set of subgraphs connecting the nodes in each part $p_i \in P$ efficiently.

Objective formulation (intuition)

- Our key idea is to use an encoding scheme
  - Imagine a sender and a receiver. Assume:
  - Both sender and receiver know graph structure G
  - Only sender knows the set of marked nodes M
  - Goal of sender:
    - transmit to the receiver the info. of which nodes are marked, using as few bits as possible.
- Why would encoding work?
  - Naïvely: encode ID of each marked node with $\log |V|$ bits
  - Better: exploit "close-by" nodes, restart for farther nodes



$2 \log |V|$ vs. $\log |V| + \log(d(u))$

Objective formulation (intuition)

- We think of encoding as
  - hopping from node to node to encode close-by nodes
  - and flying to a new node to encode farther nodes
- until all marked nodes are encoded. (hence Dot2Dot)

- Simplicity (or the description length) of connection graph T (which is a **tree**) determined by:
  - number of unmarked nodes we visit
  - how easily per visited node we can identify which edge to follow next;
    - nodes with (very) high degree make the path more complex

Objective function

DETAILS

minimize
P, T_i

$$L(P, M \mid G) = L(|P|) + \sum_i L(p_i)$$

NP-hard

(reduces from the Steiner tree problem)

- encode #partitions $L(|P|) = \log |V|$
- encode each part:

$$L(p_i) = \log |V| + L(t) + \log |T| + \log \binom{|T|}{||T||}$$

root node

spanning tree t of p_i

number of marked nodes in p_i

identities of marked nodes

- encoding of tree of each part:

$$L(t) = L_{\mathbb{N}}(|t| + 1) + \log \binom{d(v_t)}{|t|} + \sum_j^{|t|} L(b(t, j))$$

#branches of node t

identities of branch nodes

recursively encode all tree nodes

Roadmap

- Problem description
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- Problem formulation
- Algorithms
  - Graph transformation
  - Finding bounded paths
  - Connected components
  - Minimum arborescences
  - Level-k trees
- Experiments

Algorithms - preliminaries

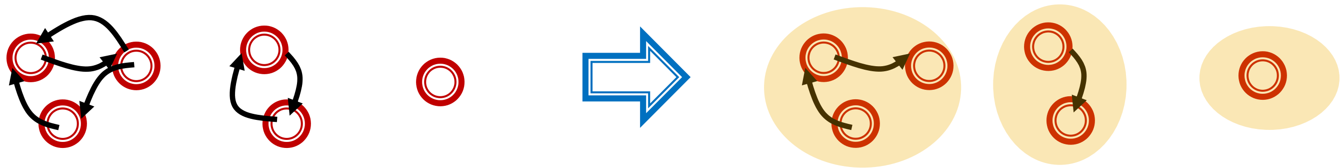
- Graph transformation
  - Given undirected unweighted G(V,E),
  - We transform it into directed weighted G'(V,E,W)
    - w(u,v) = log d(u) and w(v,u) = log d(v)

Given G', problem becomes: find *the set of trees* with minimum total cost on the marked nodes.

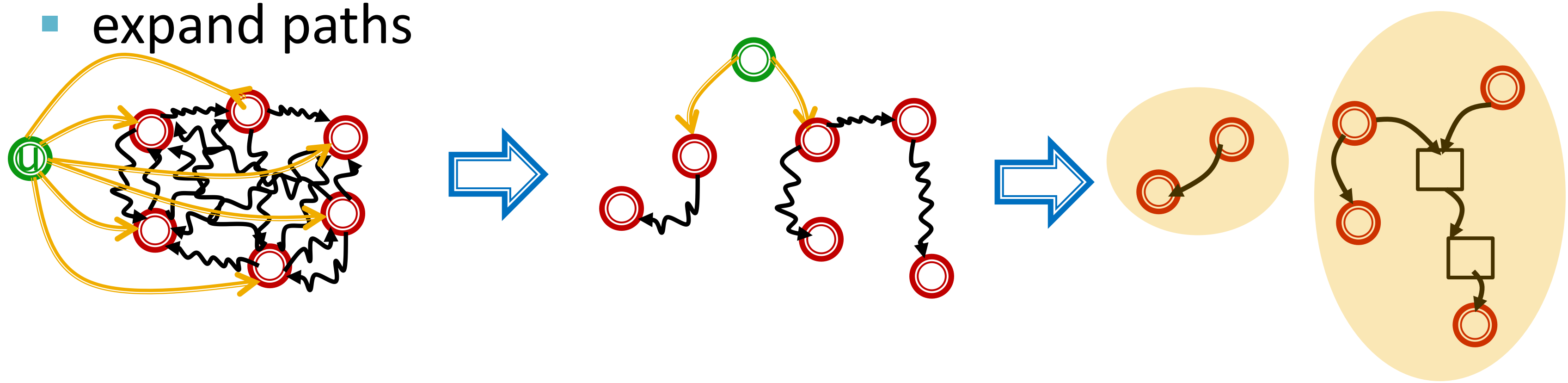
- Finding bounded-length paths
  - (multiple) short paths of length up to log |V| between marked nodes in G'
  - employ BFS-like expansion

Algorithms

- Connected components (CC)
  - find induced subgraph(s) on marked nodes in G'
  - find minimum cost directed tree(s)

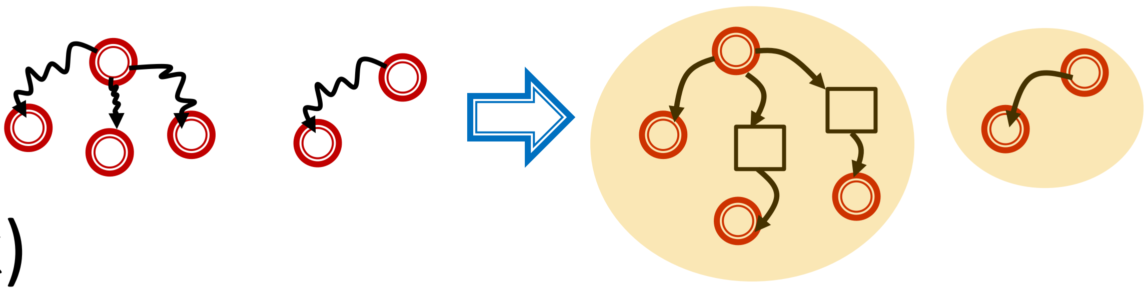


- Minimum arborescences (ARB)
  - construct transitive closure graph CG (with bounded paths)
  - add universal node u with out-edges w(u,m) = log |V|
  - find minimum cost directed tree(s), remove u
  - expand paths

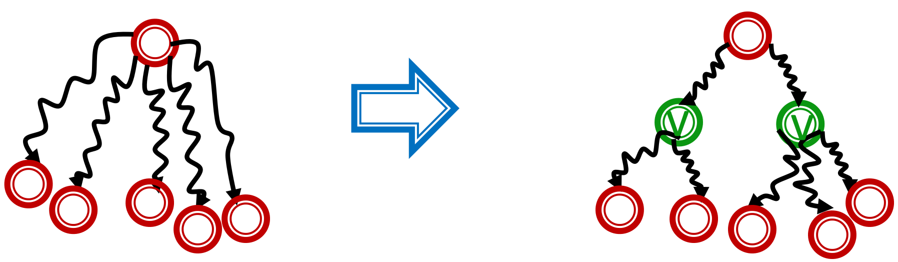


Algorithms

- Level-1 trees (L1)
  - find minimum cost depth-1 trees in CG
  - expand paths



- Level-k trees (Lk)
  - refine level-(k-1) trees by finding intermediate node v's
  - such that total cost (i.e. cost from root r to each v + costs of subtrees rooted at v's) is less



Roadmap

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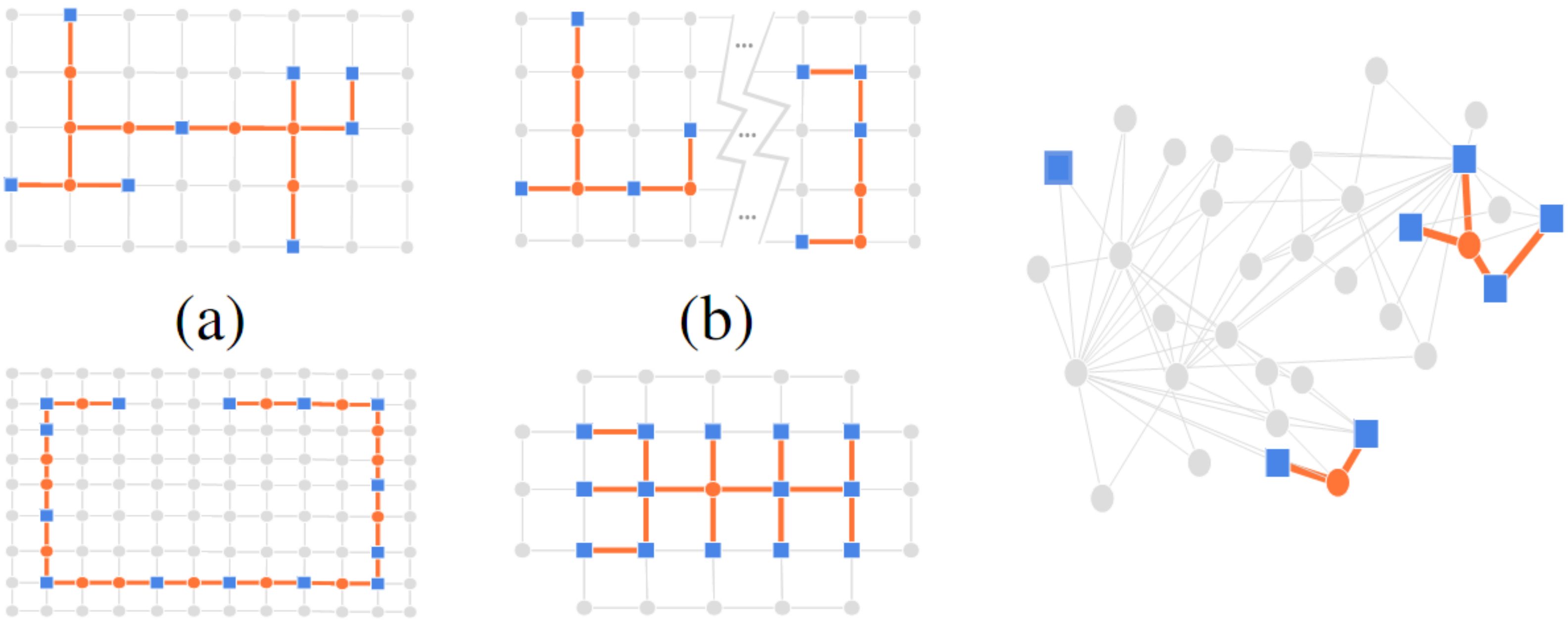
Experiments

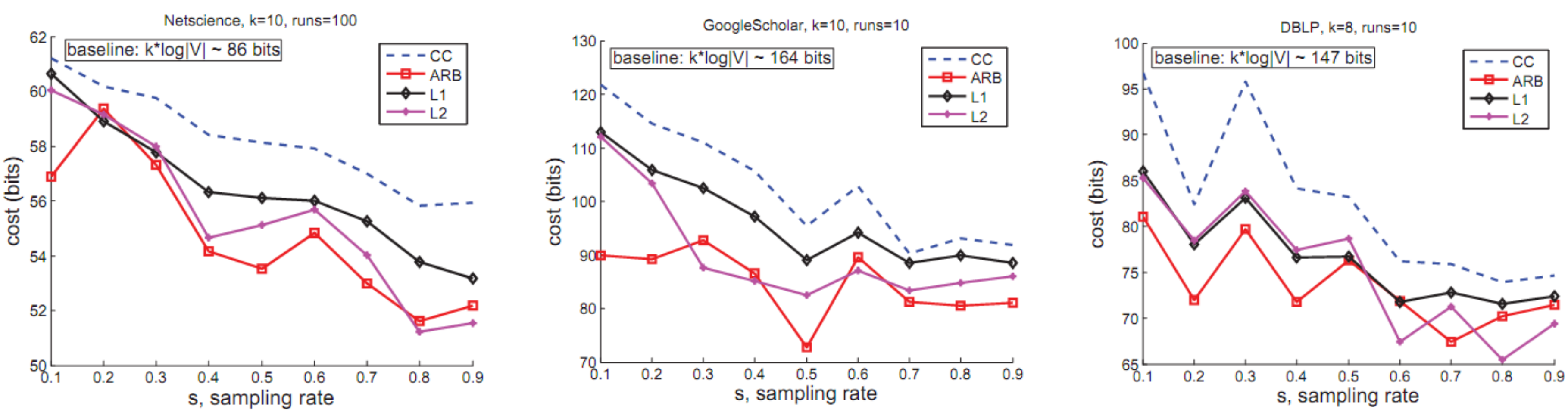
- Synthetic examples



(a) (b) (c) (d) (e)

Experiments

- Comparing the algorithms
- Real networks: Netscience, GoogleScholar, DBLP
- Random walk sampling to mark k nodes:
  - pick a random node, visit its k'<k neighbors, mark them with prob. s, pick a random node already visited

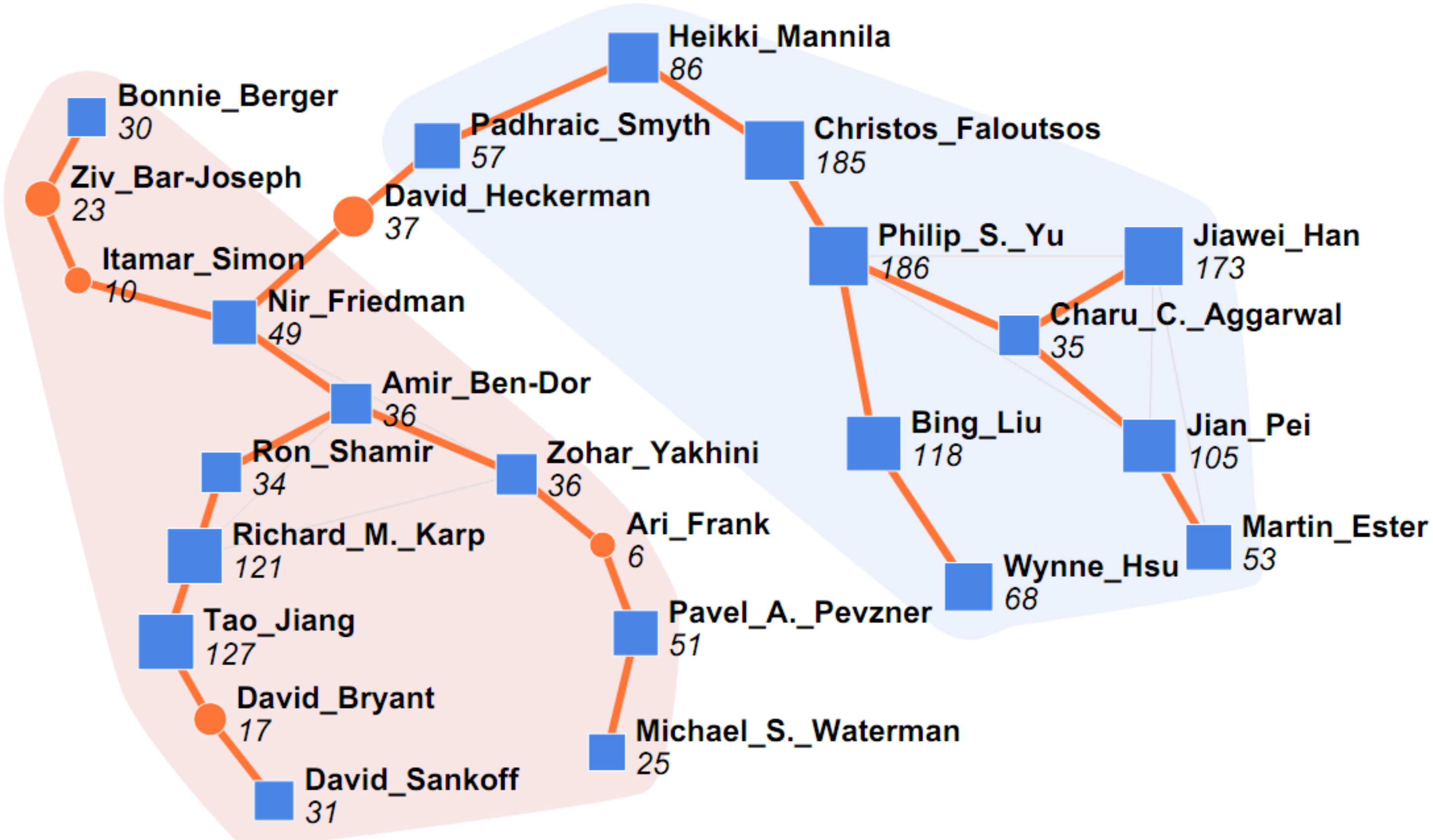


(a) *Netscience* (b) *GScholar* (c) *DBLP*

More separated ← s → More close-by

Experiments

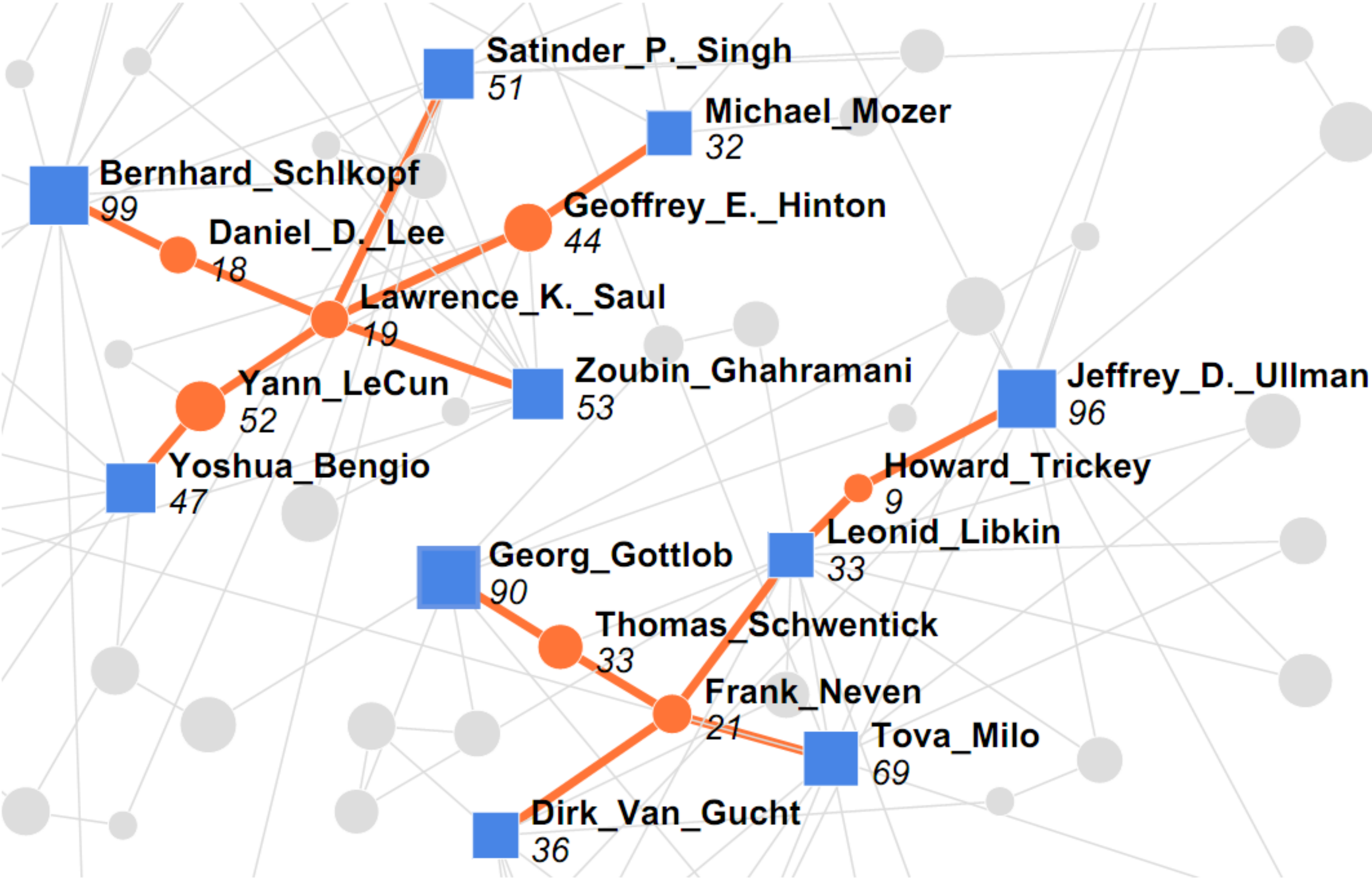
- Case studies on DBLP



(a) *DBLP*: RECOMB vs. KDD

Experiments

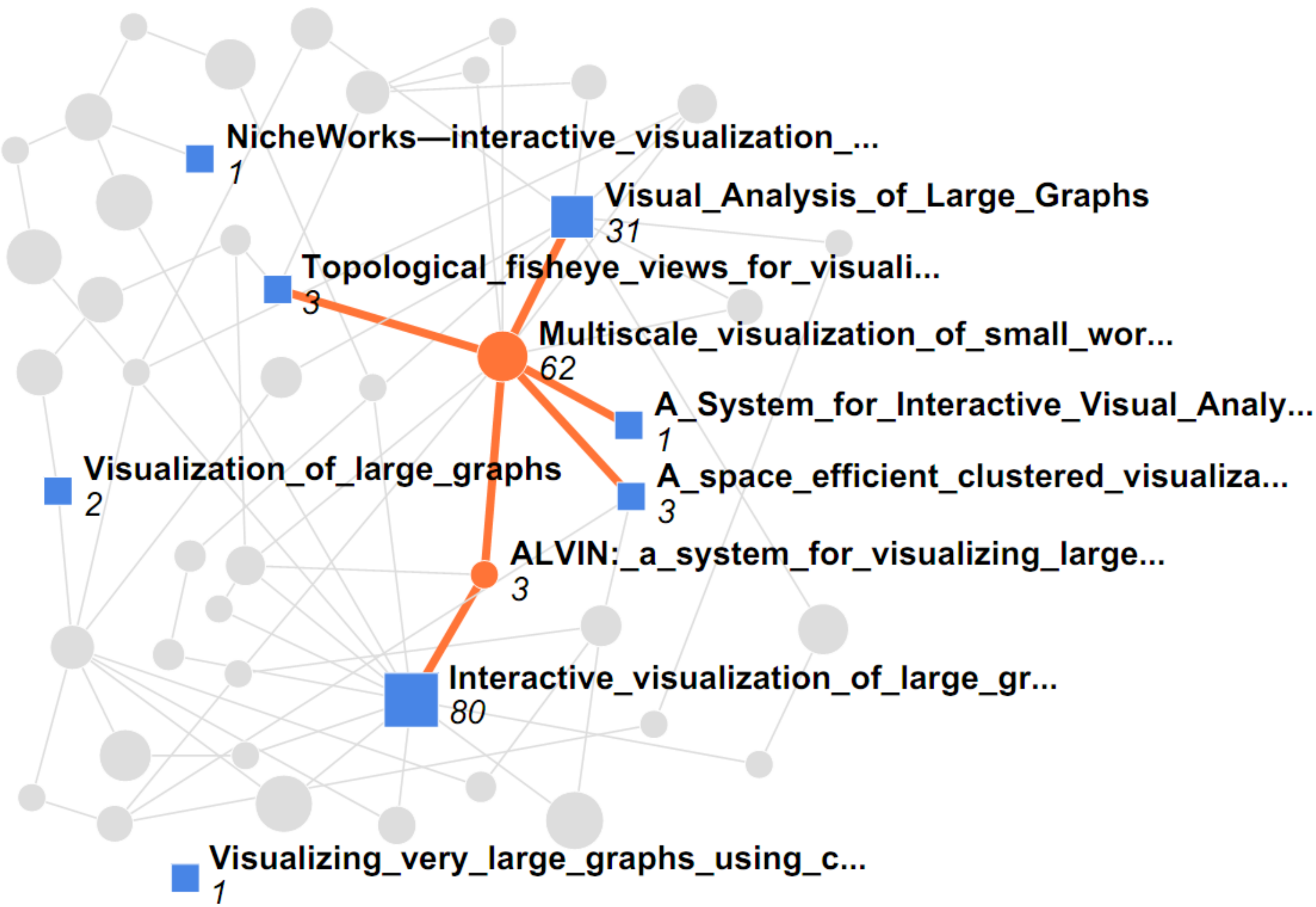
- Case studies on DBLP



(b) *DBLP*: NIPS vs. PODS

Experiments

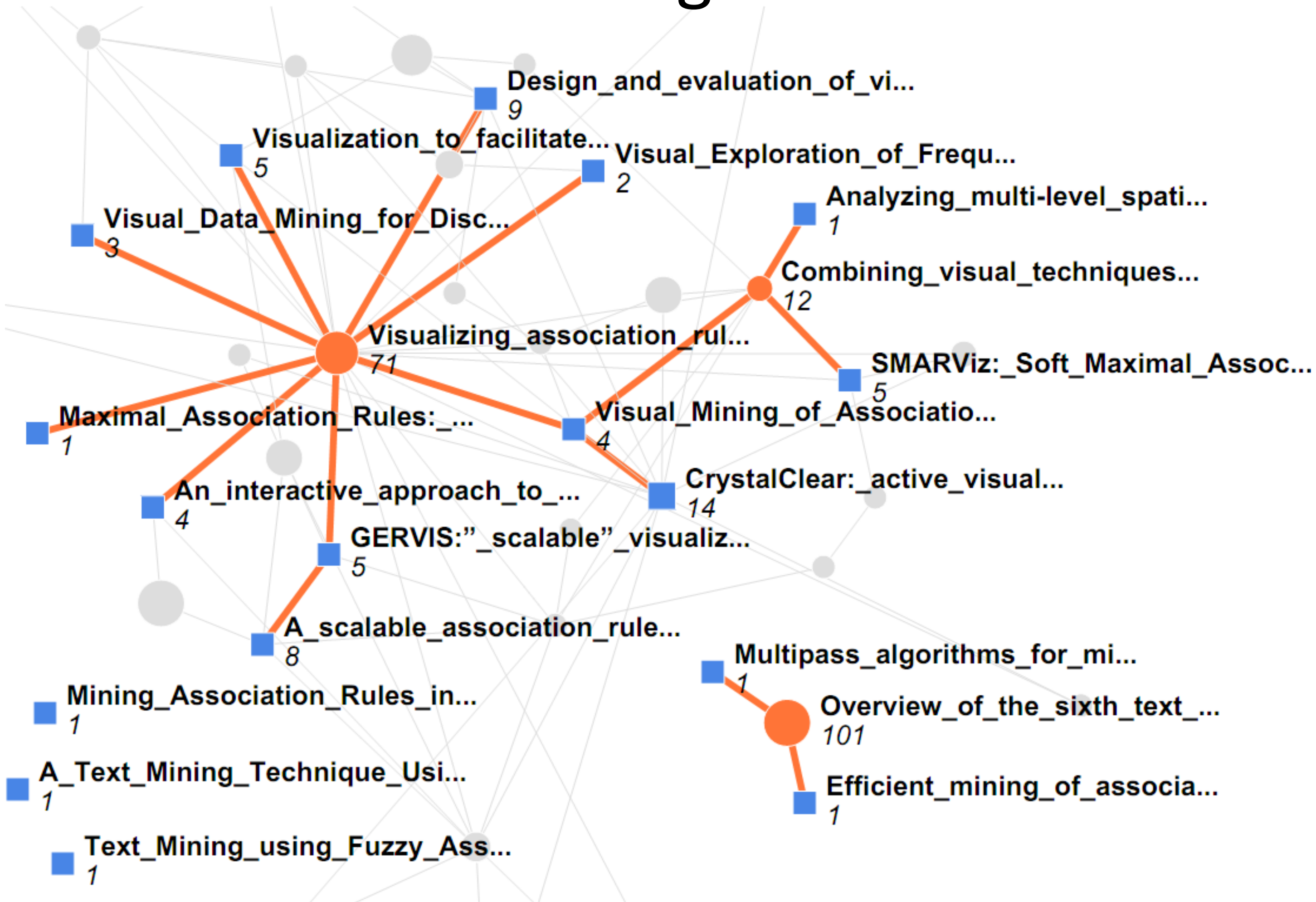
- Case studies on GoogleScholar



(a) *GScholar*: ‘large graphs’, ‘visual’

Experiments

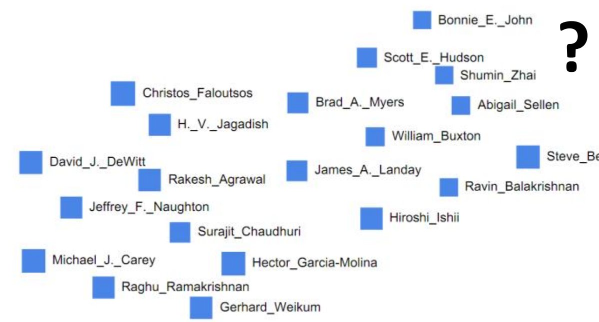
- Case studies on GoogleScholar

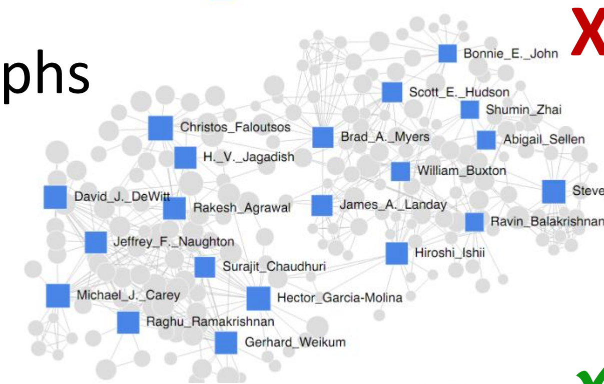


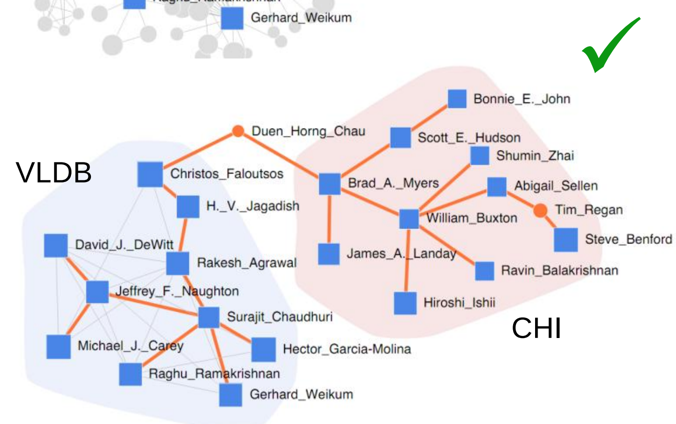
(b) *GScholar*: 'association rule', 'visual', 'text'

Summary

- **Dot2Dot**: A principled framework to "describe" a set of marked nodes in large graphs
- Many applications in the wild
  - Anomaly description/summarization
  - Query summarization
  - Understanding dynamic events on graphs
  - Understanding semantic coherence
  - Segregation studies
  - …
- MDL formulation
- NP-hardness
- Fast algorithms
- Experiments on real graphs







Thank you!

Leman Akoglu

leman@cs.stonybrook.edu

www.cs.stonybrook.edu/~leman