

# Attributed Networks: Social Circles, Summarization, Comparison

Leman Akoglu

Joint work with Bryan Perozzi  
Rashmi Raghunandan, Shruti Sridhar, Upasna Suman  
Aria Rezaei

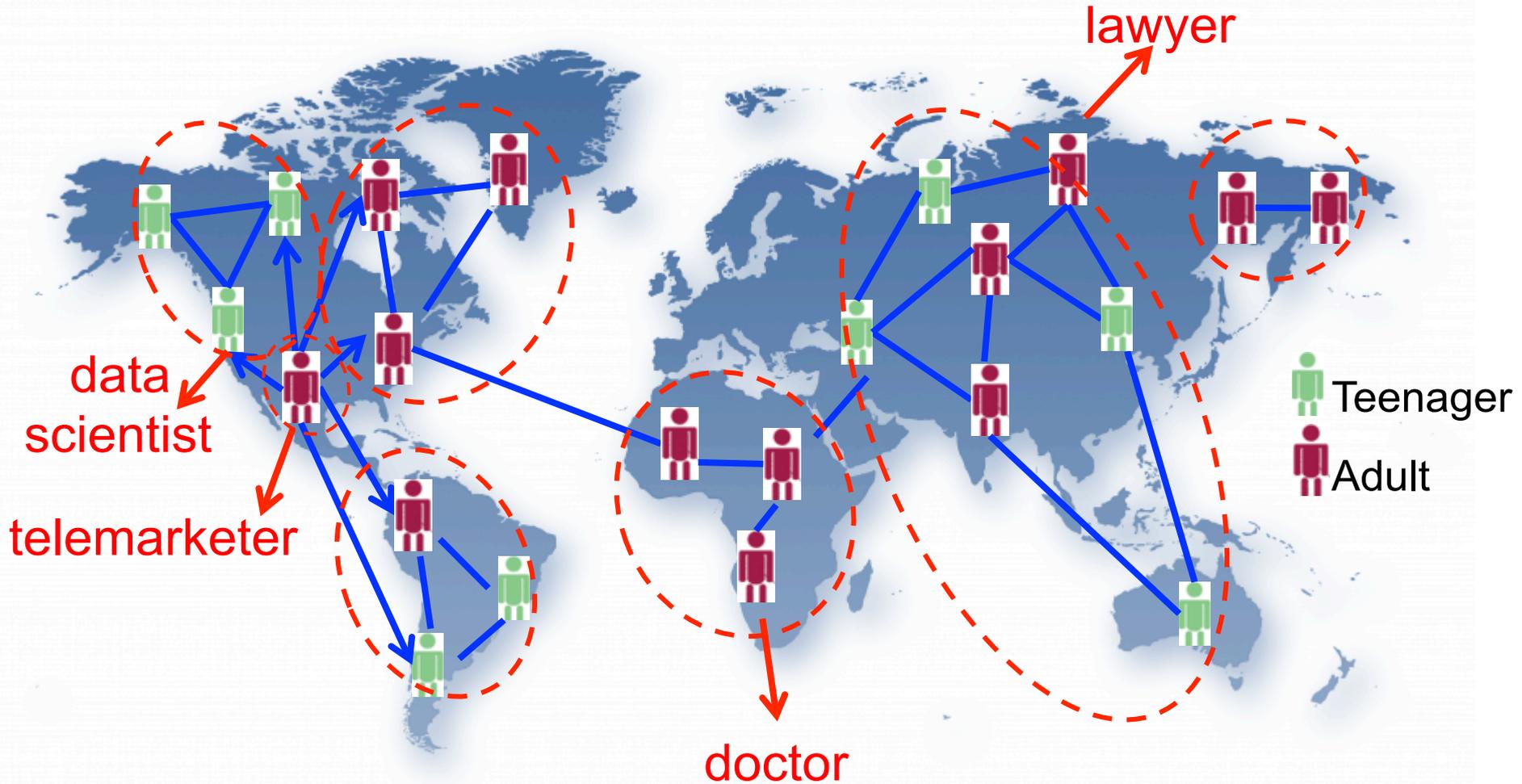
(Google Research NYC),  
(CMU),  
(Stony Brook University).

NetSci 2018 Satellite on  
Machine Learning In Network Science

June 12, 2018

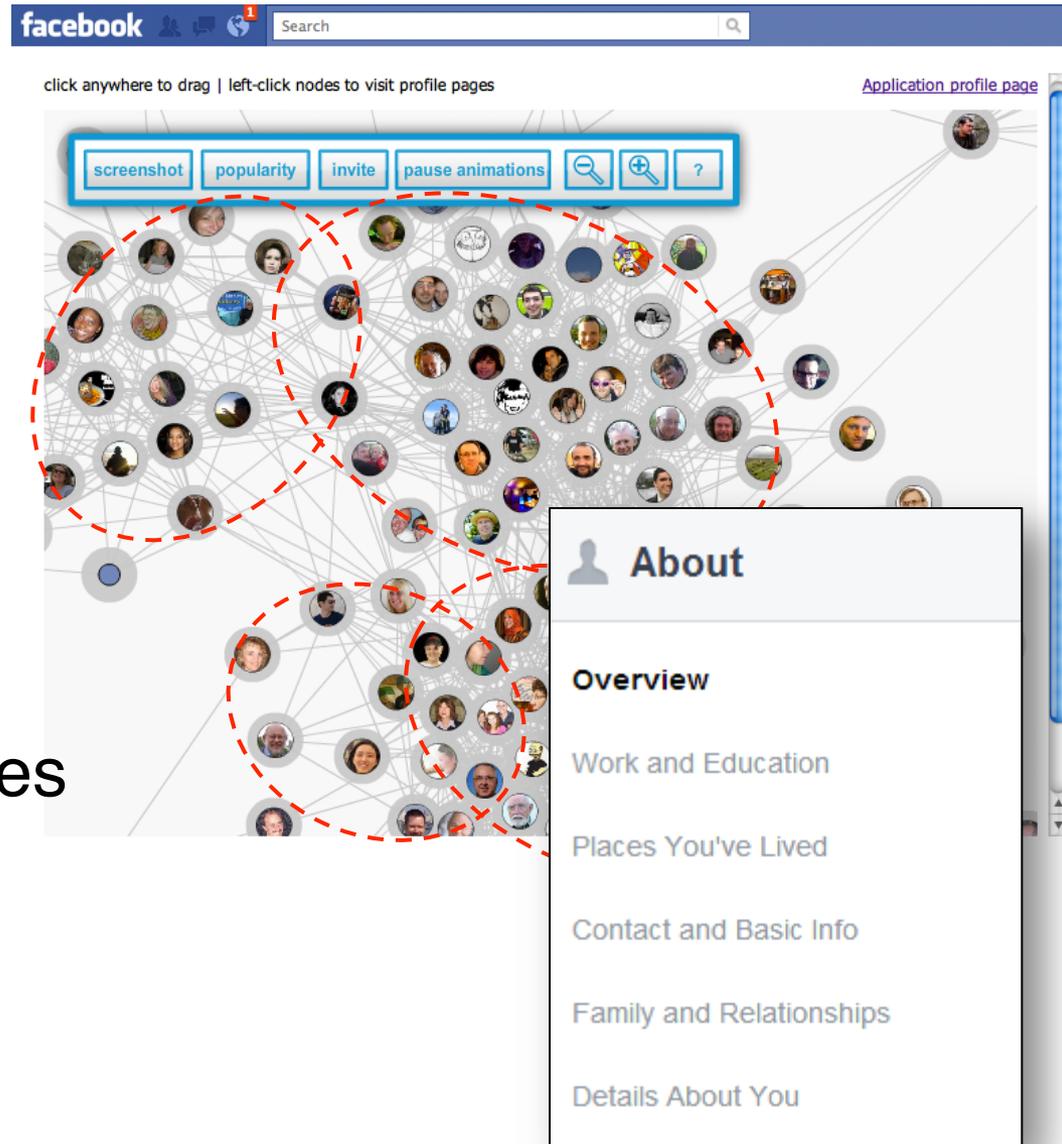
# Attributed graphs

Attributed graph: each node has 1+ properties



# Attributed networks

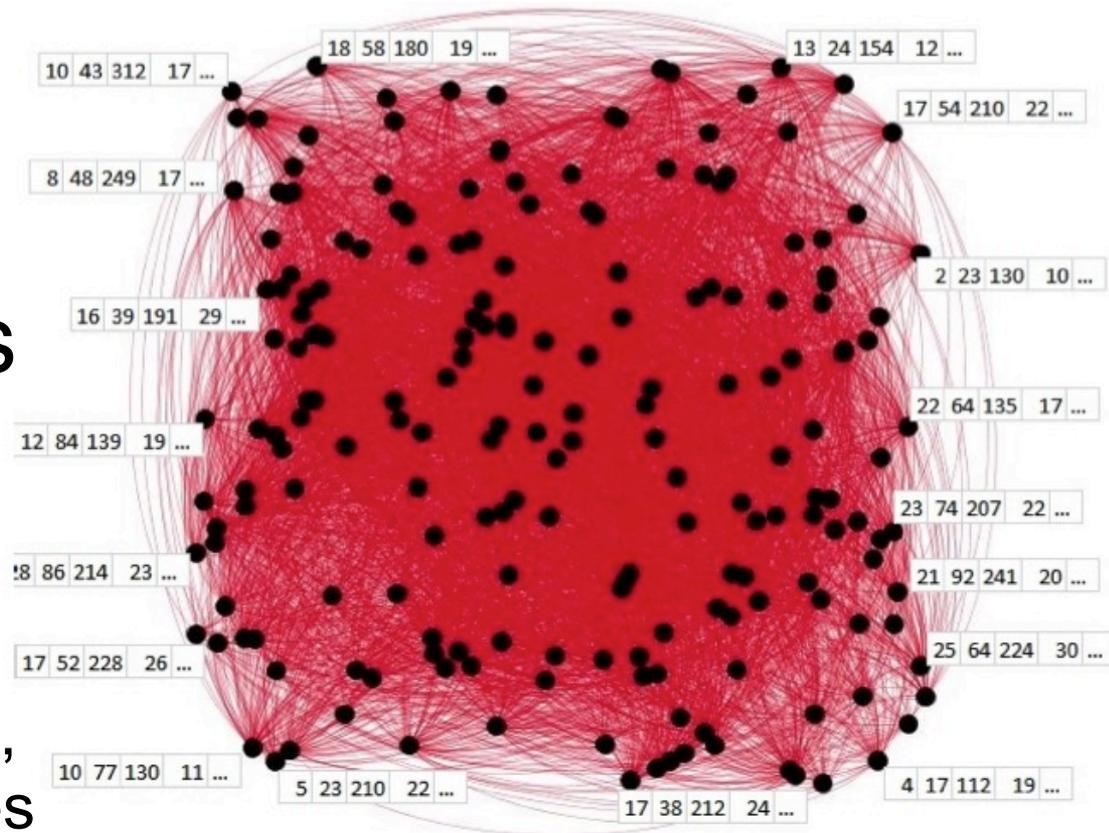
- Social networks
  - demographics, lifestyles, likes, ...
- PPI networks
  - Gene encodings
- Gene interaction networks
  - ontological properties
- Web
  - page properties
- ...



# Motivating question:

How can we **make sense** of node-attributed networks ?

- subgraphs
- summaries
- comparisons

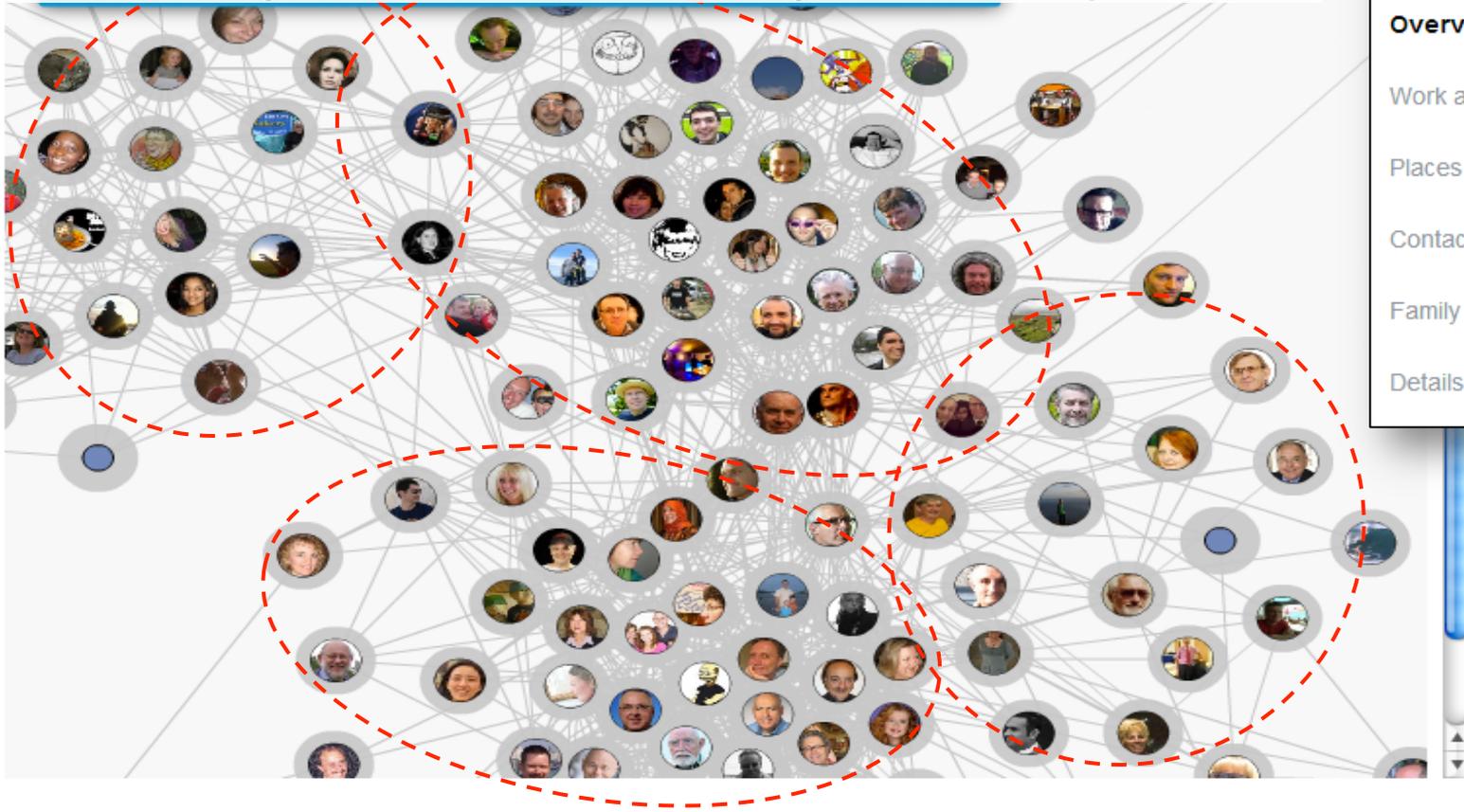


# Attributed networks

facebook

Search

Idea is “description-by-parts”:  
identifying & characterizing the **subgraphs**



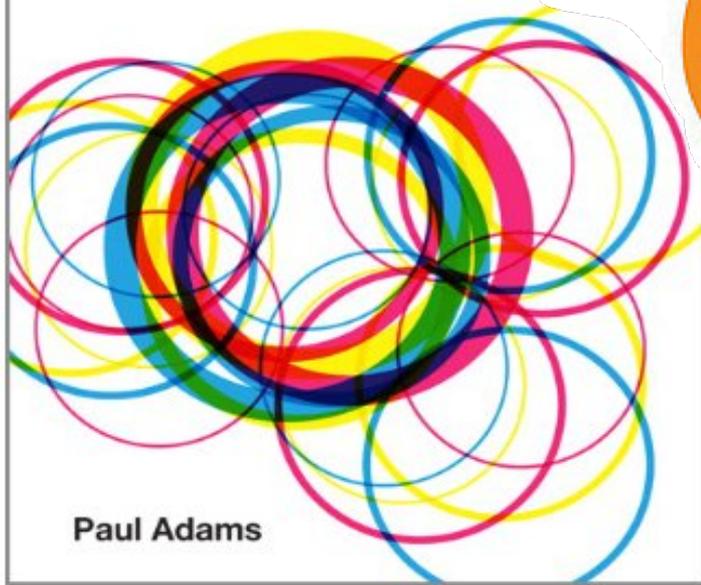
**About**

**Overview**

- Work and Education
- Places You've Lived
- Contact and Basic Info
- Family and Relationships
- Details About You

# SOCIAL CIRCLES

How offline relationships influence online behavior and what it means for design and marketing



Paul Adams

Sara  
Highschool <sup>10</sup>

Dana  
Highschool +  
Riyadh

84

Moose  
Family 50

Rula  
Family 65

Hisham  
Family  
150

Hala  
Web

100

Naseem  
Web

73

Ahmed  
Web 17

Lina  
Web

105

Ibra  
Work + Web

110

Noor  
University

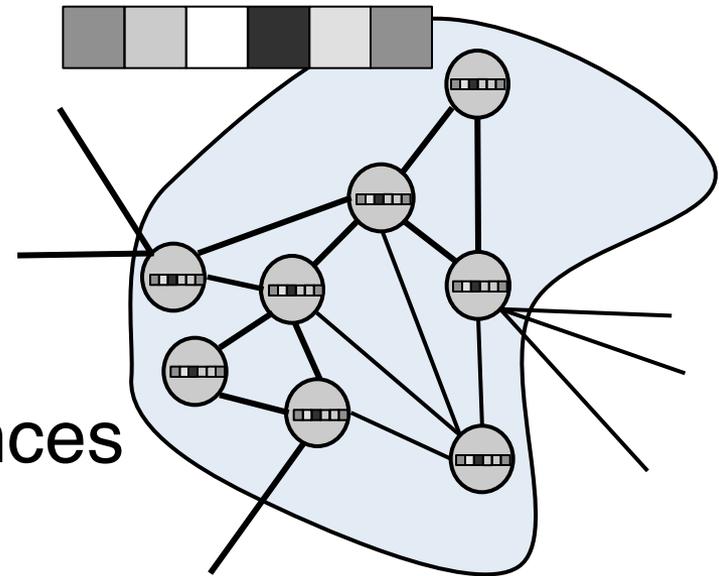
70

Yasmeen  
Work  
68

Rama  
Elementary School <sup>10</sup>

# Research questions:

- ① How to characterize & measure the **quality** of ...
- ② How to **summarize** & **interactively explore** ...
- ③ How to **characterize** differences between **classes** of ...  
... **attributed subgraphs**?



- 1) **Scalable Anomaly Ranking of Attributed Neighborhoods** SIAM SDM 2016
- 2) **Discovering Communities and Anomalies in Attributed Graphs:  
Interactive Visual Exploration and Summarization** ACM TKDD, 2018  
*Bryan Perozzi and Leman Akoglu*
- 3) **Ties That Bind - Characterizing Classes by Attributes and Social Ties**  
*Aria Rezaei, Bryan Perozzi, Leman Akoglu* WWW 2017 Companion

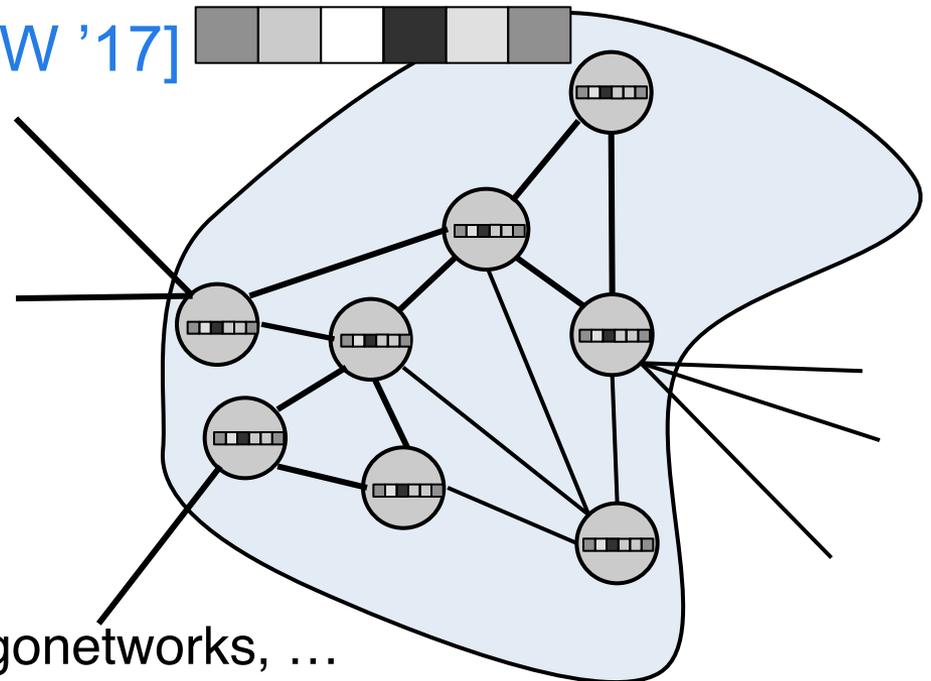
# This talk

- Attributed (sub)graphs\*

- ➔ Subgraphs [SIAM SDM'16]

  - Summarization [ACM TKDD'18]

  - Comparisons [WWW '17]

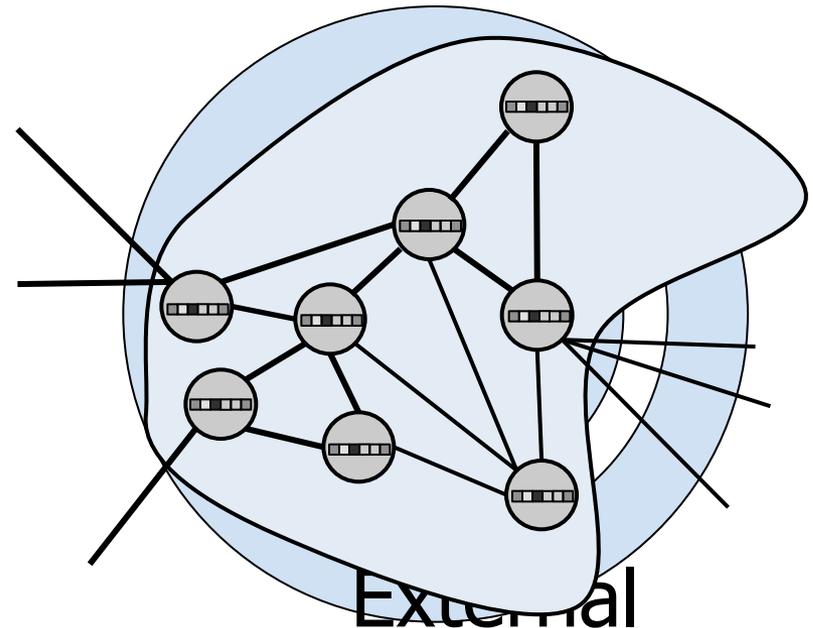


\* social circles, communities, egonetworks, ...

# What's a “good” subgraph anyway?

❖ **Given** an attributed subgraph, how to **quantify** its quality?

- ❑ Structure-only
  - Internal-only
    - ❑ average degree
  - Boundary-only
    - ❑ cut edges
  - Internal + Boundary
    - ❑ conductance
- ❑ Structure + Attributes



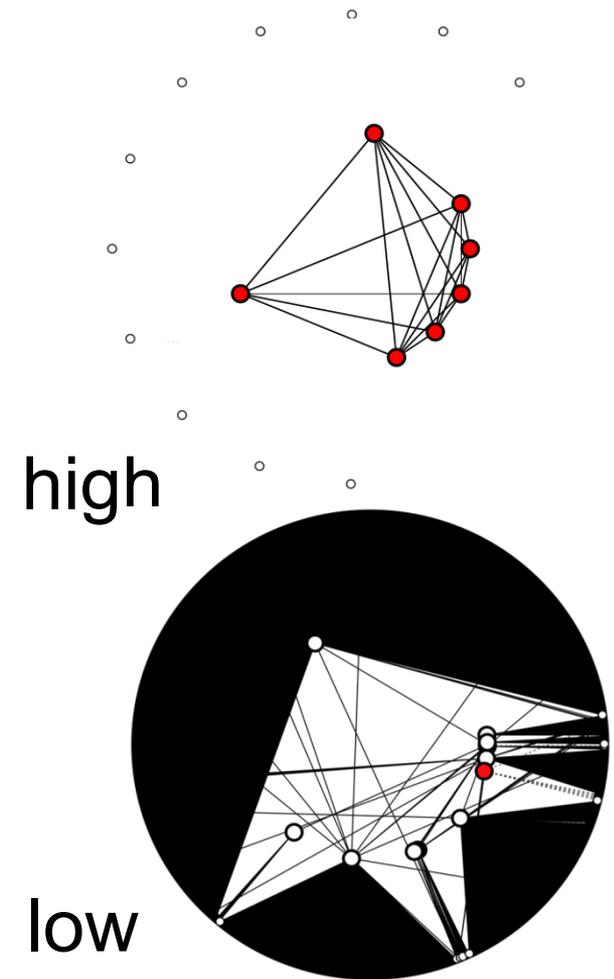
*Scalable Anomaly Ranking of Attributed Neighborhoods*

*Bryan Perozzi and Leman Akoglu*

*SIAM SDM 2016.*

# Normality (intuition)

- Given an attributed subgraph how to quantify quality?
  - Internal
    - structural density

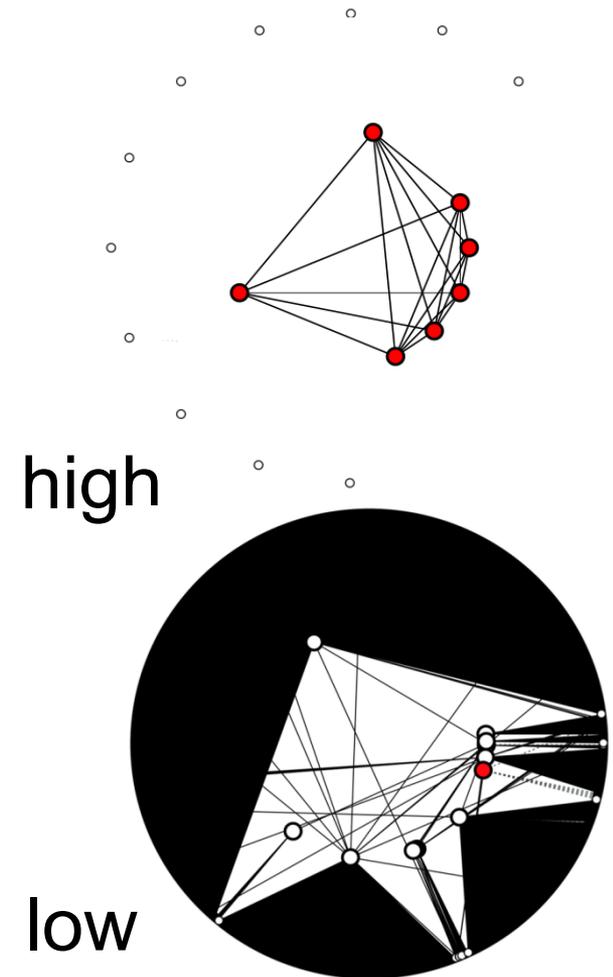
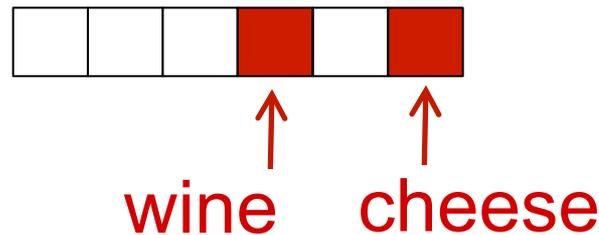


# Normality (intuition)

- Given an attributed subgraph how to quantify quality?

- Internal

- structural density AND
- attribute coherence
  - ❖ *neighborhood “focus”*



# Normality (intuition)

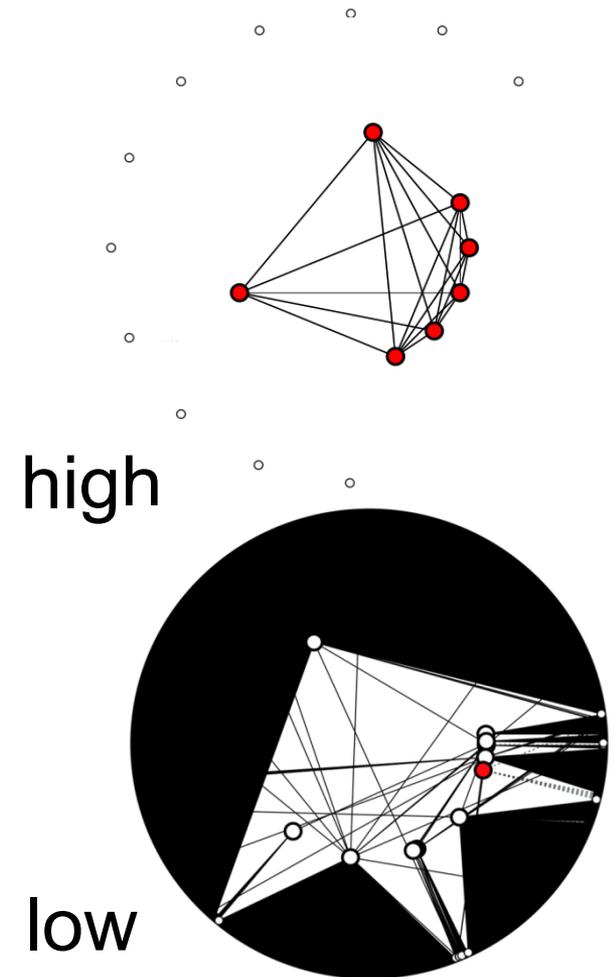
- Given an attributed subgraph how to quantify quality?

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  - ❖ *neighborhood “focus”*

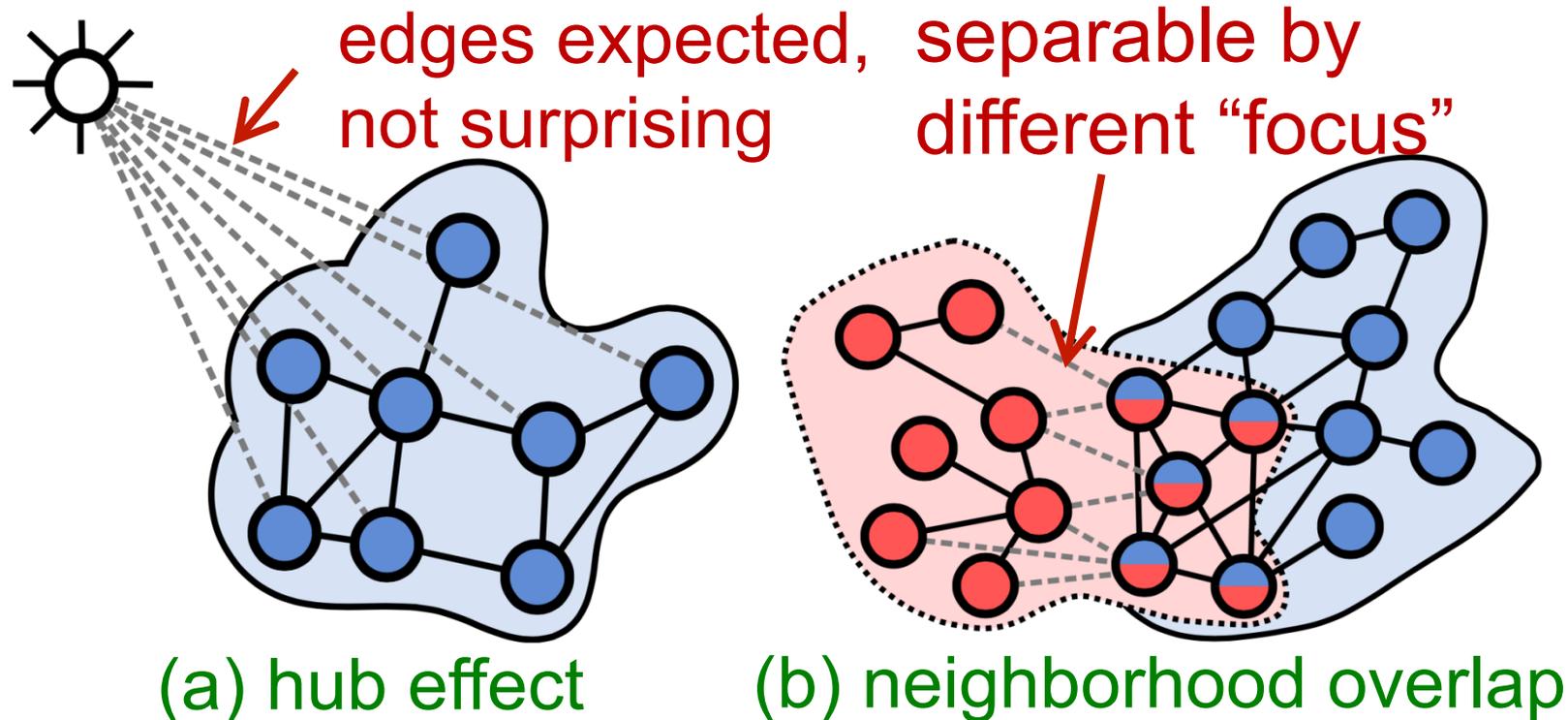
- Boundary

- structural sparsity, OR
- external separation
  - ❖ *“exoneration”*



# Normality (intuition)

- “*exoneration*” : by (a) null model, (b) attributes



- Motivation:

- no good cuts in real-world graphs [Leskovec+ '08]
- social circles overlap [McAuley+ '14]

# The measure of Normality



Null model

$$\underline{N} = \boxed{I} + E = \sum_{i \in C, j \in C} \left( A_{ij} - \frac{k_i k_j}{2m} \right) s(x_i, x_j | w)$$

internal consistency

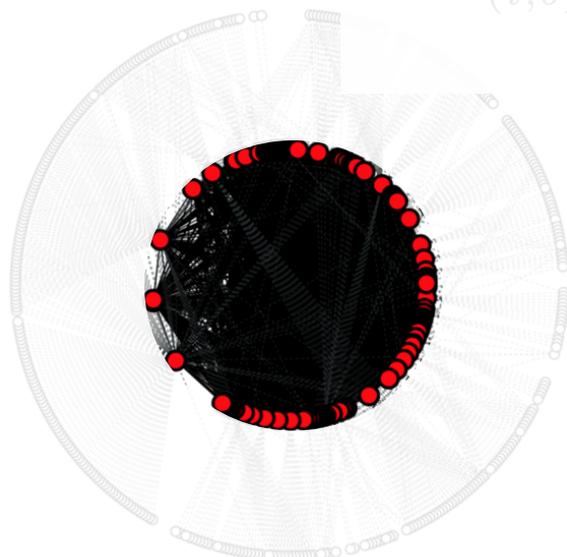
dot-product, or Kronecker's  $\delta$

"focus" vector



wine

cheese



1

# The measure of Normality

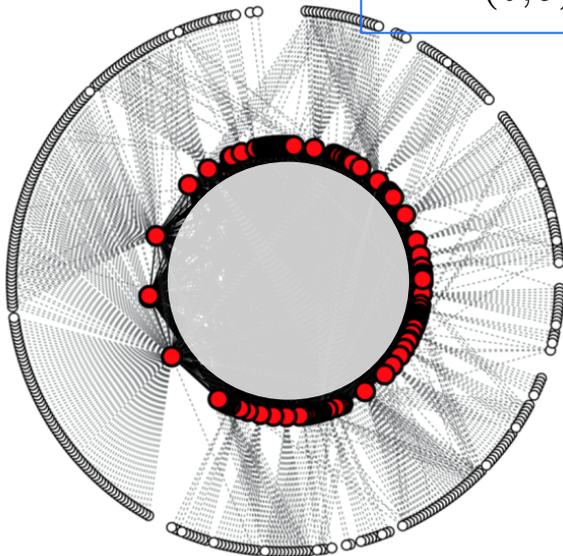


$$\underline{N} = I + \boxed{E} = \sum_{i \in C, j \in C} \left( A_{ij} - \frac{k_i k_j}{2m} \right) s(\mathbf{x}_i, \mathbf{x}_j | \mathbf{w})$$

external  
separability

$$- \sum_{\substack{i \in C, b \in B \\ (i, b) \in \mathcal{E}}} \left( 1 - \min\left(1, \frac{k_i k_b}{2m}\right) \right) s(\mathbf{x}_i, \mathbf{x}_b | \mathbf{w})$$

1



# The measure of Normality



- Given an attributed subgraph, can we find the **attribute weights**?

$$N(C) = \sum_{\substack{i \in C, j \in C, \\ i \neq j}} \left( A_{ij} - \frac{k_i k_j}{2m} \right) sim_{\mathbf{w}}(\mathbf{x}_i, \mathbf{x}_j) \\ - \sum_{\substack{i \in C, b \in B \\ (i, b) \in \mathcal{E}}} \left( 1 - \min\left(1, \frac{k_i k_b}{2m}\right) \right) sim_{\mathbf{w}}(\mathbf{x}_i, \mathbf{x}_b)$$

1

$\arg \max_{\mathbf{w}}$  **latent**  $\mathbf{w}^T$

$$\left[ \sum_{\substack{i \in C, j \in C, \\ i \neq j}} \left( A_{ij} - \frac{k_i k_j}{2m} \right) (\mathbf{x}_i \odot \mathbf{x}_j) \right. \\ \left. - \sum_{\substack{i \in C, b \in B \\ (i, b) \in \mathcal{E}}} \left( 1 - \min\left(1, \frac{k_i k_b}{2m}\right) \right) (\mathbf{x}_i \odot \mathbf{x}_b) \right]$$

2

# Optimizing Normality

Details

$$N = I + E = \sum_{i \in C, j \in C} \left( A_{ij} - \frac{k_i k_j}{2m} \right) s(\mathbf{x}_i, \mathbf{x}_j | \mathbf{w}) \\ - \sum_{\substack{i \in C, b \in B \\ (i, b) \in \mathcal{E}}} \left( 1 - \min\left(1, \frac{k_i k_b}{2m}\right) \right) s(\mathbf{x}_i, \mathbf{x}_b | \mathbf{w})$$

1

$$\max_{\mathbf{w}_C} \quad \mathbf{w}_C^T \cdot \left[ \sum_{i \in C, j \in C} \left( A_{ij} - \frac{k_i k_j}{2m} \right) s(\mathbf{x}_i, \mathbf{x}_j) \right. \\ \left. - \sum_{\substack{i \in C, b \in B \\ (i, b) \in \mathcal{E}}} \left( 1 - \min\left(1, \frac{k_i k_b}{2m}\right) \right) s(\mathbf{x}_i, \mathbf{x}_b) \right]$$

2

$$\max_{\mathbf{w}_C} \quad \mathbf{w}_C^T \cdot (\hat{\mathbf{x}}_I + \hat{\mathbf{x}}_E)$$

3

$$\text{s.t.} \quad \|\mathbf{w}_C\|_p = 1, \quad \mathbf{w}_C(f) \geq 0, \quad \forall f = 1 \dots d$$

# Optimizing Normality

Details

$$\begin{aligned} \max_{\mathbf{w}_C} \quad & \mathbf{w}_C^T \cdot \underbrace{(\hat{\mathbf{x}}_I + \hat{\mathbf{x}}_E)}_{\mathbf{x}} \\ \text{s.t.} \quad & \|\mathbf{w}_C\|_p = 1, \quad \mathbf{w}_C(f) \geq 0, \quad \forall f = 1 \dots d \end{aligned}$$

$p = 1$ :  $\mathbf{w}_C(f) = 1$  **one** attribute  $f$  with largest  $\mathbf{x}$

$p = 2$ :  $\mathbf{w}_C(f) = \frac{\mathbf{x}(f)}{\sqrt{\sum_{\mathbf{x}(i) > 0} \mathbf{x}(i)^2}}$  **all**  $f$  with positive  $\mathbf{x}$

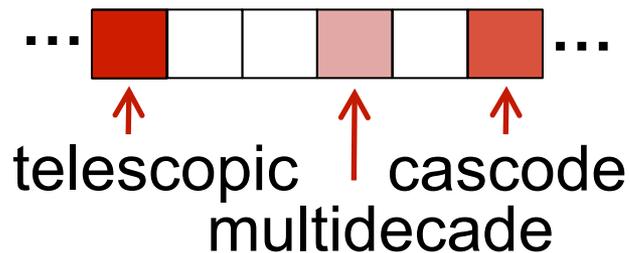
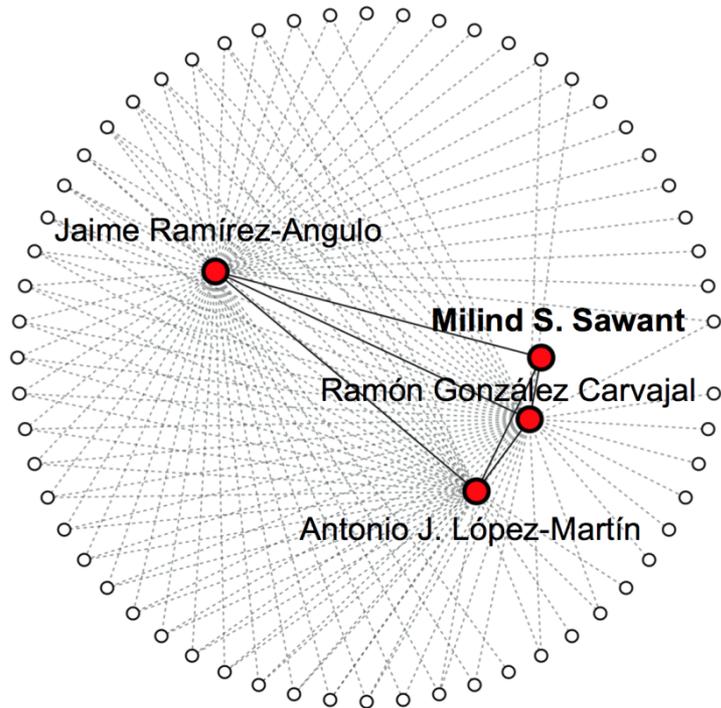
**Normality** becomes  $N = \mathbf{w}_C^T \cdot \mathbf{x} = \|\mathbf{x}_+\|_2$

**Linear in number of attributes!**

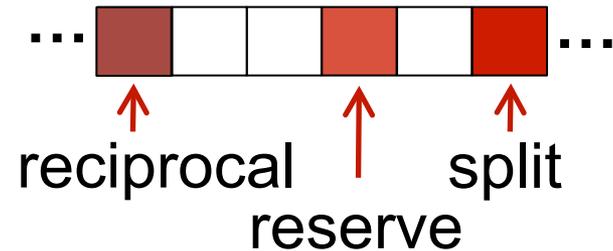
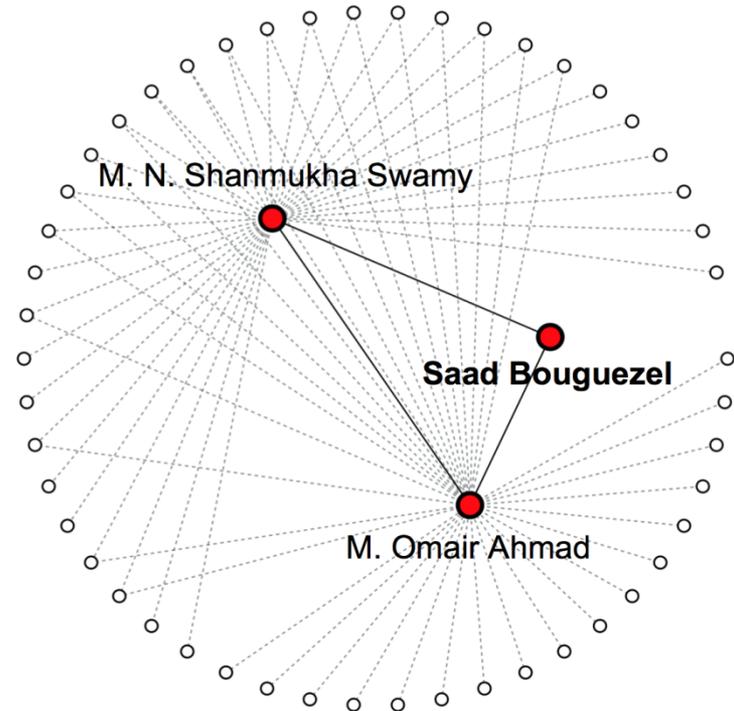
when  $p = 1$ ,  $N \in [-1, 1]$        $N \in [-1, \|\mathbf{x}_+\|_2]$  when  $p = 2$ .

# Illustrative examples

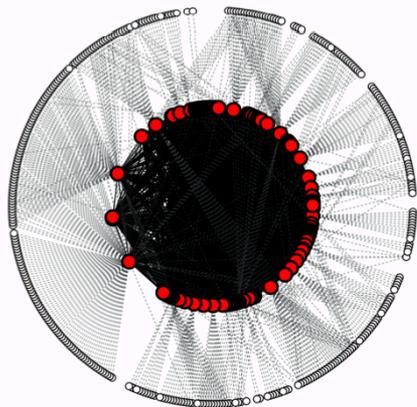
## telescopic op-amps



## split-radix FFT

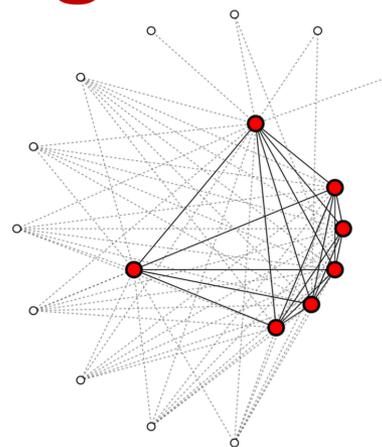


# Example neighborhoods



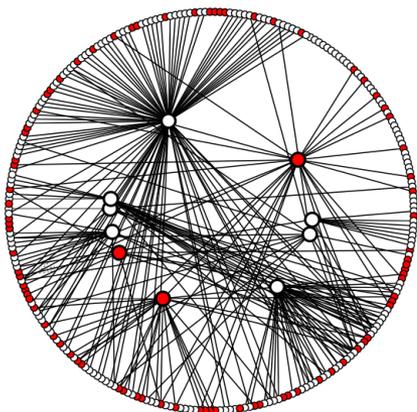
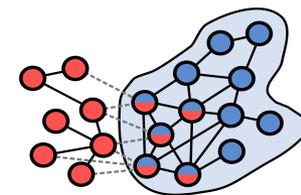
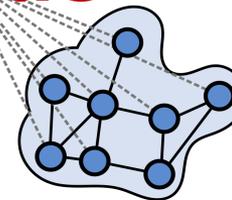
DBLP

$$L_1 = 0.979, L_2 = 2.17$$



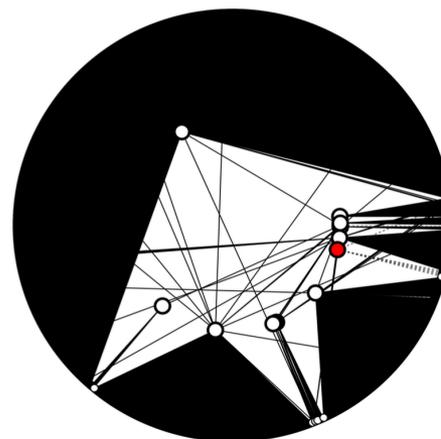
Twitter

$$L_1 = 0.724, L_2 = 1.10$$



Google+

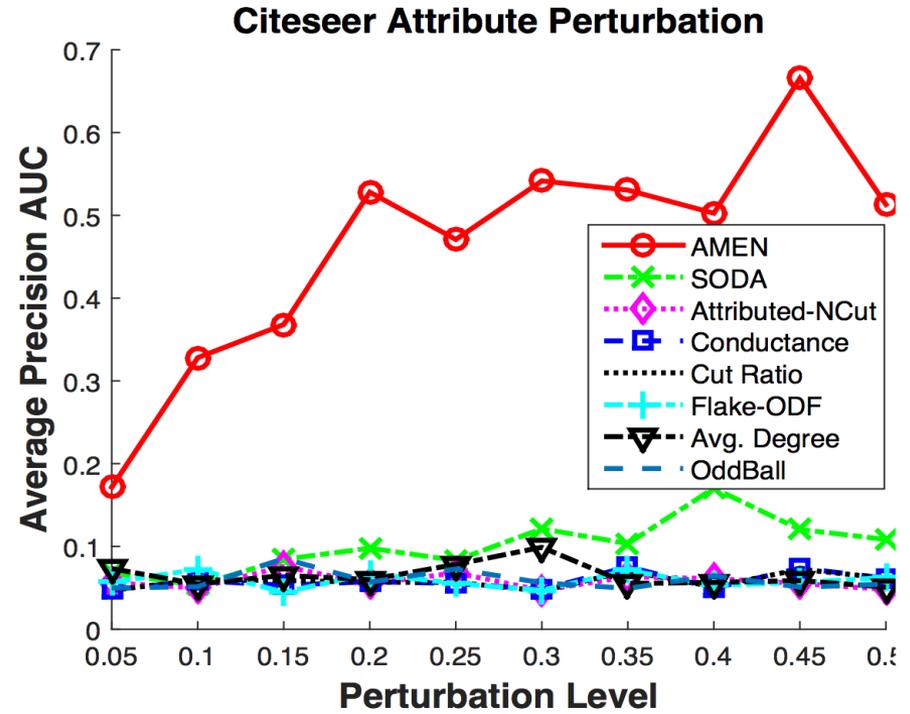
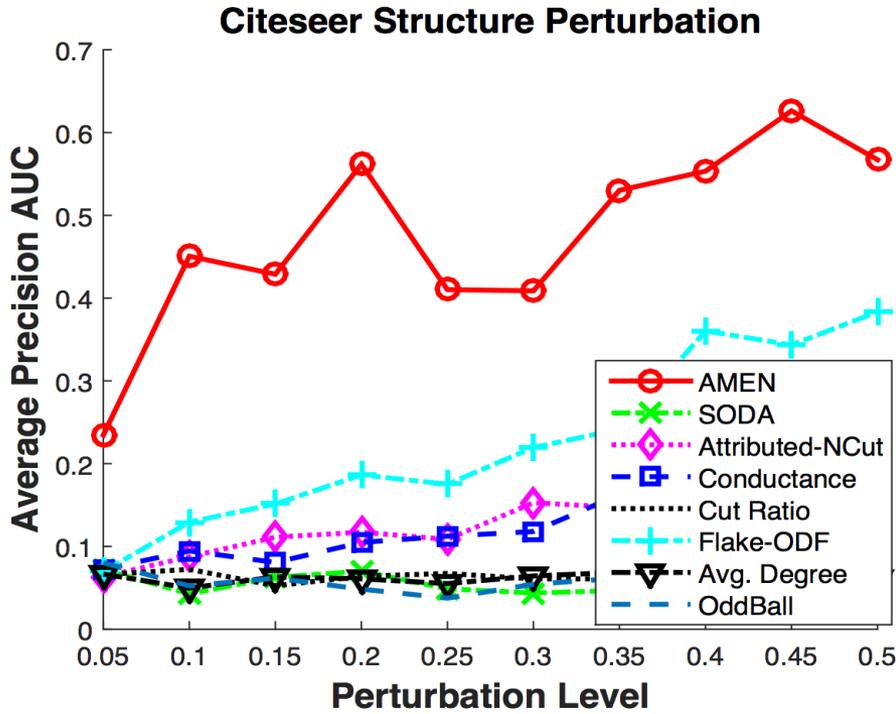
$$L_1 = L_2 = -0.873$$



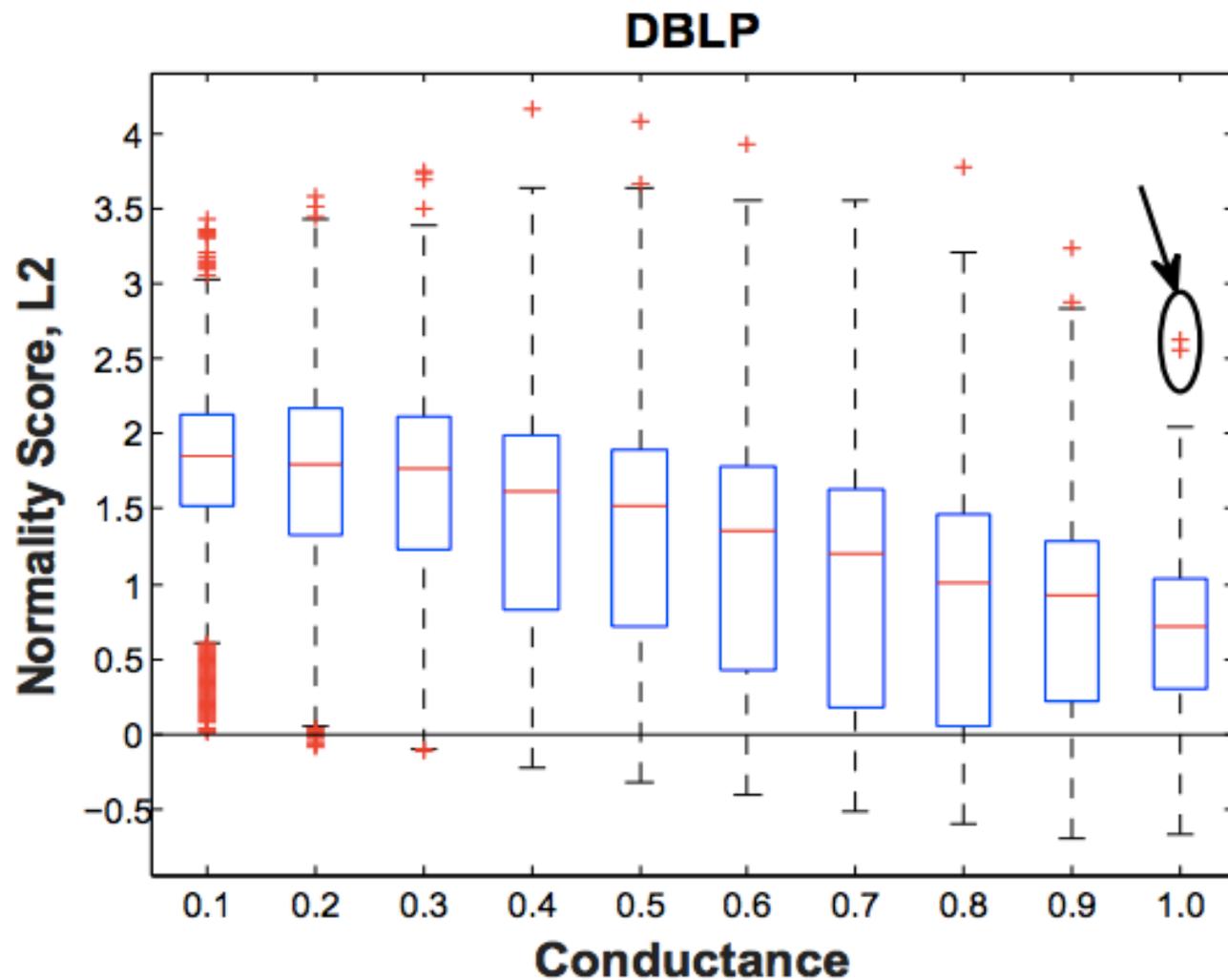
Citeseer

$$L_1 = L_2 = -0.956$$

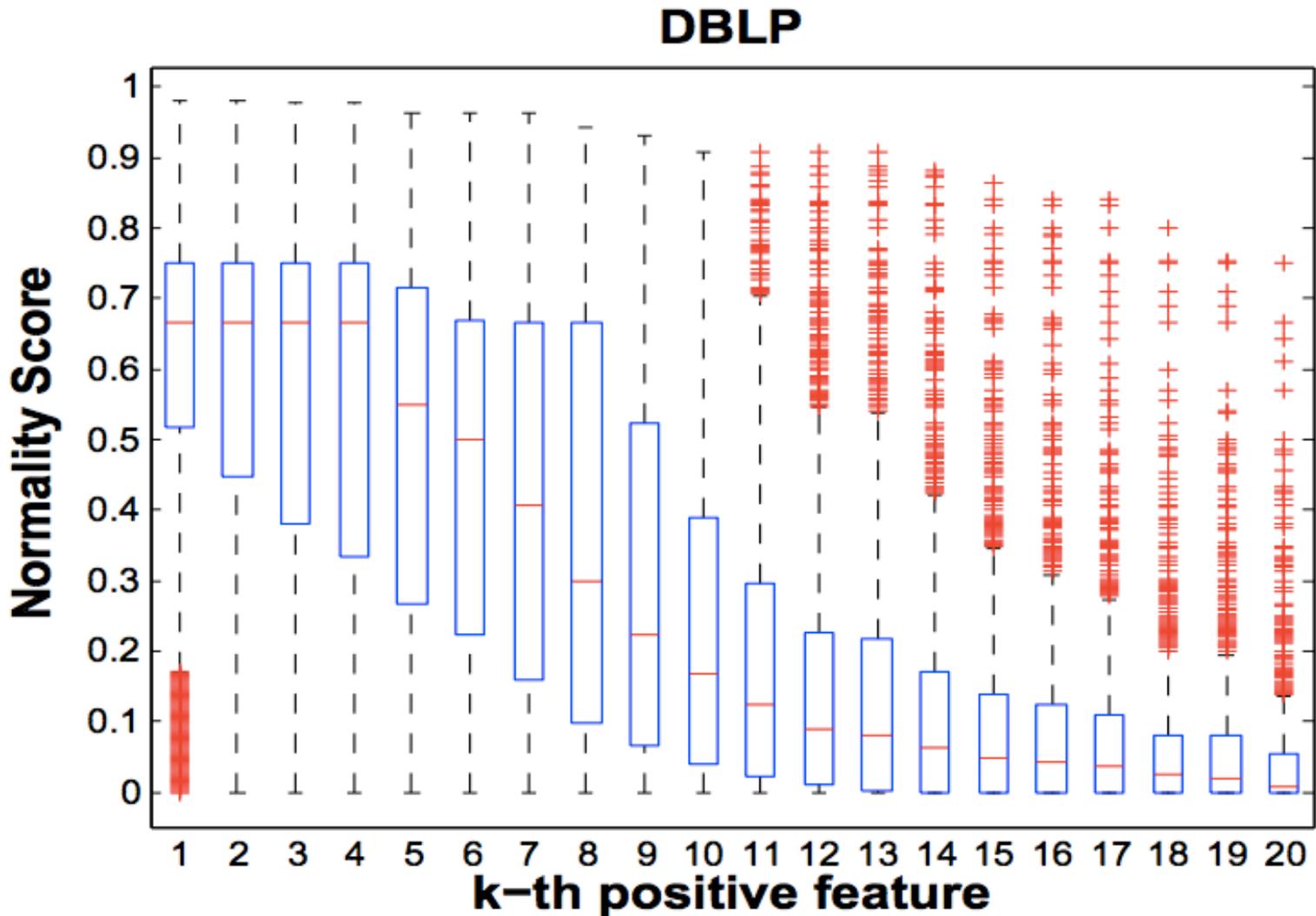
# Anomaly detection: Perturbed data



# Normality vs Conductance, DBLP



# Attribute distribution, DBLP



# Summary

A new **quality measure** for attributed subgraphs  
**normality** considers:

internal + boundary  
structure + attributes  
subgraph **focus**

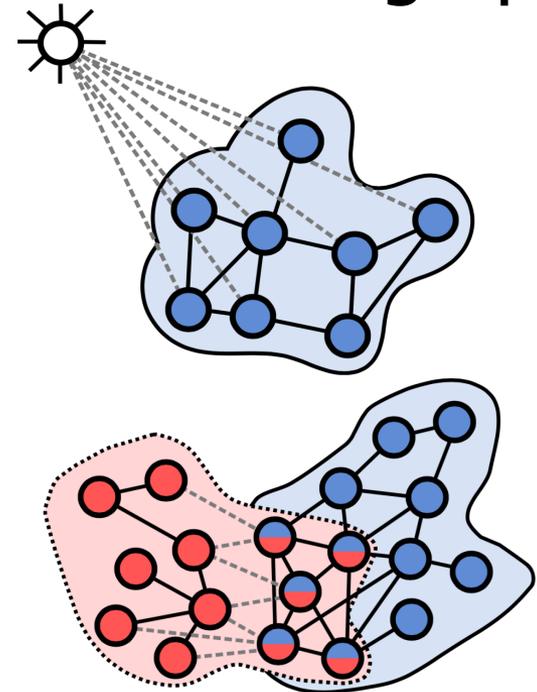
*“exoneration”*

Automatic **inference** of **focus**

via **normality** maximization

unsupervised

linear in #attributes



# Paper, code, data

- <http://www.perozzi.net/projects/amen/>

Bryan Perozzi



## Overview

- » About Me
- » Research Interests
- » Selected Publications
- » Honors and Awards
- » Press Coverage

## Publications

- » Conference & Journal
- » Workshop & Poster

## Projects

- » Anomaly Detection in Attributed Graphs

## Anomaly Ranking of Attributed Neighborhoods

Bryan Perozzi, Leman Akoglu  
May 9, 2016

Awards: **Best Paper Runner-up, SDM'16!**

### Overview

Given a graph with node attributes, what neighborhoods are anomalous? To answer this question, one needs a quality score that utilizes both structure and attributes. Popular

existing measures either quantify the structure only and ignore the attributes (e.g.,

**Scalable Anomaly Ranking of Attributed Neighborhoods**

*Bryan Perozzi and Leman Akoglu* *SIAM SDM 2016.*

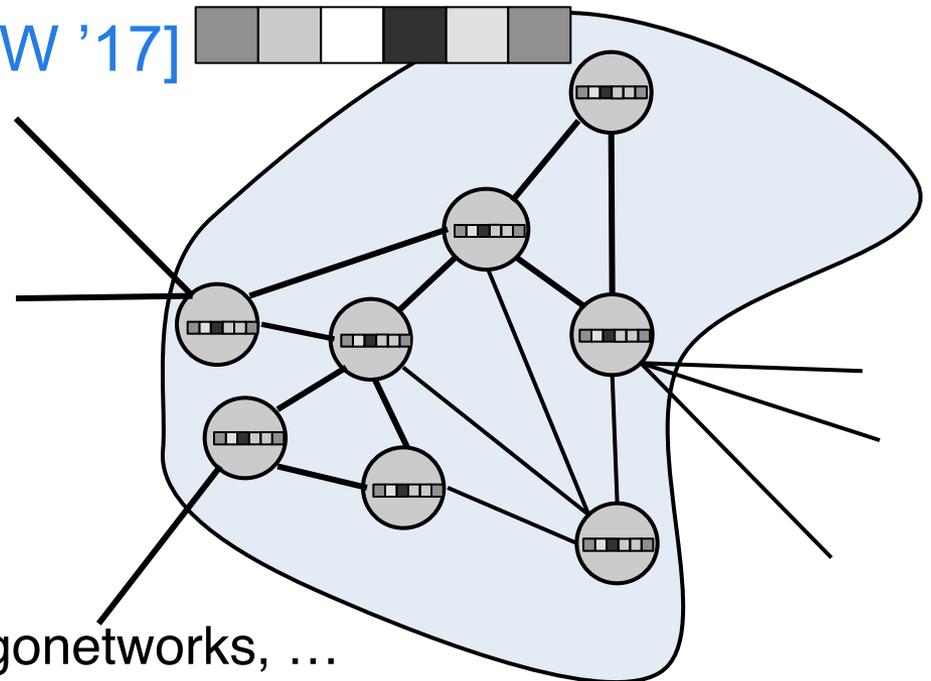
# This talk

- Attributed (sub)graphs\*

- Subgraphs [SIAM SDM'16]

- ➔ Summarization [ACM TKDD'18]

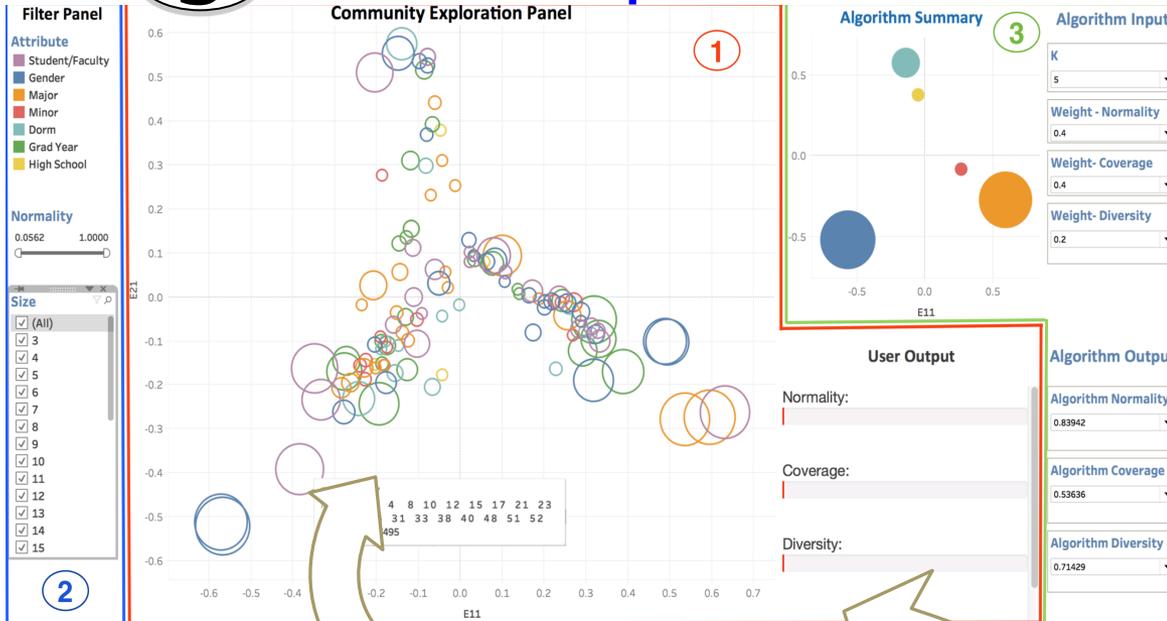
- Comparisons [WWW '17]



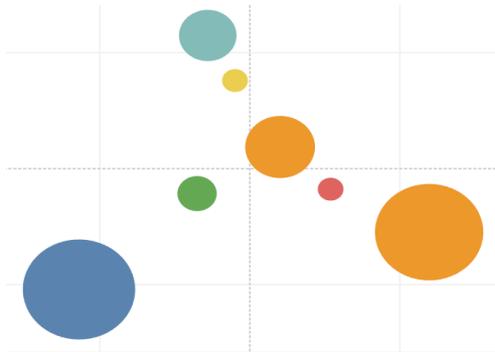
\* social circles, communities, egonetworks, ...

# Overview

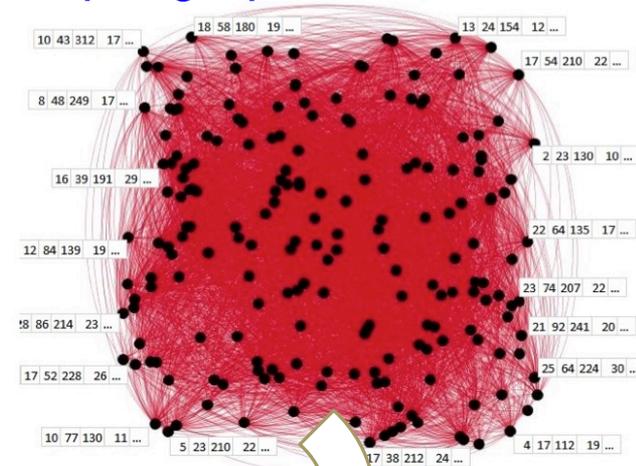
## 3 Interactive exploration



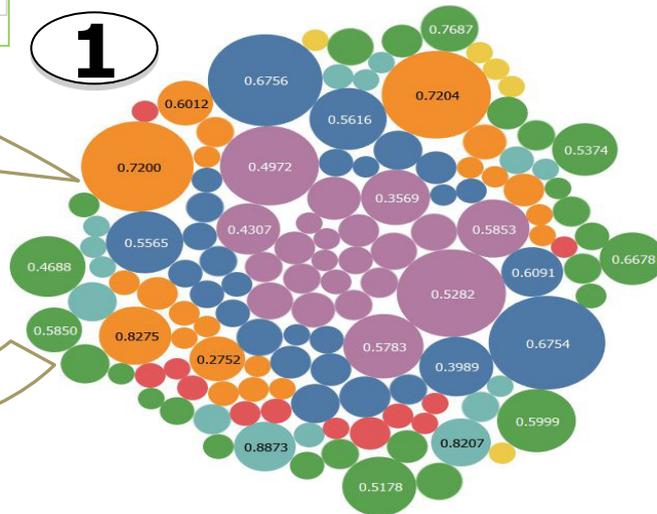
## 2 Summarization



## Input graph

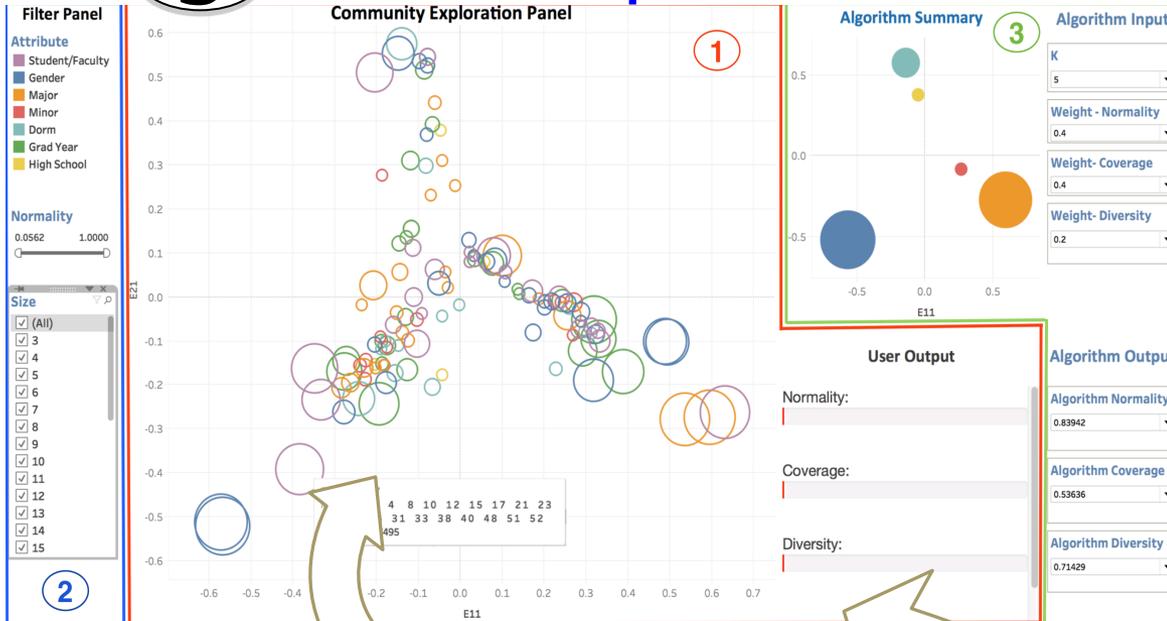


## 1 Social circle extraction

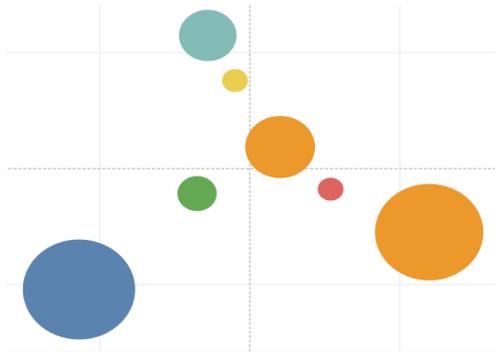


# Overview

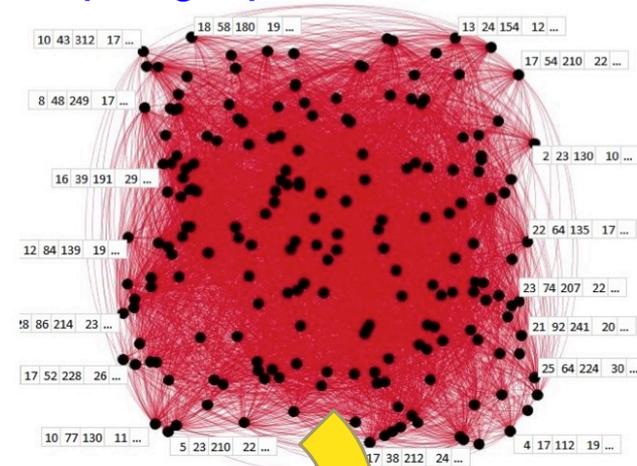
## 3 Interactive exploration



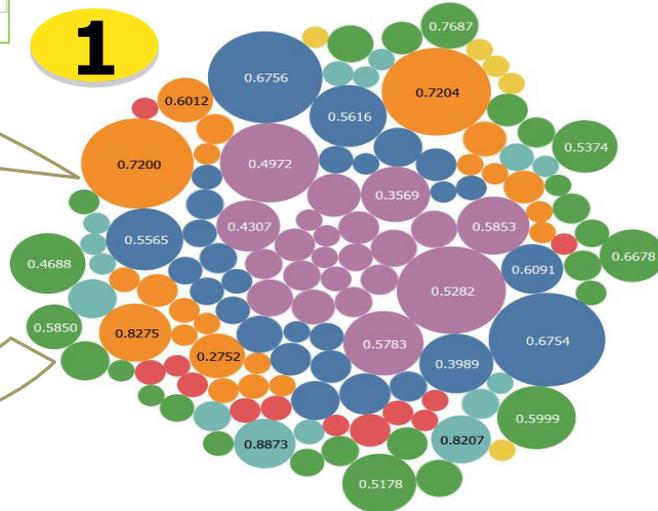
## 2 Summarization



## Input graph



## Social circle extraction



# Extracting Social Circles

- a GRASP (Greedy Randomized Adaptive Search Procedure) approach [Feo & Resende '95]

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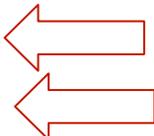
**Algorithm 1** EXTRACTATTRIBUTEDSOCIALCIRCLES

---

**Input:**  $G = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ , node attribute vectors  $\mathbf{x}_{u \in \mathcal{V}}$ ,  $T_{max}, \alpha$

**Output:** set of extracted communities  $\mathcal{C}$

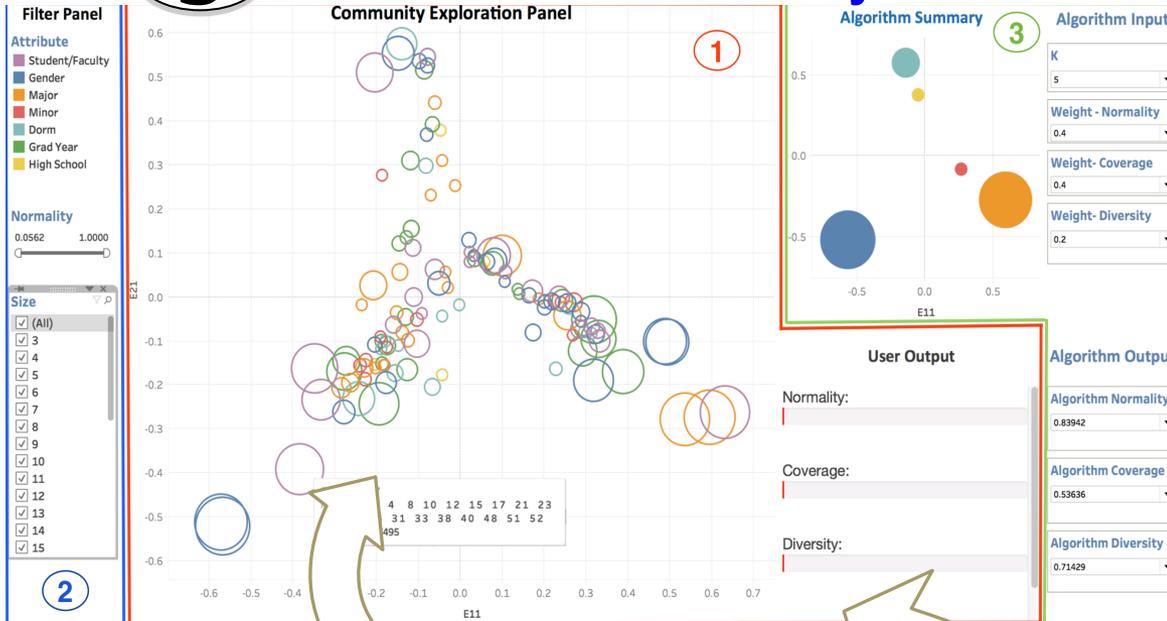
```
1:  $\mathcal{C} := \emptyset$ 
2: for each  $u \in \mathcal{V}$  do
3:   for  $t = 1 : T_{max}$  do
4:      $S :=$  CONSTRUCTION( $u, G, \alpha$ )
5:      $\mathcal{C} := \mathcal{C} \cup$  LOCALSEARCH( $S, G$ )
6:   end for
7: end for
8: return  $\mathcal{C}$ 
```



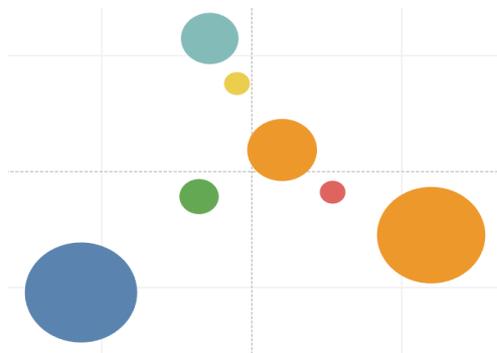
- note: one focus attribute per circle

# Overview

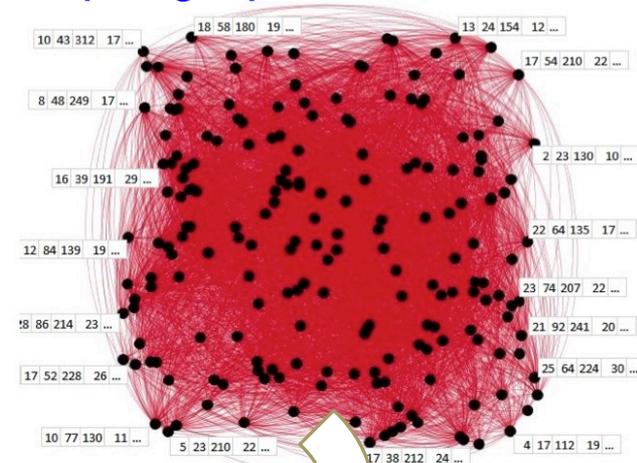
## 3 Interactive Visual Analysis



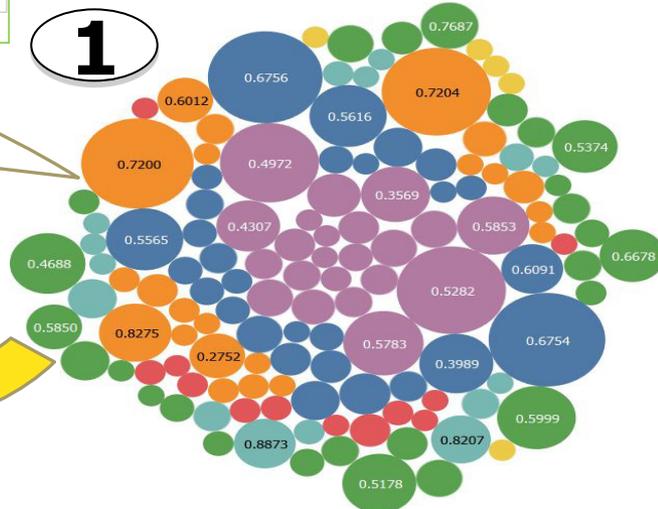
## 2 Summarization



## Input graph

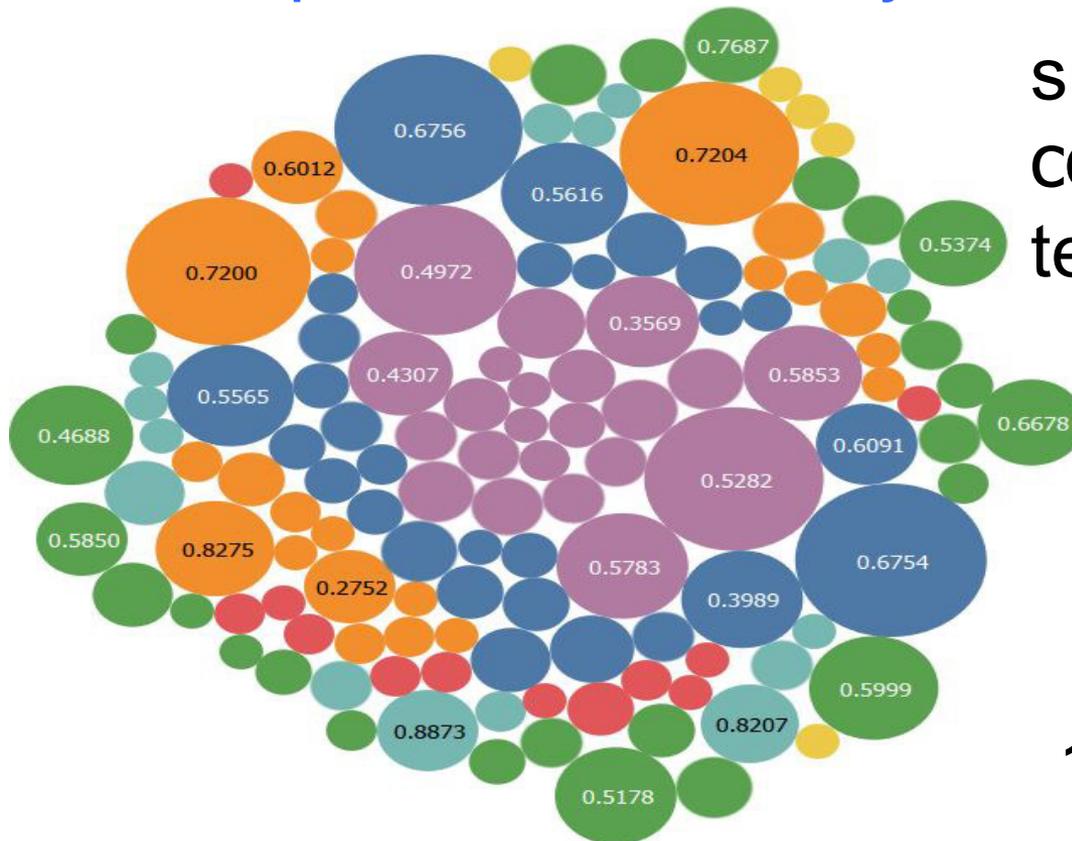


## 1 Social circle extraction



# Summarization

- Social circles: what size, quality and focus?
  - Attempt: visual summary



125 circles!

- does not reflect overlap between circles!

# Summarization



- Want a summary (a few circles):
  - high **normality**
  - well-**“cover”** the graph
  - **diverse** in ‘focus’

$$\begin{aligned} \max_{\substack{S \subseteq \mathcal{C} \\ |S|=K}} f(S) &= \alpha \text{avgnorm}(S) + \beta \text{cov}(S) + (1 - \alpha - \beta) \text{div}(S) \\ &= \alpha \frac{\sum_{C \in S} N(C)}{K} + \beta \frac{|\bigcup_{C \in S} C|}{n} + (1 - \alpha - \beta) \frac{|\bigcup_{C \in S} \mathcal{A}(C)|}{d} \end{aligned}$$

$0 \leq \alpha, \beta \leq 1$  can be interactively adjusted by users

# Summarization

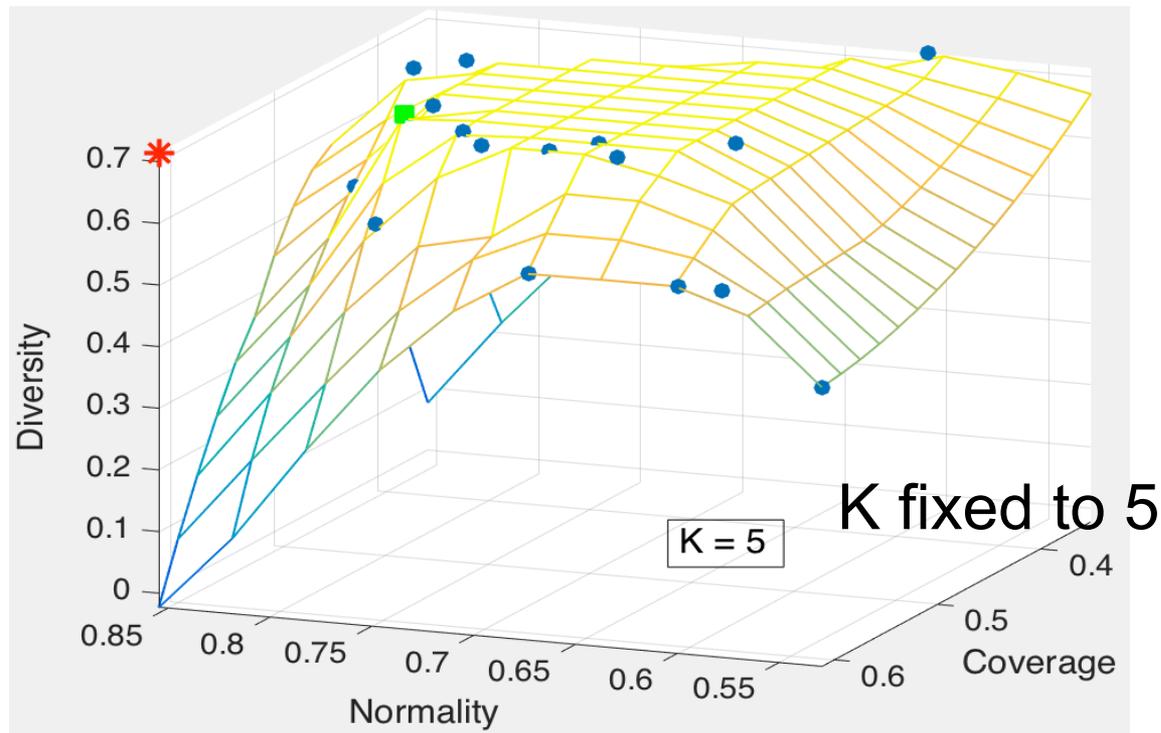


$$\max_{\substack{S \subseteq \mathcal{C} \\ |S|=K}} f(S) = \alpha \underbrace{\frac{\sum_{C \in S} N(C)}{K}}_{\text{avg. normality}} + \beta \underbrace{\frac{|\cup_{C \in S} C|}{n}}_{\text{coverage}} + (1 - \alpha - \beta) \underbrace{\frac{|\cup_{C \in S} \mathcal{A}(C)|}{d}}_{\text{diversity}}$$

- Provided  $K, n, d$  (denominators) fixed, easy to show that  $f : 2^{\mathcal{C}} \rightarrow \mathbb{R}_+$  is
  - non-negative
  - monotonic:  $A \subseteq B \subseteq \mathcal{C}, f(A) \leq f(B)$
  - submodular: for every  $A \subseteq B \subseteq \mathcal{C}$  and  $C \in \mathcal{C} \setminus B$ ,  
 $f(A \cup \{C\}) - f(A) \geq f(B \cup \{C\}) - f(B)$
- The “next-best” greedy algorithm: at least 63% of the objective value  $f(\cdot)$  of the *optimum* set.

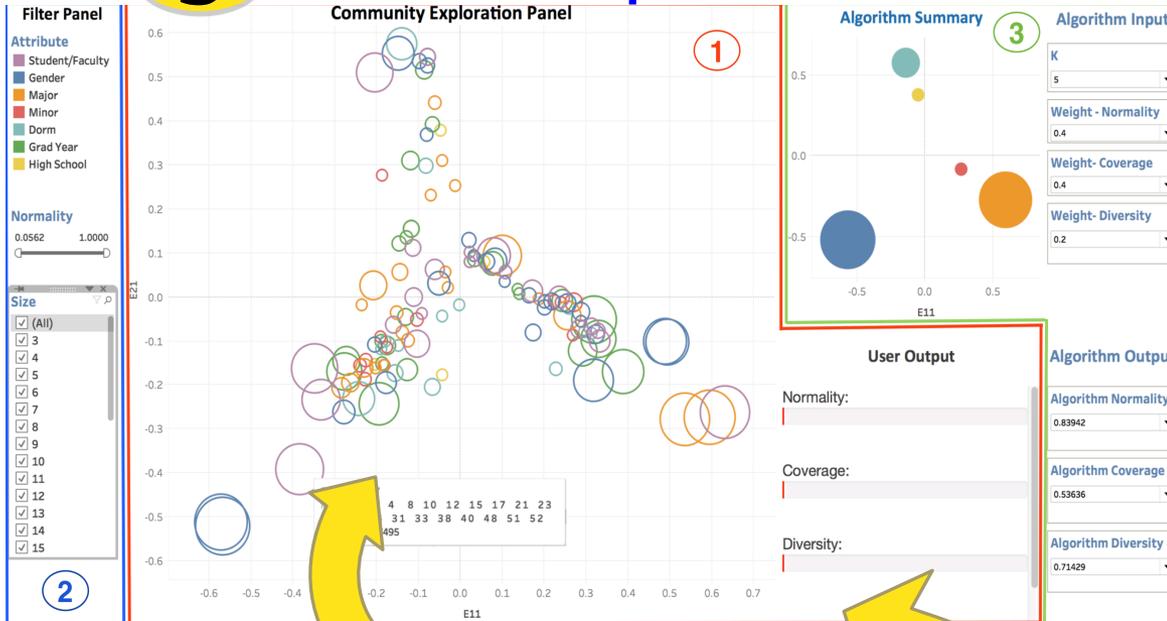
# Summarization

- surface formed by various parameter combinations  $(\alpha, \beta, 1 - \alpha - \beta)$  (blue dots)
- (green) square around the “knee”: a good trade-off between quality, coverage, and diversity

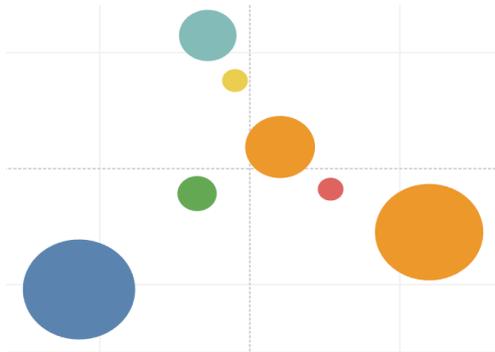


# Overview

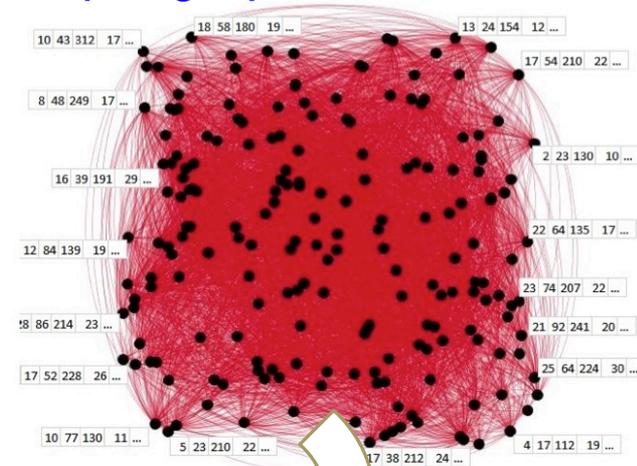
## 3 Interactive exploration



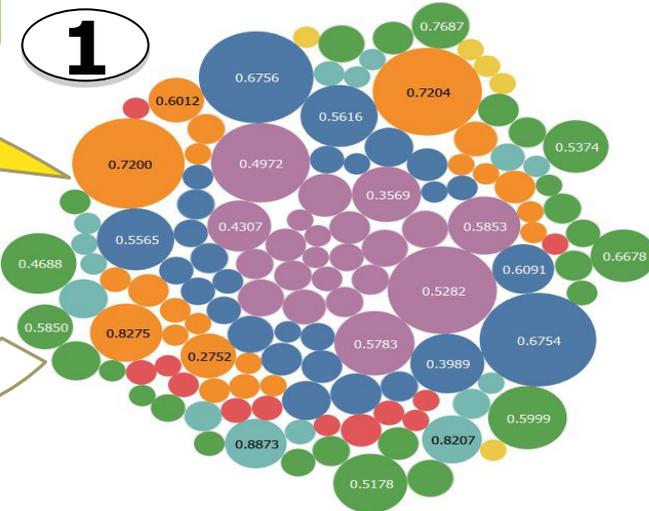
## 2 Summarization



## Input graph

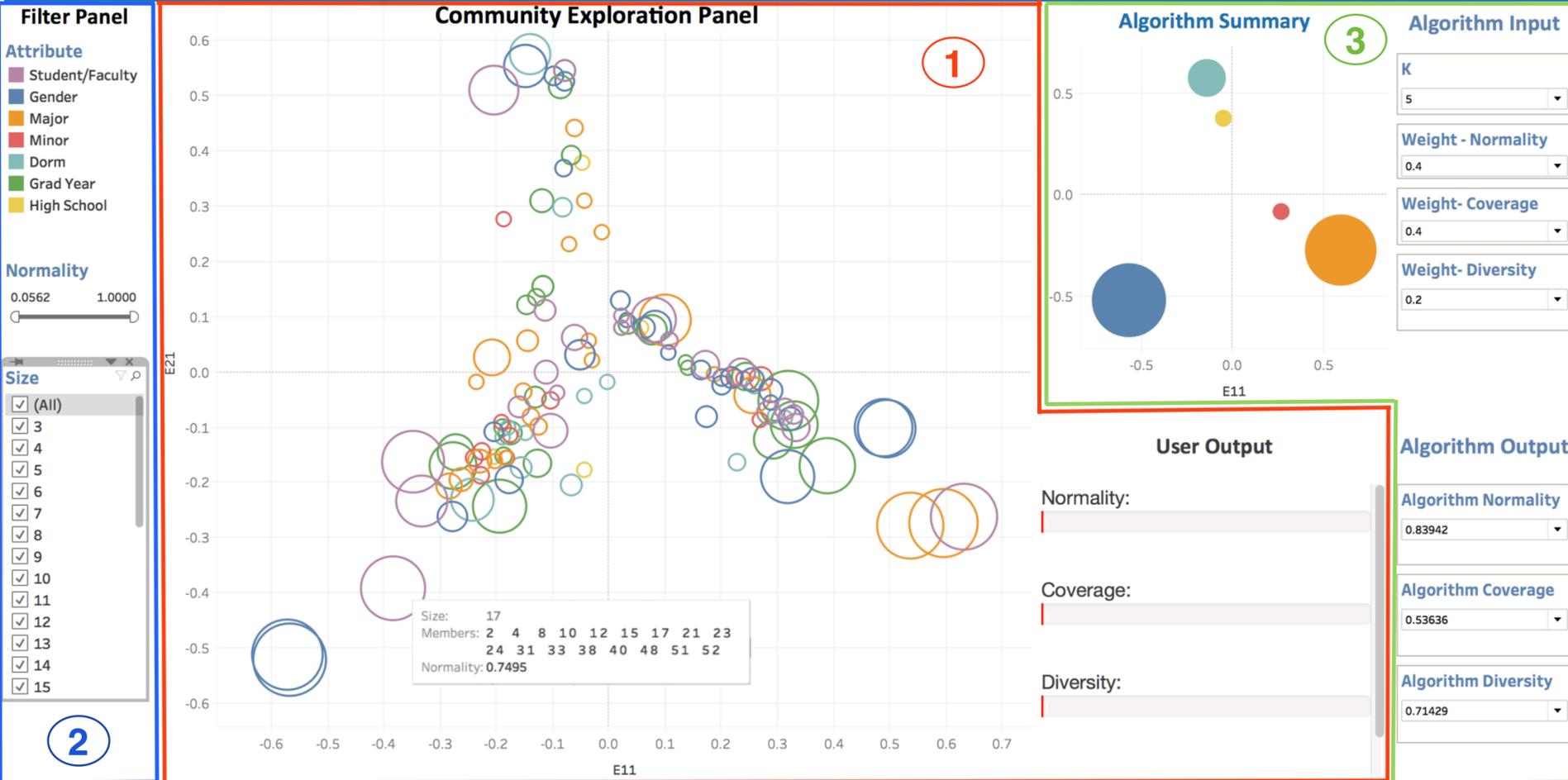


## Social circle extraction

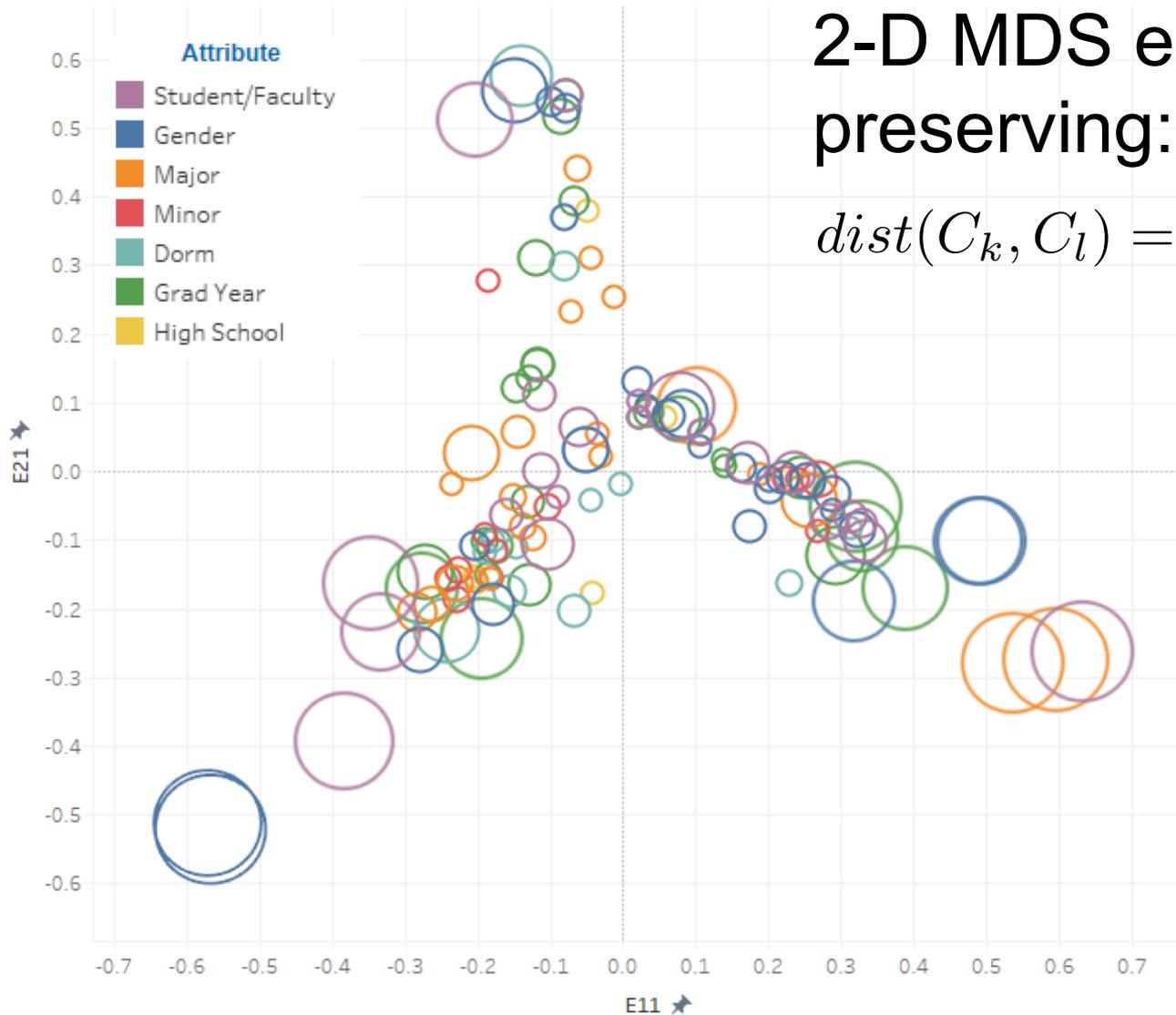


# Interactive Visual Exploration & Summarization

## Sensemaking of Attributed Social Networks



# Circle embedding

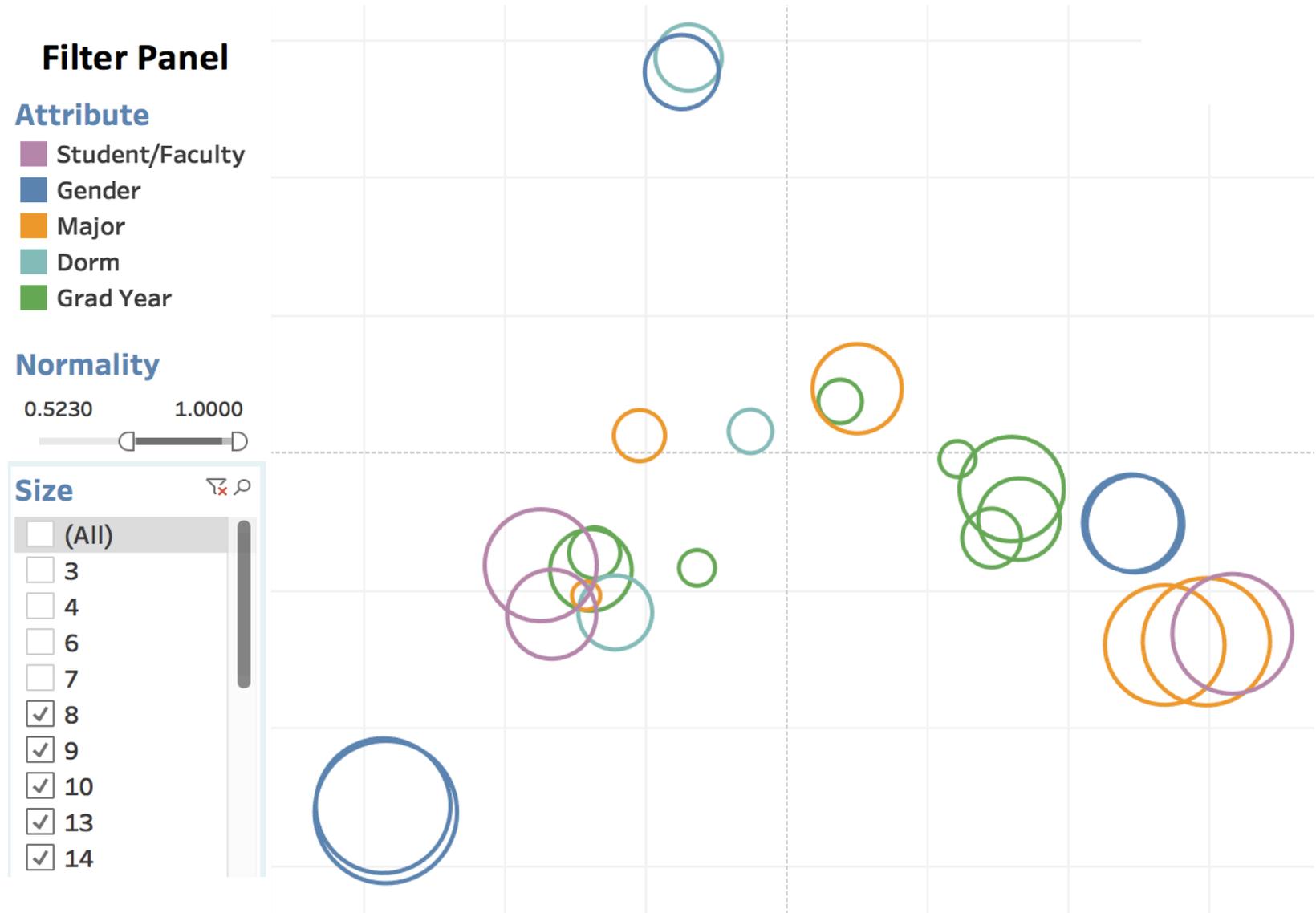


2-D MDS embedding  
preserving:

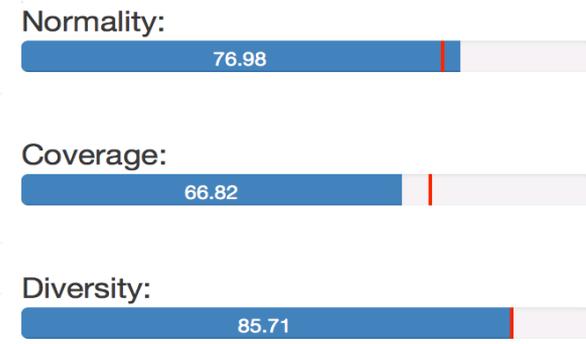
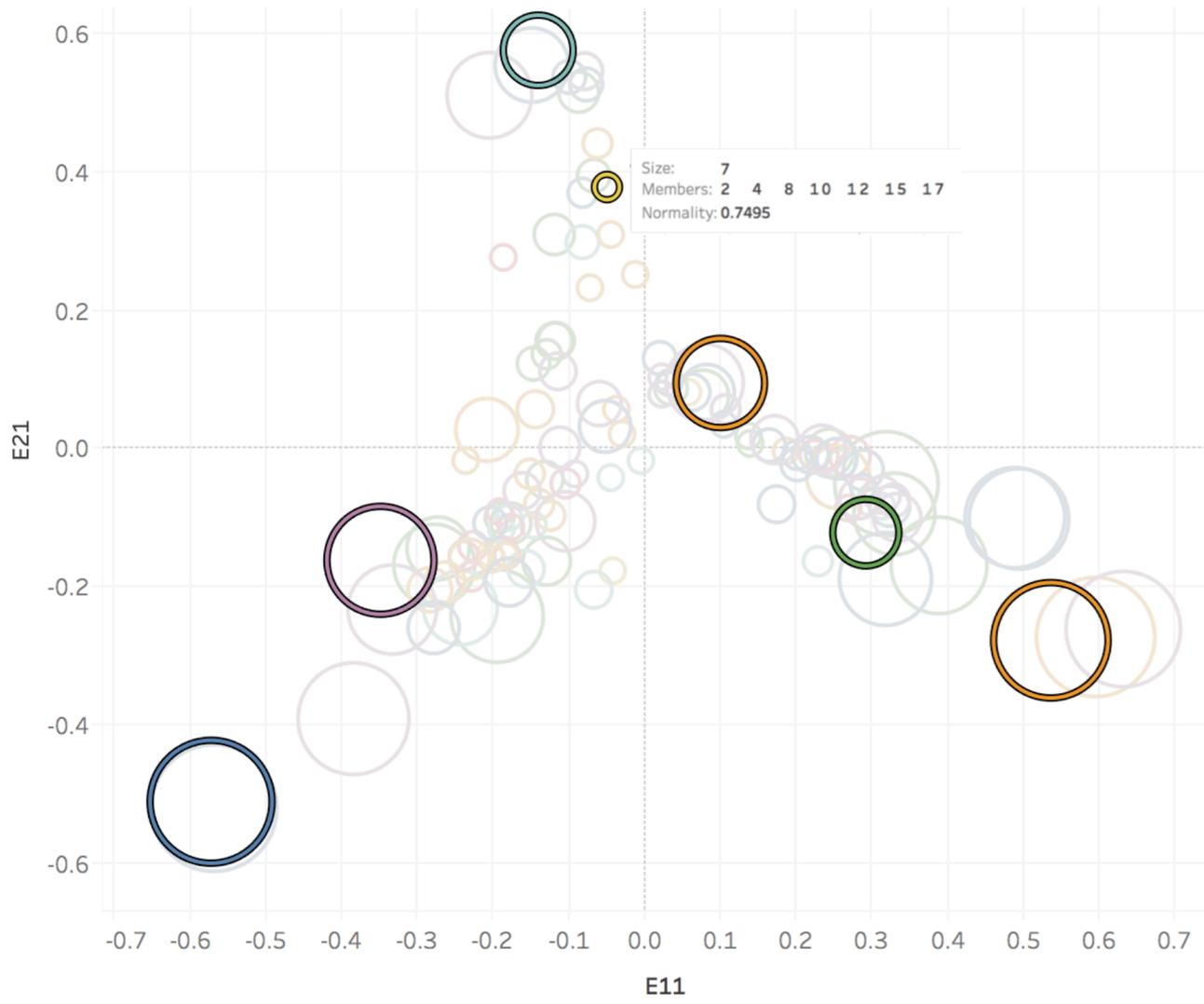
$$\text{dist}(C_k, C_l) = 1 - \frac{|C_k \cap C_l|}{\min(|C_k|, |C_l|)}$$

size  $\propto$  #nodes  
color: focus

# Interaction: Filtering

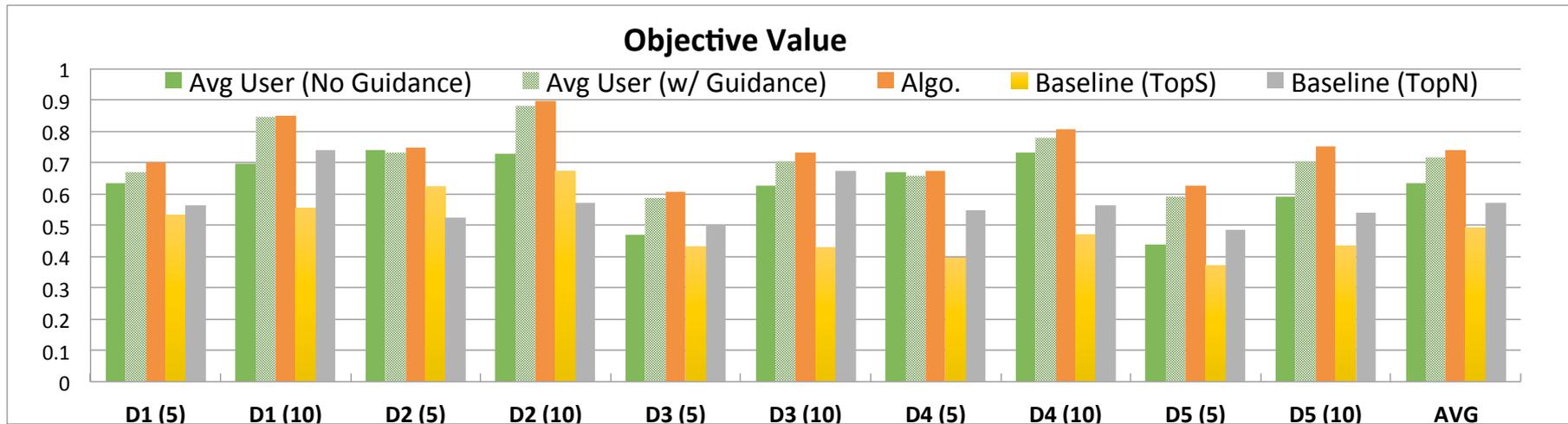


# Interaction: Circle summarization



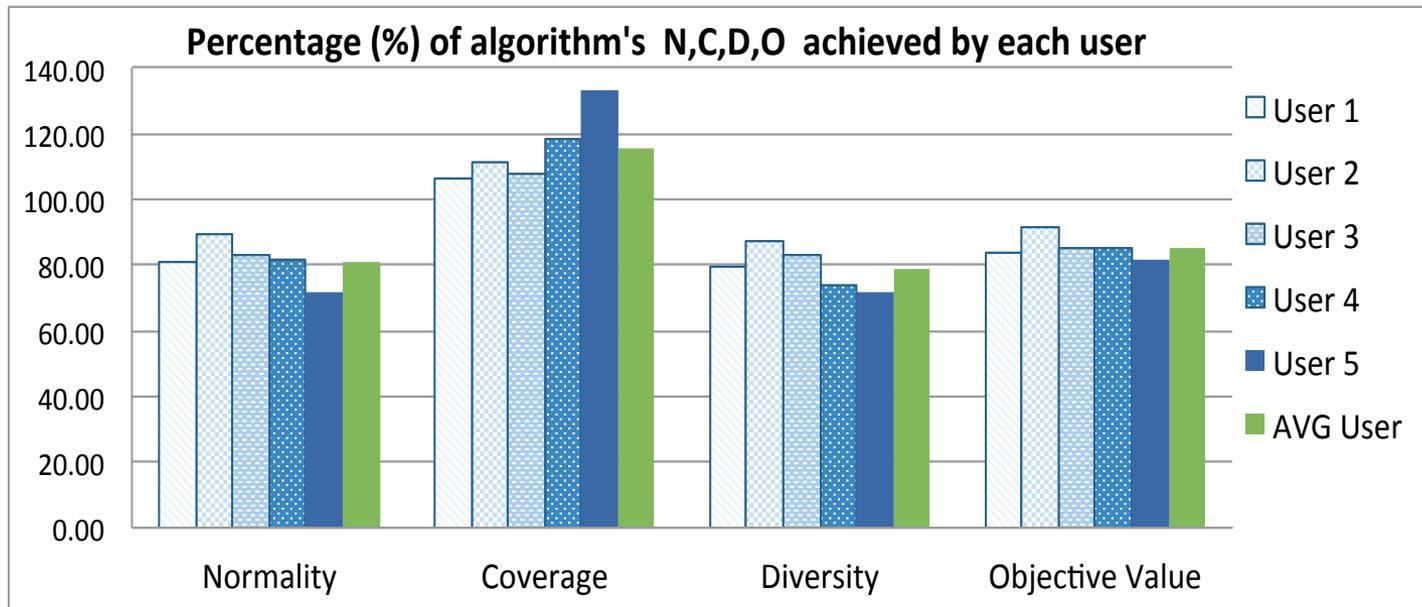
# Evaluation

**Q1) Summarization by visual exploration.** *Does interactive visualization help users construct effective summaries, as compared to strawman baselines?*



# Evaluation

**Q2)** *How close do the summaries by users **without guidance** get to the algorithm results (in terms of normality, coverage, diversity, and overall objective value)?*

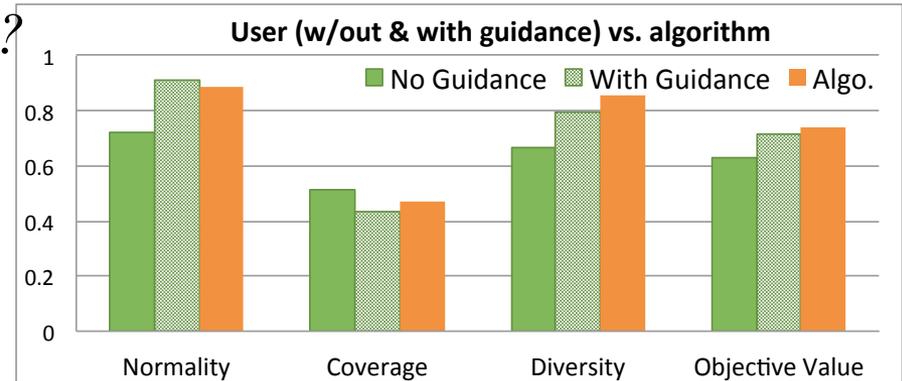


# Evaluation

## Q3) Alternative summarization by algorithmic guidance.

*How much guidance does our summarization algorithm provide users to derive alternative summaries and improve over their earlier results?*

$$100 O_{user}^{(after)} / O_{user}^{(before)}$$

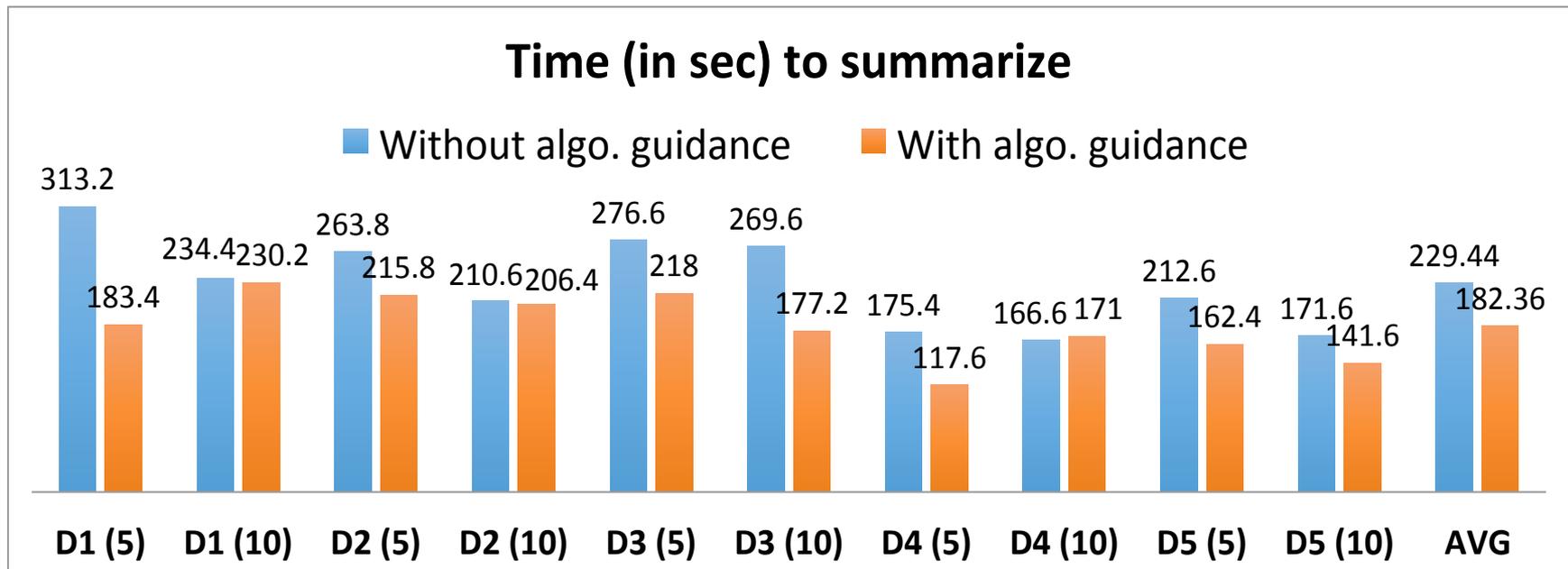


PERCENT % IMPROVEMENT IN OBJECTIVE VALUE BY EACH USER ON EACH DATA/TASK AFTER ALGORITHMIC GUIDANCE.

	D1 (5)	D1 (10)	D2 (5)	D2 (10)	D3 (5)	D3 (10)	D4 (5)	D4 (10)	D5 (5)	D5 (10)	
<b>User 1</b>	112.59	156.44	99.53	114.31	129.89	130.58	92.20	106.17	170.86	121.08	<b>123.37</b>
<b>User 2</b>	91.79	118.14	87.56	102.86	99.19	112.31	92.66	100.00	107.39	117.97	<b>102.99</b>
<b>User 3</b>	101.60	112.95	101.30	120.73	140.15	101.75	85.78	96.60	199.57	142.96	<b>120.34</b>
<b>User 4</b>	103.98	104.18	100.85	140.65	103.76	105.94	116.86	124.73	110.13	109.13	<b>112.02</b>
<b>User 5</b>	117.61	124.02	102.70	129.06	169.17	117.77	105.06	106.17	113.34	109.65	<b>119.45</b>
<b>Avg User</b>	105.51	123.15	98.39	121.52	128.43	113.67	98.51	106.73	140.26	120.16	<b>115.63</b>

# Evaluation

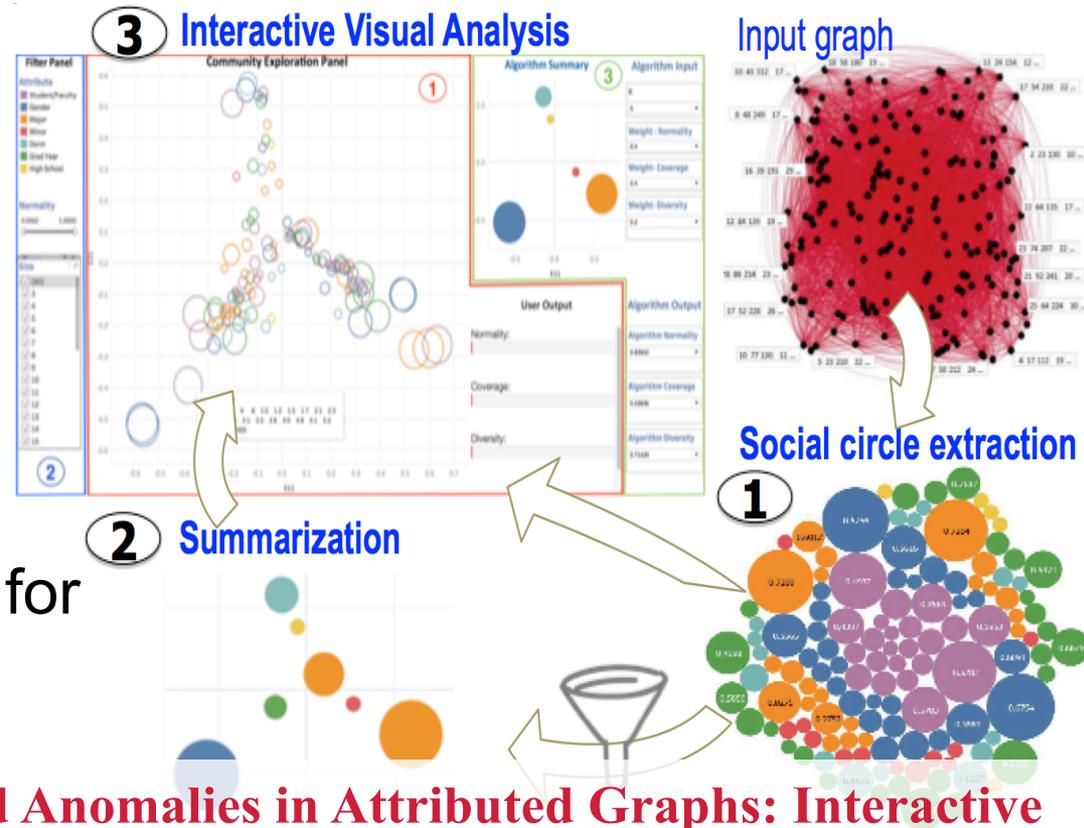
**Q4) Efficiency.** *How long does it take per user on average to construct (i) a summary without guidance, and (ii) alternative summary with guidance?*



# Summary

- An **end-to-end system** for sensemaking of node-attributed networks

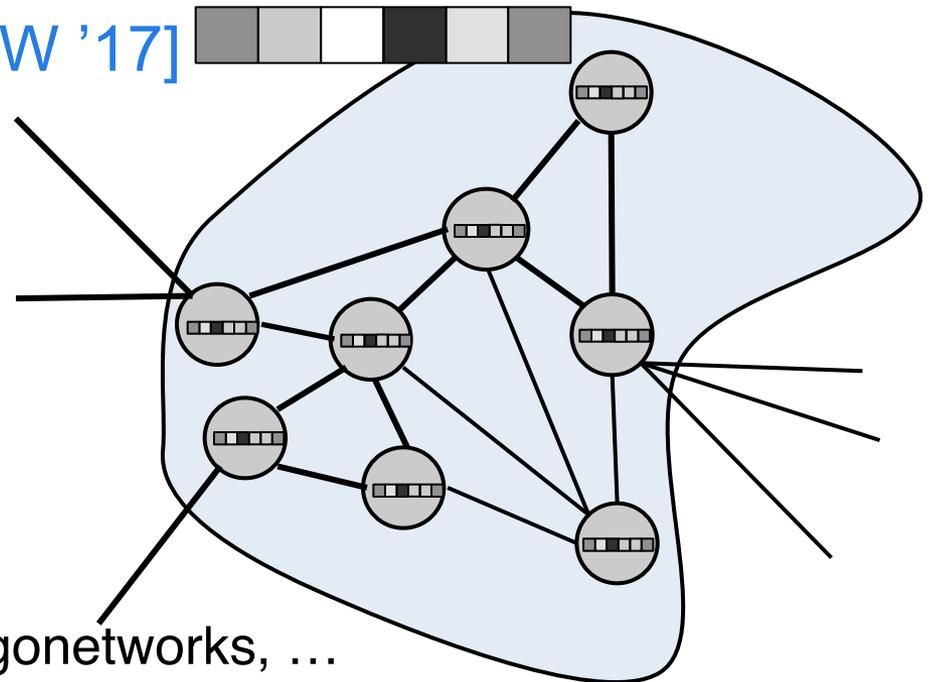
- 1. Circle extraction** based on **normality**
- 2. Summarization** wrt
  - quality,
  - coverage, and
  - diversity
- 3. Interactive interface** for
  - exploration.



**Discovering Communities and Anomalies in Attributed Graphs: Interactive Visual Exploration and Summarization** *Bryan Perozzi and Leman Akoglu*  
ACM TKDD, 2018

# This talk

- Attributed (sub)graphs\*
  - Subgraphs [SIAM SDM'16]
  - Summarization [ACM TKDD'18]
  - ➔ Comparisons [WWW '17]



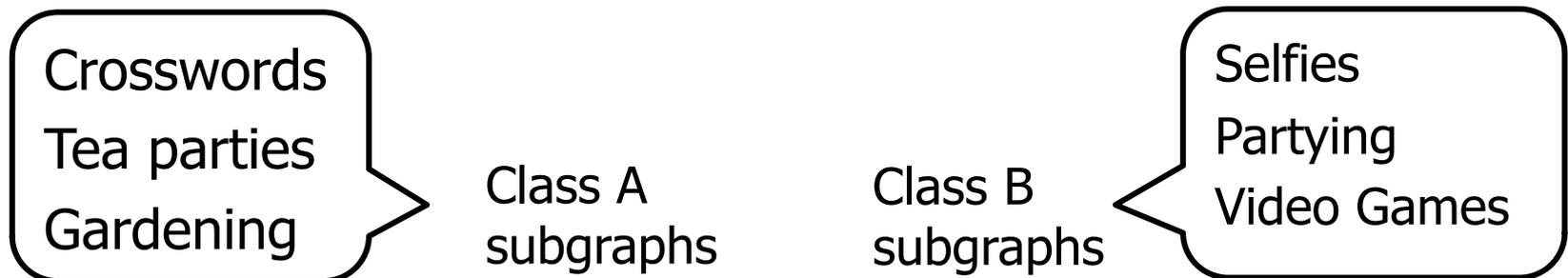
\* social circles, communities, egonetworks, ...

# Comparing attributed (sub)graphs

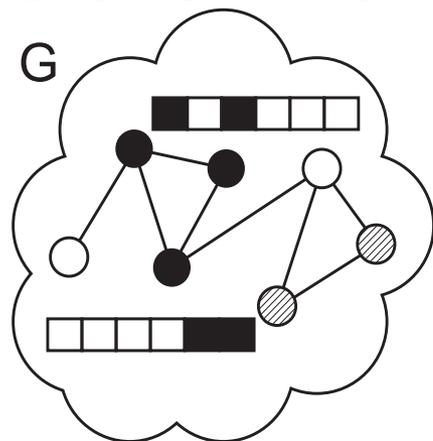
- Motivating question:

**Given** a collection of **attributed subgraphs** from different **classes**,  
how can we discover the attributes that **characterize** their **differences**?

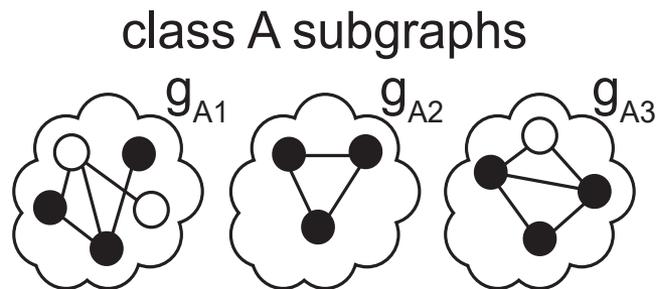
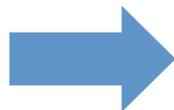
- **Hypothesis**: subgraphs from different classes exhibit *different focus attributes*



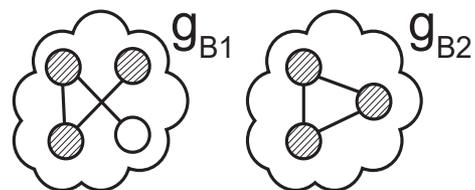
# Problem Sketch



attributed graph  
(a)



class A subgraphs



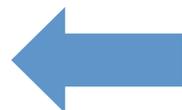
class B subgraphs

(b)



characterizing subspaces

(c)



class A	class B
$a_3$	$a_5$
$a_4$	$a_6$
$a_1$	
$a_2$	

assignment  
& ranking  
(d)

# Characterization Problem: Formal

Given

- $p$  attributed subgraphs  $g_1^+, g_2^+, \dots, g_p^+$  from class 1,  $\mathcal{S}^+$
- $n$  attributed subgraphs  $g_1^-, g_2^-, \dots, g_n^-$  from class 2,  $\mathcal{S}^-$  from graph  $G$ , and attribute vector  $\mathbf{a} \in \mathbb{R}^d$  for each node;

Find

- a partitioning of attributes to classes as  $A^+$  and  $A^-$ , where  $A^+ \cup A^- = A$  and  $A^+ \cap A^- = \emptyset$ ,
- focus attributes  $A_i^+ \subseteq A^+$  (and respective weights  $\mathbf{w}_i^+$ ) for each subgraph  $g_i^+$ ,  $\forall i$ , and
- focus attributes  $A_j^- \subseteq A^-$  (and respective weights  $\mathbf{w}_j^-$ ) for each subgraph  $g_j^-$ ,  $\forall j$ ;

such that

- total quality  $Q$  of all subgraphs is maximized, where 
$$Q = \sum_{i=1}^p q(g_i^+ | A^+) + \sum_{j=1}^n q(g_j^- | A^-);$$

Rank attributes within  $A^+$  and  $A^-$ .

# Reminder: Normality

- Normality as subgraph quality  $q$ :

$$N = w_c^T \cdot (\widehat{x}_I + \widehat{x}_X)$$

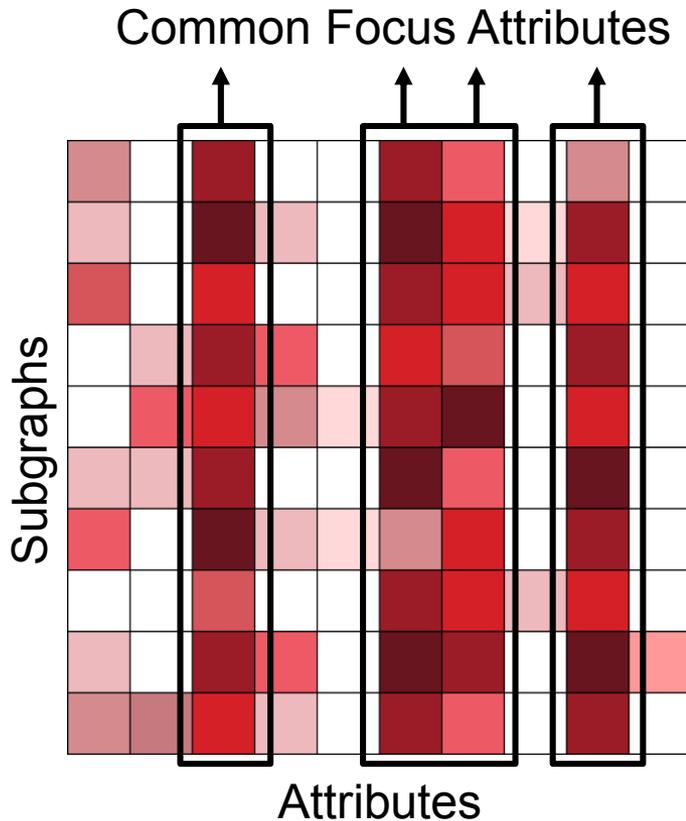
$\max_{w_c} N$  s.t.  $\|w_c\|_p = 1, w_c(a) \geq 0, \forall a = 1, \dots, d$

$L_1$  norm  $\bullet w_c(a) = 1$ , **one** attribute with largest  $\mathbf{x}$

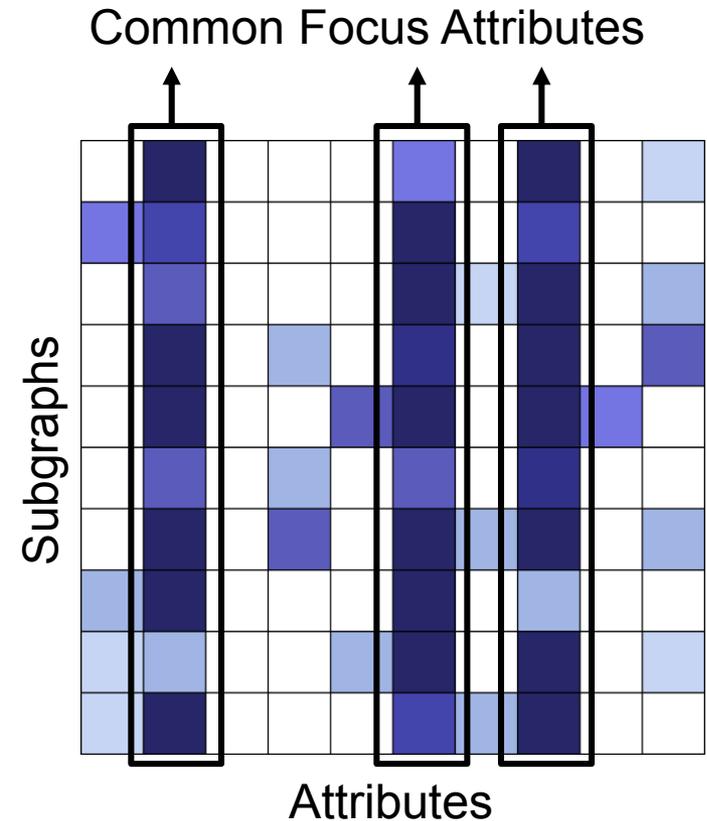
$L_2$  norm  $\bullet w_c(a) = \frac{x(a)}{\sqrt{\sum_{x(i)>0} x(i)^2}}$ , **all** attributes with positive  $\mathbf{x}$

# Splitting attributes by class: intuition

Class A



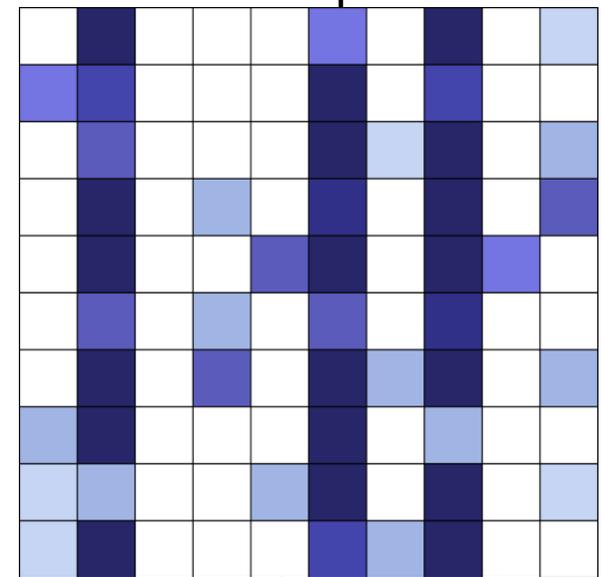
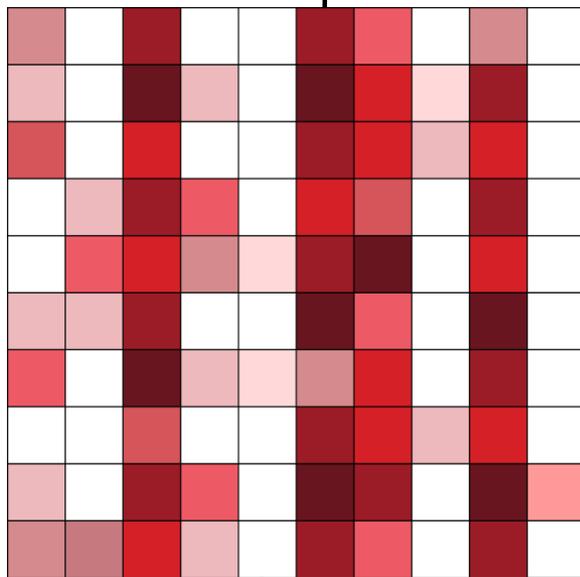
Class B



# Splitting attributes by class: intuition

- We don't want attributes that are:
  - Relevant or irrelevant to **both** classes

Highly relevant to both. Not distinguishing.

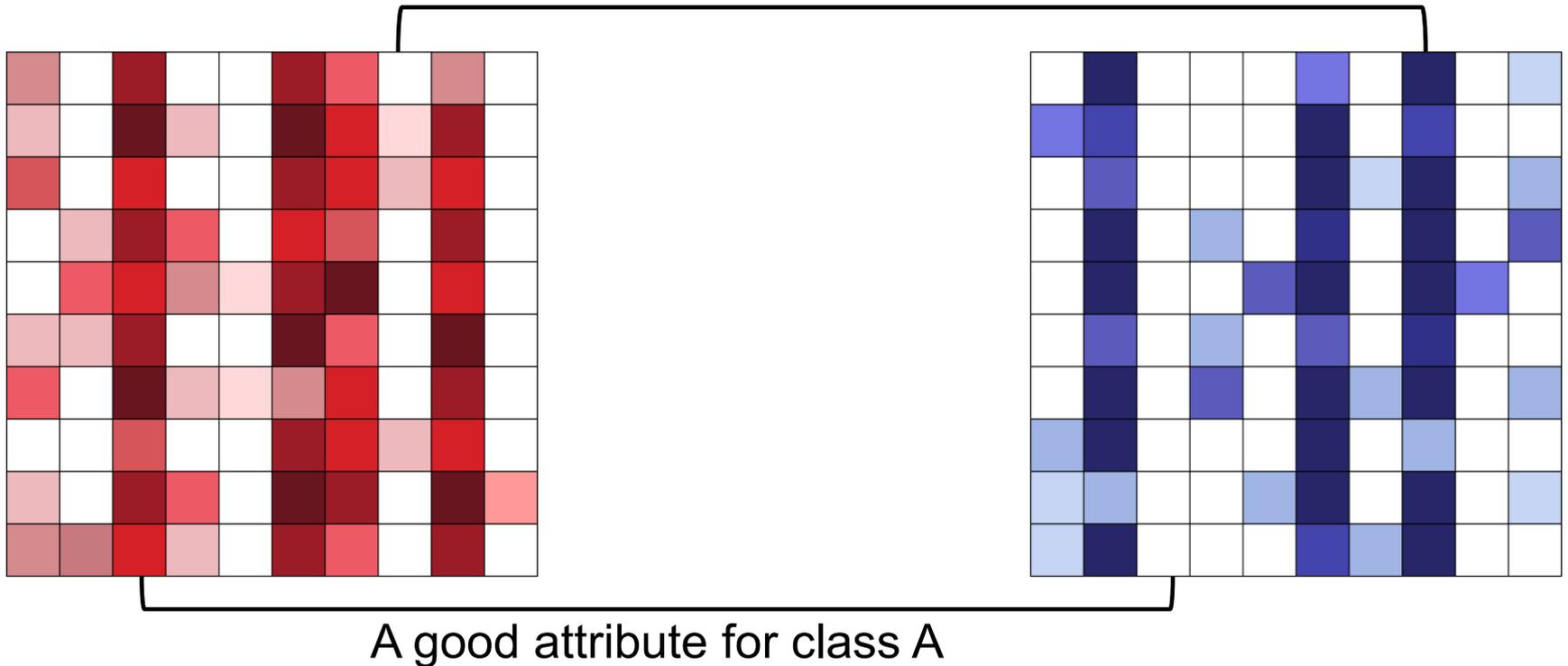


Irrelevant to both. Not Interesting.

# Splitting attributes by class: intuition

- We want attributes that are:
  - Relevant to **one** class & irrelevant to other(s)

A good attribute for class B



# Setting up the objective

Details

- Given a subset of attributes  $S$ , normality of subgraph  $g$  is

$$N(g|S) = \sqrt{\sum_{a \in S} x(a)^2} = \|x[S]\|_2$$

2-norm of  $x$  induced  
on the attribute subspace

attribute weight vector of  $g$

# Setting up the objective

Details

- Quality of an attribute split is:

$$\max_{A^+ \subseteq A, A^- \subseteq A} \frac{1}{p} \sum_{i \in S^+} \|x_i[A^+]\|_2 + \frac{1}{n} \sum_{j \in S^-} \|x_j[A^-]\|_2$$

Such that  $A^+ \cap A^- = \emptyset$

$p$  = number of subgraphs in class +  
 $n$  = number of subgraphs in class -

# Setting up the objective



- Quality of an attribute split is:

$$\max_{A^+ \subseteq A, A^- \subseteq A} \frac{1}{p} \sum_{i \in S^+} \|x_i[A^+]\|_2 + \frac{1}{n} \sum_{j \in S^-} \|x_j[A^-]\|_2$$

Such that  $A^+ \cap A^- = \emptyset$

- Rank attributes by

$p$  = number of subgraphs in class +  
 $n$  = number of subgraphs in class -

$$rc(a) = \underbrace{\frac{1}{p} \sum_{i \in S^+} x_i(a)}_{\text{Normalized contribution of } a \text{ to Class +}} - \underbrace{\frac{1}{n} \sum_{j \in S^-} x_j(a)}_{\text{Normalized contribution of } a \text{ to Class -}}$$

Normalized contribution  
of  $a$  to Class +

Normalized contribution  
of  $a$  to Class -

# Submodular Welfare Problem

Details

- Definition:

Given  $d$  items and  $m$  players having a **monotone** and **submodular** utility function  $(w_i)$  over subsets of items. Partition the  $d$  items into  $m$  **disjoint sets**  $(I_1, I_2, \dots, I_m)$  in order to maximize:

$$\sum_{i=1}^m w_i(I_i)$$

- Our quality function  $N(g|S)$  is a **monotone** and **submodular** set function.

$$w_c(I_c) = N(\mathcal{S}^{(c)} | A^{(c)}) = \frac{1}{n^{(c)}} \sum_{k \in \mathcal{S}^{(c)}} \|\mathbf{x}_k[A^{(c)}]\|_2$$

# Attribute splitting as SWP



Details

- SWP is **NP-hard**
  - First approx. factor is  $\frac{1}{2}$  [Lehmann+, 2001]
  - Improved to  $(1 - 1/e)$  [Vondrák+, 2008]
  - No better approximation unless
    - $P = NP$  [Khot+, 2008]
    - Using **exponentially-many** value queries [Mirrokni+, 2008]
- [Vondrák+, 2008] is **optimal approximation**

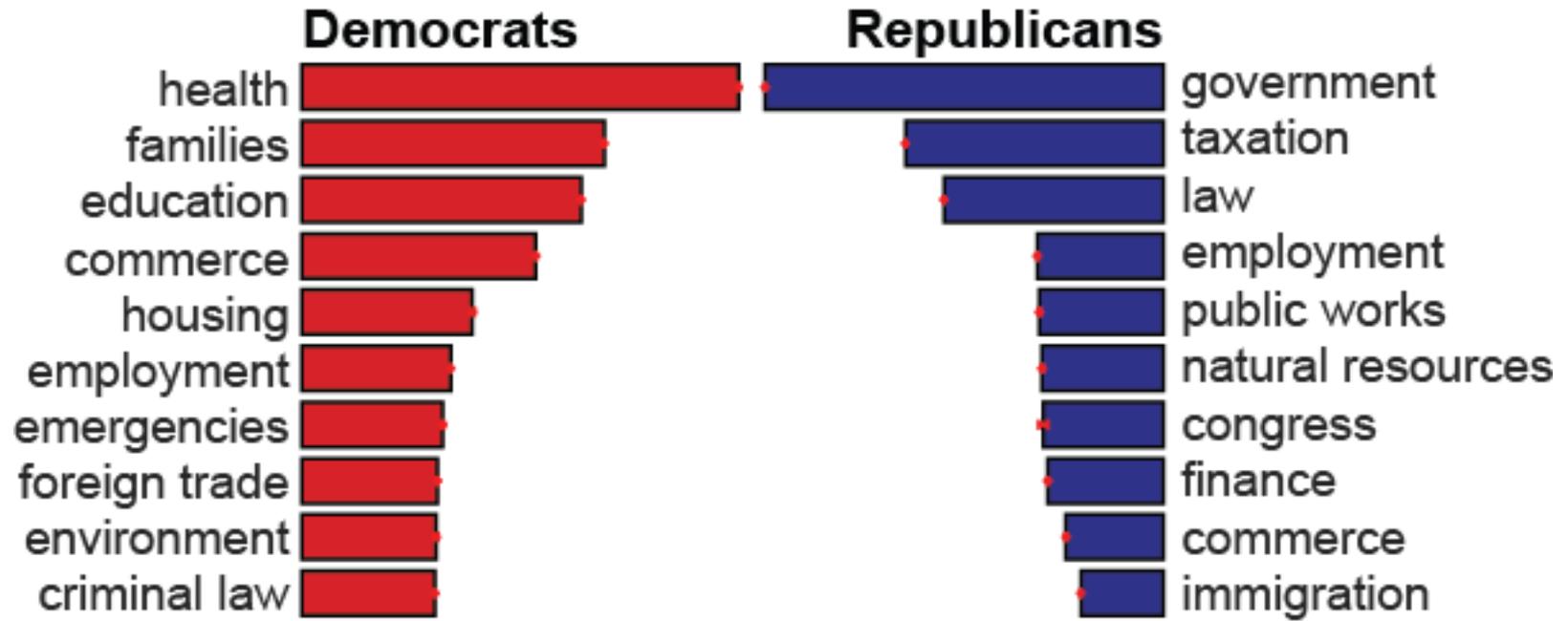
# Experiments

- Datasets
  - Congress Co-sponsorship Network
  - Amazon Co-purchase Network
  - DBLP Co-authorship Network
- Baseline (LASSO): L1-Regularized Logistic Regression
  - Positive weights are assigned to class **A**
  - Negative weights are assigned to class **B**

# Congress Co-sponsorship

- Bills in Congress
  - each bill has a set of *sponsors* & *policy area tag*
- **Attributed Graph:**
  - **Nodes:** congressmen
  - **Edges:** *co-sponsoring* a bill
  - **Attributes:** *policy areas* of bills they sponsored:
    - National Security and Armed Forces
    - Environmental Protection
    - Foreign Affairs
    - ...
- **Classes:** *party affiliation* of congressmen

# Liberal and Conservative Ideals

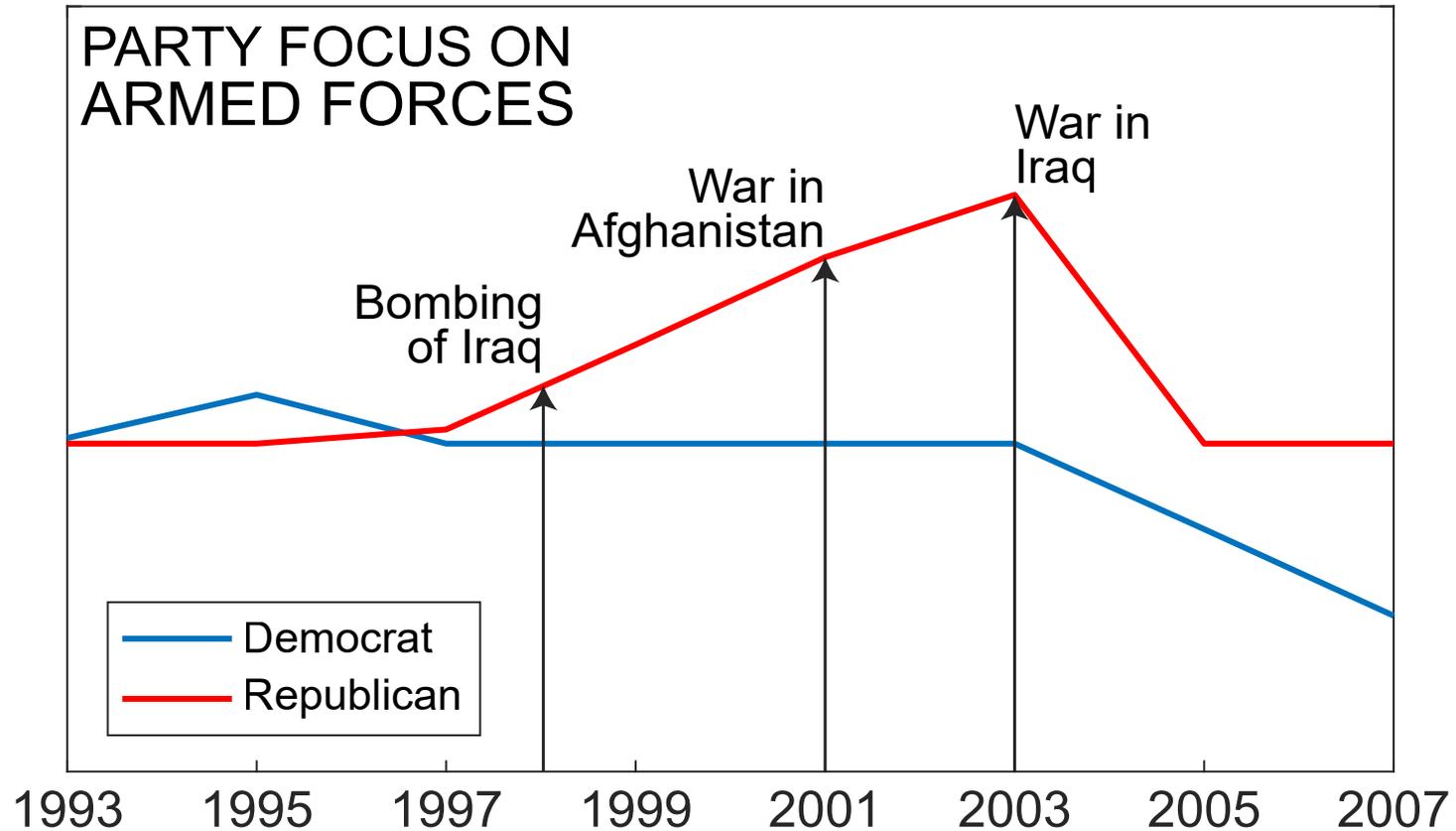


Democrats focus mostly on **social** programs

Republicans focus mostly on **governance** and **finance**

# Focus Over Time

- 13 consecutive congress two-year cycles:

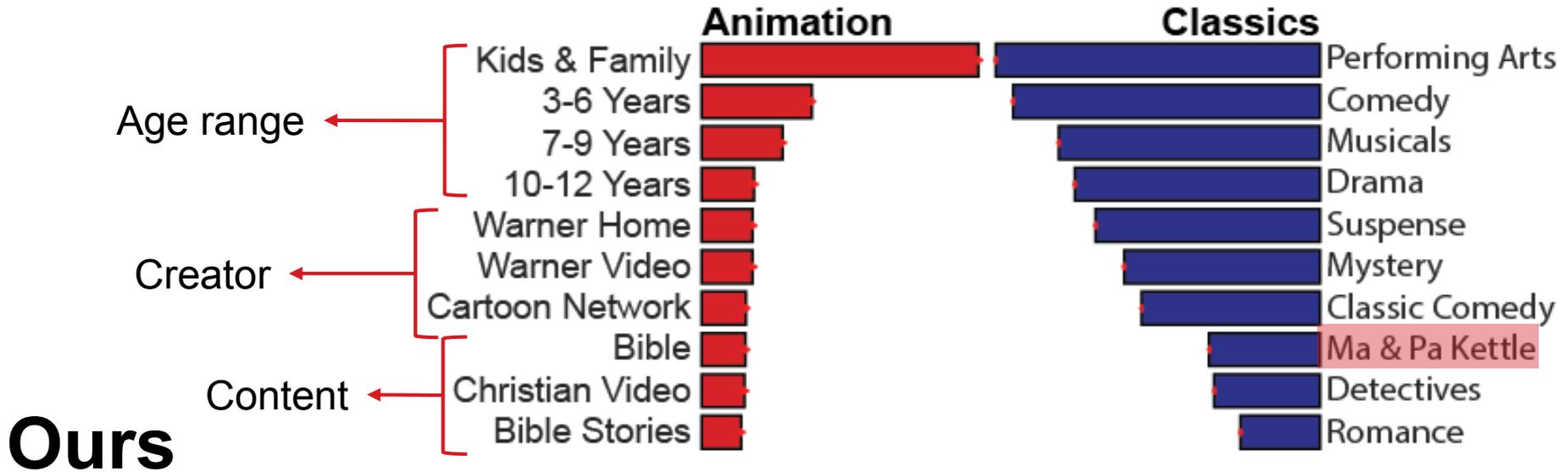


# Amazon.com Co-purchases

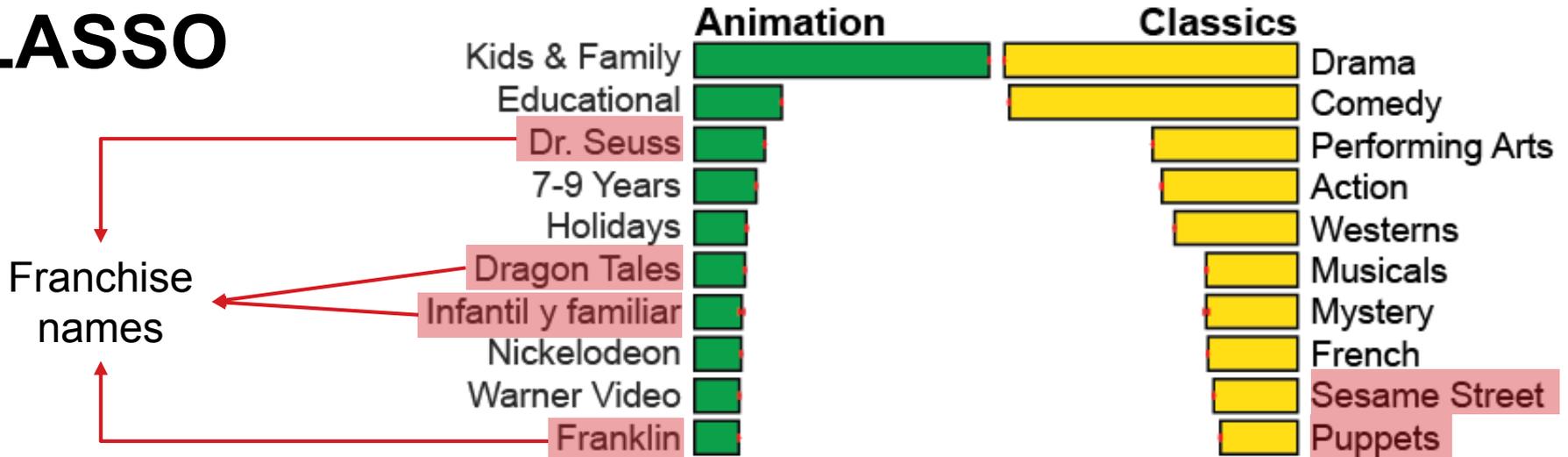
## Attributed Graph:

- **Nodes:** Amazon **videos**
- **Edges:** being **co-purchased** together
- **Attributes:**
  - Product genre (Drama, Comedy, etc.)
  - Audience age range (e.g., 10-12 years)
  - Creators (e.g. Warner Video)
  - ...

# Classes: Animation vs. Classic



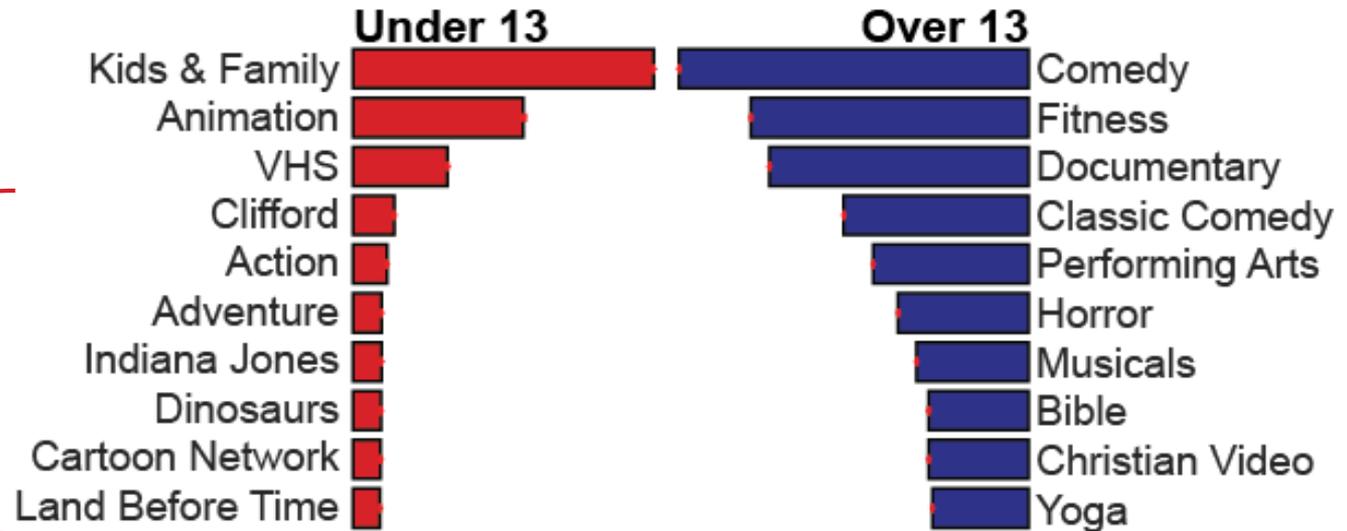
# LASSO



# Classes: Under 13 vs. Over 13

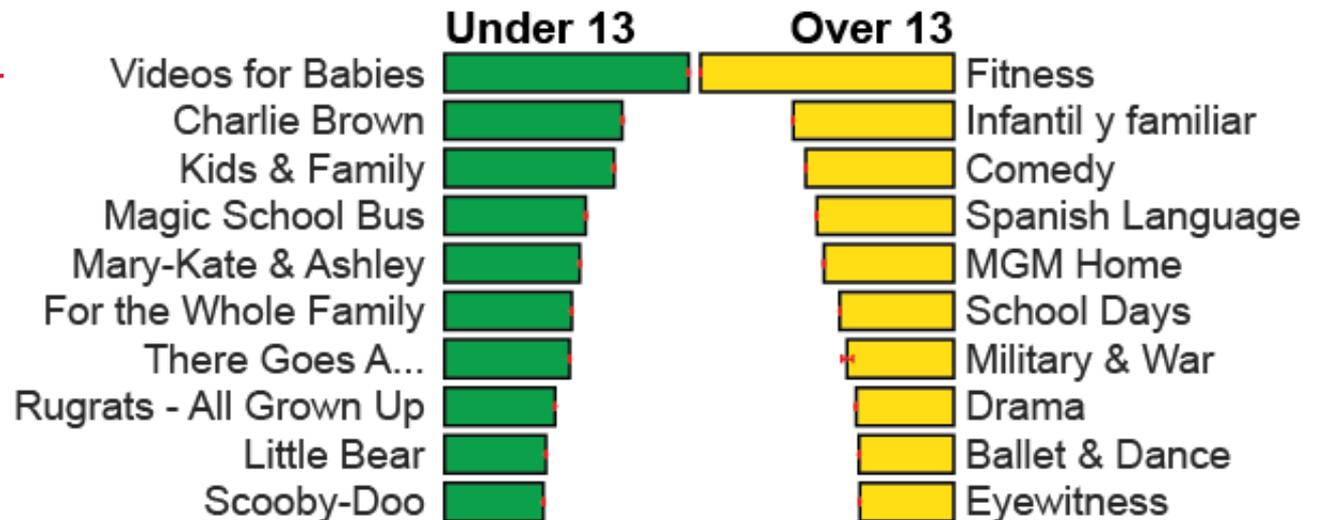
Attribute weight goes down as quality decreases

**Ours**



## LASSO

Not much differentiation



# Characterization vs. Classification

- Regularized linear classifiers (e.g. LASSO) can find
  - a sparse attribute subspace
  - coefficients for ranking
- How is our work different?

Classifiers focus on ***confidence***  
while we focus on ***support***

# Characterization vs. Classification

**Confidence**

Prob. of belonging to class  $c$  if  $a$  is observed

$$Cfd(c, a) = \Pr(c|a) = \frac{\#(c, a)}{\#(a)}$$

**Support**

Portion of nodes in class  $c$  exhibiting  $a$

$$Sup(c, a) = \frac{\#(c, a)}{\#(c)}$$

# Characterization vs. Classification

**Class  
Confidence**

→ **Relative Confidence**

$$CC(c^+, a) = \Pr(c^+|a) - \Pr(c^-|a)$$

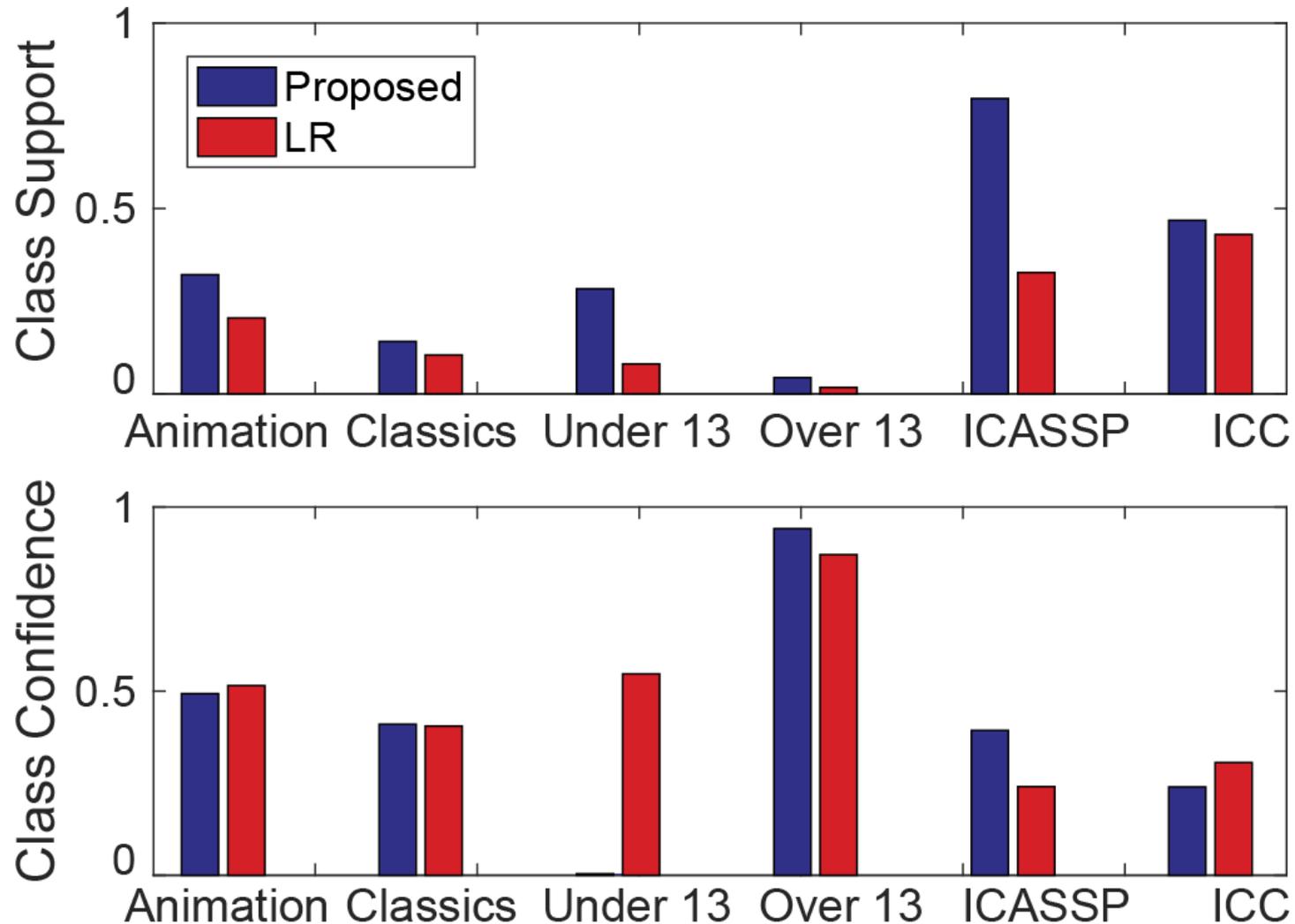
**Class  
Support**

→ **Relative Support**

$$CS(c^+, a) = \text{Sup}(c^+, a) - \text{Sup}(c^-, a)$$

Classifiers focus on **confidence**  
while we focus on **support**

# Characterization vs. Classification

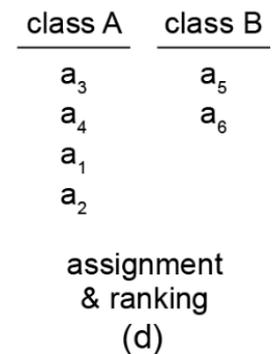
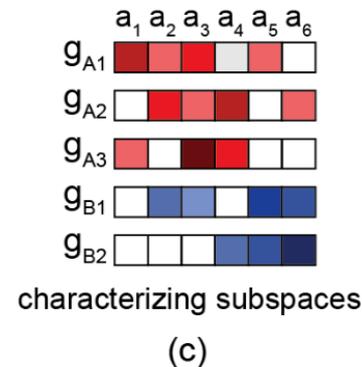
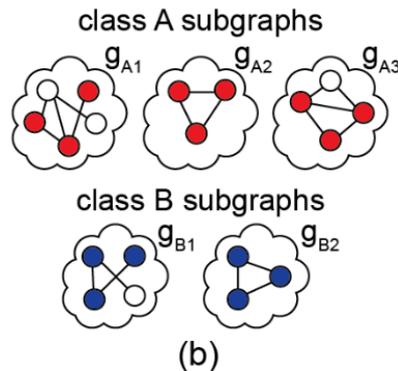
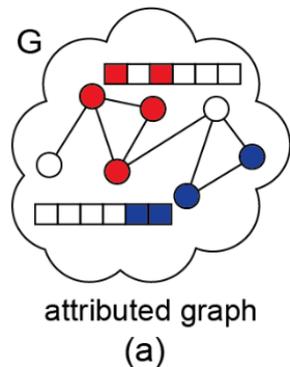


**Slides, code, data** [http://www3.cs.stonybrook.edu/~arezaei/project/amen\\_char.html](http://www3.cs.stonybrook.edu/~arezaei/project/amen_char.html)

# Characterizing Class Differences in Attributed Graphs

Aria Rezaei, Bryan Perozzi, Leman Akoglu

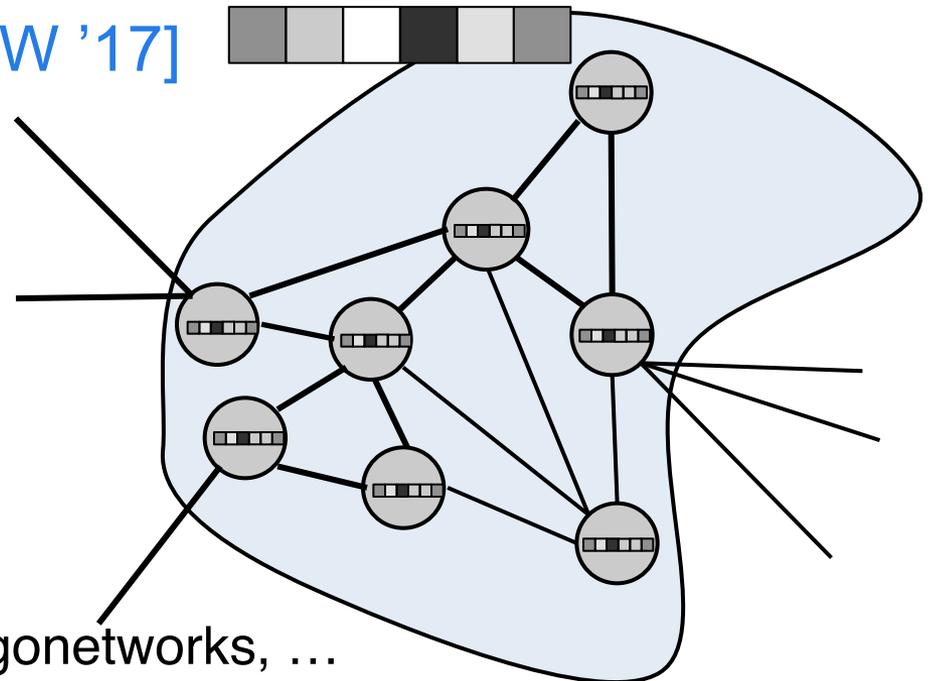
## Overview



**Ties That Bind - Characterizing Classes by Attributes and Social Ties.** *Aria Rezaei, Bryan Perozzi, Leman Akoglu.*  
WWW 2017 Companion

# This talk

- Attributed (sub)graphs\*
  - Subgraphs [SIAM SDM'16]
  - Summarization [ACM TKDD'18]
  - Comparisons [WWW '17]



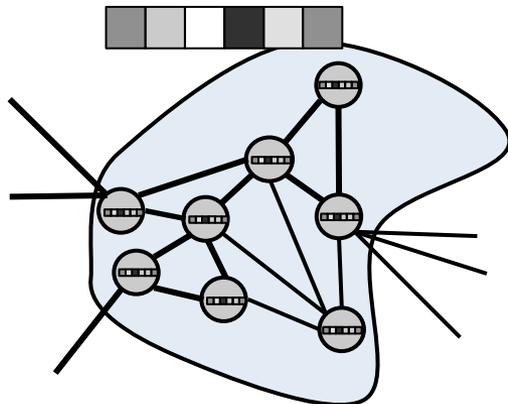
\* social circles, communities, egonetworks, ...

# References, Links to Code&Data:

- **Scalable Anomaly Ranking of Attributed Neighborhoods.**  
*Bryan Perozzi and Leman Akoglu.* SIAM SDM 2016  
<https://github.com/phanein/amen>
- **Discovering Communities and Anomalies in Attributed Graphs: Interactive Visual Exploration and Summarization.**  
*Bryan Perozzi and Leman Akoglu.* ACM TKDD, 2018  
<https://www.dropbox.com/home/Public/iSCAN>
- **Ties That Bind - Characterizing Classes by Attributes and Social Ties.** *Aria Rezaei, Bryan Perozzi, Leman Akoglu.*  
WWW 2017 Companion  
<https://github.com/rezaeiaria/AmenChar>

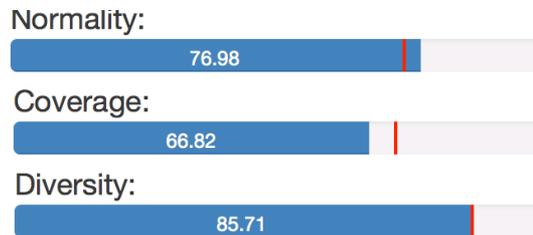
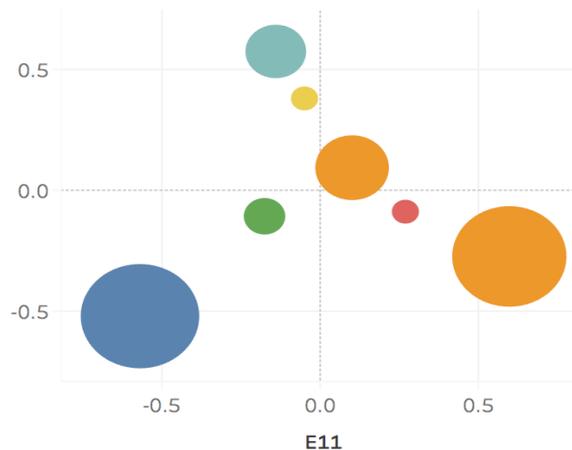
**Contact:** [lakoglu@andrew.cmu.edu](mailto:lakoglu@andrew.cmu.edu)  
[www.andrew.cmu.edu/~lakoglu](http://www.andrew.cmu.edu/~lakoglu)

### Subgraphs



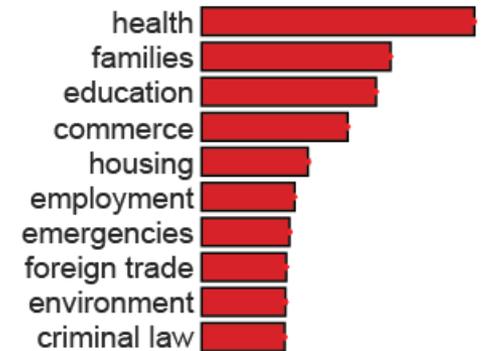
### Summarization

#### Algorithm Summary

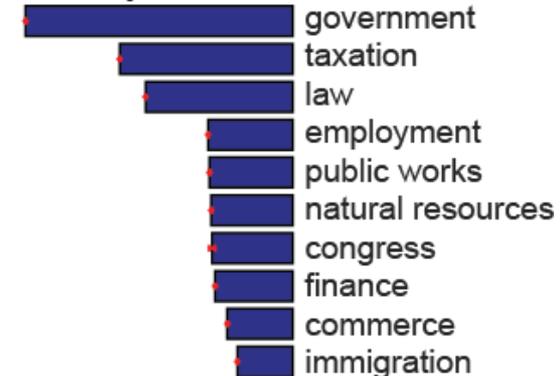


### Comparisons

#### Democrats



#### Republicans



**Thanks!**

