Constructive Logic (15-317), Spring 2022
Recitation 10: Chaining (3-30-2022)

1 Backward Chaining

Recall the rules for backward chaining:

$$\dfrac{D^- \in \Gamma^- \quad \Gamma, [D^-] \xrightarrow{f} P^-}{\Gamma^- \xrightarrow{f} P^-}\ \mathsf{focus}L$$

$$\dfrac{\Gamma^-, [D^-(X)] \xrightarrow{f} P^-}{\Gamma^-, [\forall x.\, D^-(x)] \xrightarrow{f} P^-}\ \forall L^* \qquad \dfrac{\Gamma^-, [D^-] \xrightarrow{f} P^- \quad \Gamma^- \xrightarrow{f} [G^+]}{\Gamma^-, [G^+ \supset D^-] \xrightarrow{f} P^-}\ {\supset}L$$

$$\dfrac{Q^- = P^-}{\Gamma^-, [Q^-] \xrightarrow{f} P^-}\ \mathsf{id} \qquad \begin{array}{c}\text{no rule if } Q^- \neq P^- \\ \Gamma^-, [Q^-] \xrightarrow{f} P^-\end{array}$$

$$\dfrac{\Gamma^- \xrightarrow{f} [G_1^+] \quad \Gamma^- \xrightarrow{f} [G_2^+]}{\Gamma^- \xrightarrow{f} [G_1^+ \wedge G_2^+]}\ \wedge R \qquad \dfrac{}{\Gamma^- \xrightarrow{f} [\top]}\ \top R$$

$$\dfrac{\Gamma^- \xrightarrow{f} [G(X)]}{\Gamma^- \xrightarrow{f} [\exists x. G^+(x)]}\ \exists R^* \qquad \dfrac{\Gamma^- \xrightarrow{f} P^-}{\Gamma^- \xrightarrow{f} [{\downarrow}P^-]}\ \mathsf{blur}$$

and the rules for even and odd natural numbers:

$$\dfrac{}{even(z)}\ ev_z \qquad \dfrac{odd(N)}{even(s(N))}\ ev_s \qquad \dfrac{even(N)}{odd(s(N))}\ od_s$$

Task 1. Give a proof, using the backward chaining rules, of even($s(s(z))$).

Solution 1: Define Γ_{eo} = even(z), $\forall n.$ odd(n) $\supset$ even(s(n)), $\forall n.$ even(n) $\supset$ odd(s(n)).
Our goal sequent is $\Gamma_{eo} \longrightarrow$ even(s(s(z)))).

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma_{eo}, [even(s(s(z)))] \longrightarrow even(s(s(z)))}\ id \quad \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma_{eo}, [odd(s(z))] \longrightarrow odd(s(z))}\ id \quad \dfrac{\dfrac{\dfrac{}{\Gamma_{eo}, [even(z)] \longrightarrow even(z)}\ id}{\Gamma_{eo} \longrightarrow even(z)}\ focusL}{\Gamma_{eo} \longrightarrow [even(z)]}\ blurR}{\Gamma_{eo}, [even(z) \supset odd(s(z))] \longrightarrow odd(s(z))}\ \supset L}{\Gamma_{eo}, [\forall n. even(n) \supset odd(s(n))] \longrightarrow odd(s(z))}\ \forall L}{\Gamma_{eo} \longrightarrow odd(s(z))}\ focusL}{\Gamma_{eo} \longrightarrow [odd(s(z))]}\ blurR}{\Gamma_{eo}, [odd(s(z)) \supset even(s(s(z)))] \longrightarrow even(s(s(z)))}\ \supset L}{\Gamma_{eo}, [\forall n. odd(n) \supset even(s(n))] \longrightarrow even(s(s(z)))}\ \forall L}{\Gamma_{eo} \longrightarrow even(s(s(z)))}\ focusL$$

Task 2. Give a set of inference rules for a backward chaining program factor(m, n) which determines if m evenly divides n.

Solution 2:

$$\frac{div(m,n,m)}{factor(m,n)}\,def \qquad \frac{div(m,n,d)}{div(s(m),s(n),d)}\,decr \qquad \frac{div(d,s(n),d)}{div(z,s(n),d)}\,rep \qquad \frac{}{div(z,z,d)}\,z$$

2 Forward Chaining

Task 3. Give the inference rules for a forward chaining program length(l, n) which derives the atom **no** if and only if n is not the length of list l. You may assume that n and l are ground.

Solution 3:

$$\frac{length([X \mid L],\, 0)}{no}\,zero \qquad \frac{length([X \mid L],\, N)}{length(L,\, N-1)}\,dec$$

Task 4. Recall the grammar representing natural numbers:

$$n ::= z \mid s(n)$$

Give the inference rules for the program factor(m, n) again, this time intepreted as a which derives the atom **no** if and only if m does not evenly divide n. You may assume that m and n are ground.

Solution 4:

$$\frac{factor(m,n)}{div(m,n,m)}\,def \qquad \frac{div(s(m),s(n),d)}{div(m,n,d)}\,decr \qquad \frac{div(z,s(n),d)}{div(d,s(n),d)}\,rep \qquad \frac{div(s(m),z,d)}{no}\,nz$$

Task 5. Given the fact factor($s(s(z))$, $s(s(s(s(z))))$) in the database, list all the facts that are present in a saturated database.

Solution 5:

1. div($s(s(z))$, $s(s(s(s(z))))$, $s(s(z))$)
2. div($s(z)$, $s(s(s(z)))$, $s(s(z))$)
3. div(z, $s(s(z))$, $s(s(z))$)
4. div($s(s(z))$, $s(s(z))$, $s(s(z))$)
5. div($s(z)$, $s(z)$, $s(s(z))$)
6. div(z, z, $s(s(z))$)