Constructive Logic (15-317), Spring 2022
Recitation 10: Chaining (3-30-2022)

1 Backward Chaining

Recall the rules for backward chaining:

$$\frac{D^- \in \Gamma^- \quad \Gamma, [D^-] \xrightarrow{f} P^-}{\Gamma^- \xrightarrow{f} P^-}\ \text{focus}L$$

$$\frac{\Gamma^-, [D^-(X)] \xrightarrow{f} P^-}{\Gamma^-, [\forall x.\, D^-(x)] \xrightarrow{f} P^-}\ \forall L^* \qquad \frac{\Gamma^-, [D^-] \xrightarrow{f} P^- \quad \Gamma^- \xrightarrow{f} [G^+]}{\Gamma^-, [G^+ \supset D^-] \xrightarrow{f} P^-}\ \supset L$$

$$\frac{Q^- = P^-}{\Gamma^-, [Q^-] \xrightarrow{f} P^-}\ \text{id} \qquad \begin{array}{l}\text{no rule if } Q^- \neq P^- \\ \Gamma^-, [Q^-] \xrightarrow{f} P^-\end{array}$$

$$\frac{\Gamma^- \xrightarrow{f} [G_1^+] \quad \Gamma^- \xrightarrow{f} [G_2^+]}{\Gamma^- \xrightarrow{f} [G_1^+ \wedge G_2^+]}\ \wedge R \qquad \frac{}{\Gamma^- \xrightarrow{f} [\top]}\ \top R$$

$$\frac{\Gamma^- \xrightarrow{f} [G(X)]}{\Gamma^- \xrightarrow{f} [\exists x. G^+(x)]}\ \exists R^* \qquad \frac{\Gamma^- \xrightarrow{f} P^-}{\Gamma^- \xrightarrow{f} [\downarrow P^-]}\ \text{blur}$$

and the rules for even and odd natural numbers:

$$\frac{}{even(z)}\ ev_z \qquad \frac{odd(N)}{even(s(N))}\ ev_s \qquad \frac{even(N)}{odd(s(N))}\ od_s$$

Task 1. Give a proof, using the backward chaining rules, of even($s(s(z))$).

Task 2. Give a set of inference rules for a backward chaining program factor(m, n) which determines if m evenly divides n.

2 Forward Chaining

Task 3. Give the inference rules for a forward chaining program length(l, n) which derives the atom **no** if and only if n is not the length of list l. You may assume that n and l are ground.

Task 4. Recall the grammar representing natural numbers:

$$n ::= z \mid s(n)$$

Give the inference rules for the program factor(m, n) again, this time intepreted as a which derives the atom **no** if and only if m does not evenly divide n. You may assume that m and n are ground.

Task 5. Given the fact factor($s(s(z))$, $s(s(s(s(z))))$) in the database, list all the facts that are present in a saturated database.