15-317 Lecture 25: Asynchrony

- Final exam 9th May, 17:30-20:30, HOA 160

- Asynchronous communication
- Logically justifying asynchrony
- Communication via memory
- Concurrent non-linear programs

$$\dfrac{\dfrac{\Delta \vdash P :: (x : A)}{\Delta \vdash z.\mathrm{inl}(x); P :: (z : A \oplus B)}\,\oplus R_1 \qquad \dfrac{\Delta, x:A \vdash Q :: (w : C)}{\Delta, (z : A \oplus B) \vdash \mathrm{case}\ z\ (\mathrm{inl}(x) \Rightarrow Q \mid \mathrm{inr}(y) \Rightarrow R) :: (w : C)}\,\oplus L}{\text{cut}}$$

$$\Delta \vdash z \leftarrow (z.\mathrm{inl}(x); P);\ \mathrm{case}\ z(\ldots) :: (w : C)$$

$$\hookrightarrow$$

$$x \leftarrow P; Q$$

$$\dfrac{\mathrm{proc}(z,\ z.\mathrm{inl}(x); P) \qquad \mathrm{proc}(w,\ \mathrm{case}\ z \ldots)}{\mathrm{proc}(x, P) \qquad\qquad \mathrm{proc}(w, Q)}$$

$$\left.\begin{array}{c} proc(z,\ z.inl(x);\ P) \\ \hline proc(x,\ P) \qquad msg(z,\ inl(x)) \end{array}\right\} \rightarrow \frac{proc(z,\ z.inl(x))}{msg(z,\ inl(x))}$$

$$\frac{msg(z,inl(x)) \qquad proc(w,\ case\ z\ (inl(x) \Rightarrow Q\ |\ \ldots))}{proc(w,\ Q)}$$

$$(z : inl(x);\ P) \longrightarrow (x \leftarrow P;\ z : inl(x))$$

$$\frac{proc(z,\ z : inl(x))}{msg(z,\ inl(x))} \quad \left[\frac{msg(z,\ inl(x)) \qquad proc(w,\ case\ z\ (inl(x) \Rightarrow Q) \ldots)}{proc(w,\ Q)} \right]$$

$$\frac{\dfrac{}{x : A \vdash z . inl(x) :: (z : A \oplus B)}\ \oplus R_1^0}{x : A \vdash msg(z, inl(x)) :: (z : A \oplus B)} \qquad \frac{\mathcal{E}_1 \qquad \mathcal{E}_2}{\Delta,\ z : A \oplus B \vdash case\ z \ldots :: (w : C)}\ \oplus L$$

$$\frac{}{\Delta,\ x : A \vdash \ldots :: (w : C)}\ cut$$

$$\longrightarrow R\ \mathcal{E}_1$$

$$\frac{}{x:A, y:B \vdash z.\langle x, y\rangle :: (z : A \otimes B)}\ \otimes R^{\emptyset} \qquad \frac{\Delta, x:A, y:B \vdash P :: (w : C)}{\Delta, z:A \otimes B \vdash \text{case } z \ldots :: (w : C)}\ \otimes L$$

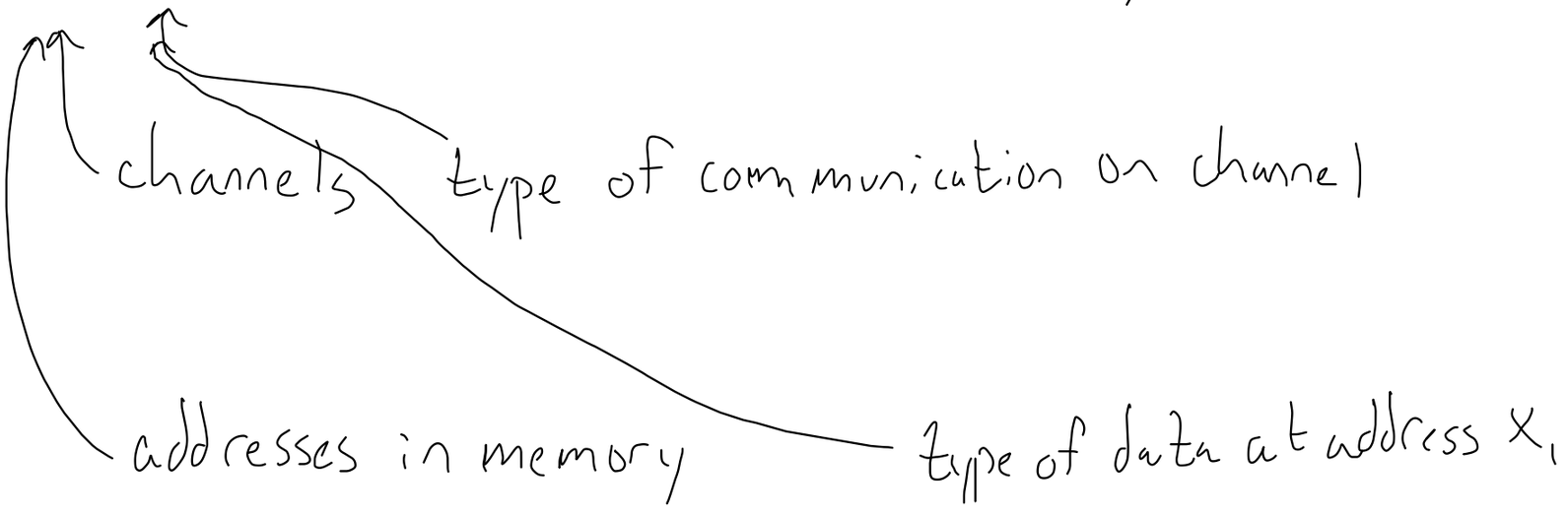
$$\frac{}{\cdot \vdash z.\langle\rangle :: (z : 1)}\ 1R^{\emptyset} \qquad \frac{\Delta \vdash P :: (w : C)}{\Delta, z:1 \vdash \text{case } z\,(\langle\rangle \Rightarrow P) :: (w : C)}\ 1L$$

$$\frac{}{}\ \&R$$

$$\frac{}{z:A \& B \vdash z.\text{inr}(x) :: (x : B)}\ \&L_2^{\emptyset}$$

$$\frac{}{x:A,\ z:A \multimap B \vdash z.\langle x, y\rangle :: (y : B)}\ \multimap L^{\emptyset}$$

$x_1 : A_1, \ldots, x_n : A_n \vdash P :: (z : C)$



$$\frac{\Delta_1 \vdash P :: (x : A) \qquad \Delta_2, x : A \vdash Q :: (z : C)}{\Delta_1, \Delta_2 \vdash x \leftarrow P ; Q :: (z : C)}\ cut$$

$\rightsquigarrow$

$$\frac{proc(z, x \leftarrow P ; Q)}{proc(a, P[a/x]) \quad cell(a, -) \quad proc(z, Q[a/x])}\ cut$$

$$\frac{}{x:A \vdash y \overset{w}{\leftarrow} x^{R} :: (y:A)}$$

$$\frac{\mathrm{proc}(y, y \leftarrow x) \quad \mathrm{cell}(x, D) \quad \mathrm{cell}(y, \_)}{\mathrm{cell}(y, D)}$$

$$\frac{}{x:A \vdash z \overset{w}{.} \mathrm{inl}(x) :: (z : A \oplus B)} \oplus R_1^{\varnothing}$$

$$\frac{\mathrm{proc}(z, z.\mathrm{inl}(x)) \quad \mathrm{cell}(z, \_)}{\mathrm{cell}(z, \mathrm{inl}(x))}$$

$$\frac{\Delta, x:A \vdash Q :: (w:C)}{\Delta, z:A \oplus B \vdash \mathrm{case}\ z^{R}\ [\mathrm{inl}(x) \Rightarrow Q \quad \ldots :: (w:C)} \oplus L$$

$$[\mathrm{inr}(y) \Rightarrow R$$

$$) \ldots$$

$$\frac{\mathrm{cell}(z, \mathrm{inl}(a)) \quad \mathrm{proc}(w, \mathrm{case}\ z\ (\mathrm{inl}(x) \Rightarrow Q \mid \ldots))}{\mathrm{proc}(w, Q[a/x])}$$

$$\frac{\Delta, x{:}A \Vdash P :: (y{:}B)}{\Delta \Vdash \mathsf{case}\ z^{w}(\langle x,y\rangle \Rightarrow P) :: (z{:}A \multimap B)}\ \multimap R$$

$$\frac{\mathsf{proc}(z, \mathsf{case}\ z\ (\dots)) \quad \mathsf{cell}(z, -)}{\mathsf{cell}(z, \langle x,y\rangle \Rightarrow P)}$$

$$\frac{}{x{:}A, z{:}A \multimap B \Vdash z^{R}.\langle x,y\rangle :: (y{:}B)}\ \multimap L^{\emptyset}$$

$$\frac{\mathsf{proc}(y, z.\langle x,y\rangle) \quad \mathsf{cell}(z, \langle u,v\rangle \Rightarrow P)}{\mathsf{proc}(y, P[x/u, y/v])}$$

- Right rules write to memory
- Left rules read

- Positive data is "small"
- Negative data is "large" (containing full stored process terms)

Structural concurrent programs

$$\frac{\Gamma \vdash P :: (x:A) \qquad \Gamma, x:A \vdash Q :: (z:C)}{\Gamma \vdash x \leftarrow P ; Q :: (z:C)}\ cut$$

$$\frac{}{\Gamma, x:A \vdash y^{w} \leftarrow x^{R} :: (y:A)}\ id$$

- Memory has one writer, but (potentially) many readers
  - $proc(z, P)$
    $cell(a, -)$
    $!cell(a, D)$

$$\frac{proc(y, y^{w} \leftarrow x^{R}) \qquad !cell(x, D) \qquad cell(y, -)}{!cell(y, D)}$$

$$\frac{proc(z, z.inl(x))\quad cell(z, -)}{!cell(z, inl(x))}$$

$$\frac{!cell(z, inl(x))\quad proc(w, case\ z\ (inl(k) \Rightarrow Q) \mid \ldots)}{[!cell(z, inl(k))]\quad proc(w, Q)}$$