

15-317 Lecture 22: Ordered Logic

- Ordered Logic
- Substructural Logic Programming

Ordered Logic

- \mathcal{P} set, allowing reuse

- Δ multiset

- Ω list

$A \rightarrow B$

$A \dashv\vdash B$

$$\frac{A \rightarrow B \quad A}{B} \rightarrow E$$

$$\frac{A \quad \vdots \quad B}{A \rightarrow B} I$$

$$\frac{\Gamma \rightarrow A \quad \Gamma, B \rightarrow C}{\Gamma, A \rightarrow B \rightarrow C} \rightarrow L$$

$$\frac{\Delta_1 \Vdash A \quad \Delta_2, A \dashv\vdash B \Vdash C}{\Delta_1, \Delta_2, A \dashv\vdash B \Vdash C} \dashv\vdash L$$

$$\begin{array}{c}
 \Omega \Vdash A \quad \Omega_L B \quad \Omega_R \Vdash C \\
 \hline
 \Omega_L \Omega (A \setminus B) \Omega_R \Vdash C
 \end{array}
 \quad \searrow L
 \quad
 \begin{array}{c}
 A \quad \Omega \Vdash B \\
 \hline
 \Omega \Vdash \underbrace{A \setminus B}
 \end{array}
 \quad \searrow R$$

\uparrow
 under
 left-implies

$$\begin{array}{c}
 \Omega \Vdash A \quad \Omega_L B \quad \Omega_R \Vdash C \\
 \hline
 \Omega_L (B/A) \Omega \Omega_R \Vdash C
 \end{array}
 \quad /L
 \quad
 \begin{array}{c}
 \Omega A \Vdash B \\
 \hline
 \Omega \Vdash B/A
 \end{array}
 \quad /R$$

\uparrow
 over
 right-implies

$$\frac{\Delta_1 \Vdash A \quad \Delta_2 \Vdash B}{\Delta_1, \Delta_2 \Vdash A \otimes B} \otimes R$$

$$\frac{\Delta, A, B \Vdash C}{\Delta, A \otimes B \Vdash C} \otimes L$$

$$\frac{\Omega_1 \Vdash A \quad \Omega_2 \Vdash B}{\Omega_1, \Omega_2 \Vdash A \bullet B} \bullet R$$

\uparrow
 fuse

$$\frac{\Omega_L A B \Omega_R \Vdash C}{\Omega_L A \bullet B \Omega_R \Vdash C} \bullet L$$

$$\frac{\Omega_2 \Vdash A \quad \Omega_1 \Vdash B}{\Omega_1, \Omega_2 \Vdash A \circ B} \circ R$$

\uparrow
 twist

$$\frac{\Omega_L B A \Omega_R \Vdash C}{\Omega_L A \circ B \Omega_R \Vdash C} \circ L$$

$$\frac{\Omega \text{ III- } A \quad \Omega \text{ III- } B}{\Omega \text{ III- } A \& B}$$

$\otimes R$

$$\frac{\Omega_L A \quad \Omega_R \text{ III- } C}{\Omega_L (A \& B) \quad \Omega_R \text{ III- } C} \otimes L$$

$$\frac{\Omega \text{ III- } A}{\Omega \text{ III- } A \oplus B}$$

$\oplus R$

$$\frac{\Omega_L A \quad \Omega_R \text{ III- } C \quad \Omega_L B \quad \Omega_R \text{ III- } C}{\Omega_L (A \oplus B) \quad \Omega_R \text{ III- } C} \oplus L$$

_____ IR
• III- I

$\frac{\Omega_2 \Omega_R \text{ III- C}}{\Omega_2 \text{ I } \Omega_R \text{ III- C}} \quad \text{IL}$

_____ TR
 $\Omega \text{ III- T}$

no TL

no $\emptyset R$

_____ $\emptyset L$
 $\Omega_2 \emptyset \Omega_R \text{ III- C}$

$$\frac{}{A \Vdash A} \text{; } d$$

$$\frac{}{C \Vdash C} :d$$

$$\frac{}{B \Vdash B} :d$$

$$B/C, C \Vdash B$$

$$A, A \setminus (B/C), C \Vdash B$$

$$A \setminus (B/C), C \Vdash A \setminus B$$

$$A \setminus (B/C) \Vdash (A \setminus B)/C$$

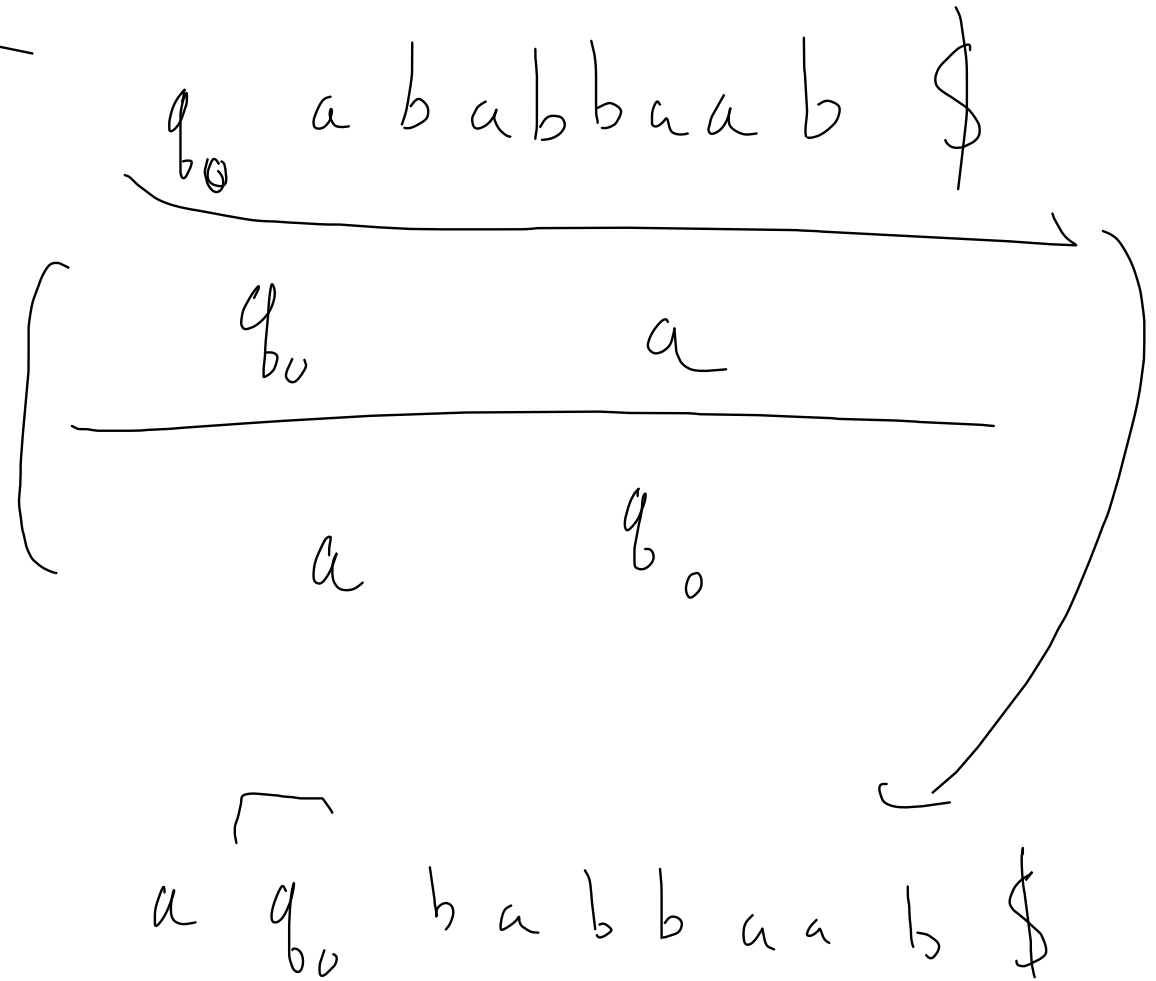
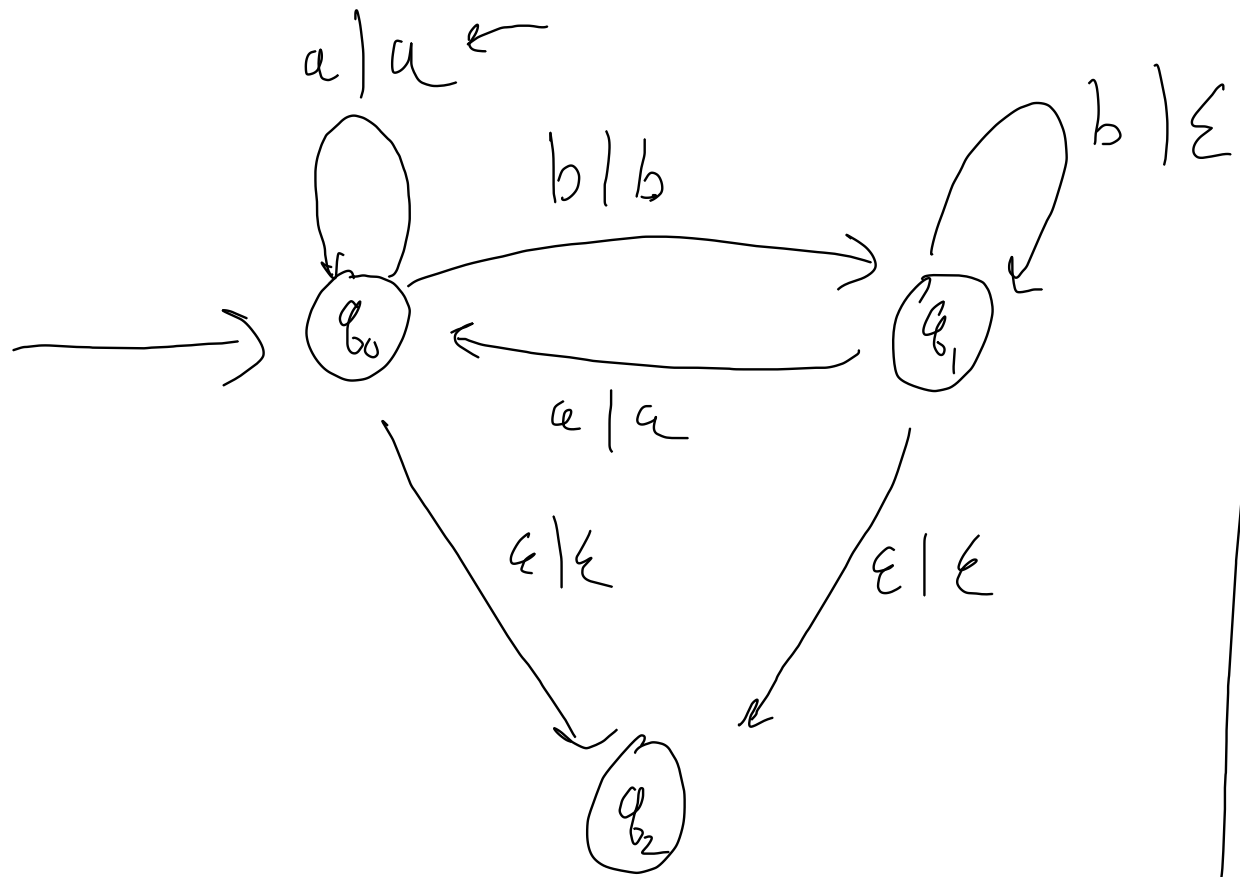
\Vdash

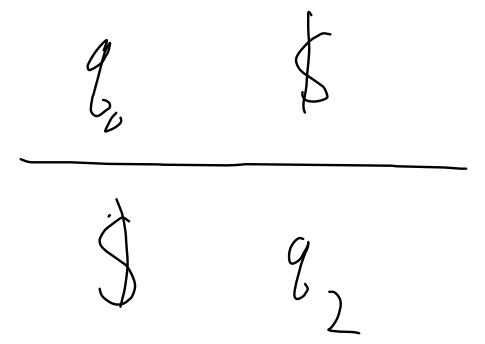
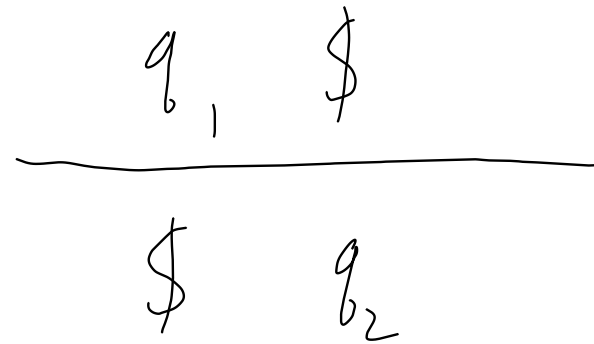
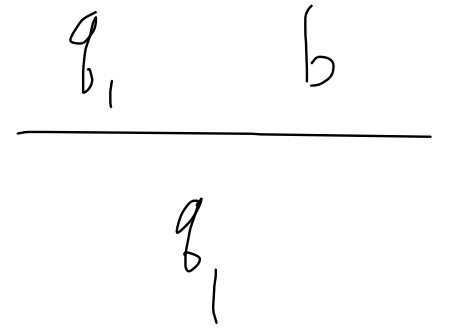
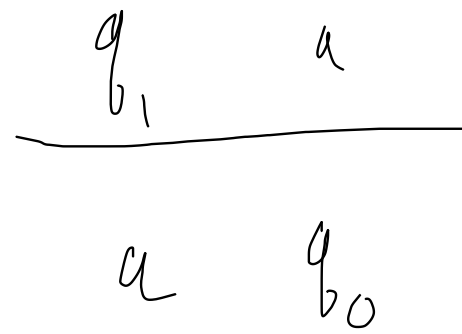
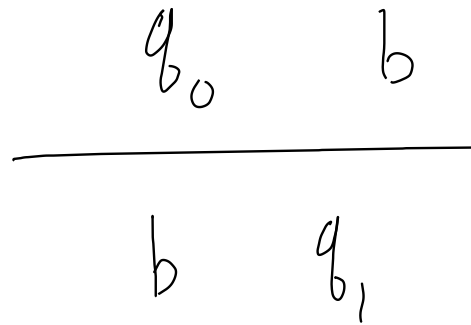
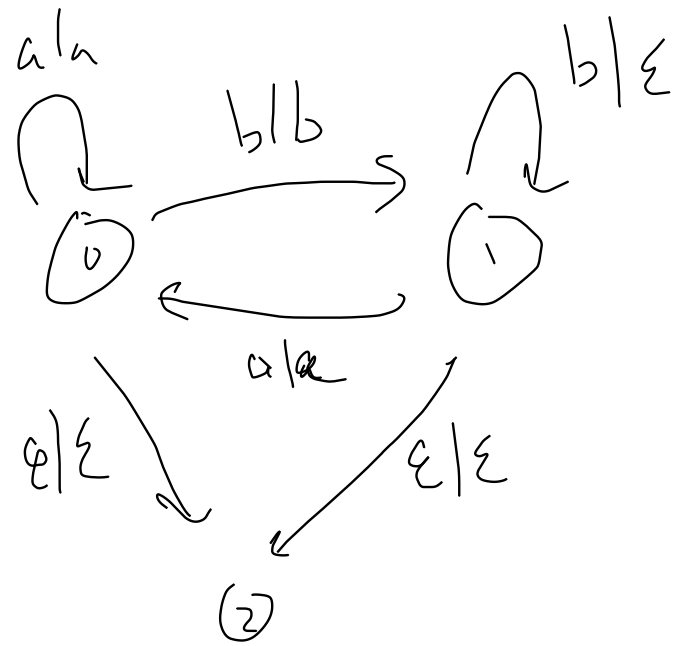
$$\frac{A \rightarrow B \quad A}{B} \rightarrow \text{E}$$

$$\frac{A \vdash B}{A \rightarrow B} \rightarrow \text{I}$$

$$\left. \begin{array}{l} \frac{A \quad A \setminus B}{B} \setminus \text{E} \\ \frac{B \setminus A \quad A}{B} \setminus \text{E} \end{array} \right\}$$

Substructural Logic Programming





q_0 a b a b b a a b \$

a q_0 b a b b a a b \$

a b q_1 a b b a a b \$

a b a q_0 b b a a b \$

a b a q_1 b a a b \$

a b a b q_0 a a b \$

a b a b a a b q_1 \$

a b a b a a b \$ q_2 \rightarrow a b a b a a b \$