15-317 Lecture 18: Forward Chaining

- Reading rules top-down
- Saturation
- Forward Chaining
- Comparing forward and backward chaining

$$\frac{edge(x,y)}{edge(y,x)}\ sym \qquad \frac{edge(x,y)}{path(x,y)}\ path_1 \qquad \frac{path(x,y) \quad path(y,z)}{path(x,z)}\ path_2$$

$$\frac{}{even(z)}\ ev_z \qquad \frac{odd(N)}{even(s\ N)}\ ev_s \qquad \frac{even(N)}{odd(s\ N)}\ odd_s$$

$even(z)$, $odd(s\ z)$, $even(s\ s\ z)$, ...

$$\frac{even(s\ N)}{odd(N)} \qquad \frac{odd(s\ N)}{even(N)} \qquad \frac{odd(z)}{C \quad (no)}$$

$even(s\ s\ z)$ X

$odd(s\ z)$ X

$even(z)$ X

$even(s\ z)$ X

$odd(z)$ X

no X

$$\Gamma_{ev} = \forall N.\ even(s\ N) \supset odd(N),\ \forall N.\ odd(s\ N) \supset even(N),$$
$$odd(z) \supset no$$

$\Gamma \longrightarrow goal$ (backward chaining)

$\Gamma_{ev},\ even(s\ s\ z) \longrightarrow no$ (forward chaining)

Backwards

$$D^- ::= P^- \mid G^+ \supset D^- \mid \dots$$

$$\Gamma^- ::= \cdot \mid \Gamma^-, D^-$$

$$G^+ ::= \downarrow P^- \mid G_1^+ \wedge G_2^+ \mid \dots$$

Forwards

$$D^- ::= \uparrow P^+ \mid G^+ \supset D^- \mid \dots$$

Database $\Gamma ::= \cdot \mid \Gamma, D^- \mid \Gamma, P^+$

$$G^+ ::= P^+ \mid G_1^+ \wedge G_2^+ \mid \dots$$

$\Gamma \xrightarrow{\mathsf{S}} G^+$ stable sequent

$\Gamma, [D^-] \xrightarrow{\mathsf{S}} G^+$ left focus

$\Gamma \xrightarrow{\mathsf{S}} [G^+]$ right focus

$$\dfrac{\Gamma, [D^-] \xrightarrow{f} G^+}{\Gamma, D^- \xrightarrow{f} G^+}\ \text{focus L} \qquad \dfrac{\Gamma \xrightarrow{f} [G^+] \qquad \Gamma, [D^-] \xrightarrow{f} G^+}{\Gamma, [G^+ \supset D^-] \xrightarrow{f} G^+}\ \supset\text{L}$$

$$\dfrac{\Gamma, p^+ \xrightarrow{f} G^+}{\Gamma, [\uparrow p^+] \xrightarrow{f} G^+}\ \text{blur L} \qquad \dfrac{}{\Gamma, p^+ \xrightarrow{f} [p^+]}\ \text{id}^+$$

$$\left[\ \dfrac{}{\Gamma, [p^-] \xrightarrow{f} p^-}\ \text{id}^- \qquad \dfrac{\Gamma \xrightarrow{f} p^-}{\Gamma \xrightarrow{f} [\downarrow p^-]}\ \text{blurR}\ \right]$$

Unnecessary for forward chaining

$$\frac{}{\Gamma_{ev}, even(s\,s\,z), odd(s\,z), even(z) \xrightarrow{f} no}$$

$$\vdots$$

$$\frac{\Gamma_{ev}, even(s\,s\,z), odd(s\,z) \xrightarrow{f} no}{\Gamma_{ev}, even(s\,s\,z), [\uparrow odd(s\,z)] \xrightarrow{f} no}\ \text{blur}L$$

$$\frac{even(s\,s\,z) = even(s\,x)}{\Gamma_{ev}, even(s\,s\,z) \xrightarrow{f} [even(s\,\underset{s\,z}{x})]}\ id^{+}$$

$$\frac{\Gamma_{ev}, even(s\,s\,z) \xrightarrow{f} [even(s\,\underset{s\,z}{x})] \qquad \Gamma_{ev}, even(s\,s\,z), [\uparrow odd(s\,z)] \xrightarrow{f} no}{\Gamma_{ev}, even(s\,s\,z), [even(s\,\underset{s\,z}{x}) \supset \uparrow odd(\underset{s\,z}{x})] \xrightarrow{f} no}\ \supset L$$

$$\frac{\Gamma_{ev}, even(s\,s\,z), [even(s\,\underset{s\,z}{x}) \supset \uparrow odd(\underset{s\,z}{x})] \xrightarrow{f} no}{\Gamma_{ev}, even(s\,s\,z), [\forall N.\, even(s\,N) \supset \uparrow odd(N)] \xrightarrow{f} no}\ \forall L$$

$$\frac{\Gamma_{ev}, even(s\,s\,z), [\forall N.\, even(s\,N) \supset \uparrow odd(N)] \xrightarrow{f} no}{\Gamma_{ev}, even(s\,s\,z) \xrightarrow{f} no}\ \text{focus}L$$

$$a^-, \downarrow a^- \supset b^-, \downarrow b^- \supset c^- = \Gamma_0$$

$$
\begin{prooftree}
\AxiomC{}
\RightLabel{id^-}
\UnaryInfC{$\Gamma_0, [c^-] \longrightarrow c^-$}
\AxiomC{}
\RightLabel{id^-}
\UnaryInfC{$\Gamma_0, [b^-] \longrightarrow b^-$}
\AxiomC{}
\RightLabel{id^-}
\UnaryInfC{$\Gamma_0, [a^-] \longrightarrow a^-$}
\RightLabel{focus L}
\UnaryInfC{$[\Gamma_0 \longrightarrow a^-]$}
\RightLabel{blur R}
\UnaryInfC{$\Gamma_0 \longrightarrow [\downarrow a^-]$}
\RightLabel{$\supset L$}
\BinaryInfC{$\Gamma_0, [\downarrow a^- \supset b^-] \longrightarrow b^-$}
\RightLabel{focus L}
\UnaryInfC{$[\Gamma_0 \longrightarrow b^-]$}
\RightLabel{blur R}
\UnaryInfC{$\Gamma_0 \longrightarrow [\downarrow b^-]$}
\RightLabel{$\supset L$}
\BinaryInfC{$\Gamma_0 \; [\downarrow b^- \supset c^-] \xrightarrow{f} c^-$}
\RightLabel{focus L}
\UnaryInfC{$[a^-, \downarrow a^- \supset b^-, \downarrow b^- \supset c^- \xrightarrow{f} c^-]$}
\end{prooftree}
$$

Backwards chaining

$\Gamma_1 = a^+, a^+ \supset \uparrow b^+, b^+ \supset \uparrow c^+$

$$\dfrac{\dfrac{\Gamma_1 \longrightarrow [a^+]}{}\,id^+ \qquad \dfrac{\dfrac{\vdots}{[\Gamma_1, b^+ \longrightarrow c^+]}\,\text{blur } L}{\Gamma_1, [\uparrow b^+] \longrightarrow c^+}}{\dfrac{\Gamma_1, [a^+ \supset \uparrow b^+] \longrightarrow c^+}{[\Gamma_1 \longrightarrow c^+]}\,\text{focus } L}\,\supset L$$

$$\vdots$$

$$\dfrac{\dfrac{}{\Gamma_1, b^+, c^+ \longrightarrow [c^+]}\,id^+}{[\Gamma_1, b^+, c^+ \longrightarrow c^+]}\,\text{focus } R$$

$$\dfrac{\Gamma \xrightarrow{f} [c^+]}{\Gamma \xrightarrow{f} c^+}\,\text{focus } R$$

Unification in forward chaining (Not recommended for HW9)

$\doteq$

$$\frac{f(\bar{t}) \doteq g(\bar{s}) \quad f \neq g}{\text{no}} \qquad \frac{f(\bar{t}) \doteq f(\bar{s})}{\bar{t} \doteq \bar{s}}$$

$$\frac{(t, \bar{t}) \doteq (s, \bar{s})}{t \doteq s} \qquad \frac{(t, \bar{t}) \doteq (s, \bar{s})}{\bar{t} \doteq \bar{s}}$$

$$\frac{() \doteq (s, \bar{s})}{\text{no}} \qquad \frac{(t, \bar{t}) \doteq ()}{\text{no}}$$

$$f(X, X) \doteq f(a(), b())$$

$$(X, X) = (a(), b())$$

$$X \doteq a()$$

$$X \doteq b()$$

$$a() \doteq X$$

$$b() \doteq X$$

$$f(a, b) \doteq f(X, X)$$

$$a() \doteq b()$$

no

$$\frac{t \doteq s}{s \doteq t}$$

$$\frac{t \doteq s \qquad s \doteq r}{t \doteq r}$$