

15-317 Lecture 17: (Backwards) Chaining

- HW 8 due, HW 9 out

- Rules as Propositions

- Chaining

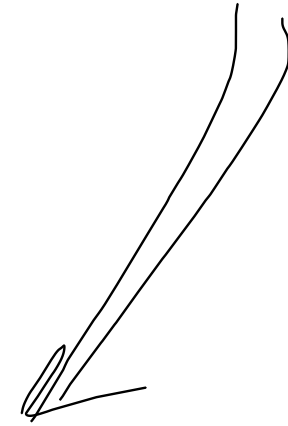
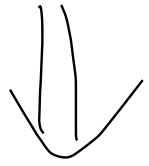
- Horn logic/Horn clauses

- Interpreting Prolog in Prolog

$$\frac{\text{inc}(e, b|e)}{\text{inc}_e}$$

$$\frac{\text{inc}(b\emptyset M, b|M)}{\text{inc}_\emptyset}$$

$$\frac{\text{inc}(M, N)}{\text{inc}(b|M, b\emptyset N)}$$



$$\text{inc}(e, b|e)$$

$$\forall M. \text{inc}(b\emptyset M, b|M)$$

Γ_{inc}

$$\forall M. \forall N. \text{inc}(M, N) \supset \text{inc}(b|M, b\emptyset N)$$

$$\Gamma_{\text{inc}} \xrightarrow{f} \text{inc}(b|e, b\emptyset b|e)$$

$$\frac{\Gamma, [A] \xrightarrow{\xi} P}{\Gamma, A \xrightarrow{\xi} P}$$

focus L

(If nothing in Γ is in focus)

$$\frac{\Gamma, [A(Y)] \xrightarrow{\xi} P}{\Gamma, [\forall x. A(x)] \xrightarrow{\xi} P}$$

$\forall L$ (Y fresh)

$$\frac{\Gamma, [B] \xrightarrow{\xi} P}{\Gamma, [A > B] \xrightarrow{\xi} P}$$

$$\frac{\Gamma \xrightarrow{\xi} [A]}{\Gamma, [A > B] \xrightarrow{\xi} P}$$

$>L$

$$\frac{\Gamma, [A > B] \xrightarrow{\xi} P}{\Gamma, [Q] \xrightarrow{\xi} P}$$

$$Q = P$$

id

$$\frac{\Gamma, [Q] \xrightarrow{\xi} P}{\Gamma \xrightarrow{\xi} [P]}$$

$$\frac{\Gamma \xrightarrow{\xi} P}{\Gamma \xrightarrow{\xi} [P]}$$

blur

$$x = e \quad y = b \mid e$$

$$\text{inc}(b \mid x, b \emptyset y) = \text{inc}(b \mid e, b \emptyset b \mid e)$$

$$\frac{\Gamma_{\text{inc}} \xrightarrow{f} \text{inc}(e, b \mid e)}{\text{focus L}} \text{blur}$$

$$\frac{\Gamma_{\text{inc}} \xrightarrow{f} [\text{inc}(e, b \mid e)]}{\dots}$$

$$\Gamma_{\text{inc}} [\text{inc}(b \mid x, b \emptyset y)] \xrightarrow{f} \text{inc}(b \mid e, b \emptyset b \mid e)$$

$$\Gamma_{\text{inc}} \xrightarrow{f} [\text{inc}(x, y)] \text{L}$$

$$\Gamma_{\text{inc}} [\text{inc}(x, y) \supset \text{inc}(b \mid x, b \emptyset y)] \xrightarrow{f} \dots$$

AL

$$\Gamma_{\text{inc}} [\forall N \text{inc}(x, N) \supset \dots] \xrightarrow{f} \dots$$

AL

$$\Gamma_{\text{inc}} [\dots \supset \text{inc}(b \mid e, b \emptyset b \mid e)] \xrightarrow{f} \text{inc}(b \mid e, b \emptyset b \mid e)$$

focus L

$$\Gamma_{\text{inc}} \xrightarrow{f} \text{inc}(b \mid e, b \emptyset b \mid e)$$

- Invertible right rules \rightarrow negative
 - Invertible left rules \rightarrow positive
- Horn Logic /
Horn clauses

Program formulas $D^- ::= \frac{P^- \mid G^+ \supset D^- \mid \forall x. D^-(x)}{D_1^- \wedge D_2^- \mid T \mid \cancel{G^+}}$

Programs $\Gamma^- ::= \bullet \mid \Gamma^-, D^-$

Goals $G^+ ::= \frac{\downarrow P^- \mid G_1^+ \wedge G_2^+ \mid T \mid \exists x. G^+(x)}{G_1^+ \vee G_2^+ \mid \perp \mid \cancel{P^+} \mid \downarrow D^-}$

$\Gamma^- \xrightarrow{\text{S}} \downarrow P^-$ stable sequent

$\Gamma^-, [D^-] \xrightarrow{\text{S}} \downarrow P^-$ left focus

$\Gamma^- \xrightarrow{\text{S}} [G^+]$ right focus

$\Gamma^- \xrightarrow{\text{S}} [G^+]$
 $\Gamma^- \xrightarrow{\text{S}} G^+$ focus R

$\Gamma^- \xrightarrow{\text{S}} [G_1^+]$ $\Gamma^- \xrightarrow{\text{S}} [G_2^+]$

 $\Gamma^- \xrightarrow{\text{S}} [G_1^+, \wedge^+ G_2^+]$ $\wedge R$

$\Gamma^- \xrightarrow{\text{S}} [T]$ $\top R$

$\Gamma^- \xrightarrow{\text{S}} [G(Y)]$

 $\Gamma^- \xrightarrow{\text{S}} [\exists x. G^+(x)]$ $\exists R$