15-317 Lecture 14: Logic Programming

- Rules (+ strategy) as programs
- Predicate logic
- Proof search as computation
- Answer substitutions
- Modes

Predicates

- Atomic prop'ns are predicates $p(t_1, \ldots, t_n)$.
- p is a **predicate symbol** of arity n
- $t_1, \ldots, t_n$ are terms and **arguments** to p

$\underbrace{z()}_{z}$ $\quad s(n)$

$even(n)$:

$$\frac{}{even(z)}\ evz$$

$$\frac{even(N)}{even(s(s(N)))}\ evs$$

$$\dfrac{\dfrac{\times}{\mathrm{even}(s\ z)}}{\mathrm{even}(s(s(s(z))))}\ \mathrm{evs}$$

$$\dfrac{\dfrac{\dfrac{}{\mathrm{even}(z)}\ \mathrm{evz}}{\mathrm{even}\ (s(s(z)))}\ \mathrm{evs}}{\mathrm{even}(s(s(s(s(z)))))}\ \mathrm{evs}$$

Prolog

goal-directed proof search

Datalog

forward reasoning proof search

Addition

$plus(M, N, P)$ "$M + N = P$"

$0 + n = n$

$(m+1) + n = (m+n) + 1$

$$\frac{plus(M, N, P)}{plus(s\,M, N, s\,P)}\ ps$$

$$\frac{}{plus(z, N, N)}\ pz$$

$$\cfrac{\cfrac{}{plus(z,\ s\,z,\ R')}\ evz}{plus(s\,z,\ s\,z,\ R)}\ evs$$

evz (where $N = s\,z$ and $N = R'$) $R' = s\,z$

evs (where $R = s\,R'$) $\boxed{R = s\,s\,z}$

$$\dfrac{\dfrac{\begin{array}{c} X \\ \mathrm{plus}(M_2,\ s\,z,\ z) \end{array}}{\mathrm{plus}(M_1,\ s\,z,\ s\,z)}\ \mathrm{ps}\quad M_1 = s\,M_2}{\mathrm{plus}(M,\ s\,z,\ s\,s\,z)}\ \mathrm{ps}\quad M = s\,M_1$$

$$\dfrac{}{\mathrm{plus}(M_1,\ s\,z,\ s\,z)}\ \mathrm{pz} \qquad M_1 = z$$

$$\boxed{M = s\,z}$$

Modes

- Write + for inputs, − for outputs.
  - Addition: plus(+M, +N, −P)
  - Subtraction: plus(−M, +N, +P)
  - Sum checking: plus(+M, +N, +P)

    Also subtraction: plus(+M, −N, +P)

    plus(−M, −N, −P)

$$\frac{plus(M, N, P)}{plus(sM, N, sP)}\ ps \qquad \frac{}{plus(Z, N, N)}\ pz$$

$plus(+, +, -)$ well-moded

case pz:

N is given/known by + in second slot,
so N is known in the third slot

Case ps:

sM, N are known, so M, N are known.
By i.h, P is known, and so is sP

$plus(-,-,-)$

Case pz:

M is known to be z

N, P are not known

$plus(+,-,+)$

pz:

M, P are known

Because N=P, N is known

ps:

$M = s\,M'$ $\quad P = s\,P'$

By i.h. we get a known N from

$plus(M', N, P')$

Subgoal order

$$\frac{}{times(z, N, z)}\ t_z \qquad \frac{times(M, N, P) \quad plus(P, N, Q)}{times(sM, N, Q)}\ t_s$$

$$P = sP', \quad Q = sQ', \quad \ldots$$

$$plus(P', ssz, Q')$$

$$P = z \qquad \frac{}{plus(z, ssz, Q)}\ p_z \quad Q = ssz$$

$$\frac{\dfrac{}{times(z, ssz, P)}\ t_z \qquad plus(P, ssz, Q)}{times(sz, ssz, Q)}\ t_s$$

$$\boxed{Q = ssz}$$

Prolog Notation

$$\frac{J_1, \quad \ldots \quad J_n}{J}$$

In prolog $\quad J \;{:}{-}\; J_1, \ldots, J_n.$

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plus(s(M), N, s(P)) :- plus(M, N, P).
plus(z(), N, N).
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Unification

goal: plus(s z, s z, R)

rule: plus(s M, N, s P) :- plus(M, N, P)

- Unification involves finding a substitution of terms for variables (M, N, P, R) such that two things become syntactically equal

e.g. plus(s z, s z, R) = plus(s M, N, s P)

M = z   R = s P

N = s z

$$\text{plus}(z, N, N) = \text{plus}(z, s\,z, R)$$

$$\boxed{N = s\,z}$$

$$\text{plus}(z, s\,z, s\,z) = \text{plus}(z, s\,z, R)$$

$$\boxed{R = s\,z}$$