

15-317 Lecture 5: Verifications and uses

- Verifications $A \uparrow$
 - Uses/Assumptions $A \downarrow$
 - Easier proof search
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- Global soundness + completeness
 - Relating $A \uparrow$, $A \downarrow$, $A \text{ true}$
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- Normal/Neutral proof terms
- Counting normal proofs/terms

Verifications

- Meaning of a connective given by intro rules
- The meaning of a proposition is its set of verifications
- A verification is a restricted form of proof
- A verification only looks at subformulas of the initial prop.

$$(A \supset B) \wedge C$$

Subformulas

- $A \supset B$

- C

- A

- B

New judgement $A \uparrow$

"A has a verification"

- A is true and we have evidence in a specific form for that

Conjunction

$$\frac{A \uparrow \quad B \uparrow}{A \wedge B \uparrow} \wedge I$$

Implication

$$\frac{\overline{A \text{ true}}^u \quad \vdots \quad B \text{ true}}{A \supset B \text{ true}} \supset I^u$$

$$\frac{\overline{A \downarrow}^u \quad \vdots \quad B \uparrow}{A \supset B \uparrow} \supset I^u$$

E lim rules

$$\frac{A \wedge B \downarrow}{A \downarrow} \wedge E_1 \quad \frac{A \wedge B \downarrow}{B \downarrow} \wedge E_2$$

Implication

$$\frac{A \supset B \downarrow \quad A \uparrow}{B \downarrow} \supset E$$

$$\frac{\frac{((A \supset A) \supset B) \downarrow}{A \supset A \uparrow} \supset I^a}{((A \supset A) \supset B) \supset B \uparrow} \supset I^u$$

Disjunction

$$\frac{A \uparrow}{A \vee B \uparrow} \text{VI}_1$$

$$\frac{B \uparrow}{A \vee B \uparrow} \text{VI}_2$$

$$\frac{\begin{array}{cc} \text{---} \neg \text{---} \vee \\ A \downarrow & B \downarrow \\ \vdots & \vdots \\ A \vee B \downarrow & C \uparrow & C \uparrow \end{array}}{C \uparrow} \text{VE}^{\neg, \vee}$$

Truth/Falsity

$$\frac{\text{---}}{T \uparrow} \text{TI}$$

$$\frac{F \downarrow}{C \uparrow} \text{FE}$$

Atomic Propositions

- Propositions which cannot be broken down

$$\frac{P \downarrow}{P \uparrow} \text{IP}$$

~~$$\frac{P \supset P \downarrow}{P \supset P \uparrow}$$~~

Global soundness and completeness

- Completeness

$A \downarrow$

\vdots

for any A

$A \uparrow$

- Soundness

If $A \uparrow$ and $A \downarrow$
 $C \uparrow$ then $C \uparrow$

Other properties of \uparrow/\downarrow

- If $A \uparrow$ then A true

- If $A \downarrow$ then A true

(Because each \uparrow/\downarrow proof
can be turned into a "true"
by replacing \uparrow and \downarrow with "true")

- If A true then $A \uparrow$

- If A true then $A \downarrow$? ~~X~~

Counting normal terms/proofs

$$\frac{\frac{\frac{\quad}{A \downarrow} \quad}{A \uparrow} \quad \downarrow \uparrow}{\quad} \supset I^u}{A \supset A \uparrow}$$

$$A \supset (A \supset A) \uparrow$$

$$f_n \quad u \Rightarrow u$$