Formal Languages, Automata and Computation

Regular Expressions

Carnegie Mellon University in Qatar

Summary

- Nondeterminism
  - Clone the FA at choice points

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  - Guess and verify

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- NFAs are equivalent to DFAs

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  - Multiple transitions from a state with the same input symbol
  - ϵ-transitions
- NFAs are equivalent to DFAs
  - Determinization procedure builds a DFA with up to 2^k states for an NFA with k states.

Closure Theorems

Theorem

The class of regular languages is closed under the union *operation.*

Closure Theorems

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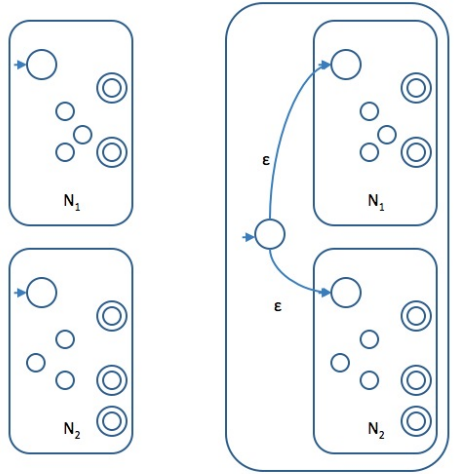
Proof Idea Based on NFAs



CLOSURE THEOREMS

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The class of regular languages is closed under the concatenation operation.

Closure Theorems

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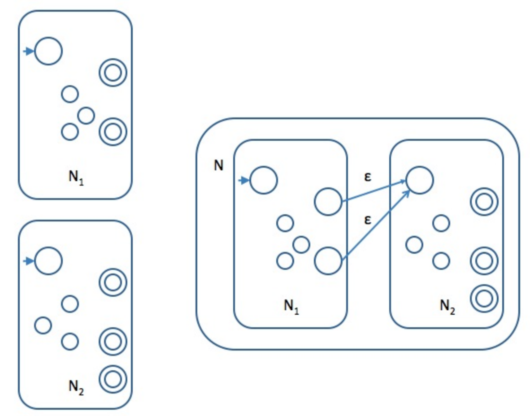

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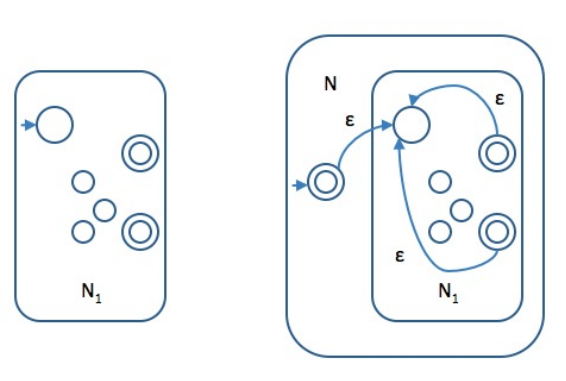

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- DFAs are finite descriptions of (finite or infinite) sets of strings
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- Regular Expressions provide an algebraic expression framework to describe the same class of strings
- Thus, DFAs and Regular Expressions are equivalent.

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- In $(\ldots)$, the parenthesis can be deleted
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- $R^+ = RR^*$ and R^k for k-fold concatenation are useful shorthands.

Regular Expression Examples

Regular Expression		Regular Language
0^*10^*	$\rightarrow$	

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$(\mathbf{0}^*\mathbf{1}\mathbf{0}^*\mathbf{1})^*\mathbf{0}^*$	$\rightarrow$	$\{\omega \mid n_1(\omega)$ is even$\}$

- All strings with at least one pair of consecutive 0s

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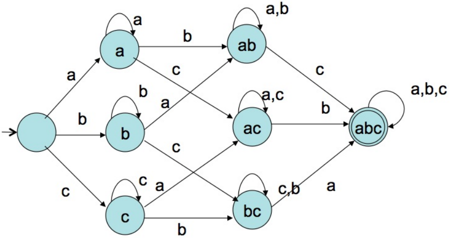

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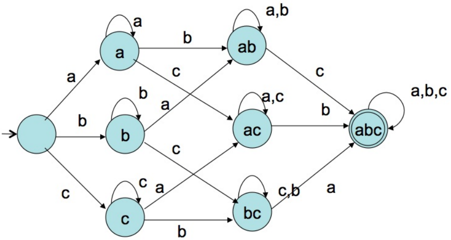

- DFAs and REs may need different ways of looking at the problem.
  - For the DFA, you count symbols
  - For the RE, you enumerate all possible patterns

RE IDENTITIES

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- Note that we do not have explicit operators for intersection or complementation!

DIGRESSION: REs IN REAL LIFE

- Linux/Unix Shell, Perl, Awk, Python all have built in regular expression support for pattern matching functionality
- See `http://www.wdvl.com/Authoring/Languages/Perl/PerlfortheWeb/perlintro2_table1.html`

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- Substring matches in a string!
- Search for and download *Regex Coach* to learn and experiment with regular expression matching

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Proof Idea

Inductively convert a given regular expression to an NFA.

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ϕ	

Converting REs to NFAs: Basis Cases

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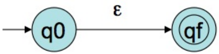
ϵ	

CONVERTING REs TO NFAs: BASIS CASES

Regular Expression **Corresponding NFA**

ϕ

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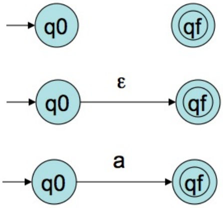
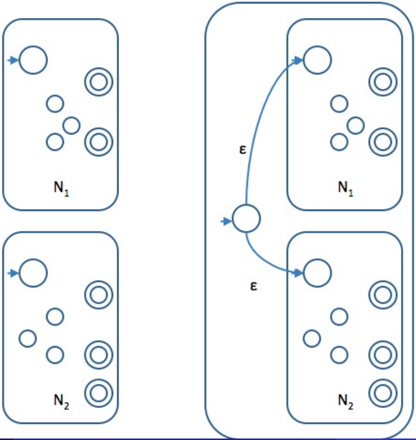
a for $a \in \Sigma$



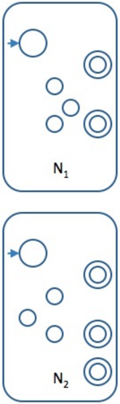
Union

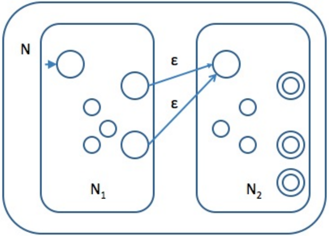
- Let N_1 and N_2 be NFAs for R_1 and R_2 respectively. Then the NFA for $\mathbf{R_1} \cup \mathbf{R_2}$ is



Concatenation

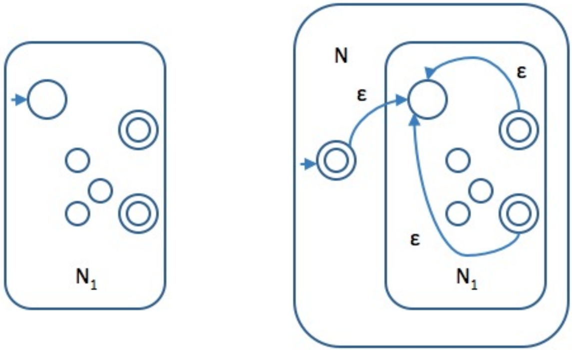
- Let N_1 and N_2 be NFAs for R_1 and R_2 respectively. Then the NFA for $\mathbf{R_1R_2}$ is





Star

- Let N be NFAs for R. Then the NFA for $\mathbf{R}^*$ is



RE TO NFA CONVERSION EXAMPLE

- Let's convert $(\mathbf{a} \cup \mathbf{b})^* \mathbf{aba}$ to an NFA.

RE to NFA to DFA

- Regular Expression $\rightarrow$ NFA (possibly with ϵ-transitions)

RE TO NFA TO DFA

- Regular Expression $\rightarrow$ NFA (possibly with ϵ-transitions)
- NFA $\rightarrow$ DFA via determinization

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EQUIVALENCE WITH FINITE AUTOMATA

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PROOF IDEA

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Proof Idea

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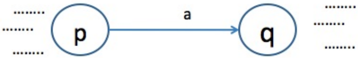
- Generalized NFAs (GNFA)
- Iteratively eliminate states of the GNFA one by one, until only two states and a single generalized transition is left.

Generalized Transitions

- DFAs have single symbols as transition labels



  - If you are in state p and the next input symbol matches a, go to state q

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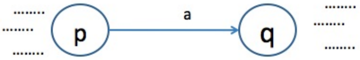
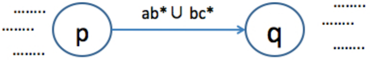
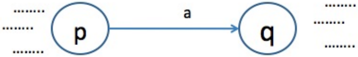


  - If you are in state p and the next input symbol matches a, go to state q
- Now consider



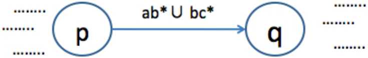
GENERALIZED TRANSITIONS

- DFAs have single symbols as transition labels



  - If you are in state p and the next input symbol matches a, go to state q

- Now consider



  - If you are in state p and a prefix of the remaining input matches the regular expression $\mathbf{ab}^* \cup \mathbf{bc}^*$ then go to state q

Generalized Transitions and NFA

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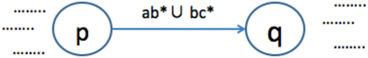

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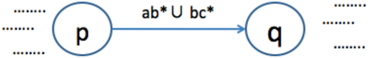

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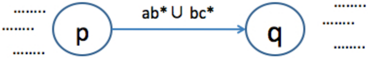

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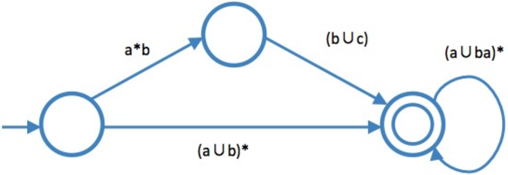
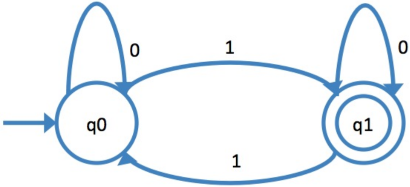


  - In fact, all standard DFA transitions are generalized transitions with regular expressions of a single symbol!

Generalized Transitions

- Consider the 2-state DFA



$\downarrow$

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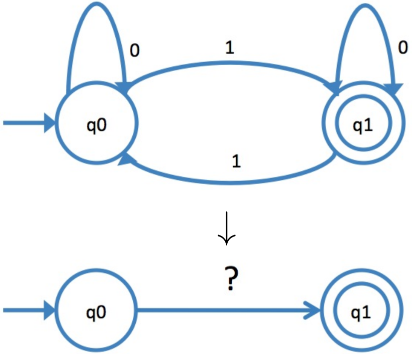

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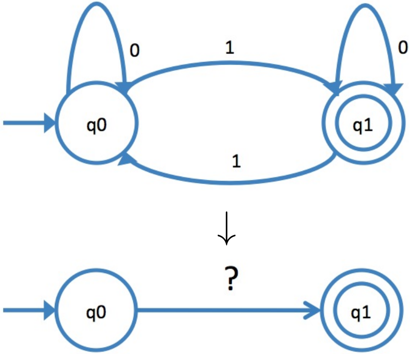

- **0*****1** takes the DFA from state q_0 to q_1

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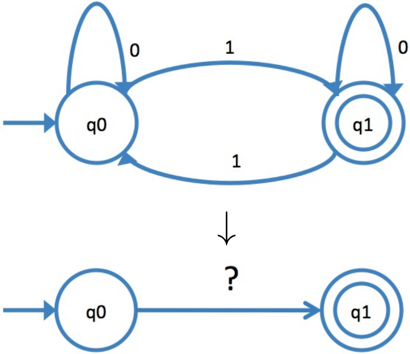

- $\mathbf{0}^*\mathbf{1}$ takes the DFA from state q_0 to q_1
- $(\mathbf{0} \cup \mathbf{10}^*\mathbf{1})^*$ takes the machine from q_1 back to q_1

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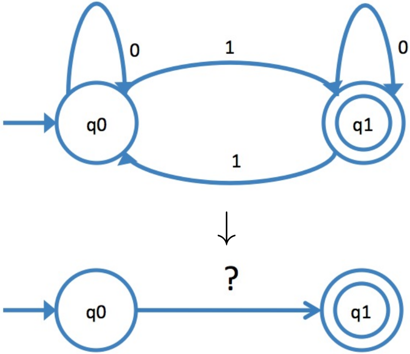

- $\mathbf{0}^*\mathbf{1}$ takes the DFA from state q_0 to q_1
- $(\mathbf{0} \cup \mathbf{10}^*\mathbf{1})^*$ takes the machine from q_1 back to q_1
- So $\mathbf{?} = \mathbf{0}^*\mathbf{1}(\mathbf{0} \cup \mathbf{10}^*\mathbf{1})^*$ represents all strings that take the DFA from state q_0 to q_1

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  - with only two states: one start and one accept

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- then we can “read” the regular expression from the label of the generalized transition (as in the example above)