Formal Languages, Automata and Computation

Turing Machines

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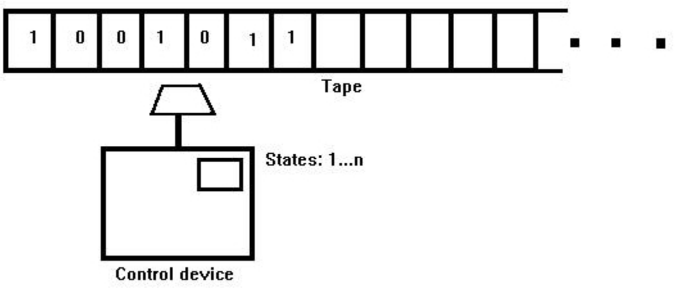
Turing Machines-Synopsis

- We now turn to a much more powerful model of computation called Turing Machines (TM).
- TMs are similar to a finite automaton, but a TM has an unlimited and unrestricted memory.
- A TM is a much more accurate model of a general purpose computer.
- Bad News: Even a TM can not solve certain problems.
- Such problems are beyond theoretical limits of computation.

Turing Machines



Turing Machines vs Finite Automata

- A TM can both read from the tape and write on the tape.
- The read-write head can move both to the left (L) and to the right (R).
- The tape is infinite (to the right).
- The states for rejecting and accepting take effect immediately (not at the end of input.)

How does a TM compute?

- Consider $B = \{w\#w \mid w \in \{0,1\}^*\}$.
- The TM starts with the input on the tape.

$\underline{0}$ 1 1 0 0 0 # 0 1 1 0 0 0 ⊔ ⊔ ⊔⊔

X $\underline{1}$ 1 0 0 0 # 0 1 1 0 0 0 ⊔ ⊔ ⊔⊔

$\rightarrow\rightarrow \cdots$

X 1 1 0 0 0 # $\underline{X}$ 1 1 0 0 0 ⊔ ⊔ ⊔⊔

$\leftarrow\leftarrow \cdots$

$\underline{X}$ 1 1 0 0 0 # X 1 1 0 0 0 ⊔ ⊔ ⊔⊔

X $\underline{X}$ 1 0 0 0 # X 1 1 0 0 0 ⊔ ⊔ ⊔⊔

$\rightarrow\rightarrow \cdots$

X X X X X X # X X X X X X $\underline{⊔}$ ⊔ ⊔⊔ **ACCEPT**

Formal Definition of a Turing Machine

A TM is 7-tuple $M = (Q, \Sigma, \Gamma, \delta, q_0, q_{accept}, q_{reject})$ where Q, Σ, Γ are all finite sets.

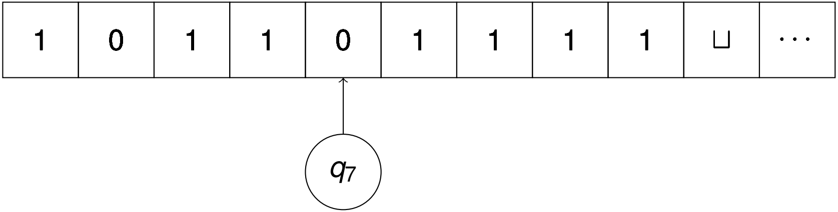
1. Q is the set of states,
2. Σ is the input alphabet (**blank symbol** $\sqcup \notin \Sigma$),
3. Γ is the tape alphabet ($\sqcup \in \Gamma$ and $\Sigma \subset \Gamma$),
4. $\delta : Q \times \Gamma \to Q \times \Gamma \times \{L, R\}$ is the state transition function,
5. $q_0 \in Q$ is the start state,
6. $q_{accept} \in Q$ is the accept state,
7. $q_{reject} \in Q$ is the reject state and $q_{reject} \neq q_{accept}$

How does a TM Compute?

- M receives its input $w = w_1 w_2 \cdots w_n$ on the leftmost n squares on the tape. The rest of the tape is blank.
- The head starts on the leftmost square on the tape.
- The first blank symbol on the tape marks the end of the input.
- The computation proceeds according to δ.
- The head of M never moves left of the beginning of the tape (stays there!)
- The computation proceeds until M enters either q_{accept} or q_{reject}, when it halts.
- *M may go on forever, never halting!*

Configuration of a TM

- As a TM proceeds with its computation, the state changes, the tape changes, the head moves.
- We capture each step of a TM computation, by the notion of a configuration.



- The machine is in state q_7, $u = 1011$ is to the left of the head, $v = 01111$ is under and to the right of the head. Tape has $uv = 101101111$ on it.
- We represent the configuration by $1011q_701111$.

Configurations

- Configuration C_1 yields ($\Rightarrow$)configuration C_2 if TM can legally go from C_1 to C_2.
- $ua\ q_i\ bv \Rightarrow u\ q_j\ acv$ if $\delta(q_i, b) = (q_j, c, L)$
- $ua\ q_i\ bv \Rightarrow uac\ q_j\ v$ if $\delta(q_i, b) = (q_j, c, R)$
- If the head is at the left end, $q_ibv \Rightarrow q_jcv$ if the transition is left-moving.
- If the head is at the left end, $q_ibv \Rightarrow cq_jv$ if the transition is right-moving.
- Think of a configuration as the contents of memory and a transition as an instruction.

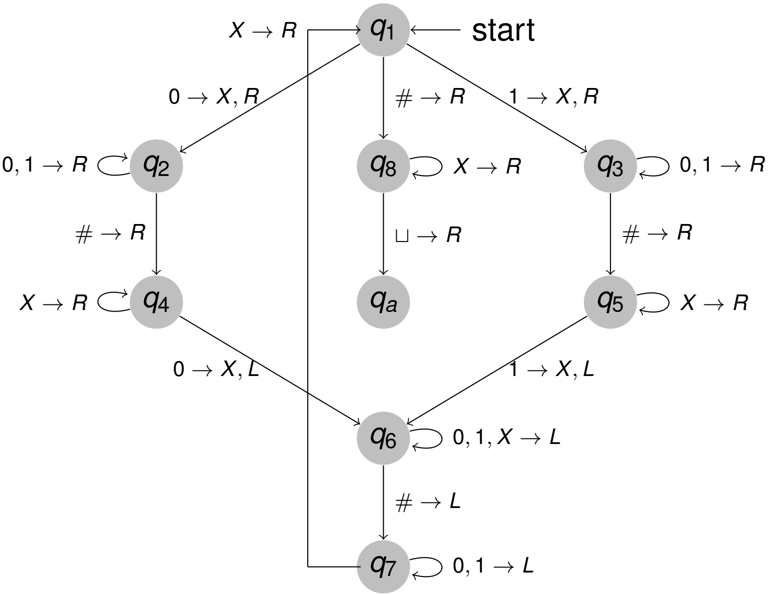
Configurations

- The start configuration is $q_0 w$.
- $uq_{accept}v$ is an accepting configuration,
- $uq_{reject}v$ is a rejecting configuration.
- Accepting and rejecting configurations are halting configurations.

Accepting Computation

- A TM M accepts input w if a sequence of configurations $C_1, C_2, \cdots, C_k$ exists, where
  1. C_1 is the start configuration of M in input w,
  2. $C_i \Rightarrow C_{i+1}$, and
  3. C_k is an accepting configuration.
- $L(M)$ is the set of strings w recognized by M.
- A language L is Turing-recognizable if some TM recognizes it (also called Recursively enumerable)
- A TM is called a decider if it halts on all inputs.
- A language is Turing-decidable if some TM decides it (also called Recursive)
- Every decidable language is Turing recognizable!

Example TM-1



start
q_1
$X \rightarrow R$
$0 \rightarrow X, R$
$\# \rightarrow R$
$1 \rightarrow X, R$
q_2
$0, 1 \rightarrow R$
q_8
$X \rightarrow R$
q_3
$0, 1 \rightarrow R$
$\# \rightarrow R$
$\sqcup \rightarrow R$
$\# \rightarrow R$
q_4
$X \rightarrow R$
q_a
q_5
$X \rightarrow R$
$0 \rightarrow X, L$
$1 \rightarrow X, L$
q_6
$0, 1, X \rightarrow L$
$\# \rightarrow L$
q_7
$0, 1 \rightarrow L$

Example TM-1

- Let us see how this TM operates on input 001101#001101