# TECHNICAL APPENDIX

## 1. Correcting Solar Time

In the central chapters of this book we have assumed that if the apparent solar day is defined as the time interval between one local noon and the next, then the time required by the stars to complete their diurnal circles is invariably just 4 minutes (more accurately 3 minutes 56 seconds) shorter than this solar day. But as noted in a footnote to Chapter 1, this is not quite the case. If the intervals between successive local noons are perfectly regular, then the stars must move at an irregular rate. Conversely, if the stars complete successive diurnal circles in equal intervals of time, then the lengths of successive solar days must vary. This fact was recognized in antiquity, at least by the time of Ptolemy and very probably before. To understand it let us assume, as the ancients did, that the apparent motion of the stars is perfectly regular, so that the stars provide a fundamental time scale. We shall then discover two distinct reasons why the intervals between the instants when the sun achieves its maximum daily elevation must vary.

The first cause of the irregularity of apparent solar time is the variation in the rate at which the sun seems to move through the zodiacal constellations. As we discovered in Chapter 2, the sun moves more rapidly along the ecliptic from autumnal to vernal equinox than from vernal to autumnal. In its daily race with the stars the sun therefore seems to lose ground more rapidly in winter than in summer, so that if time is measured by the stars, the sun must take longer to regain maximum elevation during the winter than it requires in summertime. It follows that the apparent solar day should be longest in midwinter and shortest in midsummer, and this would be the case if another cause of irregularity did not intervene.

The second source of the apparent solar day's variability is the angle at which the ecliptic intersects the equator on the celestial sphere. To understand its effect look again at Figure 13, Chapter 1, and imagine that equally spaced lines of celestial longitude are drawn on the sphere, just as lines of longitude are drawn on any terrestrial globe. For the sake of simplicity, assume in addition that the sun's apparent motion along the ecliptic is perfectly regular and at the rate of 1° along the great circle per day. It then turns out that, because the ecliptic is tilted with respect to the equator, the net *eastward* motion of the sun varies from one day to the next. When the



sun is at or near one of the solstices, its apparent motion with respect to the stars is very nearly parallel to the celestial equator. In addition, it is moving on a part of the sphere where the lines of longitude are somewhat closer together than they are at the equator. As a result the net eastward motion of the sun is somewhat more than 1° *of celestial longitude* per day, and the celestial sphere must therefore turn westward through slightly more than 361° in order to carry the sun from maximum elevation to maximum elevation. At the equinoxes the situation is quite different. There the lines of celestial longitude have their maximum spacing on the sphere. Furthermore, the sun's constant total motion is to the northeast or southeast rather than due east, and it therefore does not move eastward as much as 1° a day. As a result the celestial sphere need not rotate through quite 361° to return the sun to maximum elevation. This effect, considered alone, makes the apparent solar day longest at the solstices and shortest at the equinoxes.

In order to correct for these two irregularities modern civilizations have adopted a time scale known as mean solar time, whose fundamental time unit is the *average* length of the apparent solar day. On this time scale the stars do, by definition, move perfectly regularly, completing their diurnal circles in just 23 hours 56 minutes 4.091 seconds. But the scale that makes the stars regular makes the sun irregular. For example, the sun's maximum elevation rarely occurs at local noon, mean solar time. The time kept by sundials, the only instruments that directly measure apparent solar time, does not pass at the same rate as the time kept by our watches or announced by time signals on the radio. During December and January, when both the effects discussed above act to shorten the apparent solar day, the interval between successive maximum elevations of the sun is very nearly 0.5 minute less than the mean solar day. Furthermore, the effect of this small discrepancy is cumulative – apparent time runs slower than mean time for many days in a row – so that at one season of the year the sun reaches maximum elevation (apparent noon) almost 20 minutes before mean solar noon. At other seasons apparent time runs faster than mean time, and over the years the two stay together. But they are rarely together during any one day. In order to keep accurate time by the sun it is therefore necessary to correct the sundial by using a table, or a diagram like the one shown in Figure 53.

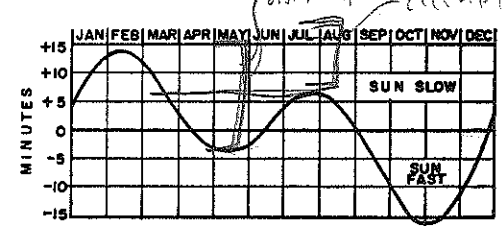



Figure 53. A graph of the equation of time, indicating the annual variation of the difference between mean and apparent solar time.

The preceding discussion of time employs the apparent motion of the stars as its standard of regularity. Clearly this choice of standard is arbitrary, at least from a logical viewpoint. Logically we might equally well have chosen the sun's apparent motion as our standard of regularity and shown that on the corresponding time scale the stars move at a continually varying rate. But the choice of the sun as a standard of regularity would be immensely inconvenient to both science and civil life. The diagram of Figure 53 would have to be applied to clocks and watches rather than to sundials. Astronomers and physicists would be forced to describe the earth's axial rotation as occurring at a constantly varying rate. The stellar standard avoids these awkwardnesses. It is well adapted to all civil and most scientific functions.

Yet it has not turned out to be quite adequate for science, at least not for scientific theory; the time scale implicit in Newton's Laws of Motion does not quite correspond to the stellar standard. From Newton's Laws, as they are now understood, it is possible to show that the earth's axial rotation is being gradually slowed by effects like tidal friction and that, as a result, the apparent stellar motions are very gradually slowing down. Either the Laws or the stellar standard must therefore be adjusted, and considerations of scientific convenience suggest the search for a new standard. To date the theoretical inadequacy of the stellar standard is without practical significance. But it has an immense importance to science, and it has therefore led scientists to a renewed search, which continues actively today, for a clock that will conform to the time scale of scientific theory more accurately than the celestial machine itself.

## 2. Precession of the Equinoxes

A second technical simplification introduced in the body of this book was the neglect of the precession of the equinoxes. This is the effect, mentioned briefly in Chapter 1, that results in a slow motion of the celestial pole through the stars. If we had been concerned only with naked-eye observations made during the lifetime of a single astronomer, our simplification would have been entirely appropriate – naked-eye observations cannot disclose its inaccuracy unless they are made at widely separated points in time. But observations made, for example, two centuries apart show that, while the stars themselves retain constant relative positions, the celestial pole about which they move gradually shifts its position among them at a rate just over 0.5° per century. Observations repeated over far longer periods disclose the pattern of this precessional motion; as the centuries pass the pole of the heavens moves gradually through the stars in a circle, completing one revolution every 26,000 years. The center of this circle is the pole of the ecliptic – the point at which a diameter perpendicular to the plane of the ecliptic intersects the celestial sphere – and the radius of the circle is just 23½°, the same as the angle in which the celestial equator intersects the ecliptic on the sphere of the stars (Figure 54*a*).



The precessional motion seems to have been noticed first by the Hellenistic astronomer Hipparchus during the second century B.C., and, though not widely known at first, it was discussed by a number of subsequent astronomers, including Ptolemy. Most of Ptolemy's Moslem successors described some form of the effect, and by adding a ninth sphere to the ancient system they succeeded in explaining it physically. Their most popular explanation is indicated diagramatically in Figure 54*b*, which shows the three outermost spheres of the set; *N* and *S* are the north and south celestial poles, and the exterior sphere rotates westward about them, once every 23 hours 56 minutes, as the sphere of the stars had rotated in the older system. The next

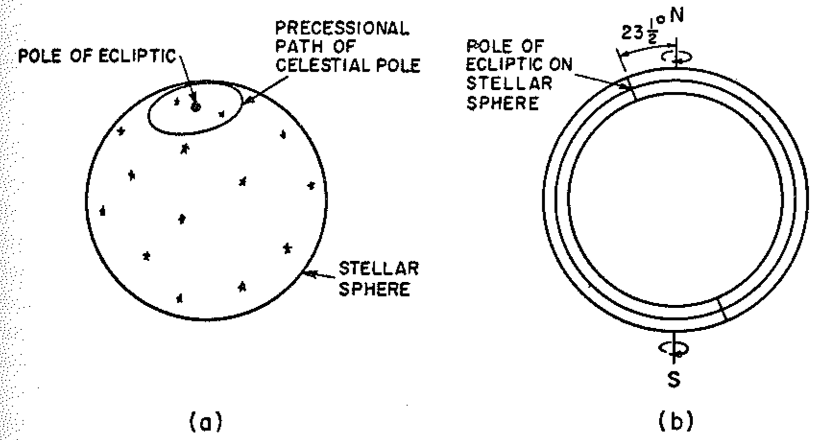



Figure 54. The precession of the equinoxes. Diagram (*a*) shows the circle on the celestial sphere around which the celestial pole moves once in every 26,000 years. The center of the circle is the pole of the ecliptic, and all points on the circle are just 23½° from this center. Diagram (*b*) shows how the Moslems explained precession with the aid of a ninth sphere, the outer sphere in the drawing. This ninth sphere rotates once every 23 hours 56 minutes, as the sphere of the stars had rotated in the older eight-sphere systems. The eighth sphere, on which the stars are set, rotates about its own poles once in 26,000 years, thus slowly changing the position of the celestial pole among the stars. Inside the eighth sphere is the sphere of Saturn, which encloses the remaining planetary spheres as in the older systems.

sphere, the middle one in the diagram, is the sphere that carries the stars, and it is joined to the outermost sphere by an axis which passes through the poles of the ecliptic on the sphere of the stars and through two points 23½° from the poles on the outer sphere. This new sphere of the stars is whirled around daily by the outermost sphere (this accounts for the diurnal stellar circles). In addition it has a slow motion of its own, one rotation every 26,000 years, which gradually changes the relations between the individual stars and the celestial poles. The innermost of the three spheres is Saturn's, and it is drawn as a thick shell to allow space for the epicyclic components of Saturn's motion. By itself this thick sphere, connected to the sphere of the stars by an axis through the poles of the ecliptic, accounts for Saturn's average circular motion through the stars.

In the context of ancient and medieval astronomical thought this ninth-sphere explanation of precession seems both simple and natural. In fact, it compares relatively well with the Copernican explanation – a gradual conical motion of the earth's axis which, during the course of 26,000 years, is directed successively to all the points on a circle of radius 23½° about the pole of the ecliptic. Until Newton explained precession as a physical consequence of the moon's gravitational attraction for the equatorial bulge of the earth, both Copernican and Ptolemaic astronomers required one extra and physically superfluous motion in order to account for it.* Precession has, therefore, no logical bearing upon the transition from an earth-centered to a sun-centered universe.

Historically, however, the problem of explaining precession had a significant role in inaugurating the Copernican Revolution. It helped to make Ptolemaic astronomy seem monstrous. The observational consequences of precession are very small even when observations extend over several centuries, and a small error in the data can therefore result in a radical change in the description of the over-all phenomenon. Both Hipparchus and Ptolemy described precession in a way qualitatively equivalent to the one represented by Figure 54, but many of their contemporaries denied the existence of the effect entirely or else described it quite differently. Particularly in the Moslem world a number of divergent descriptions of precession were prevalent. There was no agreement about its rate – in fact, many astronomers believed that the rate varied. In addition, there was an important school which believed that even the direction of precession changed periodically, an effect known as trepidation. Brahe's observations were required before astronomers could again recognize the true simplicity of the phenomenon. Copernicus himself did not improve the situation in the slightest. He added extra circles to his system in order to account for the gradual change in the precessional rate and for other nonexistent effects. But though Copernicus did not improve the account of precession given by ancient and medieval astronomers, he was immensely concerned to do so, and that concern provided an important impetus to astronomical reform. In

* Copernicus himself did not require an extra motion to account for precession, because he had already introduced one in another connection. He used an annual conical motion to keep the earth's axis parallel to itself throughout the year (Figure 31*b*), and he could therefore explain precession by giving this conical motion a period very very slightly less than a year. But Copernicus' successors, who thought that a single orbital motion would keep the earth's axis perpetually in alignment, did need an additional conical motion with a period of 26,000 years in order to explain the changing position of the celestial pole.



Copernicus' day an adequate account of precession was the principal prerequisite for the most pressing problem of practical astronomy, the reform of the Julian calendar.

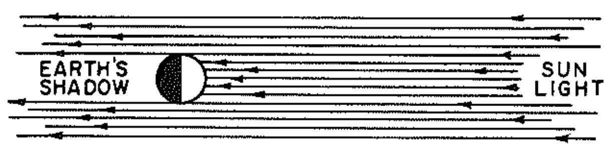
To discover the effect of precession upon the design of calendars, return once more to Figure 54. As the diagram shows, the position of the ecliptic upon the sphere of the stars is fixed once and for all. But though the changing positions of the celestial poles do not affect the ecliptic, they do change the position of the celestial equator and therefore of the equinoxes, the points at which the ecliptic and the celestial equator intersect. During the precessional period, 26,000 years, each of the equinoxes moves slowly and steadily around the ecliptic at the rate of about 1⅜° per century. Therefore, the length of time required by the sun to move once around the ecliptic (the so-called sidereal year) is not quite the same as the length of time it requires to move on the ecliptic from vernal equinox to vernal equinox (the tropical year). The latter, which is more than 20 minutes shorter than the former, is vastly more difficult to measure, because it refers the sun's motion to an imaginary and moving point rather than to a fixed star. But the tropical year is the year of the seasons, and it is this that must be measured with precision before an accurate long-term calendar can be designed. Copernicus' concern with the calendar therefore led him to a serious study of precession, and thus to an intimate knowledge of that aspect of astronomy about which Ptolemaic astronomers were in the greatest disagreement. It is the problem of precession which underlies Copernicus' remark that "the mathematicians . . . cannot even explain or observe the constant length of the seasonal year" (p. 137), and it is this remark which heads his list of motives for innovation.

### 3. Phases of the Moon and Eclipses

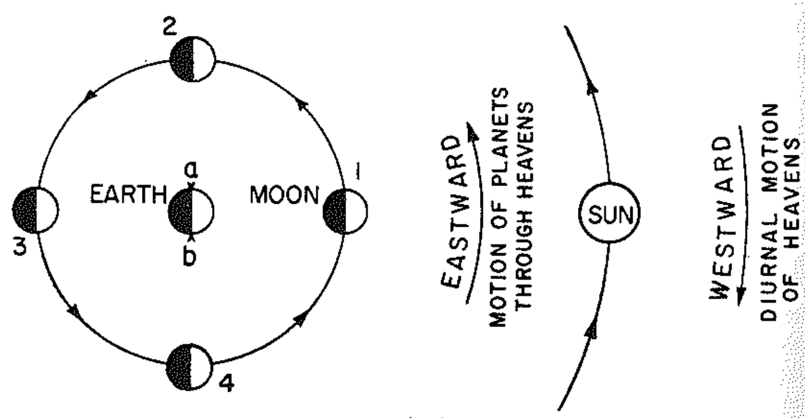
Because it is identical with the modern explanation, the ancients' account of the cause of the moon's phases played no role in the Copernican Revolution, and it could therefore be omitted from the earlier chapters of this book. But the phases of the moon play a direct role in the ancient measurements of the dimensions of the universe, and these measurements, as we have repeatedly noted, helped make the ancient two-sphere universe seem concrete and real to scientists and nonscientists alike. Besides, the ancient explanation of phases, as well as the correlated explanation of eclipses, provides an important additional illustration of the scientific adequacy of the ancient world view.

The explanation with which we are concerned was well known in Greece by the end of the fourth century B.C., though it may have originated considerably earlier. With the acceptance of the two-sphere universe came the larger and less well documented assumption that all the celestial wanderers were spheres as well. In part this assumption derived from analogy to the spherical shape of the earth and heavens, and in part from the conception of the perfection of the spherical shape and its appropriateness to the perfect heavens. More direct, though incomplete, evidence was provided by the observed cross sections of the sun and moon. Now if the moon is a sphere, a distant sun can illuminate only one-half of its surface (Figure 55*a*), and the fraction of this illuminated hemisphere visible to an observer will necessarily vary with his position. An observer on the sun would see the entire hemisphere at all times; an observer on the earth looking toward the moon when it lay between him and the sun would see none of the illuminated hemisphere whatsoever. It follows that the portion of the moon's surface clearly visible to a terrestrial observer must depend upon the relative positions of the sun, the moon, and the earth.

Four *relative* positions of the sun and moon at four equally spaced periods during the lunar month are shown in Figure 55*b*, which portrays the earth-centered orbits of the sun and moon in the plane of the ecliptic. (Since only relative positions are significant in discussions of the moon's phases, the diagram can readily be adapted to a sun-centered universe.)




(a)




(b)

Figure 55. The ancient (and modern) explanation of the moon's phases. Diagram (*a*) indicates that only half of the surface of a sphere is illuminated by the rays of the distant sun. Diagram (*b*) shows the portion of this illuminated hemisphere visible to a terrestrial observer for various *relative* positions of the sun, earth, and moon. Position 1 is new moon; 2 is the waxing half moon; 3, full moon; and 4, the waning half moon.



A westward rotation of the entire diagram, excepting the central earth, accounts for the diurnal motion of the sun and moon, so that an observer at *a* sees the sun just setting and one at *b* sees it rising. Only the eastward orbital motions of the sun and moon are motions with respect to the diagram. When the moon is at position 1 in the diagram it rises with the sun, but, since its dark side is pointed toward the earth, it can scarcely be seen by a terrestrial observer. This is the position of new moon. Slightly more than a week later the moon's rapid orbital motion has carried it 90° east of the slow-moving sun where it appears, relative to the sun, in position 2. It now rises at noon and is near the zenith at sunset. Only half of the disk is clearly visible from the earth, so that this is the position of first quarter. After another week or a bit more, the moon is full and rises as the sun sets (position 3). Third quarter is shown at position 4, corresponding to a moon that rises around midnight and is near the zenith at sunrise.

The diagram used in deciphering the moon's phases can also be used in the explanation of eclipses: as the moon moves from position 2 to position 4, it may pass through the earth's shadow, in which case it grows dim and is eclipsed. If the moon always appeared on the ecliptic, it would be eclipsed each time it reached position 3, but, since it continually wanders north and south, the full moon, earth, and sun rarely lie on a straight line. Full moon must lie close to the ecliptic for a lunar eclipse, and this cannot happen more than twice a year and seldom happens that often. Solar eclipses occur whenever the moon, at position 1, casts its shadow on the earth, and this happens relatively frequently, at least twice a year. Yet solar eclipses are rarely seen by terrestrial observers. The moon's shadow on the earth is extremely small, and an observer must be in the shadow to see the eclipse. Besides, the moon rarely blocks off more than a small fraction of the sun's disk. Therefore, an observer at any one location can seldom see even a partial eclipse of the sun and may never see a total one. For him it will be a rare, striking, and sometimes terrifying phenomenon.

### 4. Ancient Measurements of the Universe

One of the most interesting technical applications of ancient astronomy was its use in the determination of cosmological distances and sizes which could not be measured directly, that is, by ordinary measuring sticks. These distance measurements illustrate the world view's fruitfulness with greater immediacy than most of its other applications, because the mathematical operations upon which they depend lose all physical significance unless certain essential elements in the conceptual scheme are true. For example, whether the earth is a flat disk or a sphere, the stars do appear to move in diurnal circles, and techniques that describe this apparent motion are therefore useful whatever their conceptual basis. But only if the earth is really a sphere can it be said to have a circumference that can be determined from the observations of the skies discussed below.

The first reference to measurements of the earth's circumference appears in Aristotle's writings, so that such measurements were probably made by the middle of the fourth century B.C. But we know only the results of these earliest measurements, not the method employed; the first measurement of which we have a relatively complete, though second-hand, account is the one made by Eratosthenes, the librarian of the great manuscript collection at Alexandria, during the third century B.C. Eratosthenes measured the angle *a* (Figure 56) between the rays of the noon sun and a vertical

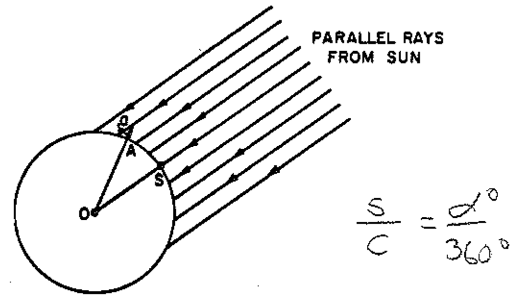



Figure 56. Eratosthenes' measurement of the earth's circumference. If *S* is due south of *A* on the earth's surface, then the distance *AS* must be the same fraction of the earth's circumference as the angle *a* is of 360°.

gnomon located at Alexandria, *A*, on a day when the noon sun was directly overhead at Syene, *S*, a second Egyptian city located 5000 stades due south of Alexandria. This angle he found to be 1/50 of a full circle (or 7 1/5°). Since all the rays striking the surface of the earth from the very distant sun may be considered parallel, the angle *a*, which is the sun's distance from the zenith at Alexandria, is equal to the angle *AOS* subtended by *S* and *A* at the center of the earth, *O*. Furthermore, since this angle is just 1/50 of a circle, the distance from Alexandria to Syene must be 1/50 of the circumference of the earth, and the total circumference must be 50 × (distance from Alexandria to Syene) = 50 × 5000 = 250,000 stades. Most modern students believe that Eratosthenes' figure is approximately 5 percent lower than the result given by modern measurement (24,000 miles), but unfortunately it is impossible to be sure. The length of the unit "stade" used by Eratosthenes is unknown, and the known location of Alexandria and Syene cannot be used to define the unit, because both the "5000" and the "1/50" used in the computation above have clearly been "rounded off" to make the report easier to read.

A second group of measurements was made during the second century B.C. by Aristarchus of Samos, now more famous for his anticipation of the Copernican system. He estimated the distance to and the sizes of the sun and moon in terms of the angle *MES* subtended by the centers of the sun



and moon at the earth when the moon is exactly half full (Figure 57). Since the moon can be half full only if the angle *EMS* subtended by the earth and the sun at the moon is exactly a right angle, the size of *MES* must completely determine the shape of the right triangle whose vertices are the moon, the earth, and the sun. Aristarchus' measurement gave *MES* = 87°, which corresponded to a triangle in which *ES*:*EM*::19:1. Ac-

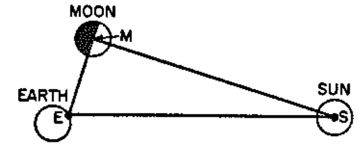



Figure 57. Aristarchus' measurement of the relative distances from the earth to the moon and the sun. When the moon is exactly half full, the angle *EMS* must be just 90°. Therefore a measurement of the angle *MES* will determine the ratio of *EM* to *ES*, that is, the ratio of the moon's distance from the earth to that of the sun.

cordingly, he reported that the sun was 19 times as far from the earth as the moon and that, since the moon and the sun subtend the same angle at the earth (Figure 58), it was also 19 times as large.

Modern measurements, made by quite different techniques and with the aid of telescopes, indicate that Aristarchus' ratio was too small by a factor of more than twenty; the ratio *ES*:*EM* is very nearly 400:1, not 19:1. This discrepancy arises from the measurement of the angle *MES* which should be 89° 51′, rather than 87°. In practice that measurement is very difficult, particularly with the instruments known to have been available to Aristarchus. The precise centers of the sun and moon are very hard to determine; in addition it is difficult to be sure when the moon is *just* half full. Given these problems, Aristarchus seems to have chosen the smallest angle compatible with his uncertain observations, presumably in order to keep the resulting ratio credible. Similar considerations must have motivated his successors, for,

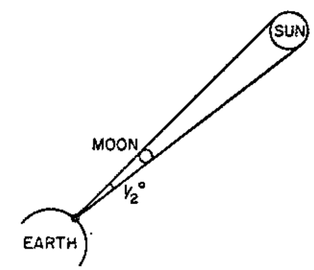



Figure 58. The large but distant sun and the smaller but nearby moon subtend the same angle at the earth's surface.

though appreciably improved, estimates of the relative distances to the sun and the moon remained too small throughout antiquity and the Middle Ages.

The preceding measurements yield only the ratios of astronomical distances, but by an immensely ingenious argument Aristarchus was able to convert them to absolute magnitudes, that is, he was able to determine the diameter of and distances to the sun and the moon in stades. His results were derived from observations of a lunar eclipse of maximum duration, an eclipse during which the moon lies squarely on the ecliptic and therefore passes through the very center of the earth's shadow. First he measured the time that elapsed between the instant when the edge of the moon first entered the shadow and the instant when the moon was totally obscured for the first time. This figure he compared with the length of time during which the moon was totally obscured, and he thus discovered that the period of total obscurity was approximately the same length as the period required for the moon to enter into the earth's shadow. He concluded that the breadth of the earth's shadow in the region where it is crossed by the moon is very nearly twice the diameter of the moon itself.

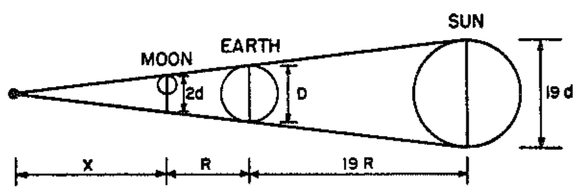



Figure 59. Aristarchus' construction for computing the absolute distances to the moon and sun in terms of the observations made during a lunar eclipse.

Figure 59 shows the astronomical configuration which Aristarchus analyzed. In the diagram the moon is shown immediately after it has fully entered the shadow of the earth. The diameter of the moon is $d$ (an unknown) and the diameter of the earth's shadow at the moon is therefore $2d$; the diameter of the earth is $D$ (known in stades from Eratosthenes' measurement of the circumference of the earth); and the distance from the moon to the earth is $R$ (again an unknown to be determined). Finally, since the sun's diameter and distance from the earth are just 19 times the moon's, the diameter of the sun's disk is just $19d$ and its distance from the earth is just $19R$. So Aristarchus' problem, and ours, is to determine $d$ and $R$, the unknown distances, in terms of the earth's diameter, $D$, a quantity whose value in stades has already been determined.

The diagram shows immediately three similar triangles whose bases are of length $2d$, $D$, and $19d$, and whose altitudes are respectively $x$ (an unknown), $x + R$, and $x + 20R$. (Actually the bases of the three triangles are very slightly shorter than the diameters with which they have been equated above, but if the triangles are extremely acute, as they are, this discrepancy is too small to affect the result.) The ratio of the altitude to the base of the smallest triangle must be the same as that of the largest, or,

$$\frac{x}{2d} = \frac{x + 20R}{19d}.$$

Multiplying both sides of the equation by $38d$ yields a new equation: $19x = 2x + 40R$, so that $x = 40R/17$. In other words, the earth's shadow extends beyond the moon for a distance about 2⅓ times the distance from the earth to the moon.

Comparing the smallest of the triangles with the triangle of intermediate size gives another equation, from which $d$ may be determined. The first comparison gives:

$$\frac{x}{2d} = \frac{x + R}{D}.$$

Substituting $40R/17$ for $x$ and multiplying both sides by $17/R$ yields:

$$\frac{20}{d} = \frac{40 + 17}{D}.$$

From the last equation, $d = 20D/57 = 0.35D$. That is, the diameter of the moon is just greater than one-third the diameter of the earth, and since the sun's diameter is just 19 times the moon's, the sun must have just over 6⅔ times the diameter of the earth.

Since $D$, the diameter of the earth, is known, the actual sizes of the sun and moon are given by the computation above. Their distances can be determined by a small additional computation. Because both the sun and the moon subtend an angle of 0.5° at the earth, each could be placed 720 times on the circumference of a full (360°) circle with its center at the earth. The distance of the moon from the earth must therefore be the radius of a circle whose circumference is 720 times the moon's diameter, now known, and the sun's distance must be just 19 times as great. Since the circumference of a circle is $2\pi$ times its radius, the moon's distance from the earth must be just over 40 diameters of the earth and the distance to the sun should be approximately 764 earth diameters.

The methods employed in these computations are brilliant, typifying the very best efforts of Greek scientists, but the numerical results, particularly those concerning the sun, are uniformly inaccurate because of the initial error in the determination of the angular separation of the sun and the half moon. Modern measurements give the moon's diameter as just over one-fourth the diameter of the earth and its distance as approximately 30 earth diameters, neither of which is far from the values computed by Aristarchus. But the sun's diameter is now thought to be almost 110 times that of the earth and the distance to the sun is about 12,000 earth diameters, both very much larger than Aristarchus supposed. Though various corrections to Aristarchus' measurements were made during antiquity and though the possibility of significant error in the measured distance to the sun was often recognized, all ancient and medieval estimates of this cosmological dimension remained vastly too small.

Because it depends only upon the relative positions of earth, moon, and sun, Aristarchus' techniques for determining size and distance can be applied with equal accuracy or inaccuracy in the Ptolemaic and Copernican universes. The ancient determinations of astronomical dimensions could, therefore, have no direct role in the Copernican Revolution. But they did have several indirect ones, all of which helped to strengthen the Ptolemaic system. The possibility of making astronomical measurements illustrated the great fruitfulness of the Aristotelian-Ptolemaic universe. In addition, the results of the measurements helped to make the ancient cosmology seem real by increasing the concreteness with which its structure was specified. Finally, and most important, the measurement of the distance to the moon provided an astronomical yardstick which, during the Middle Ages, was used to provide an indirect measure of the size of the entire universe.

As indicated early in Chapter 3, medieval cosmologists often supposed that each crystalline shell was just thick enough to contain the epicycle of its planet and that the shells as a group nested so that they filled all of space. Using these hypotheses mathematical astronomers were able to determine the relative sizes and thicknesses of all the shells. These relative dimensions were then converted to earth diameters, stades, or miles, by using Aristarchus' method of determining the distance to the moon's sphere. A typical set of the cosmological dimensions that resulted was included in the original discussion. It indicates the detail with which the universe was investigated and understood by pre-Copernican scientists.

# REFERENCES


**Chapter 1. The Ancient Two-Sphere Universe**

1. Sir Thomas L. Heath, *Greek Astronomy*, Library of Greek Thought (London: Dent, 1932), pp. 5–7.
2. Benjamin Jowett, *The Dialogues of Plato*, 3rd ed. (London: Oxford University Press, 1892), III, 452–453.

**Chapter 2. The Problem of the Planets**

1. Vitruvius, *The Ten Books on Architecture*, trans. M. H. Morgan (Cambridge: Harvard University Press, 1926), pp. 261–262.
2. Sir Thomas L. Heath, *Aristarchus of Samos* (Oxford: Clarendon Press, 1913), p. 140.

**Chapter 3. The Two-Sphere Universe in Aristotelian Thought**

1. Aristotle, *On the Heavens*, trans. W. K. C. Guthrie, The Loeb Classical Library (Cambridge: Harvard University Press, 1939), p. 91 ($279^a6$–17).
2. *Ibid.*, pp. 243–253 ($296^b8$–$298^a13$).
3. Sir Thomas L. Heath, *Greek Astronomy*, Library of Greek Thought (London: Dent, 1932), pp. 147–148.
4. Aristotle, *On the Heavens*, p. 345 ($310^b2$–5).
5. Aristotle, *Physics*, trans. P. H. Wickstead and F. M. Cornford, The Loeb Classical Library (Cambridge: Harvard University Press, 1929), I, 331 ($213^a31$–34).
6. Aristotle, *On the Heavens*, pp. 23–25 ($270^b1$–24).
7. Jean Piaget, *The Child's Conception of Physical Causality*, trans. Marjorie Gabain (London: Kegan Paul, Trench, Trubner, 1930), pp. 110–111.
8. Heinz Werner, *Comparative Psychology of Mental Development*, rev. ed. (Chicago: Follett, 1948), pp. 171–172.
9. Aristotle, *Physica*, trans. R. P. Hardie and R. K. Gaye, in *The Works of Aristotle*, II (Oxford: Clarendon Press, 1930), $208^b8$–22.

**Chapter 4. Recasting the Tradition: Aristotle to the Copernicans**

1. St. Augustine, *Works*, ed. Marcus Dods (Edinburgh: Clark, 1871–77), IX, 180–181.
2. St. Thomas Aquinas, *Commentaria in libros Aristotelis De caelo et mundo*, in *Sancti Thomae Aquinatis . . . Opera Omnia*, III (Rome: S. C. de Propaganda Fide, 1886), p. 24. My translation.
3. St. Thomas Aquinas, *The "Summa Theologica,"* Part I, Questions L–LXXIV, trans. Fathers of the English Dominican Province, 2nd ed. (London: Burns Oates & Washbourne, 1922), p. 225 (Q. 68, Art. 3).