

uum) and Brownian motion—both involve the explanation of observable effects in terms of molecular causes.

Professor Putnam rejects the positivist conception of theoretical terms according to which their meaning is given by their relation to the observational terms of the theory. On his approach the relation between the terms and the world are determined by the history of theorizing about that kind of thing. He is particularly concerned to reject the conclusions of Feyerabend and others that distinct theories have no point of contact because their expressions mean different things. Putnam rejects this conclusion, which he describes as "idealist," meaning that it construes the world as our creation, in favor of the realism according to which our theories may well talk about unobservable items even when the theories are radically wrong about their nature.

Professor Fine is dissatisfied with both realist and anti-realist positions. He is in agreement with the rejection of a Feyerabend position which regards truth as dependent upon theory; moreover, he finds van Fraassen's position unnatural, depending as it does on an expectation that science give an unequivocal and noncircular answer to what is observable but then *uniformly* suspend belief beyond that point. Fine's own "natural ontological attitude" does not, in his words, depend on interpreting science providing any philosophical interpretation of what science is doing. Whether this solves the problems or merely refuses to face them is a question we leave to the reader.

One apparent consequence of the positivist view of theories is that the meaning of theoretical terms is given via their relations in the axioms to observational terms. This leads to the conclusion that if the theory is changed, the meanings of the theoretical terms must also change, a conclusion that has seemed to many critics to create difficulties. Professor Paul Feyerabend has been one of the leading critics of the logical empiricist view that observational laws are deduced from theories. He claims that this presupposes that the laws and the theories never have conflicting empirical consequences and that the meaning

of the terms in the laws does not change when the new theory is formulated. Feyerabend calls these two conditions the "consistency condition" and "the condition of meaning invariance," and he argues that neither is actually satisfied by scientific practice. He also argues that it would be unreasonable for the scientist to confine himself or herself to the formulation of theories that satisfy these conditions. Therefore, Feyerabend concludes that the logical empiricists were mistaken and that we should view new theories not as covering laws for earlier laws and theories but as attempts to replace these earlier laws with which they conflict.

One way of avoiding the problems which seem to follow from the change of meaning attendant on the change of theory is to argue that the reference of scientific terms does not change because it is determined by the causal history of the use of the term and by facts, perhaps as yet unknown, about the world. Professor Shapere discusses a version of this approach and argues that it is no better than the empiricist approach it is intended to supplant.

His main argument is that the causal history approach would commit scientists eventually to a dogmatic identification of a particular set of properties as the ultimate definition of a term, and thus would replace the apparent relativism of the earlier meaning change approach with an unacceptable dogmatism. His own solution to the problems, which he sketches at the end of his essay, is to argue that there is a continuity of reasons underlying the changes of meaning and that these continuities unite the apparently disparate discussions that occur over a period of time.

Professor Laudan argues against two dogmas that he believes have been widely accepted in discussions of theory change and theory evaluation. The first is that scientific progress requires that all problems solved by a previous theory must also be solved by any subsequent theory if we are to progress. The second is that to evaluate two rival theories we must be able to translate fully statements of the two theories into a neutral third language. Laudan argues that both of these dogmas are

false; he cites numerous historical examples of theories which were accepted as successors and as constituting scientific progress even though they did not solve all of the problems solved by the theory they supplanted. With regard to evaluation, he points out that if we

can simply count the number of problems solved by rival theories, then we would be able to compare their problem-solving abilities without either an exact matching of problems or a translation of the theories.

## The Classical Approach

# Rudolf Carnap THEORIES AS PARTIALLY INTERPRETED FORMAL SYSTEMS

\* \* \*

### 23. PHYSICAL CALCULI AND THEIR INTERPRETATIONS

The method described with respect to geometry can be applied likewise to any other part of physics: We can first construct a calculus and then lay down the interpretation intended in the form of semantical rules, yielding a physical theory as an interpreted system with factual content. The customary formulation of a physical calculus is such that it presupposes a logico—mathematical calculus as its basis, e.g., a calculus of real numbers in any of the forms discussed above (§18). To this basic calculus are added the specific primitive signs and the axioms, i.e., specific primitive sentences, of the physical calculus in question.

Thus, for instance, a calculus of mechanics of mass points can be constructed. Some

predicates and functors (i.e., signs for functions) are taken as specific primitive signs, and the fundamental laws of mechanics as axioms. Then semantical rules are laid down stating that the primitive signs designate, say, the class of material particles, the three spatial coordinates of a particle  $x$  at the time  $t$ , the mass of a particle  $x$ , the class of forces acting on a particle  $x$  or at a space point  $s$  at the time  $t$ . (As we shall see later [§24], the interpretation can also be given indirectly, i.e., by semantical rules, not for the primitive signs, but for certain defined signs of the calculus. This procedure must be chosen if the semantical rules are to refer only to observable properties.) By the interpretation, the theorems of the calculus of mechanics become physical laws, i.e., universal statements describing certain features of events; they constitute physical mechanics as a theory with factual content which can be tested by observations. The relation of this theory to the calculus of mechanics is entirely analogous to the relation of physical

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to mathematical geometry. The customary division into theoretical and experimental physics corresponds roughly to the distinction between calculus and interpreted system. The work in theoretical physics consists mainly in constructing calculi and carrying out deductions within them; this is essentially mathematical work. In experimental physics interpretations are made and theories are tested by experiments.

In order to show by an example how a deduction is carried out with the help of a physical calculus, we will discuss a calculus which can be interpreted as a theory of thermic expansion. To the primitive signs may belong the predicates "Sol" and "Fe", and the functors "lg", "te", and "th." Among the axioms may be A1 and A2. (Here,  $x$ ,  $\beta$  and the letter with subscripts are real number variables; the parentheses do not contain explanations as in former examples, but are used as in algebra and for the arguments of functors.)

- A1. For every  $x, t_1, t_2, l_1, l_2, T_1, T_2, \beta$  [if  $[x$  is a Sol and  $\text{lg}(x, t_1) = l_1$  and  $\text{lg}(x, t_2) = l_2$  and  $\text{te}(x, t_1) = T_1$  and  $\text{te}(x, t_2) = T_2$  and  $\text{th}(x) = \beta$ ] then  $l_2 = l_1 \times (1 + \beta \times (T_2 - T_1))$ .  
 A2. For every  $x$ , if  $[x$  is a Sol and  $x$  is a Fe] then  $\text{th}(x) = 0.000012$ .

The *customary interpretation*, i.e., that for whose sake the calculus is constructed, is given by the following semantical rules.  $\text{lg}(x, t)$  designates the length in centimeters of the body  $x$  at the time  $t$  (defined by the statement of a method of measurement);  $\text{te}(x, t)$  designates the absolute temperature in centigrades of  $x$  at the time  $t$  (likewise defined by a method of measurement);  $\text{th}(x)$  designates the coefficient of thermic expansion for the body  $x$ ; Sol designates the class of solid bodies; Fe the class of iron bodies. By this interpretation, A1 and A2 become physical laws. A1 is the law of thermic

Derivation  $D_2$ :

1.  $c$  is a Sol.
2.  $c$  is a Fe.
3.  $\text{te}(c, 0) = 300$ .
4.  $\text{te}(c, 600) = 350$ .
5.  $\text{lg}(c, 0) = 1,000$ .

Premises

expansion in quantitative form, A2 the statement of the coefficient of thermic expansion for iron. As A2 shows, a statement of a physical constant for a certain substance is also a universal sentence. Further, we add semantical rules for two signs occurring in the subsequent example: The name  $c$  designates the thing at such and such a place in our laboratory; the numerical variable  $t$  as time coordinate designates the time-point  $t$  seconds after August 17, 1938, 10:00 A.M.

Now we will analyze an example of a derivation within the calculus indicated. This derivation  $D_2$  is, when interpreted by the rules mentioned, the deduction of a prediction from premises giving the results of observations. The construction of the derivation  $D_2$  is however entirely independent of any interpretation. It makes use only of the rules of the calculus, namely, the physical calculus indicated together with a calculus of real numbers as basic calculus. We have discussed, but not written down, a similar derivation  $D_1$  (§19), which, however, made use only of the mathematical calculus. Therefore the physical laws used had to be taken in  $D_1$  as premises. But here in  $D_2$  they belong to the axioms of the calculus (A1 and A2, occurring as [6] and [10]: Any axiom or theorem proved in a physical calculus may be used within any derivation in that calculus without belonging to the premisses of the derivation, in exactly the same way in which a proved theorem is used within a derivation in a logical or mathematical calculus, e.g., in the first example of a derivation in §19 sentence (7), and in  $D_1$  (§19) the sentences which in  $D_2$  are called (7) and (13). Therefore only singular sentences (not containing variables) occur as premisses in  $D_2$ . (For the distinction between premisses and axioms see the remark at the end of §19.)

Axiom A1

6. For every  $x, t_1, t_2, l_1, l_2, T_1, T_2, \beta$  [if  $[x$  is a Sol and  $\text{lg}(x, t_1) = l_1$  and  $\text{lg}(x, t_2) = l_2$  and  $\text{te}(x, t_1) = T_1$  and  $\text{te}(x, t_2) = T_2$  and  $\text{th}(x) = \beta$ ] then  $l_2 = l_1 \times (1 + \beta \times (T_2 - T_1))$ .

Proved mathem.

7. For every  $l_1, l_2, T_1, T_2, \beta$  [ $l_2 - l_1 = l_1 \times \beta \times (T_2 - T_1)$ ] if and only if  $l_2 = l_1 \times (1 + \beta \times (T_2 - T_1))$ .

theorem:

8. For every  $x, t_1, \dots$  (as in [6])  $\dots$  [if [---] then  $l_2 - l_1 = l_1 \times \beta \times (T_2 - T_1)$ ].

(6)(7)

(1)(3)(4)(8)

Axiom A2

9. For every  $l_1, l_2, \beta$  [if  $[\text{th}(c) = \beta$  and  $\text{lg}(c, 0) = l_1$  and  $\text{lg}(c, 600) = l_2$ ] then  $l_2 - l_1 \times \beta \times (350 - 300)$ ].

(1)(2)(10)

(9)(11)(5)

10. For every  $x$ , if  $[x$  is a Sol and  $x$  is a Fe] then  $\text{th}(x) = 0.000012$ .

11.  $\text{th}(c) = 0.000012$ .

12. For every  $l_1, l_2$ , [if  $[\text{lg}(c, 0) = l_1$  and  $\text{lg}(c, 600) = l_2$ ] then  $l_2 - l_1 = 1,000 \times 0.000012 \times (350 - 300)$ ].

Proved mathem.

theorem:

13.  $1,000 \times 0.000012 \times (350 - 300) = 0.6$ .

(12)(13) Conclusion:

14.  $\text{lg}(c, 600) - \text{lg}(c, 0) = 0.6$ .

On the basis of the interpretation given before, the premisses are singular sentences concerning the body  $c$ . They say that  $c$  is a solid body made of iron, that the temperature of  $c$  was at 10:00 A.M.  $300^\circ$  abs. and at 10:10 A.M.  $350^\circ$  abs., and that the length of  $c$  at 10:00 A.M. was 1,000 cm. The conclusion says that the increase in the length of  $c$  from 10:00 to 10:10 A.M. is 0.6 cm. Let us suppose that our measurements have confirmed the premisses. Then the derivation yields the conclusion as a prediction which may be tested by another measurement.

Any physical theory, and likewise the whole of physics, can in this way be presented in the form of an interpreted system, consisting of a specific calculus (axiom system) and a system of semantical rules for its interpretation; the axiom system is, tacitly or explicitly, based upon a logico-mathematical calculus with customary interpretation. It is, of course, logically possible to apply the same method to any other branch of science as well. But practically the situation is such that most of them seem at the present time to be not yet developed to a degree which would suggest this strict form of presentation. There is an interesting and successful attempt of an axiomatization of certain parts of biology, especially genetics, by Woodger (Vol. I, No. 10). Other scientific fields which may expect to be accessible soon to this method are perhaps chemistry, eco-

nomics, and some elementary parts of psychology and social science.

Within a physical calculus the mathematical and the physical theorems, i.e.,  $C$ -true formulas, are treated on a par. But there is a fundamental difference between the corresponding *mathematical* and the *physical propositions* of the physical theory, i.e., the system with customary interpretation. This difference is often overlooked. That physical theorems are sometimes mistaken to be of the same nature as mathematical theorems is perhaps due to several factors, among them the fact that they contain mathematical symbols and numerical expressions and that they are often formulated incompletely in the form of a mathematical equation (e.g., A1 simply in the form of the last equation occurring in it). A mathematical proposition may contain only logical signs, e.g., "for every  $m, n, m + n = n + m$ ," or descriptive signs also, if the mathematical calculus is applied in a descriptive system. In the latter case the proposition, although it contains signs not belonging to the mathematical calculus, may still be provable in this calculus, e.g.,  $\text{lg}(c) + \text{lg}(d) = \text{lg}(d) + \text{lg}(c)$  ( $\text{lg}$  designates length as before). A physical proposition always contains descriptive signs because otherwise it could not have factual content; in addition, it usually contains also logical signs. Thus the difference between mathematical the-

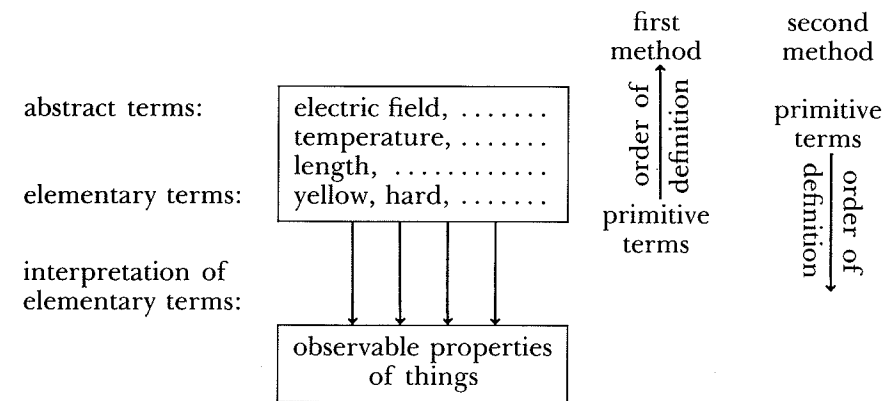
orems and physical theorems in the interpreted system does not depend upon the kinds of signs occurring but rather on the kind of truth of the theorems. The truth of a mathematical theorem, even if it contains descriptive signs, is not dependent upon any facts concerning the designata of these signs. We can determine its truth if we know only the semantical rules: Hence it is L-true. (In the example of the theorem just mentioned, we need not know the length of the body *c*). The truth of a physical theorem, on the other hand, depends upon the properties of the designata of the descriptive signs occurring. In order to determine its truth, we have to make observations concerning these designata; the knowledge of the semantical rules is not sufficient. (In the case of A2, e.g., we have to carry out experiments with solid iron bodies.) Therefore, a physical theorem, in contradistinction to a mathematical theorem, has factual content.

#### 24. ELEMENTARY AND ABSTRACT TERMS

We find among the concepts of physics and likewise among those of the whole of empirical science—differences of abstractness. Some are more elementary than others, in the sense that we can apply them in concrete cases on the basis of observations in a more direct way than others. The others are more abstract; in order to find out whether they hold in a certain case, we have to carry out a more complex procedure, which however also rests finally on observations. Between quite elementary concepts and those of high abstraction there are many intermediate levels. We shall not try to give an exact definition for “degree of abstractness”; what is meant will become sufficiently clear by the following series of sets of concepts, proceeding from elementary to abstract concepts: bright, dark, red, blue, warm, cold, sour, sweet, hard, soft (all concepts of this first set are meant as properties of things, not as sense-data); coincidence; length; length of time; mass,

velocity, acceleration, density, pressure; temperature, quantity of heat; electric charge, electric current, electric field; electric potential, electric resistance, coefficient of induction, frequency of oscillation; wave function.

Suppose that we intend to construct an interpreted system of physics—or of the whole of science. We shall first lay down a calculus. Then we have to state semantical rules of the kind SD for the specific signs, i.e., for the physical terms. (The SL-rules are presupposed as giving the customary interpretation of the logico-mathematical basic calculus.) Since the physical terms form a system, i.e., are connected with one another, obviously we need not state a semantical rule for each of them. For which terms, then, must we give rules, for the elementary or for the abstract ones? We can, of course, state a rule for any term, no matter what its degree of abstractness, in a form like this: “The term *te* designates temperature,” provided the meta-language used contains a corresponding expression (here the word *temperature*) to specify the designatum of the term in question. But suppose we have in mind the following purpose for our syntactical and semantical description of the system of physics: The description of the system shall teach a layman to understand it, i.e., to enable him to apply it to his observations in order to arrive at explanations and predictions. A layman is meant as one who does not know physics but has normal senses and understands a language in which observable properties of things can be described (e.g., a suitable part of everyday nonscientific English). A rule like “the sign *P* designates the property of being blue” will do for the purpose indicated; but a rule like “the sign *Q* designates the property of being electrically charged” will not do. In order to fulfill the purpose, we have to give semantical rules for elementary terms only, connecting them with observable properties of things. For our further discussion we suppose the system to consist of rules of this kind, as indicated in the diagram on the next page.



Now let us go back to the construction of the calculus. We have first to decide at which end of the series of terms to start the construction. Should we take elementary terms as primitive signs, or abstract terms? Our decision to lay down the semantical rules for the elementary terms does not decide this question. Either procedure is still possible and seems to have some reasons in its favor, depending on the point of view taken. The *first method* consists in taking elementary terms as primitive and then introducing on their basis further terms step by step, up to those of highest abstraction. In carrying out this procedure, we find that the introduction of further terms cannot always take the form of explicit definitions; conditional definitions must also be used (so-called reduction sentences[see Vol. I, No. 1, p. 50]). They describe a method of testing for a more abstract term, i.e., a procedure for finding out whether the term is applicable in particular cases, by referring to less abstract terms. The first method has the advantage of exhibiting clearly the connection between the system and observation and of making it easier to examine whether and how a given term is empirically founded. However, when we shift our attention from the terms of the system and the methods of empirical confirmation to the laws, i.e., the universal theorems, of the system, we get a different perspective. Would it be possible to formulate all laws of physics in elementary terms, admitting more abstract terms only as abbreviations? If so, we would have that

ideal of a science in sensationalistic form which Goethe in his polemic against Newton, as well as some positivists, seems to have had in mind. But it turns out—this is an empirical fact, not a logical necessity—that it is not possible to arrive in this way at a powerful and efficacious system of laws. To be sure, historically, science started with laws formulated in terms of a low level of abstractness. But for any law of this kind, one nearly always later found some exceptions and thus had to confine it to a narrower realm of validity. The higher the physicists went in the scale of terms, the better did they succeed in formulating laws applying to a wide range of phenomena. Hence we understand that they are inclined to choose the *second method*. This method begins at the top of the system, so to speak, and then goes down to lower and lower levels. It consists in taking a few abstract terms as primitive signs and a few fundamental laws of great generality as axioms. Then further terms, less and less abstract, and finally elementary ones, are to be introduced by definitions; and here, so it seems at present, explicit definitions will do. More special laws, containing less abstract terms, are to be proved on the basis of the axioms. At least, this is the direction in which physicists have been striving with remarkable success, especially in the past few decades. But at the present time, the method cannot yet be carried through in the pure form indicated. For many less abstract terms no definition on the basis of abstract terms

alone is as yet known; hence those terms must also be taken as primitive. And many more special laws, especially in biological fields, cannot yet be proved on the basis of laws in abstract terms only; hence those laws must also be taken as axioms.

Now let us examine the result of the interpretation if the first or the second method for the construction of the calculus is chosen. In both cases the semantical rules concern the elementary signs. In the first method these signs are taken as primitive. Hence, the semantical rules give a complete interpretation for these signs and those explicitly defined on their basis. There are, however, many signs, especially on the higher levels of abstraction, which can be introduced not by an explicit definition but only by a conditional one. The interpretation which the rules give for these signs is incomplete. This is due not to a defect in the semantical rules but to the method by which these signs are introduced; and this method is not arbitrary but corresponds to the way in which we really obtain knowledge about physical states by our observations.

If, on the other hand, abstract terms are taken as primitive—according to the second method, the one used in scientific physics—then the semantical rules have no direct relation to the primitive terms of the system but refer to terms introduced by long chains of definitions. The calculus is first constructed floating in the air, so to speak; the construction begins at the top and then adds lower and lower levels. Finally, by the semantical rules, the lowest level is anchored at the solid ground of the observable facts. The laws, whether general or special, are not directly interpreted, but only the singular sentences. For the more abstract terms, the rules determine only an *indirect interpretation*, which is—here as well as in the first method—in a certain sense incomplete. Suppose  $B$  is defined on the basis of  $A$ ; then, if  $A$  is directly interpreted,  $B$  is, although indirectly, also interpreted completely; if, however,  $B$  is directly interpreted,  $A$  is not necessarily also interpreted completely (but only if  $A$  is also definable by  $B$ ).

To give an example, let us imagine a calculus of physics constructed, according to the second method, on the basis of primitive specific signs like “electromagnetic field,” “gravitational field,” “electron,” “proton,” etc. The system of definitions will then lead to elementary terms, e.g., to  $Fe$ , defined as a class of regions in which the configuration of particles fulfills certain conditions, and  $Na$ -yellow as a class of space-time regions in which the temporal distribution of the electromagnetic fields fulfills certain conditions. Then semantical rules are laid down stating that  $Fe$  designates iron and  $Na$ -yellow designates a specified yellow color. (If “iron” is not accepted as sufficiently elementary, the rules can be stated for more elementary terms.) In this way, the connection between the calculus and the realm of nature to which it is to be applied is made for terms of the calculus which are far remote from the primitive terms.

Let us examine, on the basis of these discussions, the example of a derivation  $D_2$  (§23). The premisses and the conclusion of  $D_2$  are singular sentences, but most of the other sentences are not. Hence the premisses and the conclusion of this, as of all other derivations of the same type, can be directly interpreted, understood, and confronted with the results of observations. More of an interpretation is not necessary for a practical application of a derivation. If, in confronting the interpreted premisses with our observations, we find them confirmed as true, then we accept the conclusion as a prediction and we may base a decision upon it. The sentences occurring in the derivation between premisses and conclusion are also interpreted, at least indirectly. But we need not make their interpretation explicit in order to be able to construct the derivation and to apply it. All that is necessary for its construction are the formal rules of the calculus. This is the advantage of the method of formalization, i.e., of the separation of the calculus as a formal system from the interpretation. If some persons want to come to an agreement about the formal correctness of a given derivation, they may leave aside all differences of opinion on material questions or questions of interpretation. They simply have to examine

whether or not the given series of formulas fulfils the formal rules of the calculus. Here again, the function of calculi in empirical science becomes clear as instruments for transforming the expression of what we know or assume.

Against the view that for the application of a physical calculus we need an interpretation only for singular sentences, the following objection will perhaps be raised. Before we accept a derivation and believe its conclusion we must have accepted the physical calculus which furnishes the derivation and how can we decide whether or not to accept a physical calculus for application without interpreting and understanding its axioms? To be sure, in order to pass judgment about the applicability of a given physical calculus, we have to confront it in some way or other with observation, and for this purpose an interpretation is necessary. But we need no explicit interpretation of the axioms, nor even of any theorems. The empirical examination of a physical theory given in the form of a calculus with rules of interpretation is not made by interpreting and understanding the axioms and then considering whether they are true on the basis of our factual knowledge. Rather, the examination is carried out by the same procedure as that explained before for obtaining a prediction. We construct derivations in the calculus with premisses which are singular sentences describing the results of our observations, and with singular sentences which we can test by observations as conclusions. The physical theory is indirectly confirmed to a higher and higher degree if more and more of these predictions are confirmed and none of them is disconfirmed by observations. Only singular sentences with elementary terms can be directly tested; therefore, we need an explicit interpretation only for these sentences.

## 25. “UNDERSTANDING” IN PHYSICS

The development of physics in recent centuries, and especially in the past few decades, has more and more led to that

method in the construction, testing, and application of physical theories which we call *formalization*, i.e., the construction of a calculus supplemented by an interpretation. It was the progress of knowledge and the particular structure of the subject matter that suggested and made practically possible this increasing formalization. In consequence, it became more and more possible to forego an “intuitive understanding” of the abstract terms and axioms and theorems formulated with their help. The possibility and even necessity of abandoning the search for an understanding of that kind was not realized for a long time. When abstract, nonintuitive formulas, as, e.g., Maxwell’s equations of electromagnetism, were proposed as new axioms, physicists endeavored to make them “intuitive” by constructing a “model,” i.e., a way of representing electromagnetic micro-processes by an analogy to known macro-processes, e.g., movements of visible things. Many attempts have been made in this direction, but without satisfactory results. It is important to realize that the discovery of a model has no more than an aesthetic or didactic or at best a heuristic value, but is not at all essential for a successful application of the physical theory. The demand for an intuitive understanding of the axioms was less and less fulfilled when the development led to the general theory of relativity and then to quantum mechanics, involving the wave function. Many people, including physicists, have a feeling of regret and disappointment about this. Some, especially philosophers, go so far as even to contend that these modern theories, since they are not intuitively understandable, are not at all theories about nature but “mere formalistic constructions,” “mere calculi.” But this is a fundamental misunderstanding of the function of a physical theory. It is true a theory must not be a “mere calculus” but possess an interpretation, on the basis of which it can be applied to facts of nature. But it is sufficient, as we have seen, to make this interpretation explicit for elementary terms; the interpretation of the other terms is then indi-

rectly determined by the formulas of the calculus, either definitions or laws, connecting them with the elementary terms. If we demand from the modern physicist an answer to the question what he means by the symbol  $\Psi$  of his calculus, and are astonished that he cannot give an answer, we ought to realize that the situation was already the same in classical physics. There the physicist could not tell us what he meant by the symbol  $E$  in Maxwell's equations. Perhaps, in order not to refuse an answer, he would tell us that  $E$  designates the electric field vector. To be sure, this statement has the form of a semantical rule, but it would not help us a bit to understand the theory. It simply refers from a symbol in a symbolic calculus to a corresponding word expression in a calculus of words. We are right in demanding an interpretation for  $E$  but that will be given indirectly by semantical rules referring to elementary signs together with the formulas connecting them with  $E$ . This interpretation

enables us to use the laws containing  $E$  for the derivation of predictions. Thus we understand  $E$ , if "understanding" of an expression, a sentence, or a theory means capability of its use for the description of known facts or the prediction of new facts. An "intuitive understanding" or a direct translation of  $E$  into terms referring to observable properties is neither necessary nor possible. The situation of the modern physicist is not essentially different. He knows how to use the symbol  $\Psi$  in the calculus in order to derive predictions which we can test by observations. (If they have the form of probability statements, they are tested by statistical results of observations.) Thus the physicist, although he cannot give us a translation into everyday language, understands the symbol  $\Psi$  and the laws of quantum mechanics. He possesses that kind of understanding which alone is essential in the field of knowledge and science.

## Carl G. Hempel A LOGICAL APPRAISAL OF OPERATIONISM

Operationism, in its fundamental tenets, is closely akin to logical empiricism. Both schools of thought have put much emphasis on definite experiential meaning or import as a necessary condition of objectively significant discourse, and both have made strong efforts to establish explicit criterions of experiential significance. But logical empiricism has treated experiential import as a characteristic of statements—namely, as their susceptibility to test by experiment or observation—whereas operationism has

tended to construe experiential meaning as a characteristic of concepts or of the terms representing them—namely, as their susceptibility to operational definition.

### BASIC IDEAS OF OPERATIONAL ANALYSIS

An operational definition of a term is conceived as a rule to the effect that the term is to apply to a particular case if the performance of specified operations in that case yields a certain characteristic result. For

example, the term *harder than* might be operationally defined by the rule that a piece of mineral  $x$ , is to be called harder than another piece of mineral,  $y$ , if the operation of drawing a sharp point of  $x$  across the surface of  $y$  results in a scratch mark on the latter. Similarly, the different numerical values of a quantity such as length are thought of as operationally definable by reference to the outcomes of specified measuring operations. To safeguard the objectivity of science, all operations invoked in this kind of definition are required to be intersubjective in the sense that different observers must be able to perform "the same operation" with reasonable agreement in their results.<sup>1</sup>

P.W. Bridgman, the originator of operational analysis, distinguishes several kinds of operation that may be invoked in specifying the meanings of scientific terms.<sup>2</sup> The principal ones are (1) what he calls *instrumental operations*—these consist in the use of various devices of observation and measurement—and (2) paper-and-pencil operations, verbal operations, mental experiments, and the like—this group is meant to include, among other things, the techniques of mathematical and logical inference as well as the use of experiments in imagination. For brevity, but also by way of suggesting a fundamental similarity among the procedures of the second kind, I shall refer to them as *symbolic operations*.

The concepts of operation and of operational definition serve to state the basic principles of operational analysis, of which the following are of special importance:

(1) "Meanings are operational." To understand the meaning of a term, we must know the operational criterions of its application,<sup>3</sup> and every meaningful scientific term must therefore permit of an operational definition. Such definition may refer to certain symbolic operations and it always must ultimately make reference to some instrumental operation.<sup>4</sup>

(2) To avoid ambiguity, every scientific term should be defined by means of one unique operational criterion. Even when two different operational procedures (for

instance, the optical and the tactual ways of measuring length) have been found to yield the same results, they must still be considered as defining different concepts (for example, optical and tactual length), and these should be distinguished terminologically because the presumed coincidence of the results is inferred from experimental evidence, and it is "not safe" to forget that the presumption may be shown to be spurious by new, and perhaps more precise, experimental data.<sup>5</sup>

(3) The insistence that scientific terms should have unambiguously specifiable operational meanings serves to insure the possibility of an objective test for the hypotheses formulated by means of those terms.<sup>6</sup> Hypotheses incapable of operational test or, rather, questions involving untestable formulations, are rejected as meaningless: "If a specific question has meaning, it must be possible to find operations by which an answer may be given to it. It will be found in many cases that the operations cannot exist, and the question therefore has no meaning."<sup>7</sup>

The emphasis on "operational meaning" in scientifically significant discourse has unquestionably afforded a salutary critique of certain types of procedure in philosophy and in empirical science and has provided a strong stimulus for methodological thinking. Yet, the central ideas of operational analysis as stated by their proponents are so vague that they constitute not a theory concerning the nature of scientific concepts but rather a program for the development of such a theory. They share this characteristic with the insistence of logical empiricism that all significant scientific statements must have experiential import, that the latter consists in testability by suitable data of direct observation, and that sentences which are entirely incapable of any test must be ruled out as meaningless "pseudo hypotheses." These ideas, too, constitute not so much a thesis or a theory as a program for a theory that needs to be formulated and amplified in precise terms.

An attempt to develop an operationist theory of scientific concepts will have to deal