Reducing Belief Simpliciter to Degrees of Belief

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June 2010

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One reason why the concept of belief simpliciter is so valuable is that it occupies a *more elementary* scale of measurement than the concept of quantitative belief does.

So the really interesting question is:

Both qualitative and quantitative belief are concepts of belief. *How exactly do they relate to each other?*

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Plan of the talk:

1. Postulates on Quantitative/Qualitative Belief
2. The Representation Theorem and its Surprising Consequence
3. Applications and Extensions: A To-Do List for the Future
4. Solving a Problem

(cf. Hilpinen, *Rules of Acceptance and Inductive Logic*, 1968.
Swain, ed., *Induction, Acceptance, and Rational Belief*, 1970.

Maher, *Betting on Theories*, 1993.

Skyrms 1977, 1980 on resiliency.
Roorda 1995, Frankish 2004, Sturgeon 2008 on belief.
Snow 1998, Dubois et al. 1998 on big-stepped probabilities.)

Postulates on Quantitative/Qualitative Belief

Let W be a set of possible worlds, and let $\mathfrak{A}$ be an algebra of subsets of W (propositions) in which an agent is interested at a time.

We assume that $\mathfrak{A}$ is closed under countable unions (σ-algebra).

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P1 (Probability) $P : \mathfrak{A} \to [0, 1]$ is a probability measure on $\mathfrak{A}$.

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P2 (Countable Additivity) If $X_1, X_2, \ldots, X_n, \ldots$ are pairwise disjoint members of $\mathfrak{A}$, then

$$P(\bigcup_{n \in \mathbb{N}} X_n) = \sum_{n=1}^{\infty} P(X_n).$$

Accordingly, let *Bel* express an agent's conditional beliefs.

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- B4 (General Conjunction) If $\neg Bel(\neg X|W)$, then for $\mathcal{Y} = \{Y \in \mathfrak{A} \,|\, Bel(Y|X)\}$, $\bigcap \mathcal{Y}$ is a member of $\mathfrak{A}$, and $Bel(\bigcap \mathcal{Y}|X)$.

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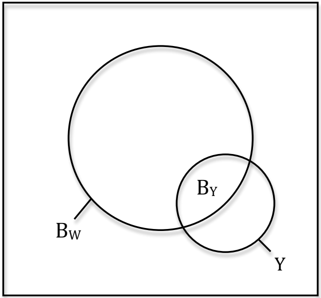
It follows: For every $X \in \mathfrak{A}$ that is consistent with the agent's beliefs there is a *strongest proposition* B_X, such that $Bel(Y|X)$ iff $Y \supseteq B_X$.

In particular, the agent believes Y iff $Y \supseteq B_W$.



B6 (Expansion) For all $Y \in \mathfrak{A}$ such that $Y \cap B_W \neq \varnothing$: $B_Y = Y \cap B_W$.

This postulate is contained in the qualitative theory of belief revision (AGM 1985, Gärdenfors 1988).

Finally, we make quantitative and qualitative belief compatible with each other:

Let $0 \leq r < 1$:

BP1^r (Likeliness) For all $Y \in \mathfrak{A}$ such that $Y \cap B_W \neq \varnothing$ and $P(Y) > 0$:
For all $Z \in \mathfrak{A}$, if $Bel(Z|Y)$, then $P(Z|Y) > r$.

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Definition

(P-Stability^r) For all $X \in \mathfrak{A}$:

X is P-stable^r iff for all $Y \in \mathfrak{A}$ with $Y \cap X \neq \varnothing$ and $P(Y) > 0$: $P(X|Y) > r$.

So P-stable^r propositions have stably high probabilities under salient suppositions. (Examples: All X with $P(X) = 1$; $X = \varnothing$; and *many* more!)

The Representation Theorem and its Surprising Consequence

Theorem

Let Bel be a class of ordered pairs of members of a σ-algebra $\mathfrak{A}$, and let $P : \mathfrak{A} \to [0,1]$. Then the following two statements are equivalent:

I. *P and Bel satisfy* P1, B1–B6, *and* $\mathrm{BP1}^r$.

II. *P satisfies* P1, *and there is a (uniquely determined) $X \in \mathfrak{A}$, such that X is a non-empty P-stable^r proposition, and:*

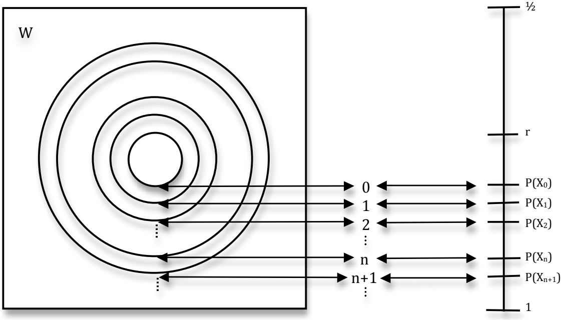
- *For all $Y \in \mathfrak{A}$ such that $Y \cap X \neq \varnothing$, for all $Z \in \mathfrak{A}$:*

$$Bel(Z \mid Y) \text{ if and only if } Z \supseteq Y \cap X$$

(and hence, $B_W = X$).

This neither presupposes P2 nor $r \geq \frac{1}{2}$.

With P2 and $r \geq \frac{1}{2}$ one can prove: The class of P-stable^r propositions X in $\mathfrak{A}$ with $P(X) < 1$ is *well-ordered* with respect to the subset relation.



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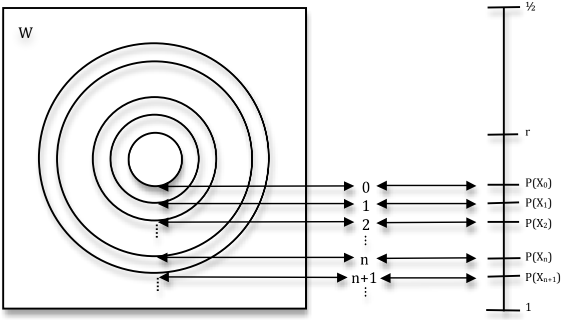

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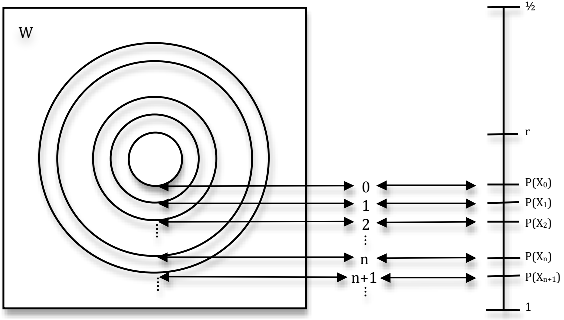

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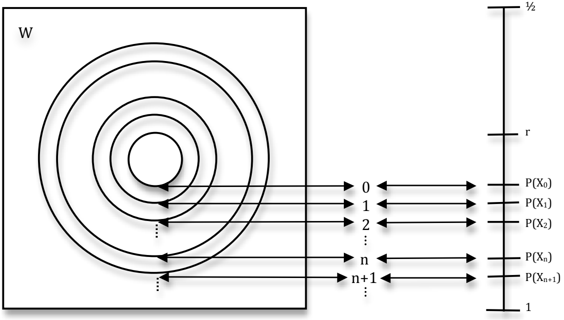

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The next postulate entails, amongst others, that there is a least X s.t. $P(X) = 1$:

BP2 (Zero Supposition) For all $Y \in \mathfrak{A}$: If $P(Y) = 0$ and $Y \cap B_W \neq \emptyset$, then $B_Y = \emptyset$.

Finally, we postulate:

BP3 (Maximality)

Among all classes Bel' of ordered pairs of members of $\mathfrak{A}$, such that P and Bel' jointly satisfy P1–P2, B1–B6, BP1^r, BP2 (with 'Bel'' replacing 'Bel'), the class Bel is the *largest* with respect to the class of beliefs.

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But now $Bel (= Bel^r_P)$ can actually be *defined explicitly* in terms of P and $r \geq \frac{1}{2}$:

Definition

Let $P : \mathfrak{A} \to [0, 1]$ be a countably additive probability measure on a σ-algebra $\mathfrak{A}$, such that there exists a least set of probability 1 in $\mathfrak{A}$.
Let X_{least} be the least non-empty P-stable^r proposition in $\mathfrak{A}$ (which exists).

Then we say for all $Y \in \mathfrak{A}$ and $\frac{1}{2} \leq r < 1$:

$Bel^r_P(Y)$ (i.e., Y is believed to a cautiousness degree of r as given by P) iff $Y \supseteq X_{least}$.

One can prove that a similar result holds even when all postulates are generalized to *suppositions that may contradict an agent's current beliefs*.

That is: Take P1 and P2, add *full* AGM belief revision, make them compatible as before, and voilà: *full* conditional belief is definable explicitly in terms of P!

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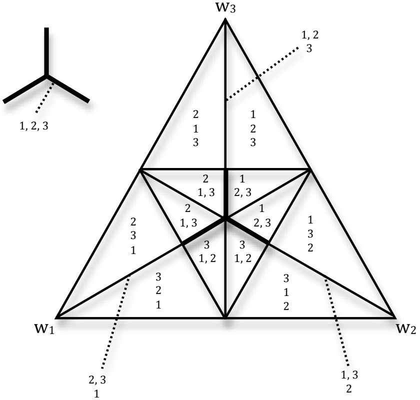
Semantically, this means that every *P* determines a *sphere system* of worlds:



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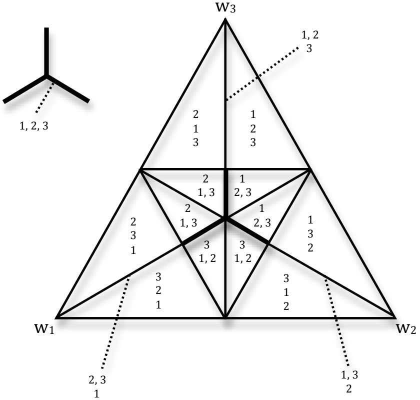

And almost all P over finite W have a least P-stable^r set X_{least} with $P(X_{least}) < 1$!

Applications and Extensions: A To-Do List for the Future

- *Lottery Paradox:* Given a uniform measure P on a finite set W of worlds, W is the only P-stable^r set with $r \geq \frac{1}{2}$; so only W is to be believed then.

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- *Preface Paradox:* What one can*not* have (with $X_i \approx$ 'page i is error-free'):

$$Bel(X_1), \ldots, Bel(X_n), Bel(\neg X_1 \vee \ldots \vee \neg X_n).$$

What one *can* have is a different version of Fallibilism:

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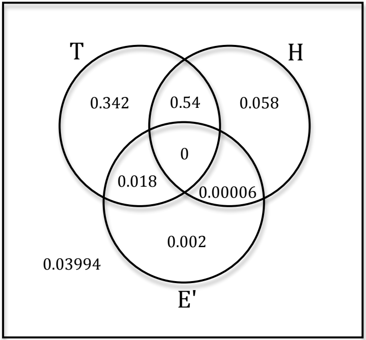
- *Conditionalization on Zero Sets:*

  P^*, with $P^*(Y|X) = P(Y|B_X)$, determines a Popper function.

  cf. van Fraassen (1995), Arló-Costa & Parikh (2004) on "belief cores".

- *John Dorling's (1979) "Duhemian" Example:*



E': Observational result for the secular acceleration of the moon.
T: Relevant part of Newtonian mechanics.
H: Auxiliary hypothesis that tidal friction is negligible.

$P(T|E') = 0.8976$, $P(H|E') = 0.003$.

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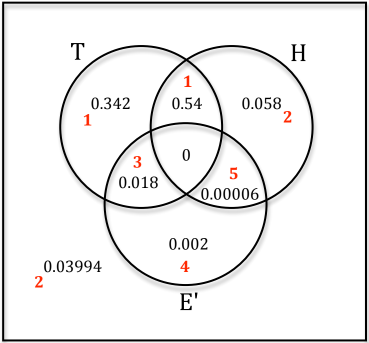

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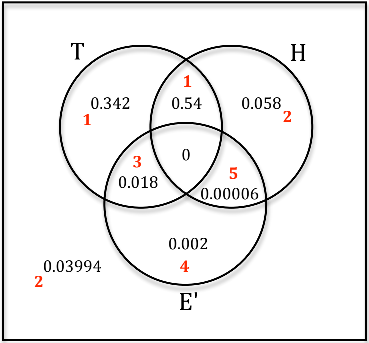

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... scientists always conducted their serious scientific debates in terms of finite qualitative subjective probability assignments to scientific hypotheses (Dorling 1979).

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    Simply let it be high enough so that $Bel^r_{P'}(E)$!

- *Indicative Conditionals:*

  > *If two people are arguing 'If p will q?' and are both in doubt as to p, they are adding p hypothetically to their stock of knowledge and arguing on that basis about q... We can say that they are fixing their degrees of belief in q given p.*
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But when is $X \rightarrow Y$ acceptable *simpliciter*?

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Let $X_1 \rightarrow Y_1, \ldots, X_n \rightarrow Y_n \therefore A \rightarrow B$ be *valid* iff for all P, $r \geq \frac{1}{2}$, if $X_1 \rightarrow Y_1, \ldots, X_n \rightarrow Y_n$ are acceptable w.r.t. P and r, so is $A \rightarrow B$.

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The resulting logic is exactly E. Adams' logic of conditionals! E.g.:

$$\frac{X \rightarrow Y, X \rightarrow Z}{X \rightarrow (Y \wedge Z)} \text{ (And)} \qquad \frac{X \rightarrow Z, Y \rightarrow Z}{(X \vee Y) \rightarrow Z} \text{ (Or)}$$

$$\frac{(X \wedge Y) \rightarrow Z, X \rightarrow Y}{X \rightarrow Z} \text{ (Cautious Cut)} \qquad \frac{X \rightarrow Y, X \rightarrow Z}{(X \wedge Y) \rightarrow Z} \text{ (Cautious M.)}$$

- *Subjunctive Conditionals:* For each world $w \in W$, let Ch_w be the chance measure of w (at a fixed time). Then it is plausible that Ch_w and '*truth of* $X \Box\!\!\rightarrow Y$ *at* w' taken together satisfy the analogues of our postulates.

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  This yields a plausible semantics for counterfactuals, its logic being the system V, and Hawthorne's and Hájek's recent probabilistic worries about the truth of ordinary counterfactuals are undermined.

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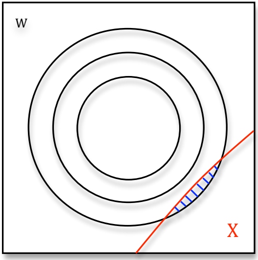
More applications: Bayesian statistics, preference aggregation, vagueness,… ?

One promising future topic in these areas might thus be: A reunification of *logical* and *probabilistic* accounts of inductive reasoning in this or in other ways.

Solving a Problem

A challenge to the theory:

- Intuitively, Expansion/Revision can be problematic:



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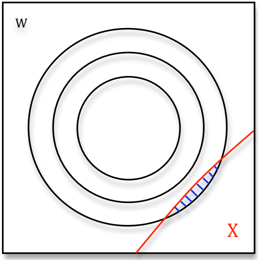

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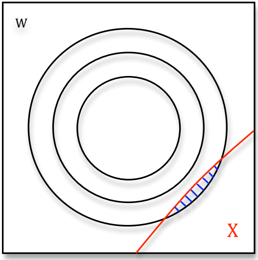

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In both cases, the solution is to make qualitative belief relativized to *partitions* (which are employed by Levi, Skyrms,... anyway):

Possible: $Bel^r_{P,\{Z_j\}}(Y_1 \vee Y_2 \vee \ldots \vee Y_n \,|\, X),\ \neg Bel^r_{P,\{Z'_j\}}(Y_1 \vee Y_2 \vee \ldots \vee Y_n \,|\, X)$