the former are not independent of experience, whereas in the latter they are.

Clearly, we are back to the 'independence of experience' condition. And for Kitcher, that independence has to do with whether, given different 'lives', one could have the same knowledge. A Native American living on the Klamath River will know of salmon, whereas a person who has never had any experience with rivers probably will not. It is simply a case of a posteriori knowledge. The same is not true for a priori knowledge. Any person with experiences sufficient for coming to know the deductive process can come to know and engage in the deductive process, whether or not the process has been explicitly called to mind, or used. A priori knowledge is not, then, innate knowledge, as Kitcher points out.

Finally, we have Kitcher's proposal for a priori knowledge. Citing directly from the article, it specifies two conditions, one for a priori knowledge proper and one for a priori warrants:

1. *S* knows a priori that *p* if and only if *S* knows that *p* and *S*'s belief that *p* was produced by a process that is an a priori warrant for it.
2. α is an a priori warrant for *S*'s belief that *p* if and only if α is a process such that, given any life *e*, sufficient for *S* for *p*, then:
   - a: Some process of the same type could produce in *S* a belief that *p*.
   - b: If a process of the same type were to produce in *S* a belief that *p*, then it would warrant *S* in believing that *p*.
   - c: If a process of the same type were to produce in *S* a belief that *p*, then *p*.

A Pragmatic Conception of The A Priori

C. I. LEWIS

The conception of the a priori points to two problems which are perennial in philosophy: the part played in knowledge by the mind itself, and the possibility of 'necessary truth' or of knowledge 'independent of experience'. But traditional conceptions of the a priori have proved untenable. That the mind approaches the flux of immediacy with some godlike foreknowledge of principles which are legislative for experience, that there is any natural light or any innate ideas, it is no longer possible to believe.

Nor shall we find the clues to the a priori in any compulsion of the mind to incontrovertible truth or any peculiar kind of demonstration which establishes first principles. All truth lays upon the rational mind the same compulsion to belief; as Mr Bosanquet has pointed out, this character belongs to all propositions or judgments once their truth is established.

The difficulties of the conception are due, I believe, to two mistakes: whatever is a priori is necessary, but we have misconstrued the relation of necessary truth to mind; and the a priori is independent of experience, but in so taking it, we have misunderstood its relation to empirical fact. What is a priori is necessary truth not because it compels the mind's acceptance, but precisely because it

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does not. It is given experience, brute fact, the a posteriori element in knowledge which the mind must accept willy-nilly. The a priori represents an attitude in some sense freely taken, a stipulation of the mind itself, and a stipulation which might be made in some other way if it suited our bent or need. Such truth is necessary as opposed to contingent, not as opposed to voluntary. And the a priori is independent of experience not because it prescribes a form which the data of sense must fit, or anticipates some preestablished harmony of experience with the mind, but precisely because it prescribes nothing to experience. That is a priori which is true, no matter what. What it anticipates is not the given, but our attitude toward it: it concerns the uncompelled initiative of mind or, as Josiah Royce would say, our categorical ways of acting.

The traditional example of the a priori par excellence is the laws of logic. These cannot be derived from experience since they must first be taken for granted in order to prove them. They make explicit our general modes of classification. And they impose upon experience no real limitation. Sometimes we are asked to tremble before the spectre of the 'alogical', in order that we may thereafter rejoice that we are saved from this by the dependence of reality upon mind. But the 'alogical' is pure bogy, a word without a meaning. What kind of experience could defy the principle that everything must either be or not be, that nothing can both be and not be, or that if x is y and y is z, then x is z? If anything imaginable or unimaginable could violate such laws, then the ever-present fact of change would do it every day. The laws of logic are purely formal; they forbid nothing but what concerns the use of terms and the corresponding modes of classification and analysis. The law of contradiction tells us that nothing can be both white and not-white, but it does not and cannot tell us whether black is not-white, or soft or square is not-white. To discover *what contradicts what* we must always consult the character of experience. Similarly the law of the excluded middle formulates our decision that whatever is not designated by a certain term shall be designated by its negative. It declares our purpose to make, for every term, a complete dichotomy of experience, instead—as we might choose—of classifying on the basis of a tripartite division into opposites (as black and white) and the middle ground between the two. Our rejection of such tripartite division represents only our penchant for simplicity.

Further laws of logic are of similar significance. They are principles of procedure, the parliamentary rules of intelligent thought and speech. Such laws are independent of experience because they impose no limitations whatever upon it. They are legislative because they are addressed to ourselves—because definition, classification, and inference represent no operations of the objective world, but only our own categorical attitudes of mind.

And further, the ultimate criteria of the laws of logic are pragmatic. Those who suppose that there is, for example, *a* logic which everyone would agree to if he understood it and understood himself are more optimistic than those versed in the history of logical discussion have a right to be. The fact is that there are several logics, markedly different, each self-consistent in its own terms and such that whoever using it, if he avoids false premises, will never reach a false conclusion. Mr Russell, for example, bases *his* logic on an implication relation such that if twenty sentences be cut from a newspaper and put in a hat, and then two of these be drawn at random, one of them will certainly imply the other, and it is an even bet that the implication will be mutual. Yet upon a foundation so remote from ordinary modes of inference the whole structure of *Principia Mathematica* is built. This logic—and there are others even more strange—is utterly consistent and the results of it entirely valid. Over and above all questions of consistency, there are issues of logic which cannot be determined—nay, cannot even be argued—except on pragmatic grounds of conformity to human bent and intellectual convenience. That we have been blind to this fact, itself

reflects traditional errors in the conception of the a priori.

We may note in passing one less important illustration of the a priori—the proposition 'true by definition'. Definitions and their immediate consequences, analytic propositions generally, are necessarily true, true under all possible circumstances. Definition is legislative because it is in some sense arbitrary. Not only is the meaning assigned to words more or less a matter of choice—that consideration is relatively trivial—but the manner in which the precise classifications which definition embodies shall be effected is something not dictated by experience. If experience were other than it is, the definition and its corresponding classification might be inconvenient, fantastic, or useless, but it could not be false. Mind makes classifications and determines meanings; in so doing it creates the a priori truth of analytic judgements. But that the manner of this creation responds to pragmatic considerations is so obvious that it hardly needs pointing out.

If the illustrations so far given seem trivial or verbal, that impression may be corrected by turning to the place which the a priori has in mathematics and in natural science. Arithmetic, for example, depends in toto upon the operation of counting or correlating, a procedure which can be carried out at will in any world containing identifiable things—even identifiable ideas—regardless of the further characters of experience. Mill challenged this a priori character of arithmetic. He asked us to suppose a demon sufficiently powerful and maleficent so that every time two things were brought together with two other things, this demon should always introduce a fifth. The implication which he supposed to follow is that under such circumstances $2 + 2 = 5$ would be a universal law of arithmetic. But Mill was quite mistaken. In such a world we should be obliged to become a little clearer than is usual about the distinction between arithmetic and physics; that is all. If two black marbles were put in the same urn with two white ones, the demon could take his choice of colors, but it would be evident that there were more black marbles or more white ones than were put in. The same would be true of all objects in any wise identifiable. We should simply find ourselves in the presence of an extraordinary physical law, which we should recognize as universal in our world, that whenever two things were brought into proximity with two others, an additional and similar thing was always created by the process. Mill's world would be physically most extraordinary. The world's work would be enormously facilitated if hats or locomotives or tons of coal could be thus multiplied by anyone possessed originally of two pairs. But the laws of mathematics would remain unaltered. It is because this is true that arithmetic is a priori. Its laws prevent *nothing*; they are compatible with anything which happens or could conceivably happen in nature. They would be true in any possible world. Mathematical addition is not a physical transformation. Physical changes which result in an increase or decrease of the countable things involved are matters of everyday occurrence. Such physical processes present us with phenomena in which the purely mathematical has to be separated out by abstraction. Those laws and those laws only have necessary truth which we are prepared to maintain, no matter what. It is because we shall always separate out that part of the phenomenon not in conformity with arithmetic and designate it by some other category—physical change, chemical reaction, optical illusion—that arithmetic is a priori.

The a priori element in science and in natural law is greater than might be supposed. In the first place, all science is based upon definitive concepts. The formulation of these concepts is, indeed, a matter determined by the commerce between our intellectual or our pragmatic interests and the nature of experience. Definition is classification. The scientific search is for such classification as will make it possible to correlate appearance and behaviour, to discover law, to penetrate to the 'essential nature' of things in order that behaviour may become predictable. In other words, if definition is

unsuccessful, as early scientific definitions mostly have been, it is because the classification thus set up corresponds with no natural cleavage and does not correlate with any important uniformity of behaviour. A name itself must represent *some* uniformity in experience or it names nothing. What does not repeat itself or recur in intelligible fashion is not a thing. Where the definitive uniformity is a clue to other uniformities, we have successful scientific definition. Other definitions cannot be said to be false; they are merely useless. In scientific classification the search is, thus, for *things worth naming*. But the naming, classifying, defining activity is essentially prior to investigation. We cannot interrogate experience in general. Until our meaning is definite and our classification correspondingly exact, experience cannot conceivably answer our questions.

In the second place, the fundamental laws of any science—or those treated as fundamental—are a priori because they formulate just such definitive concepts or categorical tests by which alone investigation becomes possible. If the lightning strikes the railroad track at two places, A and B, how shall we tell whether these events are simultaneous?

> We . . . require a definition of simultaneity such that this definition supplies us with the method by means of which . . . [we] can decide whether or not both the lightning strokes occurred simultaneously. As long as this requirement is not satisfied, I allow myself to be deceived as a physicist (and of course the same applies if I am not a physicist), when I imagine that I am able to attach a meaning to the statement of simultaneity. . . .
>
> After thinking the matter over for some time you then offer the following suggestions with which to test simultaneity. By measuring along the rails, the connecting line AB should be measured up and an observer placed at the mid-point M of the distance AB. This observer should be supplied with an arrangement (e.g. two mirrors inclined at 90 degrees) which allows him visually to observe both places A and B at the same time. If the observer perceives the two flashes at the same time, then they are simultaneous.
>
> I am very pleased with this suggestion, but for all that I cannot regard the matter as quite settled, because I feel constrained to raise the following objection: 'Your definition would certainly be right, if I only knew that the light by means of which the observer at M perceives the lightning flashes travels along the length A—M with the same velocity as along the length B—M. But an examination of this supposition would only be possible if we already had at our disposal the means of measuring time. It would thus appear as though we were moving here in a logical circle.'
>
> After further consideration you cast a somewhat disdainful glance at me—and rightly so—and you declare: 'I maintain my previous definition, nevertheless, because in reality it assumes absolutely nothing about light. There is only *one* demand to be made of the definition of simultaneity, namely, that in every real case it must supply us with an empirical decision as to whether or not the conception which has to be defined is fulfilled. . . . That light requires the same time to traverse the path A—M as for the path B—M is in reality *neither a supposition nor a hypothesis* about the physical nature of light, but a *stipulation* which I can make of my own free will in order to arrive at a definition of simultaneity.' . . . We are thus led also to a definition of 'time' in physics.[1]

As this example from the theory of relativity well illustrates, we cannot even ask the questions which discovered law would answer until we have first by a priori stipulation formulated definitive criteria. Such concepts are not verbal definitions, nor classifications merely; they are themselves laws which prescribe a certain uniformity of behaviour to whatever is thus named. Such definitive laws are a priori; only so can we enter upon the investigation by which further laws are sought. Yet it should also be pointed out that such a priori laws are subject to abandonment if the structure which is built upon them does not succeed in simplifying our interpretation of phenomena. If, in the illustration given, the relation 'simultaneous with', as defined, should not prove transitive—if event A should prove simultaneous with B, and B with C, but not A with C—this definition would certainly be rejected.

And thirdly, there is that a priori element in science—as in other human affairs—which constitutes the criteria of the real as opposed to the unreal in experience. An object itself is a uniformity. Failure to behave

in certain categorical ways marks it as unreal. Uniformities of the type called 'natural law' are the clues to reality and unreality. A mouse which disappears where no hole is, is no real mouse; a landscape which recedes as we approach is but illusion. As the queen remarked in the episode of the wishing-carpet: 'if this were real, then it would be a miracle. But miracles do not happen. Therefore I shall wake presently.' That the uniformities of natural law are the only reliable criteria of the real is inescapable. But such a criterion is *ipso facto* a priori. No conceivable experience could dictate the alteration of a law so long as failure to obey that law marked the content of experience as unreal.

This is one of the puzzles of empiricism. We deal with experience: What any reality may be which underlies experience, we have to learn. What we desire to discover is natural law, the formulation of those uniformities which obtain amongst the real. But experience as it comes to us contains not only the real but all the content of illusion, dream, hallucination, and mistake. The *given* contains both real and unreal, confusingly intermingled. If we ask for uniformities of this unsorted experience, we shall not find them. Laws which characterize all experience, of real and unreal both, are non-existent and would in any case be worthless. What we seek are the uniformities of the *real*; but *until we have such laws, we cannot sift experience and segregate the real.*

The obvious solution is that the enrichment of experience, the separation of the real from the illusory or meaningless, and the formulation of natural law all grow up together. If the criteria of the real are a priori, that is not to say that no conceivable character of experience would lead to alteration of them. For example, spirits cannot be photographed. But if photographs of spiritistic phenomena, taken under properly guarded conditions, should become sufficiently frequent, this a priori dictum would be called in question. What we should do would be to redefine our terms. Whether 'spook' was spirit or matter, whether the definition of 'spirit' or of 'matter' should be changed—all this would constitute one interrelated problem. We should reopen together the question of definition or classification, of criteria for this sort of real, and of natural law. And the solution of one of these would mean the solution of all. Nothing could *force* a redefinition of spirit or of matter. A sufficiently fundamental relation to human bent, to human interests, would guarantee continuance unaltered even in the face of unintelligible and baffling experiences. In such problems, the mind finds itself uncompelled save by its own purposes and needs. I *may* categorize experience as I will; but *what* categorical distinctions will best serve my interests and objectify my own intelligence? What the mixed and troubled experience shall be—that is beyond me. But what I shall do with it—that is my own question, when the character of experience is sufficiently before me. I am coerced only by my own need to understand.

It would indeed be inappropriate to characterize as a priori a law which we are wholly prepared to alter in the light of further experience, even though in an isolated case we should discard as illusory any experience which failed to conform. But the crux of the situation lies in this: beyond such principles as those of logic, which we seem fully prepared to maintain no matter what, there must be further and more particular criteria of the real prior to any investigation of nature whatever. We cannot even interrogate experience without a network of categories and definitive concepts. And we must further be prepared to say what experimental findings will answer what questions, and how. Without tests which represent anterior principle, there is no question which experience could answer at all. Thus the most fundamental laws in any category—or those which we regard as most fundamental—are a priori, even though continued failure to render experience intelligible in such terms might result eventually in the abandonment of that category altogether. Matters so comparatively small as the behaviour of Mercury and of starlight passing the sun's limb may, if there be persistent failure to bring them

within the field of previously accepted modes of explanation, result in the abandonment of the independent categories of space and time. But without the definitions, fundamental principles, and tests of the type which constitute such categories, no experience whatever could prove or disprove anything. And to that mind which should find independent space and time absolutely necessary conceptions, no possible experiment could prove the principles of relativity. 'There must be some error in the experimental findings, or some law not yet discovered', represents an attitude which can never be rendered impossible. And the only sense in which it could be proved unreasonable would be the pragmatic one of comparison with another method of categorical analysis which more successfully reduced all such experience to order and law.

At the bottom of all science and all knowledge are categories and definitive concepts which represent fundamental habits of thought and deep-lying attitudes which the human mind has taken in the light of its total experience. But a new and wider experience may bring about some alteration of these attitudes, even though by themselves they dictate nothing as to the content of experience, and no experience can conceivably prove them invalid.

Perhaps some will object to this conception on the ground that only such principles should be designated a priori as the human mind *must* maintain, no matter what; that if, for example, it is shown possible to arrive at a consistent doctrine of physics in terms of relativity even by the most arduous reconstruction of our fundamental notions, then the present conceptions are by that fact shown not to be a priori. Such objection is especially likely from those who would conceive the a priori in terms of an absolute mind or an absolutely universal human nature. We should readily agree that a decision by popular approval or a congress of scientists or anything short of such a test as would bring to bear the full weight of human capacity and interest would be ill-considered as having to do with the a priori. But we wish to emphasize two facts: first, that in the field of those conceptions and principles which have altered in human history, there are those which could neither be proved nor disproved by any experience, but represent the uncompelled initiative of human thought—that without this uncompelled initiative no growth of science, nor any science at all, would be conceivable; and second, that the difference between such conceptions as are, for example, concerned in the decision of relativity versus absolute space and time, and those more permanent attitudes such as are vested in the laws of logic, there is only a difference of degree. The dividing line between the a priori and the a posteriori is that between principles and definitive concepts which *can* be maintained in the face of all experience and those genuinely empirical generalizations which *might* be proven flatly false. The thought which both rationalism and empiricism have missed is that there are principles, representing the initiative of mind, which impose upon experience no limitations whatever, but that such conceptions are still subject to alteration on pragmatic grounds when the expanding boundaries of experience reveal their infelicity as intellectual instruments.

Neither human experience nor the human mind has a character which is universal, fixed, and absolute. 'The human mind' does not exist at all save in the sense that all humans are very much alike in fundamental respects, and that the language habit and the enormously important exchange of ideas has greatly increased our likeness in those respects which are here in question. Our categories and definitions are peculiarly social products, reached in the light of experiences which have much in common, and beaten out, like other pathways, by the coincidence of human purposes and the exigencies of human co-operation. Concerning the a priori there need be neither universal agreement nor complete historical continuity. Conceptions, such as those of logic, which are least likely to be affected by the opening of new ranges of experience, represent the most stable of our categories; but

none of them is beyond the possibility of alteration.

Mind contributes to experience the element of order, of classification, categories, and definition. Without such, experience would be unintelligible. Our knowledge of the validity of these is simply consciousness of our own fundamental ways of acting and our own intellectual intent. Without this element, knowledge is impossible, and it is here that whatever truths are necessary and independent of experience must be found. But the commerce between our categorical ways of acting, our pragmatic interests, and the particular character of experience is closer than we have realized. No explanation of any one of these can be complete without consideration of the other two.

Pragmatism has sometimes been charged with oscillating between two contrary notions: the one, that experience is 'through and through malleable to our purpose'; the other, that facts are 'hard' and uncreated by the mind. We here offer a mediating conception: through all our knowledge runs the element of the a priori, which is indeed malleable to our purpose and responsive to our need. But throughout, there is also that other element of experience which is 'hard', 'independent', and unalterable to our will.

NOTES

1. Albert Einstein, *Relativity: The Special and General Theory*, trans. R. W. Lawson (New York, 1920), pp. 26–8; italics are the author's.

The A Priori

A. J. AYER

The view of philosophy which we have adopted may, I think, fairly be described as a form of empiricism. For it is characteristic of an empiricist to eschew metaphysics, on the ground that every factual proposition must refer to sense-experience. And even if the conception of philosophizing as an activity of analysis is not to be discovered in the traditional theories of empiricists, we have seen that it is implicit in their practice. At the same time, it must be made clear that, in calling ourselves empiricists, we are not avowing a belief in any of the psychological doctrines which are commonly associated with empiricism. For, even if these doctrines were valid, their validity would be independent of the validity of any philosophical thesis. It could be established only by observation, and not by the purely logical considerations upon which our empiricism rests.

Having admitted that we are empiricists, we must now deal with the objection that is commonly brought against all forms of empiricism: the objection, namely, that it is impossible on empiricist principles to account for our knowledge of necessary truths. For, as Hume conclusively showed, no general proposition whose validity is subject to the test of actual experience can ever be logically certain. No matter how often it is verified in practice, there still remains the possibility that it will be confuted on some future occasion. The fact that a law has been substantiated in $n - 1$ cases affords no logical guarantee that it will be substantiated in

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the *n*th case also, no matter how large we take *n* to be. And this means that no general proposition referring to a matter of fact can ever be shown to be necessarily and universally true. It can at best be a probable hypothesis. And this, we shall find, applies not only to general propositions, but to all propositions which have a factual content. They can none of them ever become logically certain. This conclusion, which we shall elaborate later on, is one which must be accepted by every consistent empiricist. It is often thought to involve him in complete scepticism; but this is not the case. For the fact that the validity of a proposition cannot be logically guaranteed in no way entails that it is irrational for us to believe it. On the contrary, what is irrational is to look for a guarantee where none can be forthcoming; to demand certainty where probability is all that is obtainable. We have already remarked upon this, in referring to the work of Hume. And we shall make the point clearer when we come to treat of probability, in explaining the use which we make of empirical propositions. We shall discover that there is nothing perverse or paradoxical about the view that all the 'truths' of science and common sense are hypotheses; and consequently that the fact that it involves this view constitutes no objection to the empiricist thesis.

Where the empiricist does encounter difficulty is in connection with the truths of formal logic and mathematics. For whereas a scientific generalization is readily admitted to be fallible, the truths of mathematics and logic appear to everyone to be necessary and certain. But if empiricism is correct no proposition which has a factual content can be necessary or certain. Accordingly, the empiricist must deal with the truths of logic and mathematics in one of the two following ways: he must say either that they are not necessary truths, in which case he must account for the universal conviction that they are; or he must say that they have no factual content, and then he must explain how a proposition which is empty of all factual content can be true and useful and surprising.

If neither of these courses proves satisfactory, we shall be obliged to give way to rationalism. We shall be obliged to admit that there are some truths about the world which we can know independently of experience; that there are some properties which we can ascribe to all objects, even though we cannot conceivably observe that all objects have them. And we shall have to accept it as a mysterious inexplicable fact that our thought has this power to reveal to us authoritatively the nature of objects which we have never observed. Or else we must accept the Kantian explanation which, apart from the epistemological difficulties which we have already touched on, only pushes the mystery a stage further back.

It is clear that any such concession to rationalism would upset the main argument of my book. For the admission that there were some facts about the world which could be known independently of experience would be incompatible with our fundamental contention that a sentence says nothing unless it is empirically verifiable. And thus the whole force of our attack on metaphysics would be destroyed. It is vital, therefore, for us to be able to show that one or other of the empiricist accounts of the propositions of logic and mathematics is correct. If we are successful in this, we shall have destroyed the foundations of rationalism. For the fundamental tenet of rationalism is that thought is an independent source of knowledge, and is moreover a more trustworthy source of knowledge than experience; indeed some rationalists have gone so far as to say that thought is the only source of knowledge. And the ground for this view is simply that the only necessary truths about the world which are known to us are known through thought and not through experience. So that if we can show either that the truths in question are not necessary or that they are not 'truths about the world' we shall be taking away the support on which rationalism rests. We shall be making good the empiricist contention that there are no 'truths of reason' which refer to matters of fact.

The course of maintaining that the truths

of logic and mathematics are not necessary or certain was adopted by Mill. He maintained that these propositions were inductive generalizations based on an extremely large number of instances. The fact that the number of supporting instances was so very large accounted, in his view, for our believing these generalizations to be necessarily and universally true. The evidence in their favour was so strong that it seemed incredible to us that a contrary instance should ever arise. Nevertheless, it was in principle possible for such generalizations to be confuted. They were highly probable, but, being inductive generalizations, they were not certain. The difference between them and the hypotheses of natural science was a difference in degree and not in kind. Experience gave us very good reason to suppose that a 'truth' of mathematics or logic was true universally; but we were not possessed of a guarantee. For these 'truths' were only empirical hypotheses which had worked particularly well in the past; and, like all empirical hypotheses, they were theoretically fallible.

I do not think that this solution of the empiricist's difficulty with regard to the propositions of logic and mathematics is acceptable. In discussing it, it is necessary to make a distinction which is perhaps already enshrined in Kant's famous dictum that, although there can be no doubt that all our knowledge begins with experience, it does not follow that it all arises out of experience.[1] When we say that the truths of logic are known independently of experience, we are not of course saying that they are innate, in the sense that we are born knowing them. It is obvious that mathematics and logic have to be learned in the same way as chemistry and history have to be learned. Nor are we denying that the first person to discover a given logical or mathematical truth was led to it by an inductive procedure. It is very probable, for example, that the principle of the syllogism was formulated not before but after the validity of syllogistic reasoning had been observed in a number of particular cases. What we are discussing, however, when we say that logical and mathematical truths are known independently of experience, is not a historical question concerning the way in which these truths were originally discovered, nor a psychological question concerning the way in which each of us comes to learn them, but an epistemological question. The contention of Mill's which we reject is that the propositions of logic and mathematics have the same status as empirical hypotheses; that their validity is determined in the same way. We maintain that they are independent of experience in the sense that they do not owe their validity to empirical verification. We may come to discover them through an inductive process; but once we have apprehended them we see that they are necessarily true, that they hold good for every conceivable instance. And this serves to distinguish them from empirical generalizations. For we know that a proposition whose validity depends upon experience cannot be seen to be necessarily and universally true.

In rejecting Mill's theory, we are obliged to be somewhat dogmatic. We can do no more than state the issue clearly and then trust that his contention will be seen to be discrepant with the relevant logical facts. The following considerations may serve to show that of the two ways of dealing with logic and mathematics which are open to the empiricist, the one which Mill adopted is not the one which is correct.

The best way to substantiate our assertion that the truths of formal logic and pure mathematics are necessarily true is to examine cases in which they might seem to be confuted. It might easily happen, for example, that when I came to count what I had taken to be five pairs of objects, I found that they amounted only to nine. And if I wished to mislead people I might say that on this occasion twice five was not ten. But in that case I should not be using the complex sign '$2 \times 5 = 10$' in the way in which it is ordinarily used. I should be taking it not as the expression of a purely mathematical proposition, but as the expression of an empirical generalization, to the effect that whenever I

counted what appeared to me to be five pairs of objects I discovered that they were ten in number. This generalization may very well be false. But if it proved false in a given case, one would not say that the mathematical proposition '$2 \times 5 = 10$' had been confuted. One would say that I was wrong in supposing that there were five pairs of objects to start with, or that one of the objects had been taken away while I was counting, or that two of them had coalesced, or that I had counted wrongly. One would adopt as an explanation whatever empirical hypothesis fitted in best with the accredited facts. The one explanation which would in no circumstances be adopted is that ten is not always the product of two and five.

To take another example: if what appears to be a Euclidean triangle is found by measurement not to have angles totalling 180 degrees, we do not say that we have met with an instance which invalidates the mathematical proposition that the sum of the three angles of a Euclidean triangle is 180 degrees. We say that we have measured wrongly, or, more probably, that the triangle we have been measuring is not Euclidean. And this is our procedure in every case in which a mathematical truth might appear to be confuted. We always preserve its validity by adopting some other explanation of the occurrence.

The same thing applies to the principles of formal logic. We may take an example relating to the so-called law of excluded middle, which states that a proposition must be either true or false, or, in other words, that it is impossible that a proposition and its contradictory should neither of them be true. One might suppose that a proposition of the form '*x* has stopped doing *y*' would in certain cases constitute an exception to this law. For instance, if my friend has never yet written to me, its seems fair to say that it is neither true nor false that he has stopped writing to me. But in fact one would refuse to accept such an instance as an invalidation of the law of excluded middle. One would point out that the proposition 'My friend has stopped writing to me' is not a simple proposition, but the conjunction of the two propositions 'My friend wrote to me in the past' and 'My friend does not write to me now'; and, furthermore, that the proposition 'My friend has not stopped writing to me' is not, as it appears to be, contradictory to 'My friend has stopped writing to me', but only contrary to it. For it means 'My friend wrote to me in the past, and he still writes to me.' When, therefore, we say that such a proposition as 'My friend has stopped writing to me' is sometimes neither true nor false, we are speaking inaccurately. For we seem to be saying that neither it nor its contradictory is true. Whereas what we mean, or anyhow should mean, is that neither it nor its apparent contradictory is true. And its apparent contradictory is really only its contrary. Thus we preserve the law of excluded middle by showing that the negating of a sentence does not always yield the contradictory of the proposition originally expressed.

There is no need to give further examples. Whatever instance we care to take, we shall always find that the situations in which a logical or mathematical principle might appear to be confuted are accounted for in such a way as to leave the principle unassailed. And this indicates that Mill was wrong in supposing that a situation could arise which would overthrow a mathematical truth. The principles of logic and mathematics are true universally simply because we never allow them to be anything else. And the reason for this is that we cannot abandon them without contradicting ourselves, without sinning against the rules which govern the use of language, and so making our utterances self-stultifying. In other words, the truths of logic and mathematics are analytic propositions or tautologies. In saying this we are making what will be held to be an extremely controversial statement, and we must now proceed to make its implications clear.

The most familiar definition of an analytic proposition, or judgment, as he called it, is that given by Kant. He said[2] that an analytic judgement was one in which the predicate *B* belonged to the subject *A* as to

something which was covertly contained in the concept of *A*. He contrasted analytic with synthetic judgements, in which the predicate *B* lay outside the subject, *A*, although it did stand in connection with it. Analytic judgements, he explains, 'add nothing through the predicate to the concept of the subject, but merely break it up into those constituent concepts that have all along been thought in it, although confusedly.' Synthetic judgements, on the other hand, 'add to the concept of the subject a predicate which has not been in any wise thought in it, and which no analysis could possibly extract from it.' Kant gives 'all bodies are extended' as an example of an analytic judgement, on the ground that the required predicate can be extracted from the concept of 'body', 'in accordance with the principle of contradiction'; as an example of a synthetic judgement, he gives 'all bodies are heavy'. He refers also to '7 + 5 = 12' as a synthetic judgement, on the ground that the concept of twelve is by no means already thought in merely thinking the union of seven and five. And he appears to regard this as tantamount to saying that the judgement does not rest on the principle of contradiction alone. He holds, also, that through analytic judgements our knowledge is not extended as it is through synthetic judgements. For in analytic judgements 'the concept which I already have is merely set forth and made intelligible to me.'

I think that this is a fair summary of Kant's account of the distinction between analytic and synthetic propositions, but I do not think that it succeeds in making the distinction clear. For even if we pass over the difficulties which arise out of the use of the vague term 'concept', and the unwarranted assumption that every judgement, as well as every German or English sentence, can be said to have a subject and a predicate, there remains still this crucial defect. Kant does not give one straightforward criterion for distinguishing between analytic and synthetic propositions; he gives two distinct criteria, which are by no means equivalent. Thus his ground for holding that the proposition '7 + 5 = 12' is synthetic is, as we have seen, that the subjective intension of '7 + 5' does not comprise the subjective intension of '12'; whereas his ground for holding that 'all bodies are extended' is an analytic proposition is that it rests on the principle of contradiction alone. That is, he employs a psychological criterion in the first of these examples, and a logical criterion in the second, and takes their equivalence for granted. But, in fact, a proposition which is synthetic according to the former criterion may very well be analytic according to the latter. For, as we have already pointed out, it is possible for symbols to be synonymous without having the same intensional meaning for anyone; and accordingly, from the fact that one can think of the sum of seven and five without necessarily thinking of twelve, it by no means follows that the proposition '7 + 5 = 12' can be denied without self-contradiction. From the rest of his argument, it is clear that it is this logical proposition, and not any psychological proposition, that Kant is really anxious to establish. His use of the psychological criterion leads him to think that he has established it, when he has not.

I think that we can preserve the logical import of Kant's distinction between analytic and synthetic propositions, while avoiding the confusions which mar his actual account of it, if we say that a proposition is analytic when its validity depends solely on the definitions of the symbols it contains, and synthetic when its validity is determined by the facts of experience. Thus, the proposition 'There are ants which have established a system of slavery' is a synthetic proposition. For we cannot tell whether it is true or false merely by considering the definitions of the symbols which constitute it. We have to resort to actual observation of the behavior of ants. On the other hand, the proposition 'either some ants are parasitic or none are ' is an analytic proposition. For one need not resort to observation to discover that there either are or are not ants which are parasitic. If one knows what is the function of the words 'either', 'or', and 'not' then one can see that any proposition of the form 'Either

p is true or *p* is not true' is valid, independently of experience. Accordingly, all such propositions are analytic.

It is to be noticed that the proposition 'Either some ants are parasitic or none are' provides no information whatsoever about the behavior of ants, or, indeed, about any matter of fact. And this applies to all analytic propositions. They none of them provide any information about any matter of fact. In other words, they are entirely devoid of factual content. And it is for this reason that no experience can confute them.

When we say that analytic propositions are devoid of factual content, and consequently that they say nothing, we are not suggesting that they are senseless in the way that metaphysical utterances are senseless. For, although they give us no information about any empirical situation, they do enlighten us by illustrating the way in which we use certain symbols. Thus if I say, 'Nothing can be coloured in different ways at the same time with respect to the same part of itself', I am not saying anything about the properties of any actual thing; but I am not talking nonsense. I am expressing an analytic proposition, which records our determination to call a colour expanse which differs in quality from a neighbouring colour expanse a different part of a given thing. In other words, I am simply calling attention to the implications of a certain linguistic usage. Similarly, in saying that if all Bretons are Frenchmen, and all Frenchmen Europeans, then all Bretons are Europeans, I am not describing any matter of fact. But I am showing that in the statement that all Bretons are Frenchmen, and all Frenchmen Europeans, the further statement that all Bretons are Europeans is implicitly contained. And I am thereby indicating the convention which governs our usage of the words 'if' and 'all'.

We see, then, that there is a sense in which analytic propositions do give us new knowledge. They call attention to linguistic usages, of which we might otherwise not be conscious, and they reveal unsuspected implications in our assertions and beliefs. But we can see also that there is a sense in which they may be said to add nothing to our knowledge. For they tell us only what we may be said to know already. Thus, if I know that the existence of May Queens is a relic of tree-worship, and I discover that May Queens still exist in England, I can employ the tautology 'if *p* implies *q*, and *p* is true, *q* is true' to show that there still exists a relic of tree-worship in England. But in saying that there are still May Queens in England, and that the existence of May Queens is a relic of tree-worship, I have already asserted the existence in England of a relic of tree-worship. The use of the tautology does, indeed, enable me to make this concealed assertion explicit. But it does not provide me with any new knowledge, in the sense in which empirical evidence that the election of May Queens had been forbidden by law would provide me with new knowledge. If one had to set forth all the information one possessed, with regard to matters of fact, one would not write down any analytic propositions. But one would make use of analytic propositions in compiling one's encyclopaedia, and would thus come to include propositions which one would otherwise have overlooked. And, besides enabling one to make one's list of information complete, the formulation of analytic propositions would enable one to make sure that the synthetic propositions of which the list was composed formed a self-consistent system. By showing which ways of combining propositions resulted in contradictions, they would prevent one from including incompatible propositions and so making the list self-stultifying. But in so far as we had actually used such words as 'all' and 'or' and 'not' without falling into self-contradiction, we might be said already to know what was revealed in the formulation of analytic propositions illustrating the rules which govern our usage of these logical particles. So that here again we are justified in saying that analytic propositions do not increase our knowledge.

The analytic character of the truths of formal logic was obscured in the traditional logic through its being insufficiently formal-

ized. For in speaking always of judgements, instead of propositions, and introducing irrelevant psychological questions, the traditional logic gave the impression of being concerned in some specially intimate way with the workings of thought. What it was actually concerned with was the formal relationship of classes, as is shown by the fact that all its principles of inference are subsumed in the Boolean class-calculus, which is subsumed in its turn in the propositional calculus of Russell and Whitehead.[3] Their system, expounded in *Principia Mathematica*, makes it clear that formal logic is not concerned with the properties of men's minds, much less with the properties of material objects, but simply with the possibility of combining propositions by means of logical particles into analytic propositions, and with studying the formal relationship of these analytic propositions, in virtue of which one is deducible from another. Their procedure is to exhibit the propositions of formal logic as a deductive system, based on five primitive propositions, subsequently reduced in number to one. Hereby the distinction between logical truths and principles of inference, which was maintained in the Aristotelian logic, very properly disappears. Every principle of inference is put forward as a logical truth and every logical truth can serve as a principle of inference. The three Aristotelian 'laws of thought', the law of identity, the law of excluded middle, and the law of non-contradiction, are incorporated in the system, but they are not considered more important than the other analytic propositions. They are not reckoned among the premises of the system. And the system of Russell and Whitehead itself is probably only one among many possible logics, each of which is composed of tautologies as interesting to the logician as the arbitrarily selected Aristotelian 'laws of thought'.[4]

A point which is not sufficiently brought out by Russell, if indeed it is recognized by him at all, is that every logical proposition is valid in its own right. Its validity does not depend on its being incorporated in a system, and deduced from certain propositions which are taken as self-evident. The construction of systems of logic is useful as a means of discovering and certifying analytic propositions, but it is not in principle essential even for this purpose. For it is possible to conceive of a symbolism in which every analytic proposition could be seen to be analytic in virtue of its form alone.

The fact that the validity of an analytic proposition in no way depends on its being deducible from other analytic propositions is our justification for disregarding the question whether the propositions of mathematics are reducible to propositions of formal logic, in the way that Russell supposed.[5] For even if it is the case that the definition of a cardinal number as a class of classes similar to a given class is circular, and it is not possible to reduce mathematical notions to purely logical notions, it will still remain true that the propositions of mathematics are analytic propositions. They will form a special class of analytic propositions, containing special terms, but they will be none the less analytic for that. For the criterion of an analytic proposition is that its validity should follow simply from the definition of the terms contained in it, and this condition is fulfilled by the propositions of pure mathematics.

The mathematical propositions which one might most pardonably suppose to be synthetic are the propositions of geometry. For it is natural for us to think, as Kant thought, that geometry is the study of the properties of physical space, and consequently that its propositions have factual content. And if we believe this, and also recognize that the truths of geometry are necessary and certain, then we may be inclined to accept Kant's hypothesis that space is the form of intuition of our outer sense, a form imposed by us on the matter of sensation, as the only possible explanation of our a priori knowledge of these synthetic propositions. But while the view that pure geometry is concerned with the physical space was plausible enough in Kant's day, when the geometry of Euclid was the only geometry known, the subsequent invention of non-Euclidean geometries has shown it to be mistaken. We

see now that the axioms of a geometry are simply definitions, and that the theorems of a geometry are simply the logical consequences of these definitions.[6] A geometry is not in itself about physical space; in itself it cannot be said to be 'about' anything. But we can use a geometry to reason about physical space. That is to say, once we have given the axioms a physical interpretation, we can proceed to apply the theorems to the objects which satisfy the axioms. Whether a geometry can be applied to the actual physical world or not, it is an empirical question which falls outside the scope of the geometry itself. There is no sense, therefore, in asking which of the various geometries known to us are false and which are true. In so far as they are all free from contradiction, they are all true. What one can ask is which of them is the most useful on any given occasion, which of them can be applied most easily and most fruitfully to an actual empirical situation. But the proposition which states that a certain application of a geometry is possible is not itself a proposition of that geometry. All that the geometry itself tells us is that if anything can be brought under the definitions, it will also satisfy the theorems. It is therefore a purely logical system, and its propositions are purely analytic propositions.

It might be objected that the use made of diagrams in geometrical treatises shows that the geometrical reasoning is not purely abstract and logical, but depends on our intuition of the properties of figures. In fact, however, the use of diagrams is not essential to completely rigorous geometry. The diagrams are introduced as an aid to our reason. They provide us with a particular application of the geometry, and so assist us to perceive the more general truth that the axioms of the geometry involve certain consequences. But the fact that most of us need the help of an example to make us aware of those consequences does not show that the relation between them and the axioms is not a purely logical relation. It shows merely that our intellects are unequal to the task of carrying out very abstract processes of reasoning without the assistance of intuition. In other words, it has no bearing on the nature of geometrical propositions, but is simply an empirical fact about ourselves. Moreover, the appeal to intuition, though generally of psychological value, is also a source of danger to the geometer. He is tempted to make assumptions which are accidentally true of the particular figure he is taking as an illustration, but do not follow from his axioms. It has, indeed, been shown that Euclid himself was guilty of this, and consequently that the presence of the figure is essential to some of his proofs.[7] This shows that his system is not, as he presents it, completely rigorous, although of course it can be made so. It does not show that the presence of the figure is essential to a truly rigorous geometrical proof. To suppose that it did would be to take as a necessary feature of all geometries what is really only an incidental defect in one particular geometrical system.

We conclude, then, that the propositions of pure geometry are analytic. And this leads us to reject Kant's hypothesis that geometry deals with the form of intuition of our outer sense. For the ground for this hypothesis was that it alone explained how the propositions of geometry could be both true a priori and synthetic; and we have seen that they are not synthetic. Similarly, our view that the propositions of arithmetic are not synthetic but analytic leads us to reject the Kantian hypothesis[8] that arithmetic is concerned with our pure intuition of time, the form of our inner sense. And thus we are able to dismiss Kant's transcendental aesthetic without having to bring forward the epistemological difficulties which it is commonly said to involve. For the only argument which can be brought in favour of Kant's theory is that it alone explains certain 'facts'. And now we have found that the 'facts' which it purports to explain are not facts at all. For while it is true that we have a priori knowledge of necessary propositions, it is not true, as Kant supposed, that any of these necessary propositions are synthetic. They are without exception analytic propositions, or, in other words, tautologies.

We have already explained how it is that

these analytic propositions are necessary and certain. We saw that the reason why they cannot be confuted in experience is that they do not make any assertion about the empirical world. They simply record our determination to use words in a certain fashion. We cannot deny them without infringing the conventions which are presupposed by our very denial, and so falling into self-contradiction. And this is the sole ground of their necessity. As Wittgenstein puts it, our justification for holding that the world could not conceivably disobey the laws of logic is simply that we could not say of an unlogical world how it would look.[9] And just as the validity of an analytic proposition is independent of the nature of the external world; so it is independent of the nature of our minds. It is perfectly conceivable that we should have employed different linguistic conventions from those which we actually do employ. But whatever these conventions might be, the tautologies in which we recorded them would always be necessary. For any denial of them would be self-stultifying.

We see, then, that there is nothing mysterious about the apodeictic certainty of logic and mathematics. Our knowledge that no observation can ever confute the proposition '7 + 5 = 12' depends simply on the fact that the symbolic expression '7 + 5' is synonymous with '12', just as our knowledge that every oculist is an eye-doctor depends on the fact that the symbol 'eye-doctor' is synonymous with 'oculist'. And the same explanation holds good for every other a priori truth.

What is mysterious at first sight is that these tautologies should on occasion be so surprising, that there should be in mathematics and logic the possibility of invention and discovery. As Poincaré says: 'If all the assertions which mathematics puts forward can be derived from one another by formal logic, mathematics cannot amount to anything more than an immense tautology. Logical inference can teach us nothing essentially new, and if everything is to proceed from the principle of identity, everything must be reducible to it. But can we really allow that these theorems which fill so many books serve no other purpose than to say in a roundabout fashion "$A = A$"?'[10] Poincaré finds this incredible. His own theory is that the sense of invention and discovery in mathematics belongs to it in virtue of mathematical induction, the principle that what is true for the number 1×2, and true for $n + 1 \times 2$ when it is true for n, is true for all numbers. And he claims that this is a synthetic a priori principle. It is, in fact, a priori, but it is not synthetic. It is a defining principle of the natural numbers, serving to distinguish them from such numbers as the infinite cardinal numbers, to which it cannot be applied.[11] Moreover, we must remember that discoveries can be made, not only in arithmetic, but also in geometry and formal logic, where no use is made of mathematical induction. So that even if Poincaré were right about mathematical induction, he would not have provided a satisfactory explanation of the paradox that a mere body of tautologies can be so interesting and so surprising.

The true explanation is very simple. The power of logic and mathematics to surprise us depends, like their usefulness, on the limitations of our reason. A being whose intellect was infinitely powerful would take no interest in logic and mathematics.[12] For he would be able to see at a glance everything that his definitions implied, and accordingly, could never learn anything from logical inference which he was not fully conscious of already. But our intellects are not of this order. It is only a minute proportion of the consequences of our definitions that we are able to detect at a glance. Even so simple a tautology as '91 × 79 = 7189' is beyond the scope of our immediate apprehension. To assure ourselves that '7189' is synonymous with '91 × 79' we have to resort to calculation, which is simply a process of tautological transformation—that is, a process by which we change the form of expressions without altering their significance. The multiplication tables are rules for carrying out this process in arithmetic, just as the laws of logic are rules for the tautological transformation of sen-

tences expressed in logical symbolism or in ordinary language. As the process of calculation is carried out more or less mechanically, it is easy for us to make a slip and so unwittingly contradict ourselves. And this accounts for the existence of logical and mathematical 'falsehoods' which otherwise might appear paradoxical. Clearly the risk of error in logical reasoning is proportionate to the length and the complexity of the process of calculation. And in the same way, the more complex an analytic proposition is, the more chance it has of interesting and surprising us.

It is easy to see that the danger of error in logical reasoning can be minimized by the introduction of symbolic devices, which enable us to express highly complex tautologies in a conveniently simple form. And this gives us an opportunity for the exercise of invention in the pursuit of logical enquiries. For a well-chosen definition will call our attention to analytic truths, which would otherwise have escaped us. And the framing of definitions which are useful and fruitful may well be regarded as a creative act.

Having thus shown that there is no inexplicable paradox involved in the view that the truths of logic and mathematics are all of them analytic, we may safely adopt it as the only satisfactory explanation of their a priori necessity. And in adopting it we vindicate the empiricist claim that there can be no a priori knowledge of reality. For we show that the truths of pure reason, the propositions which we know to be valid independently of all experience, are so only in virtue of the lack of factual content. To say that a proposition is true a priori is to say that it is a tautology. And tautologies, though they may serve to guide us in our empirical search for knowledge, do not in themselves contain any information about any matter of fact.

NOTES

1. *Critique of Pure Reason*, 2nd edn., Introduction, sect. i.
2. *Critique of Pure Reason*, 2nd edn., Introduction, sect. iv and v.
3. See Karl Menger, 'Die Neue Logik', *Krise und Neuaufbau in den Exakten Wissenschaften*, pp. 94–6; and C. I. Lewis & C. H. Langford, *Symbolic Logic* (New York: Dover Publications, Inc; 1932), ch. 5.
4. Lewis and Langford, *Symbolic Logic*, ch. 7, for an elaboration of this point.
5. See Bertrand Russell, *Introduction To Mathematical Philosophy* (London: Allen & Unwin, 1960), ch. 2.
6. Cf. H. Poincaré, *La Science et l'Hypothèse*, Pt. II, ch. 3.
7. Cf. M. Black, *The Nature of Mathematics*, p. 154.
8. This hypothesis is not mentioned in the *Critique of Pure Reason*, but was maintained by Kant at an earlier date.
9. Ludwig Wittgenstein, *Tractatus Logico-Philosophicus* (London: Routledge & Paul, 1963) 3.031.
10. *La Science et l'Hypothèse*, Pt. I, ch. I.
11. Cf. B. Russell's *Introduction to Mathematical Philosophy*, ch. 3, p. 27.
12. Cf. Hans Hahn, 'Logik, Mathematik und Naturerkennen', *Einheitswissenschaft*, Pt. II, p. 18. 'Ein allwissendes Wesen braucht keine Logik und keine Mathematik'.