80-310/610 Logic and Computation

Exercise Set 6

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Exercise 1 (2.3.4) *If they interpreted the vague instructions "which terms are free" as presenting the free variables in the formula, explain in a short note what was intended by don't take off points.*

1. x *is free for* x *in* $x = x$. $(x = x)[x/x] = (x = x)$.

2. y *is free for* x *in* $x = x$. $(x = x)[y/x] = (y = y)$.

3. $x + y$ *is free for* y *in* $z = 0$. $(z = 0)[(x + y)/y] = (z = 0)$.

4. $0 + y$ *is free for* y *in* $\exists x(y = x)$. $\exists x(y = x)[(0 + y)/y] = \exists x((0 + y) = x)$.

5. $(x+y)$ *is free for* z *in* $\exists w(w+x = 0)$. $\exists w(w+x = 0)[(x+y)/z] = \exists w(w+x = 0)$

6. $(x + w)$ *is not free for* z *in* $\forall w(x + z = 0)$. $\forall w(x + z = 0)[(x + w)/z] = \forall w(x + (x + w) = 0)$.

7. $(x + y)$ *is not free for* z *in* $\forall w(x + z = 0) \wedge \exists y(z = x)$. $\forall w(x + z = 0) \wedge \exists y(z = x)[(x + y)/z] = \forall w(x + (x + y) = 0) \wedge \exists y((x + y) = x)$.

8. $x + y$ *is free for* z *in* $\forall u(u = v) \rightarrow \forall z(z = y)$. $(\forall u(u = v) \rightarrow \forall z(z = y))[(x + y)/z] = (\forall u(u = v) \rightarrow \forall z(z = y)$.

Exercise 2 (2.4.1) *Many correct answers are possible.*

1. $S(S(S(S(S(0)))))$, $S(S(0)) + S(S(S(0)))$

2. $v_{\mathcal{A}}(0) = 0$, *Suppose that* $v_{\mathcal{A}}(t) = n$. *Then* $v_{\mathcal{A}}(S(t)) = n + 1$.

3. *From the previous item, there exists* t *such that* $v_{\mathcal{A}}(t) = n$. *Suppose that* $v_{\mathcal{A}}(t) = n$. *Then* $v_{\mathcal{A}}(t + 0) = n$.

Exercise 3 (2.4.3)

1. 8

2. -4

Exercise 4 (2.4.5) *Of course there are many answers. The gist is this:*

1. $\exists x(x < y)$ *is interpreted in entailment statements as* $\forall y \exists x(x < y$. *That is false in the natural numbers because no number is strictly less than zero. The negation* $\neg \exists x(x < y) \approx \forall x(x \not< y)$ *is interpreted in entailment statements as* $\forall y \forall x(x \not< y)$, *which is again false in the natural numbers.*

2. *Let the sentence be atomic proposition* p.