

7a

$$\begin{array}{c}
 \frac{[\phi]6 \quad [\phi \rightarrow \psi]1}{\psi} \\
 \frac{[-(-\phi \vee \psi)]5 \quad \frac{\psi}{-\phi \vee \psi}}{\perp} \\
 \frac{\perp}{-\phi} \quad 6 \\
 \frac{[-(-\phi \vee \psi)]5 \quad \frac{\perp}{-\phi \vee \psi}}{\perp} \quad 5 \\
 \frac{\perp}{-\phi \vee \psi} \quad 5 \\
 \hline
 \end{array}
 \qquad
 \begin{array}{c}
 \frac{[-\phi]4 \quad [\phi]2 \quad \frac{[-\psi]3 \quad [\psi]4}{\perp}}{[-\phi \vee \psi]1 \quad \perp} \quad 4 \\
 \frac{\perp}{\psi} \quad 3 \\
 \frac{\psi}{\phi \rightarrow \psi} \quad 2 \\
 \hline
 \end{array}$$

$$\frac{\perp}{(\phi \rightarrow \psi) \leftrightarrow (-\phi \vee \psi)} \quad 1$$

7b

$$\frac{[-((\phi \rightarrow \psi) \vee (\psi \rightarrow \phi))]1 \quad \frac{[\phi \rightarrow \psi]2}{(\phi \rightarrow \psi) \vee (\psi \rightarrow \phi)}}{\perp}$$

$$\frac{\perp}{-(\phi \rightarrow \psi)} \quad 2 \quad \frac{[\psi]3}{\phi \rightarrow \psi}$$

$$\frac{\perp}{(\psi \rightarrow \phi)} \quad 3$$

$$\frac{(\phi \rightarrow \psi) \vee (\psi \rightarrow \phi)}{\perp}$$

$$\frac{\perp}{(\phi \rightarrow \psi) \vee (\psi \rightarrow \phi)} \quad 1$$