Van Dalen Exercise 1.5.2

By contraposition.

Suppose: $\{\phi 1, \ldots, \phi n\} \vdash \bot$.

So there exists derivation:

$$\boxed{\begin{array}{c}\phi 1, \ldots, \phi n\\ D\\ \bot\end{array}}$$

So there exists derivation:

$$\boxed{\dfrac{\begin{array}{c}\dfrac{\phi 1\ \&, \ldots, \&\ \phi n}{\phi 1, \ldots, \phi n}\ \&E\text{ iterated}\\ D\\ \bot\end{array}}{\phi 1\ \&, \ldots, \&\ \phi n \longrightarrow \bot}\ \longrightarrow I}$$

So $\vdash\ -(\phi 1\ \&\ \ldots\ \&\ \phi n)$

Suppose: $\vdash\ -(\phi 1\ \&\ \ldots\ \&\ \phi n)$.

So there exists derivation:

$$\boxed{\begin{array}{c}D\\ (\phi 1\ \&\ \ldots\ \&\ \phi n) \longrightarrow \bot\end{array}}$$

So there exists derivation:

$$\boxed{\dfrac{\dfrac{\dfrac{\begin{array}{c}D\\ (\phi 1\ \&\ \ldots\ \&\ \phi n) \longrightarrow \bot\end{array} \qquad \dfrac{[(\phi 1\ \&\ \ldots\ \&\ \phi n\text{-}1)] \quad [\phi n]}{(\phi 1\ \&\ \ldots\ \&\ \phi n)}}{\bot}}{\phi n \longrightarrow \bot}}{(\phi 1\ \&\ \ldots\ \&\ \phi n\text{-}1) \longrightarrow (\phi n \longrightarrow \bot)}}$$

So $\vdash\ (\phi 1\ \&\ \ldots\ \&\ \phi n-1) \longrightarrow -\phi n$.

Suppose: $\vdash\ (\phi 1\ \&\ \ldots\ \&\ \phi n-1) \longrightarrow -\phi n$.

So there exists derivation:

$$\boxed{\begin{array}{c}D\\ (\phi 1\ \&\ \ldots\ \&\ \phi n-1) \longrightarrow (\phi n \longrightarrow \bot)\end{array}}$$

So there exists derivation:

$$\boxed{\dfrac{\dfrac{\begin{array}{c}D\\ (\phi 1\ \&\ \ldots\ \&\ \phi n-1) \longrightarrow (\phi n \longrightarrow \bot)\end{array} \qquad \dfrac{\phi 1\ \ \ldots\ \ \phi n-1}{\phi 1\ \&\ \ldots\ \&\ \phi n-1}}{(\phi n \longrightarrow \bot) \qquad \phi n}}{\bot}}$$

So $\{\phi 1, \ldots, \phi n\} \vdash \bot$.