

Exercise set 10

$x$  does not occur free in  $\perp$  or any uncanceled hyp in derivation of  $\perp$        $x$  is free for  $x$  in  $P(x)$ .

$$\frac{\frac{[Ex P(x)] \quad \frac{\frac{[Ax -P(x)] \quad -P(x) \quad [P(x)]}{\perp}}{EE}}{\perp}}{-Ex P(x)}}{-Ex P(x) \leftrightarrow Ax -P(x)}$$

$$\frac{\frac{[-Ex P(x)] \quad \frac{[P(x)]}{Ex P(x)}}{\perp}}{-P(x)}}{Ax -P(x)} \quad EI$$

2.9.5

$$\frac{\frac{\frac{[P(x)]}{AxP(x)} \quad [AxP(x) \rightarrow Q]}{Q}}{P(x) \rightarrow Q}}{Ex(P(x) \rightarrow Q)} \quad \frac{\frac{[AxP(x)]}{P(x)}}{Ex P(x)} \quad [Ex(P(x) \rightarrow Q)]}{Q}}{(AxP(x) \rightarrow Q)}$$

2.9.6       $Ex(P(x) \rightarrow Q) \leftrightarrow (AxP(x) \rightarrow Q)$       substitutions OK because  $x$  not free in  $Q$

$$\frac{\frac{[P] \quad [P \rightarrow ExQ(x)]}{ExQ(x)}}{Q(x)}}{P \rightarrow Q(x)} \quad \frac{\frac{[P \rightarrow Q(x)] \quad [P]}{Q(x)}}{ExQ(x)}}{ExQ(x)} \quad EE$$

2.9.7       $Ex(P \rightarrow Q(x)) \leftrightarrow (P \rightarrow ExQ(x))$       tricky: have to add existential quantifier before applying EE, else  $x$  is free in conclusion

$$\frac{[EyEx P(x, y)] \quad \frac{\frac{[Ex P(x, y)] \quad \frac{[P(x, y)]}{Ey P(x, y)} EI}{Ey P(x, y)} EE}{ExEy P(x, y)} EI}{ExEy P(x, y)} EE$$

2.9.8      symmetrical to other case       $ExEy P(x, y) \leftrightarrow EyEx P(x, y)$