Exercise set 10

2.8.3.1.i

x is free for x in P(x) ---> Q(x) x is free for x in P(x)

$$\cfrac{\cfrac{\cfrac{[Ax\ (P(x)\ \text{--->}\ Q(x))]}{P(x)\ \text{--->}\ Q(x)}\qquad\cfrac{[Ax\ P(x)]}{P(x)}}{\cfrac{Q(x)}{Ax\ Q(x)}\ x\ \text{not free in any uncanceled hyp of }Q(x)}}{Ax\ P(x)\ \text{--->}\ Ax\ Q(x)}}{Ax\ (P(x)\ \text{--->}\ Q(x))\ \text{--->}\ (Ax\ P(x)\ \text{--->}\ Ax\ Q(x))}$$

2.8.3.1.v

$$\cfrac{\cfrac{\cfrac{\cfrac{[Ax\ Ay\ P(x,\ y)]}{Ay\ P(x,\ y)}\ x\text{ is free for }x\text{ in }Ay\ P(x,\ y)}{P(x,\ x)}\ x\text{ is free for }x\text{ in }P(x,\ y)}{Ax\ P(x,\ x)}\ x\text{ not free in any uncanceled hyp of }P(x,\ x)}{Ax\ Ay\ P(x,\ y)\ \text{--->}\ Ax\ P(x,\ x)}$$

2.8.3.1.vi

$$\cfrac{\cfrac{\cfrac{[Ax\ P(x)\ \&\ Ax\ Q(x)]}{P(x)\ \&\ Q(x)}}{P(x)}\qquad\cfrac{\cfrac{[Ax\ P(x)\ \&\ Ax\ Q(x)]}{P(x)\ \&\ Q(x)}}{Q(x)}}{\cfrac{P(x)\ \&\ Q(x)}{Ax\ (P(x)\ \&\ Q(x))}}\qquad\cfrac{\cfrac{\cfrac{[Ax\ (P(x)\ \&\ Q(x))]}{P(x)\ \&\ Q(x)}}{Ax\ P(x)}\qquad\cfrac{\cfrac{[Ax\ (P(x)\ \&\ Q(x))]}{P(x)\ \&\ Q(x)}}{Ax\ Q(x)}}{Ax\ P(x)\ \&\ Ax\ Q(x)}$$

$$Ax\ (P(x)\ \&\ Q(x))\ \text{<--->}\ Ax\ P(x)\ \&\ Ax\ Q(x) \qquad \text{All substitution restrictions met.}$$

2.8.3.1.vii

$$\cfrac{\cfrac{\cfrac{[(P\ \text{-->}\ Ax\ Q(x))]\qquad[P]}{Q(x)}}{P\ \text{-->}\ Q(x)}}{Ax\ (P\ \text{-->}\ Q(x))}\qquad\cfrac{\cfrac{\cfrac{\cfrac{[Ax\ (P\ \text{-->}\ Q(x))]}{P\ \text{-->}\ Q(x)}\qquad[P]}{Q(x)}}{Ax\ Q(x)}}{(P\ \text{-->}\ Ax\ Q(x))}$$

$$Ax\ (P\ \text{-->}\ Q(x))\ \text{<--->}\ (P\ \text{-->}\ Ax\ Q(x))$$

Note, the hypothesis that x is not free in P licenses both applications of AI.