Mathematics, Matter and Method

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machine of a certain description and start it scanning that tape, the machine would never halt. In a previous paper, I showed that an arbitrary statement† of set theory – even one that quantifies over sets of unbounded rank – can be paraphrased by a possibility statement.

(4) The main question we must speak to is simply, *what is the point*? Given that one can either take modal notions as primitive and regard talk of mathematical existence as derived, or the other way around, what is the advantage to taking the modal notions as the basic ones? It seems to us that there are two advantages to starting with the modal concepts. One advantage is purely mathematical. Construing set talk, etc., as talk about possible or impossible structures puts problems in a different focus. In particular, different axioms are evident. It is not my intention to discuss these purely mathematical advantages here. The

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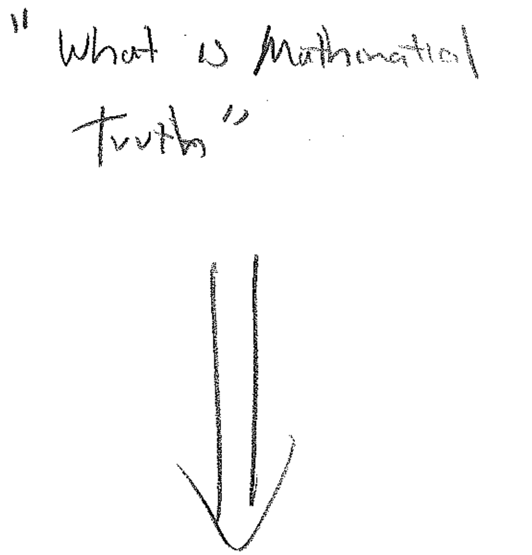

Let us return now to the topic of realism. Realism with respect to empirical science rests on two main kinds of arguments, which we may classify loosely as negative arguments and positive arguments. Negative arguments are to the effect that various reductive or operationalist philosophies are just unsuccessful. One tries to show that various attempts to reinterpret scientific statements as highly derived statements about sense data or measurement operations or whatever are unsuccessful, or hopelessly vague, or require the redescription of much ordinary scientific discovery as 'meaning stipulation' in an implausible way, or

† *Ibid.*

something of that kind, with the aim of rendering it plausible that most scientific statements are best not philosophically reinterpreted at all. The positive argument for realism is that it is the only philosophy that doesn't make the success of science a miracle. That terms in mature scientific theories typically refer (this formulation is due to Richard Boyd), that the theories accepted in a mature science are typically approximately true, that the same term can refer to the same thing even when it occurs in different theories – these statements are viewed by the scientific realist not as necessary truths but as part of the only scientific explanation of the success of science, and hence as part of any adequate scientific description of science and its relations to its objects.

I believe that the positive argument for realism has an analogue in the case of mathematical realism. Here too, I believe, realism is the only philosophy that doesn't make the success of the science a *miracle*.

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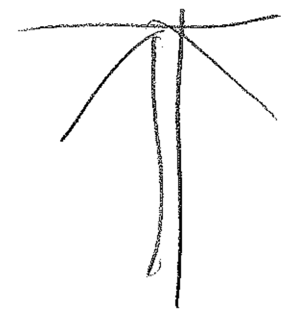

no interpretation under which most of mathematics is *true*, if we are really just writing down strings of symbols at random, or even by trial and error, what are the chances that our theory would be consistent, let alone mathematically fertile?

Let us be careful, however. If this argument has force and I believe it does, it is not quite an argument for mathematical realism. The argument says that the consistency and fertility of classical mathematics is evidence that it – or most of it – *is true under some interpretation*. But the interpretation might not be a *realist* interpretation. Thus Bishop might say, 'indeed, most of classical mathematics is true under some