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Meaning and Necessity

A Study in Semantics and Modal Logic

By

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PREFACE TO THE FIRST EDITION

The main purpose of this book is the development of a new method for the semantical analysis of meaning, that is, a new method for analyzing and describing the meanings of linguistic expressions. This method, called *the method of extension and intension*, is developed by modifying and extending certain customary concepts, especially those of class and property. The method will be contrasted with various other semantical methods used in traditional philosophy or by contemporary authors. These other methods have one characteristic in common: They all regard an expression in a language as a name of a concrete or abstract entity. In contradistinction, the method here proposed takes an expression, not as naming anything, but as possessing an intension and an extension.

This book may be regarded as a third volume of the series which I have called "Studies in Semantics", two volumes of which were published earlier. However, the present book does not presuppose the knowledge of its predecessors but is independent. The semantical terms used in the present volume are fully explained in the text. The present method for defining the L-terms (for example, 'L-true', meaning 'logically true', 'analytic') differs from the methods discussed in the earlier *Introduction to Semantics*. I now think that the method used in this volume is more satisfactory for languages of a relatively simple structure.

After meaning analysis, the second main topic discussed in this book is *modal logic*, that is, the theory of modalities, such as necessity, contingency, possibility, impossibility, etc. Various systems of modal logic have been proposed by various authors. It seems to me, however, that it is not possible to construct a satisfactory system before the meanings of the modalities are sufficiently clarified. I further believe that this clarification can best be achieved by correlating each of the modal concepts with a corresponding semantical concept (for example, necessity with L-truth). It will be seen that this method also leads to a clarification and elimination of certain puzzles which logicians have encountered in connection with modalities. In the Preface to the second volume of "Studies in Semantics," I announced my intention to publish, as the next volume, a book on modal logic containing, among other things, syntactical and semantical systems which combine modalities with quantification. The present book, however, is not as yet the complete fulfilment of that promise: it contains

B. MEANING POSTULATES

1. The Problem of Truth Based upon Meaning

Philosophers have often distinguished two kinds of truth: the truth of some statements is logical, necessary, based upon meaning, while that of other statements is empirical, contingent, dependent upon the facts of the world. The following two statements belong to the first kind:

(1) 'Fido is black or Fido is not black'

(2) 'If Jack is a bachelor, then he is not married'

In either case it is sufficient to understand the statement in order to establish its truth; knowledge of (extra-linguistic) facts is not involved. However, there is a difference. To ascertain the truth of (1), only the meanings of the logical particles ('is', 'or', 'not') are required; the meanings of the descriptive (i.e., nonlogical) words ('Fido', 'black') are irrelevant (except that they must belong to suitable types). For (2), on the other hand, the meanings of some descriptive words are involved, viz., those of 'bachelor' and 'married'.

Quine[1] has recently emphasized the distinction; he uses the term 'analytic' for the wider kind of statement to which both examples belong, and 'logically true' for the narrower kind to which (1) belongs but not (2). I shall likewise use these two terms for the explicanda. But I do not share Quine's skepticism; he is doubtful whether an explication of analyticity, especially one in semantics, is possible, and even whether there is a sufficiently clear explicandum, especially with respect to natural languages.

It is the purpose of this paper to describe a way of explicating the concept of analyticity, i.e., truth based upon meaning, in the framework of a semantical system, by using what we shall call *meaning postulates*. This simple way does not involve any new idea; it is rather suggested by a common-sense reflection. It will be shown in this paper how the definitions of some concepts fundamental for deductive and inductive logic can be reformulated in terms of postulates.[2]

Our explication, as mentioned above, will refer to semantical language

[1] W. V. Quine [Dogmas], especially pp. 23 f.

[2] This paper presupposes the explication of logical truth, which will be indicated in § 2, and that of the distinction between logical and descriptive constants (compare [I], § 13). Our present task is only to solve the additional problem involved in the explication of analyticity.

systems, not to natural languages. It shares this character with most of the explications of philosophically important concepts given in modern logic, e.g., Tarski's explication of truth. It seems to me that the problems of explicating concepts of this kind for natural languages are of an entirely different nature.[3]

2. Meaning Postulates

Our discussion refers to a semantical language system $\mathfrak{L}$ of the following kind. $\mathfrak{L}$ contains the customary connectives, individual variables with quantifiers, and as descriptive signs individual constants ('*a*', '*b*', etc.) and primitive descriptive predicates (among them '*B*', '*M*', '*R*', and '*Bl*', for the properties Bachelor, Married, Raven, and Black, respectively). The following statements in $\mathfrak{L}$ correspond to the two earlier examples:

(3) '$Bl\ a \vee \sim Bl\ a$'

(4) '$B\ b \supset \sim M\ b$'

Suppose that the customary truth-tables for the connectives are laid down for $\mathfrak{L}$ (in the form of rules of truth or satisfaction) but that *no* rules of designation for the descriptive constants are given (hence the meanings of the four predicates mentioned above are *not* incorporated into the system). Before we state meaning postulates, let us see what can be done without them, on the basis of semantical rules of the customary kinds. First let us define the L-truth of a sentence $\mathfrak{S}_i$ of $\mathfrak{L}$ as an explicatum for logical truth (in the narrow sense). We may use as definiens any one of the subsequent four formulations (5a) to (5d); they are equivalent to one another (provided they are applicable to $\mathfrak{L}$). Insertions in square brackets refer to example (3).

(5a) The open logical formula corresponding to $\mathfrak{S}_i$ [e.g., '$fx \vee \sim fx$'] is universally valid (i.e., satisfied by all values of the free variables). (Here it is presupposed that $\mathfrak{L}$ contains corresponding variables for all descriptive constants.)

(5b) The universal logical statement corresponding to $\mathfrak{S}_i$ [e.g., '$(f)(x)(fx \vee \sim fx)$'] is true. (Here it is presupposed that $\mathfrak{L}$ has variables with quantifiers corresponding to all descriptive constants.)

[3] The great difficulties and complications of any attempt to explicate logical concepts for natural languages have been clearly explained by Benson Mates in [Analytic] and by Richard Martin in [Analytic]. Both articles offer strong arguments against the view held by Quine [Dogmas] and Morton G. White [Analytic] that there is no clear distinction between analytic and synthetic.

(5c) $\mathfrak{S}_i$ is satisfied by all values of the descriptive constants occurring. [The ranges of values for '*Bl*' and '*a*' here are the same as those for '*f*' and '*x*', respectively, in (5a).]

(5d) $\mathfrak{S}_i$ holds in all state-descriptions. (A state-description is a conjunction containing for every atomic statement either it or its negation but not both, and no other statements. Here it is presupposed that $\mathfrak{L}$ contains constants for all values of its variables and, in particular, individual constants for all individuals of the universe of discourse.)

Each of these formulations presupposes, of course, that rules for the system $\mathfrak{L}$ are given which determine the concepts involved, e.g., rules of formation (determining the forms of open formulas and statements, i.e., closed formulas), rules for the range of values of all variables and for (5c) also analogous rules for the range of values for all descriptive constants,[4] and for (5d) rules determining those state-descriptions in which any given statement holds. Form (5d) is quite convenient if $\mathfrak{L}$ has the required form. Form (5c) imposes the least restrictions on $\mathfrak{L}$.

The other concepts can easily be defined on the basis of L-truth. Thus L-falsity, L-implication, and L-equivalence may be defined by the L-truth of $\sim\mathfrak{S}_i$, $\mathfrak{S}_i \supset \mathfrak{S}_j$, and $\mathfrak{S}_i \equiv \mathfrak{S}_j$, respectively.

The definition of L-truth in $\mathfrak{L}$, in any one of the four alternative forms, covers example (3) but obviously not (4). In order to provide for (4), we lay down the following *meaning postulate:*

$$(\mathrm{P}_1)\ `(x)(Bx \supset \sim Mx)'$$

Even now we do not give rules of designation for '*B*' and '*M*'. They are not necessary for the explication of analyticity, but only for that of factual (synthetic) truth. But postulate P_1 states as much about the meanings of '*B*' and '*M*' as is essential for analyticity, viz., the incompatibility of the two properties. If logical relations (e.g., logical implication or incompatibility) hold between the intended meanings of the primitive predicates of a system, then the explication of analyticity requires that postulates for all such relations are laid down. The term 'postulate' seems suitable for this purpose; it has sometimes been used in a similar sense.[5] (This usage is not the same as the more frequent one according to which 'postulate' is synonymous with 'axiom'.)

Suppose that the author of a system wishes the predicates '*B*' and '*M*' to designate the properties Bachelor and Married, respectively. How does

[4] Compare [Syntax], § 34c.

[5] See, for example, J. Cooley, *Primer of Formal Logic* (1942), p. 153.

he know that these properties are incompatible and that therefore he has to lay down postulate P_1? This is not a matter of knowledge but of decision. His knowledge or belief that the English words 'bachelor' and 'married' are always or usually understood in such a way that they are incompatible may influence his decision if he has the intention to reflect in his system some of the meaning relations of English words. In this particular case, the influence would be relatively clear, but in other cases it would be much less so.

Suppose he wishes the predicates '*Bl*' and '*R*' to correspond to the words 'black' and 'raven'. While the meaning of 'black' is fairly clear, that of 'raven' is rather vague in the everyday language. There is no point for him to make an elaborate study, based either on introspection or on statistical investigation of common usage, in order to find out whether 'raven' always or mostly entails 'black'. It is rather his task to make up his mind whether he wishes the predicates '*R*' and '*Bl*' of his system to be used in such a way that the first logically entails the second. If so, he has to add the postulate

$$(\mathrm{P}_2)\ `(x)(Rx \supset Bl\ x)'$$

to the system, otherwise not.

Suppose the meaning of '*Bl*', viz., Black, is clear to him. Then the two procedures between which he has to choose may be formulated as follows: (1) he wishes to give to '*R*' a meaning so strong that it cannot possibly be predicated of any non-black thing; (2) he gives to '*R*' a certain (weaker) meaning; although he may believe that all things to which '*R*' applies are black so that he would be greatly surprised if he found one that was not black, the intended meaning of '*R*' does not by itself rule out such an occurrence. Thus we see that it cannot be the task of the logician to prescribe to those who construct systems what postulates they ought to take. They are free to choose their postulates, guided not by their beliefs concerning facts of the world but by their intentions with respect to the meanings, i.e., the ways of use of the descriptive constants.

Suppose that certain meaning postulates have been accepted for the system $\mathfrak{L}$. Let $\mathfrak{P}$ be their conjunction. Then the concept of analyticity, which applies to both examples (3) and (4), can now be explicated. We shall use for the explicatum the term 'L-true with respect to $\mathfrak{P}$' and define it as follows:

(6) A statement $\mathfrak{S}_i$ in $\mathfrak{L}$ is L-true with respect to $\mathfrak{P}$ $=_{\mathrm{Df}}$ $\mathfrak{S}_i$ is L-implied by $\mathfrak{P}$ (in $\mathfrak{L}$).[6]

[6] The term 'L-true with respect to $\mathfrak{P}$' is simply a special case of the relative L-terms which I have used elsewhere; see [Probability], D20–2.

The definiens could, of course, also be formulated as "$\mathfrak{P} \supset \mathfrak{S}_i$ is L-true (in $\mathfrak{L}$)" or "$\mathfrak{S}_i$ holds in all state-descriptions in which $\mathfrak{P}$ holds" (the latter presupposes that L-truth in $\mathfrak{L}$ is defined by (5d)).

The definitions of the other L-concepts with respect to $\mathfrak{P}$ in terms of L-truth with respect to $\mathfrak{P}$ are analogous to the earlier definitions and therefore need not be stated here. The following theorem can be seen to result immediately:

(7) Each of the following conditions (a) to (d) is a sufficient and necessary condition for $\mathfrak{S}_i$ L-implying $\mathfrak{S}_j$ with respect to $\mathfrak{P}$:

(a) $\mathfrak{P}$ L-implies $\mathfrak{S}_i \supset \mathfrak{S}_j$

(b) $\mathfrak{P} \supset (\mathfrak{S}_i \supset \mathfrak{S}_j)$ is L-true

(c) $P \bullet \mathfrak{S}_i \supset \mathfrak{S}_j$ is L-true

(d) $P \bullet \mathfrak{S}_i$ L-implies $\mathfrak{S}_j$

An alternative way, differing merely in the form of systematization but leading to the same results, would be as follows. Let $\mathfrak{L}$ be the original system without meaning postulates. The system $\mathfrak{L}'$ is constructed out of $\mathfrak{L}$ by adding the meaning postulates $\mathfrak{P}$. Then we define:

(8) $\mathfrak{S}_i$ is L-true in $\mathfrak{L}' =_{\mathrm{Df}} \mathfrak{S}_i$ is L-implied by $\mathfrak{P}$ in $\mathfrak{L}$.

L-truth in $\mathfrak{L}'$ is then the explicatum for analyticity.

If L-truth in $\mathfrak{L}$ is defined by (5d), then the following definitions could take the place of (8):

(9) The state-descriptions in $\mathfrak{L}' =_{\mathrm{Df}}$ those state-descriptions in $\mathfrak{L}$ in which $\mathfrak{P}$ holds.

(10) $\mathfrak{S}_i$ is L-true in $\mathfrak{L}' =_{\mathrm{Df}} \mathfrak{S}_i$ holds in every state-description in $\mathfrak{L}'$.

The other L-concepts in $\mathfrak{L}'$ are then defined in terms of L-truth in $\mathfrak{L}'$ in the same way as before. If, for example, $\mathfrak{P}$ contains the postulates P_1 and P_2 mentioned earlier, then the following results would hold in $\mathfrak{L}'$: '$B\,b \supset \sim M\,b$' and '$R\,a \supset Bl\,a$' are L-true; '$B\,b \bullet M\,b$' and '$R\,a \bullet \sim Bl\,a$' are L-false; '$B\,b$' L-implies '$\sim M\,b$', and '$R\,a$' L-implies '$Bl\,a$'; '$R\,a \bullet Bl\,a$' is L-equivalent to '$R\,a$'.

3. Meaning Postulates for Relations

Suppose that among the primitive predicates there are also some with two or more arguments designating two- or more-place relations, and that one of these predicates possesses some structural properties in virtue of its meaning. For example, let 'W' be a primitive predicate designating the relation Warmer. Then 'W' is transitive, irreflexive, and hence asymmetric in virtue of its meaning. Therefore the statements '$Wab \bullet Wbc \bullet \sim Wac$', '$Wab \bullet Wba$', and '$Waa$' are false due to their meanings. The same holds for state-descriptions which contain one of these statements as subconjunctions; hence they do not represent possible cases. This difficulty was discovered by John G. Kemeny[7] and Yehoshua Bar-Hillel[8] independently. It is more serious than that due to logical dependencies between two or more one-place predicates, as it cannot be avoided by simply replacing dependent by independent predicates with the same expressive power.

There are two ways of overcoming the difficulty. The first, which maintains the requirement of the logical independence of all atomic statements, consists in avoiding primitive relations entirely or at least those of the customary kinds.[9]

The second way abandons the requirement of independence. It admits dependent primitives including relational ones, but restricts state-descriptions to those which represent possible cases, by stating meaning postulates or other equivalent rules. This way was first proposed by Kemeny.[10] In comparison with the first way, the second has the disadvantage of needing a new semantical concept (either 'directly L-true', i.e., 'meaning postulate', or 'directly L-false' in an alternative procedure), defined by enumeration in each semantical system or taken as primitive in general semantics. Another disadvantage is the more complicated form of theorems and computations of values of the degree of confirmation in inductive logic. For these reasons, Bar-Hillel and I previously did not pursue the second way any further.[11] On the other hand, it has the advantage of giving more freedom in the choice of primitives.

In the previous example of the predicate 'W', we could lay down the following postulates (a) for transitivity and (b) for irreflexivity; then the statement (c) of asymmetry is L-true with respect to these two postulates:

(11) (a) '$(x)(y)(z)\ (Wxy \bullet Wyz \supset Wxz)$'

(b) '$(x) \sim Wxx$'

(c) '$(x)(y)\ (Wxy \supset \sim Wyx)$'

[7] J. G. Kemeny, review of [Probability] in *Journal of Symbolic Logic*, 16 (1951), 205–7.

[8] Y. Bar-Hillel, "A Note on State-Descriptions", *Philosophical Studies*, 2 (1951), 72–75. Compare my reply, "The Problem of Relations in Inductive Logic", *ibid.*, 75–80.

[9] Some possibilities of this are outlined in my paper mentioned in the preceding footnote.

[10] See footnote 7. The procedure was carried out by Kemeny in "Extension of the Methods of Inductive Logic", *Philosophical Studies*, 3 (1952), 38–42, and in "A Logical Measure Function", *Journal of Symbolic Logic*, 18 (1953), 289–308. (These two articles were not known to me when I wrote the present paper.)

[11] See Bar-Hillel, *op. cit.*, p. 74, "the third possibility".

If we admit the form of semantical rules which we have called meaning postulates, we find that other customary kinds of rules may be construed as special kinds of meaning postulates. This holds, for example, for explicit definitions (if written as statements in the object language with '$\equiv$' or '$=$') and for contextual definitions. Likewise, the two or more formulas of a so-called recursive definition of an arithmetical functor may be regarded as meaning postulates. In this case, the label 'postulate' is perhaps even more appropriate than the customary one of 'definition'. The formulas serve not merely for an introduction of an abbreviating notation, since the new functor is not eliminable in all contexts. Further, the reduction-sentences which I proposed earlier for the introduction of disposition predicates[12] may be construed as meaning postulates.

[A bilateral reduction-sentence '$(x)\ [Q_1x \supset (Q_3x \equiv Q_2x)]$' for '$Q_3$' may simply be taken as a postulate, since it has no synthetic consequences in terms of the original predicates 'Q_1' and 'Q_2'. This is, however, in general not possible for the formulas of a reduction pair, e.g., '$(x)\ [Q_1x \supset (Q_2x \supset Q_3x)]$' ($\mathfrak{S}_1$) and '$(x)\ [Q_4x \supset (Q_5x \supset \sim Q_3x)]$' ($\mathfrak{S}_2$), since they together imply the synthetic statement '$(x) \sim (Q_1x \bullet Q_2x \bullet Q_4x \bullet Q_5x)$' ($\mathfrak{S}_3$). Here, we must take as postulate the weaker statement $\mathfrak{S}_3 \supset \mathfrak{S}_1 \bullet \mathfrak{S}_2$, which has no synthetic consequences.]

4. Meaning Postulates in Inductive Logic

A few brief remarks may be made here concerning the consequences of the use of meaning postulates for inductive logic. Let $\mathfrak{m}$ be any regular measure-function for the system $\mathfrak{L}$, and $\mathfrak{c}$ be the confirmation-function based upon $\mathfrak{m}$ (i.e., $\mathfrak{c}(h,e) = \mathfrak{m}(e \bullet h)/\mathfrak{m}(e)$). Let $\mathfrak{m}'$ be a function for the state-descriptions in $\mathfrak{L}$ fulfilling the following three conditions:

(12) (a) For any state-description k in $\mathfrak{L}$ in which $\mathfrak{P}$ does not hold, $\mathfrak{m}'(k) = 0$.

(b) For any state-description k in $\mathfrak{L}$ in which $\mathfrak{P}$ holds, $\mathfrak{m}'(k)$ is proportional to $\mathfrak{m}(k)$; say, $\mathfrak{m}'(k) = K\mathfrak{m}(k)$.

(c) The sum of the $\mathfrak{m}'$-values for all state-descriptions in $\mathfrak{L}$ is 1.

It is easily seen that, for any regular function $\mathfrak{m}$, there is one and only one function $\mathfrak{m}'$ of this kind. We find from (b) and (c) that K must be $1/\mathfrak{m}(\mathfrak{P})$. Since for the state-descriptions in $\mathfrak{L}'$, $\mathfrak{m}'$ has positive values (according to (9) and (12)(b)) whose sum is 1, $\mathfrak{m}'$ may be regarded as the regular function for $\mathfrak{L}'$ corresponding to $\mathfrak{m}$ for $\mathfrak{L}$.

[12] "Testability and Meaning", *Philosophy of Science*, Vols. 3 and 4 (1936 and 1937); reprinted by Whitlock's Inc., New Haven, Connecticut, 1950; see §§ 8–10.

Let $\mathfrak{m}'$ be applied to other statements in the customary way, and let the function $\mathfrak{c}'$ for $\mathfrak{L}'$ be based upon $\mathfrak{m}'$ (i.e., $\mathfrak{c}'(h,e) = \mathfrak{m}'(e \bullet h)/\mathfrak{m}'(e)$). Then $\mathfrak{c}'$ may be regarded as the regular confirmation-function for $\mathfrak{L}'$ corresponding to $\mathfrak{c}$ for $\mathfrak{L}$. The following results are easily obtained:

(13) For any state-description k in $\mathfrak{L}'$ (which is a state-description in $\mathfrak{L}$ in which $\mathfrak{P}$ holds), $\mathfrak{m}'(k) = \mathfrak{m}(k)/\mathfrak{m}(\mathfrak{P})$.

(14) For any statement j, $\mathfrak{m}'(j) = \mathfrak{m}(\mathfrak{P} \bullet j)/\mathfrak{m}(\mathfrak{P}) = \mathfrak{c}(j,\mathfrak{P})$.

(15) For any statements h and e, where e is not L-false in $\mathfrak{L}'$ (and hence $\mathfrak{P} \bullet e$ is not L-false in $\mathfrak{L}$), $\mathfrak{c}'(h,e) = \mathfrak{m}'(e \bullet h)/\mathfrak{m}'(e)$
$= \mathfrak{m}(\mathfrak{P} \bullet e \bullet h)/\mathfrak{m}(\mathfrak{P} \bullet e) = \mathfrak{c}(h,\mathfrak{P} \bullet e)$.

We see that the degree of confirmation in a system with postulates $\mathfrak{P}$ has in each case the same value as that obtained in the original system by adding $\mathfrak{P}$ to the evidence. This is analogous to the earlier result, according to which $\mathfrak{S}_i$ L-implies $\mathfrak{S}_j$ in $\mathfrak{L}'$ if and only if $\mathfrak{S}_i \bullet \mathfrak{P}$ L-implies $\mathfrak{S}_j$ in $\mathfrak{L}$ (compare (7)(d)). With the help of (15), general theorems concerning regular confirmation functions for systems with meaning postulates can easily be obtained from the known theorems for systems without postulates. However, if primitive relations occur and postulates are laid down for structural properties of these relations, then the computation of values of a particular function, e.g., $\mathfrak{c}^*$, will in many cases become even more complicated than in a system with the same primitives but without postulates.