Winning by Default: Why is There So Little Competition in Government Procurement?

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Abstract

In government procurement auctions, eligibility requirements are often imposed and, perhaps not surprisingly, contracts generally have a small number of participating bidders. To understand the effects of the restrictions of competition on the total cost of government procurement, we develop, identify, and estimate a principal-agent model in which the government selects a contractor to undertake a project. We consider three reasons why restricting entry could be beneficial to the government: by decreasing bid processing and solicitation costs, by increasing the chance of selecting a favored contractor and consequently reaping benefits from the favored contractor, and by excluding ex-ante less efficient contracts, which may intensify competition in terms of bid behavior. Using our estimates, we quantify the effects of the eligibility restrictions on the total cost of procurement.

1 Introduction

In recent ten years, the market for the United States federal government procurement is worth over $460 billion annually, which constitutes about 18% of the annual federal government spending. Despite its vast size, the extent of competition for a procurement contract is not very intense. Among the procurement contracts awarded during 2004–2012, about 43% of them were awarded under either limited or no competition. Even when a contract was open to full competition, attracting only one bidder for the contract was not uncommon. In this paper, we develop, identify, and estimate a procurement model to better understand the extent of competition observed in the data.

There are two important institutional features of federal government procurement that have received relatively little attention from the literature. First, for each procurement contract, the extent and method by which the contract will be competed is chosen by contracting

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officers who are hired by the government. The regulations allow them to eliminate bidders from consideration, although full and open competition is encouraged. As a result, the number of bids is relatively small; in fact, winning a contract by default is not uncommon in federal government procurement.

Second, a sealed-bid auction is not always a dominant procedure to choose a contractor, depending on the nature of the products or services to be procured. An alternative solicitation procedure is by negotiation, through which the proposals submitted by contractors are evaluated, negotiated, and selected. After the request for proposals is posted, the qualified contractors can submit their proposals, which will be reviewed in detail to determine which proposals are within a competitive range. Discussions may then be carried out with the contractors within the competitive range, and the contractor will be selected whose proposal is found to be most advantageous to the procuring agency. During the discussions, the contract terms are prices are considered together, which determines the winner.

In this paper, we construct a principal-agent framework that incorporates these features. In our model, the principal (the procurer) chooses the extent of competition, i.e., the eligibility conditions and the expected number of bidders, and offers a menu of contracts to the participating contractors (the agents) with a hidden type (cost). A contract chosen by a contractor is interpreted as a bid, which reflects the competitive proposal evaluation procedure. This procedure can be more profitable to the procurer than a sealed-bid auction when the number of bidders is very small.

The procurer may choose to decrease the extent of competition for three reasons. First, bid processing and solicitation costs may increase as the number of bidders increases. These costs include the time cost of waiting to receive more bids and the cost of reading the proposals and assessing various attributes of the contractors. Second, decreasing competition may increase the chance that a favored contractor will win the contract. If the procurer receives positive benefits from hiring a favored contractor, the procurer may want to decrease the extent of competition. Note that this favoritism is not necessarily related to corruption, because these positive benefits may be due to better quality of work, which cannot be formally measured or verified. Third, an unrestricted competition does not necessarily guarantee a lower expected amount of price to the winning contractor. By excluding ex-ante less efficient contractors, the procurer may incentivize the remaining bidders to bid more aggressively.

Using the federal procurement contracts awarded during 2004–2012, we nonparametrically estimate the model following our identification argument. Using the estimates, we conduct counter-factual analyses to decompose the effects of the three sources of entry restrictions. We also quantify the effects of the eligibility restrictions on the total cost of procurement.

The rest of the paper is organized as follows. We describe the model in Section 2, and then introduce our data and present descriptive analyses in Section 3. The identification of the model follows in Section 4, which we closely follow to nonparametrically estimate our model. We show the estimation results in Section 5. Section 6 concludes.

1.1 Related Literature

Our paper is related to the large literature on procurement and auctions. One strand of the literature explains why less competition does not necessarily lower the payoff of the auctioneer in independent private value auctions. Li and Zheng (2009) show that when the number of bidders is endogenously determined, the equilibrium bidding behavior can become
less aggressive as the number of potential bidders increases. Krasnokutskaya and Seim (2011) study a bid preference program, and Athey et al. (2013) compare a set-asides program and the bid subsidy program. Both papers show the importance of allowing endogenous entry when assessing restrictive competition policies.

An important contribution of our paper is that we build and estimate a model where the procurer is assumed to optimally choose the extent of competition, considering favoritism and bid processing and solicitation costs. In this regard, Bandiera et al. (2009) and Coviello et al. (2014) are closely related to our paper. Bandiera et al. (2009) develop a formal framework for distinguishing active waste and passive waste in the total government cost of procurement, and separately estimate them exploiting a policy experiment in Italy’s public procurement system. Active waste entails utility for the public decision makers, part of which is related to favoritism in our paper, while passive waste does not, such as bid processing and solicitation costs. Coviello et al. (2014) study government discretion on public goods provision in terms of whether or not to impose entry restrictions, and document the casual effect of increasing such discretion on procurement outcomes using a database for public procurement in Italy.

Another strand of the literature studies nonstandard contractor selection procedures, such as scoring auctions (Asker and Cantillon (2010)) or multi-attribute auctions (Krasnokutskaya et al. (2013)), where the price is not the only factor in selecting a contractor. We consider an optimal direct revelation mechanism in a competitive environment, studied by Laffont and Tirole (1987), McAfee and McMillan (1987), and Riordan and Sappington (1987). We extend their models by allowing the procurer to choose the optimal extent of competition.

Lewis and Bajari (2014) and Bajari et al. (2014) are related to our paper in that they study the price adjustments after the winning contractor is chosen and the project initiates. We distinguish the price adjustments into two categories: those incurring due to the cost changes related to the unknown type of the contractors and the rest. Lewis and Bajari (2014) study the former type of the price adjustments, and Bajari et al. (2014) study the latter. We consider both types of the price adjustment, and this is possible because we observe the reasons for contract modifications. Furthermore, although they take the contract type as exogenously given, we allow that the contract type and the winning contractor are endogenously determined.

Our paper also belongs to a literature on the identification of a principal-agent model, for example, Perrigne and Vuong (2011). In their paper, an implicit identifying assumption is that the optimal contracts are linear in costs. Their theoretical model provides the payoffs of the principal and the contractor, but it does not provide a particular contractual form that an optimal contract must follow. In our identification result, we do not impose any functional form assumptions on the optimal contracts.

Lastly, our paper is related to the political economy literature on how the federal government funds are allocated to the state or local governments or the private entities. Knight (2005) shows that members in the transportation committee secure higher project spending than do members from other districts. De Figueiredo and Silverman (2006) find that universities represented by a House or Senate Appropriations Committee member receive benefits regarding earmarks.
2 The US Federal Procurement

In this section, we describe the institutional backgrounds that motivate our modeling of the US federal procurement. We focus on how the extent of competition, solicitation procedure, contract type, and ex-post contract modifications are determined in practice.

In providing descriptive statistics regarding these institutional features, we analyze the data on federal government contracts and their modifications from the Federal Procurement Data System - Next Generation. We focus on definitive contracts that were initiated during the period of FY 2004–2012. Definitive contracts have specified terms and conditions, as opposed to indefinite delivery, indefinite quantity contracts. The former contracts tend to be much bigger in terms of payment size than the latter. For example, the amount of money that the federal government was obliged to pay for definitive contracts in FY 2010 comprise 94 percent of the total obligated amount for all contracts during the same period, $507 billion out of $540 billion.\footnote{In the FY 2010 contract data, there are about 2.8 million unique contracts. About 60\% of them are definitive contracts (over 1.7 million contracts). The average obligated amount of money for a definitive contract during the one-year period is $296,000, while that for an indefinite delivery, indefinite quantity contracts is $31,000.}

We further restrict our attention to the contracts with an expected size of $300,000 or more.\footnote{This size threshold is chosen because the contracts of an anticipated size greater than $300,000 are not expected to be reserved for small business concerns. FAR 13.003(b)(1) states that acquisitions of supplies or services that have an anticipated dollar value exceeding $3,000 but not exceeding $150,000 are reserved exclusively for small business concerns and shall be set aside. For certain supplies or services, the upper limit can be $300,000, according to FAR 2.101.}

2.1 Restriction of Competition

The full and open competition without exclusion is default in the acquisition process. However, the federal regulations specify the circumstances under which contracting officers are allowed to provide for full and open competition after excluding one or more sources.\footnote{See FAR 6.202-8 for the list of circumstances under which full and open competition after excluding one or more sources is allowed.} One such circumstance is when the contracting officers can justify that doing so could establish or maintain alternative sources for the supplies or services being acquired. In that case, they are required to provide a documentation signed by the head of the agency that describes the estimated reduction in overall costs and how the estimate was derived. In addition, contracting officers may set aside solicitations to fulfill the statutory requirements relating to small business concerns.\footnote{The statutes or the programs are section 8(a) of the Small Business Act, the Small Business Innovation Research Program, the Historically Underutilized Business Zones Act of 1997, the Veterans Benefits Act of 2003, (Economically Disadvantaged) Women-owned Small Business Program, and the Disaster Relief Act Amendments of 1974.}

Furthermore, contracting officers are also permitted to contract with no competition for various reasons, and for most cases, written justifications and approvals are required to do so.\footnote{See FAR 6.302 for the list of seven different circumstances under which no competition is allowed, and FAR 6.303-4 describes the procedures for written justifications and approvals.} International agreements or statutes may authorize or require that acquisition be made from a specified source or through another agency. Sometimes national security, unusual and compelling urgency, or public interest could be cited for a reason not to provide full
Table 1

The Extent of Competition

<table>
<thead>
<tr>
<th>Extent Competed</th>
<th>Num.</th>
<th>Total Size ($ Billion)</th>
<th>Median Size ($K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full &amp; open competition</td>
<td>91,865 (41.8%)</td>
<td>689.55 (43.8%)</td>
<td>735.29</td>
</tr>
<tr>
<td>Full &amp; open after exclusion</td>
<td>46,490 (21.2%)</td>
<td>154.13 (9.8%)</td>
<td>737.71</td>
</tr>
<tr>
<td>Not competed</td>
<td>81,127 (37.0%)</td>
<td>731.16 (46.4%)</td>
<td>690.24</td>
</tr>
<tr>
<td>Total</td>
<td>219,360 (100%)</td>
<td>1,574.83 (100%)</td>
<td>724.26</td>
</tr>
</tbody>
</table>

Note: There are in total 219,482 definitive contracts that satisfy the following two criteria: (i) initiated during FY 2004–2012 and (ii) either the anticipated size or the actual size is greater than or equal to $300,000 at the nominal value. For 122 contracts, the extent of competition is not reported. The size of the contracts is the CPI-adjusted total amount of money that the government is obligated to pay to the contractors, where CPI of December 2010 is 100.

As can be seen in Table 1, less than half of the contracts in the data were fully competed. As a result, out of $1.5 trillion spent on large contracts that initiated during the period of study, about 56% was spent on contracts with limited or no competition. For those that were not competed, the reasons should be justified. Table 2 show the various reasons why there was no competition for some contracts. The most prevalent reason in terms of the number of contracts is statutes or international agreements, accounting for about half of the contracts. However, more than 60% of the money spent on non-competed contracts paid for the contracts with unique available sources for reasons other than brand, patents, data rights, follow-on contracts, or utilities.

2.2 Solicitation Procedure and the Number of Bids

When full and open competition with or without exclusion of sources is employed, sealed bidding and negotiation are both acceptable procedures to solicit bids. Sealed bids are used if (i) time permits the solicitation, submission, and evaluation of sealed bids, (ii) the award will be made on the basis of price and other price-related factors, (iii) it is not necessary to conduct discussions with the responding contractors about their bids, and (iv) there is a reasonable expectation of receiving more than one sealed bid. When these conditions are not met, negotiated acquisitions can be used instead.\(^7\)

\(^6\)For example, FAR 6.302-7 states that full and open competition need not be provided for when the disclosure of the agency’s needs may compromise the national security unless the agency is permitted to limit the number of sources from which it solicits bids or proposals.

\(^7\)See FAR 6.4 for the conditions under which either of the two procedures is chosen. Although there are other procedures, such as two step, architect-engineer, and basic research, amongst others, sealed bidding and negotiation are the major solicitation procedures. See Table 3.
Table 2

Reasons for No Competition

<table>
<thead>
<tr>
<th>Reasons for no competition</th>
<th>Num.</th>
<th>Total Size ($ Billion)</th>
<th>Median Size ($K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Only one source available</td>
<td>22,751 (28.0%)</td>
<td>474.75 (64.9%)</td>
<td>633.65</td>
</tr>
<tr>
<td>Brand, patents, or data rights</td>
<td>561 (0.7%)</td>
<td>3.22 (0.4%)</td>
<td>559.80</td>
</tr>
<tr>
<td>Follow-on contract</td>
<td>2,590 (3.2%)</td>
<td>71.75 (9.8%)</td>
<td>691.15</td>
</tr>
<tr>
<td>Utilities</td>
<td>2,906 (3.6%)</td>
<td>4.75 (0.6%)</td>
<td>595.45</td>
</tr>
<tr>
<td>Urgency</td>
<td>4,195 (5.2%)</td>
<td>25.33 (3.5%)</td>
<td>767.23</td>
</tr>
<tr>
<td>Maintaining alternative sources</td>
<td>526 (0.6%)</td>
<td>21.65 (3.0%)</td>
<td>549.83</td>
</tr>
<tr>
<td>National security or public interest</td>
<td>609 (0.8%)</td>
<td>6.53 (0.9%)</td>
<td>1,449.91</td>
</tr>
<tr>
<td>Statute or international agreement</td>
<td>40,240 (49.6%)</td>
<td>114.79 (15.7%)</td>
<td>773.64</td>
</tr>
<tr>
<td>Simplified acquisition</td>
<td>2,106 (2.6%)</td>
<td>2.34 (0.3%)</td>
<td>507.11</td>
</tr>
<tr>
<td>Unspecified</td>
<td>4,643 (5.7%)</td>
<td>6.05 (0.8%)</td>
<td>458.94</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>81,127 (100%)</strong></td>
<td><strong>731.16 (100%)</strong></td>
<td><strong>690.24</strong></td>
</tr>
</tbody>
</table>

Note: There are in total 81,127 contracts that (i) initiated during FY 2004–2012, (ii) either the anticipated size or the actual size is greater than or equal to $300,000 at the nominal value, and (iii) were not competed. For these contracts, the reasons for no competition should be justified pursuant to FAR 6.302. The size of the contracts is the CPI-adjusted total amount of money that the government is obligated to pay to the contractors, where CPI of December 2010 is 100.

As can be seen Table 3, the most prevalent solicitation procedure is negotiation. Negotiated acquisition was used for about 60% of the definitive contracts with an expected size greater than $300,000 that were competed for award during the period of study, accounting for over $662 billion of government spending. Sealed bidding, on the other hand, was used much less frequently, accounting for less than $61 billion. In this paper, we study negotiated acquisitions.

In a negotiated acquisition, a contracting agency issues a request for proposal (RFP), upon which interested contractors submit their proposals. A typical RFP describes (i) the requirement, (ii) the anticipated terms and conditions that will apply to the contract, (iii) the information required to be in the bidder’s proposal, and (iv) the proposal evaluation criteria. RFPs can be posted at the federal business opportunities website, faxed, mailed, or presented orally. After receipt of proposals, award can be made with or without discussions. If discussions are to be conducted, the agency first determines competitive range and then negotiate with the bidders within the range. The discussions may apply to price, schedule, technical requirements, type of contract, or other terms of a proposed contract.\(^8\)

In evaluating the proposals, the agency’s objective is to select the proposal that represents the best value. Therefore, the relative importance of price in choosing the winner may vary. There are two selection processes: tradeoff and lowest price technically acceptable selection processes. A tradeoff process allows the government to accept other than the lowest priced proposal. On the other hand, the proposal with the lowest price is chosen as long as the proposal meets or exceeds the acceptability standards for non-cost factors in a lowest price technically acceptable selection process. The factors that may be considered other than price include past performance, compliance with solicitation requirements, technical excellence, management capability, personnel qualifications, and prior experience. The less definitive

\(^8\)See FAR 15 for details of the negotiated acquisition process.
Table 3

<table>
<thead>
<tr>
<th>Solicitation Procedure</th>
<th>Num.</th>
<th>Total Size ($ Billion)</th>
<th>Num. of Bids</th>
<th>Num. of Bids</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Average</td>
<td>Median</td>
</tr>
<tr>
<td>Negotiation</td>
<td>83,227</td>
<td>661.07 (78.4%)</td>
<td>8.13</td>
<td>2</td>
</tr>
<tr>
<td>Sealed bid</td>
<td>20,076</td>
<td>59.77 (7.1%)</td>
<td>6.11</td>
<td>4</td>
</tr>
<tr>
<td>Simplified acquisition</td>
<td>14,852</td>
<td>19.62 (2.3%)</td>
<td>4.82</td>
<td>2</td>
</tr>
<tr>
<td>Other procedures</td>
<td>10,460</td>
<td>71.71 (8.5%)</td>
<td>12.90</td>
<td>2</td>
</tr>
<tr>
<td>No solicitation</td>
<td>1,319</td>
<td>3.60 (0.4%)</td>
<td>2.70</td>
<td>2</td>
</tr>
<tr>
<td>Unspecified</td>
<td>8,421</td>
<td>27.90 (3.3%)</td>
<td>6.79</td>
<td>3</td>
</tr>
<tr>
<td>Total</td>
<td>138,355</td>
<td>843.68 (100.0%)</td>
<td>7.68</td>
<td>3</td>
</tr>
</tbody>
</table>

Note: There are in total 138,355 contracts that (i) initiated during FY 2004–2012, (ii) either the anticipated size or the actual size is greater than or equal to $300,000 at the nominal value, and (iii) were competed with or without exclusion of sources. "Other solicitation procedures" include two step, architect-engineer, and basic research, amongst others. The size of the contracts is the CPI-adjusted total amount of money that the government is obligated to pay to the contractors, where CPI of December 2010 is 100.

the requirement, the more development work required, or the greater the risk of unsuccessful contract performance, the more technical or past performance considerations may play a dominant role in selecting the winner, as opposed to price.

Table 3 shows the summary statistics regarding the number of bids. For all competed contracts in the data, the average number of bids is 7.7, and the median is 3. In the data, 28% of the competed contracts were awarded to a single bidder, and including non-competed contracts, only one contractor was considered for 55% of the contracts in the data. Putting it differently, $875.14 billion, over half of the total amount, was obligated to pay contractors that won a contract by default during the period of the study.

The number of bids can be affected by the efforts of the contracting agency. Prior to issuing a RFP, the agency may exchange information with industry prior to receipt of proposals. The early exchanges of information can take the form of industry conferences, public hearings, market research, one-on-one meetings with potential contractors, draft request for proposals (RFP), or request for information (RFI). Furthermore, the agency may publish a presolicitation notice that provides a general description of the scope or purpose of the acquisition and invites potential contractors to submit information. The notice contains information to permit a potential contractor to make an informed decision about whether to participate in the acquisition. The agency evaluates all responses in accordance with the criteria stated in the notice, and advises each respondent in writing either that it will be invited to participate in the resultant acquisition or, based on the information submitted, that it is unlikely to be a viable competitor.

These activities before issuing a RFP could help decrease the burden of contractors to search for a suitable contract to apply for and to prepare for their proposal. Note that these can be costly to the contracting agency. Furthermore, an additional proposal incurs an additional administrative cost of evaluating the proposal and a larger risk of receiving a bid protest. In FY 2012, the Government Accountability Office received 2,475 bid protest cases. Although the office upholds only a small number of bid protests, they may still have a big impact.9

9Federal Times reported in July 2013 on how bid protests are slowing down procurements. The article
Table 4

<table>
<thead>
<tr>
<th>Contract Type</th>
<th>Firm Fixed Price</th>
<th>Other</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Num.</td>
<td>Median Size</td>
</tr>
<tr>
<td></td>
<td>($K)</td>
<td>($K)</td>
</tr>
<tr>
<td>Any price adjustment</td>
<td>101,770 (57.1%)</td>
<td>208.23</td>
</tr>
<tr>
<td>Bilateral</td>
<td>81,795 (45.9%)</td>
<td>261.18</td>
</tr>
<tr>
<td>Unilateral</td>
<td>46,100 (25.9%)</td>
<td>280.16</td>
</tr>
<tr>
<td>No price adjustment</td>
<td>31,554 (42.9%)</td>
<td>-</td>
</tr>
<tr>
<td>Total</td>
<td>178,097 (100 %)</td>
<td>0.00</td>
</tr>
</tbody>
</table>

*Note: There are in total 128,278 contracts that (i) initiated during FY 2004–2012, and (ii) either the anticipated size or the actual size is greater than or equal to $300,000 at the nominal value. The second and the fourth columns represent the number of contracts of each type that had at least one ex-post price adjustment of each category. The third and the fifth columns show the median of the total price adjustments of each category conditional on at least one such adjustment.*

2.3 Ex-post Price Adjustment and Contract Type

The agreed-upon price at the time of award, base price, can be different from the actual price at the end of the contract, final price. The final prices are often larger than than the base prices. The median final price of a contract, $724,270, is about 1.5 times larger than the median base price, $485,580. The difference between the base and the final prices, which we call ex-post price adjustment, is nonzero for about 60% of the contracts in the data.

The ex-post price adjustment reflects the terms of the contract that allow the final price to vary with the observed outcomes of the project. Such contract terms depend on contract types, ranging from firm fixed price, in which the contractor has full responsibility for the performance costs and resulting profit or loss, to cost plus fixed fee, in which the contractor has minimal responsibility for the performance costs and the negotiated fee is fixed. In between are the various incentive contracts.

As can be seen in Table 4, even for firm fixed price contracts, the ex-post price adjustments are not uncommon, although its size or frequency is smaller or less than other contracts. Note that the frequency of bilateral price adjustments is similar across contract types, but unilateral ones are much more frequent for non firm-fixed-price contracts. A bilateral adjustment is signed by both the contractor and the contracting officer. They are used to make negotiated equitable adjustments resulting from the issuance of a change order, to definitize letter contracts, and to reflect other agreements of the parties modifying the terms of contracts. A unilateral adjustment is, on the other hand, signed only by the contracting officer. This does not require an additional agreement because it is due to the predetermined terms of a contract. Therefore, firm fixed price contracts are less likely to have unilateral price adjustments.

quoted Mary Davie, assistant commissioner of the Office of Integrated Technology Services at the General Services Administration: “We build time in our procurement now for protests. We know we are going to get protested.”

10 The ex-post price adjustment records in the data fall into one of the following categories: (i) additional work, (ii) supplemental agreement for work within scope, (iii) exercise an option, (iv) change order, (v) definitized letter contracts, all five of which are considered as bilateral in this paper, and the remaining unilateral categories, such as administrative action and various contract terminations.
In our analysis, we divide the contracts into two broad contract categories: fixed-price or variable-price. The former contracts are firm-fixed price contracts without any unilateral price-adjustment records, and the rest are categorized as the latter. We specifically use the terminology, variable-price contract, instead of commonly used cost-plus or cost-reimbursement contract. This is to make it explicit that the contracting agency does not always observe the cost of completing a project that has incurred to the contractor, and therefore, the final payment is often not based on the actual, realized cost. The government may require the contractors to disclose in writing their cost accounting practices and to comply with the Cost Accounting Standards. Table 5 shows that among the 87,485 variable-price contracts, the clause for these standards was included in about 10% of the contracts.

The contracting agency typically decide on the anticipated contract type prior to issuing a solicitation. However, particularly in negotiated procurements, selection of the contract terms can be a matter for negotiation.\footnote{See FAR 16.1 for the regulations on selecting contract types.}

2.4 Scope of the Data in the Analysis

For our analysis, we further narrow down our sample that satisfy the following criteria: (i) available for competition but commercially unavailable, (ii) solicited via negotiation, (iii) not set-asided for small business concerns, (iv) initiated and completed during the period of study, (v) performed in the U.S., (vi) expected to take longer than two weeks for completion, and (vi) without any inconsistent records on the contract. There are 45,743 contracts that satisfy all of the above criteria, totaling $290 billion. The summary statistics of the variables for the sample are provided in the following table.

3 Model

This section lays out a procurement model in which a procurer selects a contractor from multiple bidders with hidden efficiency type to undertake a project. This model captures the institutions of the U.S. federal procurement as documented in the previous section. First,
Table 6

<table>
<thead>
<tr>
<th>Summary statistics</th>
<th>Median</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prices ($K, 2010)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Base price</td>
<td>578.5</td>
<td>4,027.8</td>
<td>72,462.7</td>
<td>1.0</td>
<td>13,892,030</td>
</tr>
<tr>
<td>Final price</td>
<td>712.9</td>
<td>6,340.0</td>
<td>50,001.6</td>
<td>-8,789.3</td>
<td>4,126,789</td>
</tr>
<tr>
<td>Bilateral price change</td>
<td>0.0</td>
<td>2,569.2</td>
<td>30,653.6</td>
<td>-196,352.9</td>
<td>2,944,045</td>
</tr>
<tr>
<td>Unilateral price change</td>
<td>0.0</td>
<td>1,063.6</td>
<td>65,649.7</td>
<td>-13,392,210</td>
<td>2,129,391</td>
</tr>
<tr>
<td>Duration (Days)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Base duration</td>
<td>364</td>
<td>428.1</td>
<td>385.0</td>
<td>15</td>
<td>3,652</td>
</tr>
<tr>
<td>Final Duration</td>
<td>528</td>
<td>718.2</td>
<td>623.5</td>
<td>0</td>
<td>3,953</td>
</tr>
<tr>
<td>Bilateral duration change</td>
<td>0</td>
<td>186.2</td>
<td>393.1</td>
<td>-3,896</td>
<td>3,390</td>
</tr>
<tr>
<td>Unilateral duration change</td>
<td>0</td>
<td>89.8</td>
<td>285.8</td>
<td>-2,397</td>
<td>3,186</td>
</tr>
<tr>
<td>Contract type</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fixed price</td>
<td>1</td>
<td>0.5</td>
<td>0.5</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Competition</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Exclusion or no competition</td>
<td>1</td>
<td>0.5</td>
<td>0.5</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Number of bids</td>
<td>1</td>
<td>6.6</td>
<td>28.0</td>
<td>1</td>
<td>730</td>
</tr>
</tbody>
</table>

Note: The number of observations is 45,743. The amount of prices is CPI-adjusted, where CPI of December 2010 is 100.

The procurer chooses the extent of competition by restricting the group of contractors who are eligible to participate and by deciding the amount of search efforts to receive enough bids. The search efforts are costly, and so is restricting competition. Second, we model the negotiated acquisitions, where the contract type and the price are determined when the winner is chosen, by an indirect revelation mechanism. Third, we allow that an equilibrium contract type can be variable-price, and the final price may differ from the initially agreed price because of the terms of a variable-price contract or unexpected events that are not under the control of contractors. We characterize the equilibrium extent of competition and the selection mechanism in this model. Figure 1 presents the timeline of this model.

Figure 1
Timeline of the Procurement Process in the Model

- Project characteristics are observed
- # of participants by favor type is observed
- Signal and shock are observed
- Eligibility
- Rate of Bidder Arrival
- Menu of Contracts
- Winner
- Payment
- Participate
- Submit a Contract
3.1 Setup

For a given procurement project, the total cost of completing the project depends on the minimum expected cost \((c)\) and the ex-post cost changes due to stochastic realizations of demand or supply shocks \((\epsilon)\), as well as the the efficiency of the contractor. If the contractor is efficient, the cost is
\[
c + \epsilon,
\]
and otherwise,
\[
c + \beta + \epsilon.
\]
The realization of \(c\) and \(\epsilon\) is observed by both the procurer and the contractors at the same time: \(c\) is observed before the project is let and \(\epsilon\) is observed after the project is initiated. We assume that \(\epsilon\) is distributed independently of the efficiency of a contractor.

In completing the project, a signal, denoted by \(s\), is revealed to both the procurer and the contractor. The procurer cannot directly observe the cost of completing the project or \(\beta\), but does observe \(s\). The signal is drawn from cumulative density functions \(F\) for the efficient and \(\overline{F}\) for the inefficient, and the support for these two distributions is common. We denote the corresponding probability density functions by \(f(\cdot)\) and \(\overline{f}(\cdot)\), respectively. We consider the case where the signal is informative, i.e., \(F(s) \neq \overline{F}(s)\) for some \(s\) in its support, \(S\).

Some contractors are favored by the procurer, while others are not. We assume that procurer strictly prefers hiring a favored contractor to hiring a non-favored one, if their efficiency is the same. The subjective belief of the procurer that a contractor is efficient may vary by the favor type, which is a common knowledge.

Upon a realization of a project, the procurer determines whether to exclude non-favored contractors from competition and how many bids to receive on average. In doing so, the procurer minimizes the total expected cost of procurement, which consists of the expected transfer to the winning contractor, the bid solicitation and processing cost, and the net cost of holding an exclusive competition as opposed to a competition without entry restrictions.

Given the realized number of bidders by type, the procurer announces a menu of contracts, and the participating contractors simultaneously choose an item from the menu if the expected rent from doing so is nonnegative. When submitting their contract, the bidders do not know other bidders’ efficiency and have the same belief on the distribution as the procurer’s. However, they know whether or not each of other bidders is favored by the procurer. Given the submitted contracts, the procurer selects a contractor, who undertakes the project.

A typical contract in the menu has two components, a base price, \(p\), and a schedule of ex-post price adjustments, \(\Delta(\cdot, \cdot)\), which are contingent on the realization of the signal and the cost shock \(\epsilon\). Given the realized value of \(s\) and \(\epsilon\), the payoff to an efficient contractor under the contract is
\[
p - c + \psi(\Delta(s, \epsilon) - \epsilon),
\]
where \(\psi(\cdot)\) is a continuous function, with \(\psi(0) = 0, \psi'(0) = 1, \psi' > 0,\) and \(\psi'' < 0\). Note that \(p - c\) is fixed while \(\Delta(s, \epsilon) - \epsilon\) is variable. Due to liquidity concerns and potential adjustment costs, the variable part of the payoff is discounted, which is represented by \(\psi(\cdot)\). The payoff to an inefficient contractor is similarly defined.

Note that the procurer does not have liquidity concerns nor bear adjustment costs. Furthermore, the cost shock is independent of the efficiency of a contractor. Therefore, the
procuree minimizes her expected price by fully insuring the contractors against the cost shock. Therefore, we focus on the schedule \( r \) such that
\[
\Delta(s, \epsilon) = q(s) + \epsilon,
\]
and solve for \( q(\cdot) \). We assume that there exists a maximal penalty that the government can legally impose on contractors, \( M < 0 \). Therefore, the procuree is constrained by
\[
q(s) \geq M,
\]
for any signal \( s \). In this section, we consider the case where this legal bound on penalty is large enough so that it is not reached at optimum.\(^{12}\) Specifically,
\[
|M| \geq \left| h \left( \frac{1 - \pi}{1 - \pi \max_s \{\bar{f}(s) / f(s) \}} \right) \right|,
\]
where \( h(\cdot) \) is the inverse of the first derivative of \( \psi(\cdot) \), i.e., \( h(\psi'(q)) = q \). In the following, we first characterize the optimal selection mechanism which induces a truth-telling Bayesian Nash equilibrium, given the number of bidders by favor type. Then we solve the optimal extent of competition to minimize the expected total cost of procurement. The proofs for theorems are in Appendix.

### 3.2 Selection Mechanism

#### Symmetric Bidders
Suppose \( n \geq 1 \) bidders of the same favor type participate, and the probability that a bidder is efficient is \( \pi \in (0, 1) \). We show that it is always optimal to offer a menu of two contracts, one fixed-price contract, \( p \), and one variable-price contract, \( \{\bar{p}, q(\cdot)\} \). Given this menu, contractors will reveal their efficiency type by their contract choices; an efficient contractor will choose the fixed-price contract while an inefficient one will choose the variable-price contract.

Given this separation, procuree will select a participant that accepts the fixed-price contract, if there’s one. Otherwise, a random participant will be selected. Therefore, the probability that the fixed-price contract is selected is \( 1 - (1 - \pi)^n \). The resulting expected transfer to a winning contractor is:
\[
[1 - (1 - \pi)^n] p + (1 - \pi)^n \left[ \bar{p} + \int q(s) \bar{f}(s) ds \right].
\]
The procuree chooses \( p \) and \( \{\bar{p}, q(\cdot)\} \) to minimize the amount of the above expected transfer. Given the selection rule, a bidder believes that the probability of winning if he chooses the fixed-price contract in the menu is found by integrating the probability of winning over \( k \in \{0, \ldots, n - 1\} \) when there are \( k \) other contractors selecting the same contract:
\[
\phi(n) = \sum_{k=0}^{n-1} \binom{n-1}{k} \pi^k (1 - \pi)^{n-1-k} \frac{k + 1}{n \pi} = \frac{1 - (1 - \pi)^n}{n \pi}, \tag{1}
\]

\(^{12}\)In the Appendix, we provide a characterization of the equilibrium where the maximal penalty may be binding.
assuming that other participating contractors follow the equilibrium strategy.\footnote{The simplification of $\phi(n)$ in equation (1) is derived as follows:}

If the contractor chooses the variable-price contract instead, the winning probability, $\bar{\phi}(n)$, is the quotient of every bidder selecting the variable-price contract and the number of bidders:

$$\bar{\phi}(n) = \frac{(1 - \pi)^{n-1}}{n}.$$  

(2)

An implementable menu of contracts, $p$ and $\{\bar{p}, q(\cdot)\}$, satisfies the individual rationality and incentive constraints for each efficiency type. For the inefficient, these constraints are

$$\bar{p} + \int \psi[q(s)]\bar{f}(s)ds \geq c + \beta,$$

(3)

$$\phi(n) \left\{ \bar{p} + \int \psi[q(s)]\bar{f}(s)ds - c - \beta \right\} \geq \bar{\phi}(n) \left\{ p - c - \beta \right\},$$

(4)

where $\phi(n)$ and $\bar{\phi}(n)$ denote the winning probabilities conditional on the choice of contract, $p$ or the $\{\bar{p}, q(\cdot)\}$, as defined in equations (1) and (2), respectively. Similarly, the constraints for the efficient are

$$p \geq c,$$

(5)

$$\phi(n) \{p - c\} \geq \bar{\phi}(n) \left\{ \bar{p} + \int \psi[q(s)]\bar{f}(s)ds - c \right\}.$$  

(6)

We show that the participation constraint for the inefficient contractors, or condition (3), and the incentive compatibility constraint for the efficient contractors, condition (6), hold in equality and the incentive compatibility constraint for the inefficient contractors, or (4), holds with strict inequality at the optimum. The participation constraint for the efficient contractors, or condition (5), however, may or may not be binding at the optimum. We show that there exists a threshold of $\pi$, at or above which the participation constraint for the efficient contractors binds, defined as the unique root to:

$$\beta - \int \psi \left[ h \left( \frac{1 - \pi}{1 - \pi \bar{f}(s)/\bar{f}(s)} \right) \right] \{ f(s) - \bar{f}(s) \} ds.$$  

(7)

This unique threshold is used to characterize the optimal menu of contracts.

**Theorem 3.1** Suppose there are $n$ symmetric bidders with the probability that a bidder is efficient being $\pi \in (0, 1)$. Then there are two contracts in the optimal menu: one fixed-price contract, denoted by $p(\pi, n)$, and one variable-price contract, denoted by $\{\bar{p}(\pi), q(\cdot, \pi)\}$. There exists a unique root to (7), denoted by $\bar{\pi}$, such that the ex-post price adjustment of the
variable-price contract, $q(\cdot, \pi)$, is

\[
q(s, \pi) = h \left[ \frac{1 - \pi}{1 - \pi \overline{f}(s)/\underline{f}(s)} \right], \tag{8}
\]

for any $s \in S$ if $\pi \leq \overline{\pi}$. If $\pi > \overline{\pi}$, then $q(s, \pi) = q(s, \overline{\pi})$ for all $s \in S$. The base price of the variable-price contract is:

\[
\overline{p}(\pi) = c + \beta - \int \psi[q(s, \pi)] \overline{f}(s) ds. \tag{9}
\]

The price of the fixed-price contract is:

\[
p(\pi, n) = c + \frac{\pi (1 - \pi)^{n-1}}{1 - (1 - \pi)^n} \left( \beta - \int \psi[q(s, \pi)] \{ \overline{f}(s) - f(s) \} ds \right). \tag{10}
\]

There are a few notable features in the menu of contracts described in the above theorem. First, the expected amount of the transfer to the winning contractor is lower for the fixed-price contract than the variable-price contract; i.e., $p(\pi, n) \leq \overline{p}(\pi) + \int q(s, \pi) \overline{f}(s) ds$, for any number of bids and any $\pi$. Second, the efficient contractors enjoy a nonnegative informational rent; i.e., $p(\pi, n) \geq c$. This rent dissipates, however, as the number of bids increases or $\pi$ increases. Note that when $\pi \geq \overline{\pi}$, the individual rationality condition for the efficient binds, and hence $\overline{p}(\pi, n)$ becomes $c$ for any $n$. Third, the expected amount of the ex-post price adjustment, $\int q(s, \pi) \overline{f}(s) ds$, is not necessarily zero. This is because the ex-post price adjustment schedule is designed to punish masquerading of efficient contractors. When the likelihood ratio, $f(s)/\overline{f}(s)$, is greater (less) than one, implying that the contractor is more (less) likely to be efficient, the ex-post price adjustment, $q(s, \pi)$, is negative (positive). The price adjustment is zero if and only if the likelihood ratio is one.

In equilibrium the value of fixed-price contracts decline with the number of bids, but regardless of the number of bids, the expected utility of inefficient contractors is set at their reservation value of losing the procurement auction. This implies the variable contract does not depend on the number of bidders. To see this, note that the individual rationality constraint for an inefficient contractor binds regardless of number of bids. Given this, $\overline{p}(\pi)$ can be substituted out of the incentive compatibility constraint for the efficient contractor, yielding:

\[
\phi(n) \{ p - c \} = \overline{\phi}(n) \{ \beta - \int \psi[q(s)] [\overline{f}(s) - f(s)] ds \}.
\]

Since this is the only expression in the procurer’s objective function where $q$ appears, the optimal choice of this variable component does not depend on $n$. Together the binding individual rationality constraint and the choice of $q$ now pin down the base payment $\overline{p}(\pi)$.

To show the role of signaling, we consider the case where there is no signal to differentiate between different types of contractors. Following a result from the theory of optimal auctions, the procurer can extract some of the surplus an efficient contractor would receive via competition posed by other contractors who might be also efficient. When there are $n$ bidders, this can be achieved by offering two fixed-price contracts, $c + \frac{\pi (1 - \pi)^{n-1}}{1 - (1 - \pi)^n} \beta$ and $c + \beta$, where the former is associated with a higher chance of winning, $\phi(n)$, and the latter with a
lower chance of winning, $\overline{\varphi}(n)$.

**Asymmetric Bidders** Suppose $n_f$ favored contractors and $n_u$ non-favored ones participate. We assume that the procurer is not allowed to choose a favored, inefficient contractor if a non-favored, efficient contractor participates.\(^\text{14}\)

One implication of introducing favoritism is that inefficient and non-favored contractors will never win a project. Therefore, non-favored contractors will receive a take-it-or-leave-it offer of the fixed-price contract of $c$. On the other hand, favored contractors will be offered a menu of two contracts, which induces the separation of efficiency types as a Bayesian Nash equilibrium. The following theorem characterizes the menus of optimal contracts when both favor types participate.

**Theorem 3.2** Suppose $n_f \geq 1$ favored bidders and $n_u \geq 1$ unfavored ones participate. The probability that a favored bidder is efficient is $\pi \in (0, 1)$, and that for an unfavored one is $\pi_u \in (0, 1)$. Then a fixed-price contract, $c$, is offered to unfavored bidders. Favored bidders are offered with a menu of two contracts, consisting of one fixed-price contract, $p_f(n, n_f)$, and one variable-price contract, $\{\overline{p}_f, q_f(s)\}$. The variable-price contracts for favored contractors are identical to those in Theorem 3.1, and the price of the fixed-price contract for favored contractors is:

\[
p_f(n, n_f) = c + \frac{\pi (1 - \pi)^n - (1 - \pi)(1 - \pi_u)^{n-n_f}}{1 - (1 - \pi)^n_f} \left[ \beta - \int \psi[q(s, \pi)] \{\overline{f}(s) - f(s)\} ds \right]. \tag{11}
\]

Note that first, the participation of unfavored bidders will increase the informational rent to the favored with the total number of bidders being equal, if $\pi \geq \pi_u$. Second, when favored contractors are absent, unfavored contractors will be presented with two contracts, as in Theorem 3.1, and they will enjoy a positive rent. However, the participation of just one favored competitor will strip the unfavored of any informational rent.

**3.3 The Extent of Competition**

The procurer chooses whether to exclude unfavored contractors from participation, and given the mode of competition, exclusive or nonexclusive, she determines how many bids to attract and process on average. In doing so, she minimizes the expected *total* cost of procurement.

When a contract is competed with entry restrictions, only favored contractors are allowed to participate. Then the expected transfer for a project exclusively competed by $n$ contractors, $T_r(n)$, can be written as:

\[
T_r(n) = [1 - (1 - \pi)^n] p(\pi, n) + (1 - \pi)^n \left[ \overline{p}(\pi) + \int q(s, \pi) f(s) ds \right]. \tag{12}
\]

---

\(^{14}\)The opposite case, where the procurer can choose a favored contractor regardless of non-favored participants’ efficiency, is identical to the exclusive competition case, except that the favoritism manifests *covertly* in the non-exclusive competition. Our assumption here is that this *covert* favoritism is not allowed, reflecting the legal processes, including bid protest, in the United States. This assumption limits the extent to which the favoritism negatively affects efficiency, which may be of a larger concern in other countries. Alternatively, Burguet and Che (2004) and Celentani and Gauza (2002) allow the procurer to manipulate the quality assessment in exchange for a bribe.
$T_r(n)$ is the weighted average of the amount of transfer under the fixed-price contract and the expected amount of transfer under the variable-price contract. The probability that the fixed-price contract is chosen is the probability that at least one bidder among $n$ of them is efficient, $1 - (1 - \pi)^n$.

The expected total cost of exclusive procurement, denoted by $V_r(n)$, is:

$$V_r(n) = T_r(n) + \kappa_r n + \eta,$$

where $\kappa_r > 0$ denotes per-bidder bid processing cost. This cost includes the cost of reading the proposals, making sure that the language and terms of the proposals are unambiguous, and assessing various attributes of the contractors, as well as the time cost of delaying the initiation of the project. $\eta$ represents the risk of bid protest or the administrative or political burdens to justify exclusion of sources, net of the direct benefit that the procurer receives from favoritism. The value of $\eta$, therefore, could be negative if the direct benefit from favoritism outweighs the cost of potential bid protests and justification.

When any contractor can participate, the expected transfer to the winning contractor depends on the number of favored bidders and the number of unfavored bidders. Let $T(n, n_f)$ denote the expected transfer when $n$ bidders participate, $n_f$ of whom are favored:

$$T(n, n_f) = \left[1 - (1 - \pi)^{n_f}\right] p_f(n, n_f) + (1 - \pi)^{n_f} [1 - (1 - \pi_u)^{n - n_f}] c$$

$$+ (1 - \pi)^{n_f} (1 - \pi_u)^{n - n_f} \left[ \bar{p}_f + \int q_f(s) f(s) ds \right].$$

(13)

$T(n, n_f)$ is the weighted average of the amount of transfer under three possible cases: an efficient favored contractor wins and the procurement officer pays $p_f(n, n_f)$; an efficient unfavored contractor wins and the procurement officer pays $c$; or an inefficient favored contractor wins and takes the variable price contract.

The expected total cost of nonexclusive procurement, denoted by $V(n)$, is:

$$V(n) = \sum_{n_f = 0}^{n} \binom{n}{n_f} \rho^{n_f} (1 - \rho)^{n - n_f} T(n, n_f) + \kappa n,$$

where $\kappa_o > 0$ denotes per-bidder bid processing cost, which may differ from $\kappa_r$. Given the proportion of favored contractors being $\rho$, the number of favored contractors given $n$ bidders follows a binomial distribution with $n$ and $\rho$.

**Average Number of Bids** For a project to be implemented, at least one contractor must bid. Therefore, we consider the problem of choosing the average number of extra bids, above the necessary one bid. The realized number of extra bidders follows a Poisson distribution with the average arrival rate of choice, $\lambda_r$. Given this, the expected total cost of exclusive procurement can be written as:

$$\sum_{n=0}^{\infty} \frac{\lambda_r^n e^{-\lambda_r}}{n!} V_r(n + 1).$$

As $\lambda_r$ increases, the expected transfer to a winning contractor decreases. This is because
the probability of hiring an efficient contractor increases as the number of bidders increases, and hiring an efficient contractor is cheaper than hiring an inefficient one. Furthermore, the rent that an efficient contractor receives decreases as more contractors participate.

At the same time, an increase in \( \lambda_r \) is associated with an increase in the total bid-processing cost. Balancing this trade-off between the transfer and the bid-processing cost guarantees one unique \( \lambda_r \) that minimize the expected total procurement cost. Given the first order condition with respect to \( \lambda \), the optimal arrival rate of extra bids for an exclusive procurement, \( \lambda_r \), satisfies the following equation:

\[
\sum_{n=0}^{\infty} \frac{(n - \lambda_r)\lambda_r^{n-1}e^{-\lambda_r}}{n!} T_r(n+1) + \kappa_r = 0.
\] (14)

With a similar argument for non-exclusive procurement, the optimal arrival rate of extra bids, \( \lambda \), satisfies the following equation:

\[
\sum_{n=0}^{\infty} \frac{(n - \lambda)\lambda^{n-1}e^{-\lambda}}{n!} \sum_{n_f=0}^{n+1} \left( \frac{n+1}{n_f} \right) \rho^{n_f}(1 - \rho)^{n+1-n_f}T(n+1,n_f) + \kappa = 0.
\] (15)

**Entry Restriction** The procurer chooses to impose an entry restriction to prevent unfa-vored contractors from participating if and only if the expected total procurement cost is lower with the restrictions than without them.

Note that the procurer may prefer an exclusive competition to the alternative even when the direct benefits from favoritism are relatively small, if unfavored contractors are on average less efficient than favored ones. Allowing unfavored contractors to participate will increase the expected transfer by increasing the chance of employing an inefficient contractor and the informational rent of the favored, efficient contractors.

4 **Identification**

Identification investigates whether the model primitives can be uniquely recovered from the observed data generating process. Let us introduce two random variables for future references: \( d \) indicating that the contract is fixed-price and \( r \) indicating that the entry of the unfavored is restricted. In the ideal framework where the number of contracts is infinite, we recover (i) the joint distribution of the contract type and the extent of competition, \( \Pr(d,n,r) \), (ii) the distribution of signals, \( G_s(\cdot|d,n,r) \), (iii) the distribution of the price of fixed-price contracts, \( G_p(\cdot|n,r) \), (iv) the distribution of the base price of variable-price contracts, \( G_p(\cdot|n,r) \), (v) the distribution of the ex-post price adjustment of variable-price contracts, in absolute values, conditional on signal: \( G_{q|s}(\cdot|s,n,r) \), and (vi) the mapping of \( p \) from \( q \) and \( s \), denoted by \( p(q,s) \). Notice that the identity of the bidders that are favored by the procurer is not observed. For future reference, let us define \( \varphi_n \) as \( \Pr(d=1|n,r=1)/\Pr(d=0|n,r=1) \).

Each contract may differ by unobserved type, \( \pi \in [\pi_{\text{min}}, \pi_{\text{max}}] \equiv \Pi \subset (0,1) \), drawn from \( F_\pi(\cdot) \). This unobserved type is the probability that a favored contractor is efficient, and it also varies with other primitives of the model: the probability that an unfavored contractor is efficient (\( \pi_u \)), the cost parameters (\( c \) and \( \beta \)), and the per-bidder cost of soliciting and processing bids (\( \kappa_r \) and \( \kappa \)).
We suppose that \( \pi_u(\pi) \) is increasing in \( \pi \) and that \( \pi_{\text{min}} = \pi_u(\pi_{\text{min}}) \) and \( \pi_{\text{max}} = \pi_u(\pi_{\text{max}}) \) implying that the range of efficiencies does not vary by favorability status. For example we might set:

\[
\pi_u(\pi) \equiv \pi_{\text{min}} + (\pi_{\text{max}} - \pi_{\text{min}})^{1-\zeta}(\pi - \pi_{\text{min}})^\zeta.
\] (16)

If \( \zeta > 1 \), then the average efficiency of favored contractors is higher than that of unfavored contractors. If \( \zeta < 1 \), then the opposite is true.

We allow that the per bidder cost of soliciting and processing bids may differ depending on the entry restrictions. We assume that the cost differential, \( \kappa(\pi) - \kappa_r(\pi) \), is constant over \( \pi \). With a slight abuse of notations, let us denote the differential by \( \kappa_u \), which can be positive or negative. We assume that bid solicitation and processing costs are positive for any \( \pi \).

The model is deterministic in the prediction of whether to exclude non-favored bidders given \( \pi \). To allow that the probability that eligibility restrictions are imposed is not always degenerate, we assume that the administrative cost of imposing eligibility restrictions, \( \eta \), is stochastic, drawn from \( F_\eta \).

Thus, the model primitives can be summarized as: (i) the distribution of \( \pi \) and \( \eta \); (ii) the distribution of signal conditional on the efficiency type, \( E \) and \( F \); (ii) the liquidity cost function, \( \psi \); (iii) the cost parameters as a function of \( \pi \), \( c(\pi) \) and \( \beta(\pi) \); (iv) the per bidder bid solicitation and processing costs for exclusive competition as a function of \( \pi \), \( \kappa_r(\cdot) \); and (v) the remaining scalar parameters of the model, \( \kappa_u \), \( \zeta \), and \( \rho \).

We make the following assumptions on the unobservable variables, \( \pi \) and \( \eta \), and some of the primitives of the model.

A1. The signals are independent of \( \pi \) and \( \eta \) conditional on the efficiency type.

A2. \( \eta \) is independent of \( \pi \).

A3. \( c(\pi) \) and \( \beta(\pi) \) are nonincreasing in \( \pi \).

A4. The CDF of \( \pi \) is strictly increasing for all \( \pi \in \Pi \), and \( \pi_{\text{max}} < \pi \), where \( \pi \) is the unique root of (7).

Assumption A1 allows that the conditional signal distributions are identified from the observed signal distributions conditional on the contract type. Under A1, we identify \( E \) from the observed distribution of the signal for fixed-price contracts and \( F \) from that for variable-price contracts.

\[
\underline{E}(s) = G_s(s|d = 0); \quad \overline{F}(s) = G_s(s|d = 1).
\]

Assumption A2 allows that the distribution of \( \eta \) is identified from the conditional probability that a project of type \( \pi \) is competed exclusively. Assumption A4 guarantees that the individual rationality condition for the efficient does not bind at the optimum when the procurement is exclusively competed. This assumption can be relaxed, in which case the identification result holds for \( \pi \leq \pi \). Assumption A3, along with Assumption A4, guarantees that the contracts monotonically vary with \( \pi \), as stated in the following lemma.

**Lemma 4.1** Under Assumptions A3 and A4, the following holds for procurement with entry restrictions.
(i) The variable-price contract becomes more volatile as \( \pi \) increases for any signal \( s \):
\[
\frac{\partial |q(s, \pi)|}{\partial \pi} > 0.
\]

(ii) The fixed-price contract becomes cheaper as \( \pi \) increases for any number of bids \( n \):
\[
\frac{\partial p(\pi, n)}{\partial \pi} < 0.
\]

We first show that \( c(\cdot), \beta(\cdot), \kappa_r(\cdot) \), and the probability distribution of \( \pi \) for exclusive competition, \( f(\pi|d = 1) \), are identified off the exclusively competed contracts if \( \psi \) is known. In obtaining this result, we exploit (i) and (ii) of Lemma 4.1 where the price of the fixed-price contracts and the volatility of the variable-price contracts vary monotonically with \( \pi \).

Then we establish an identification result for \( \kappa, \zeta, \rho \), and the probability distribution of \( \pi \) for nonexclusive competition, denoted by \( f(\pi|d = 0) \). Lastly, the distribution of the idiosyncratic net cost of holding an exclusive competition, \( F_{\eta} \), is identified from the choice probability that an exclusive competition is held conditional on \( \pi \), under the independence of \( \pi \) and \( \eta \) as in Assumption A2. We discuss each of the identification argument below in order.

4.1 Identification from Exclusive Contracts

We temporarily assume that \( \psi(\cdot) \), the liquidity cost function, is known. Given \( \psi(\cdot), q(\pi, s) \) for all \((\pi, s) \in \Pi \times S\) is identified from equation (8):
\[
q(\pi, s) = h \left[ (1 - \pi) / (1 - \pi l(s)) \right].
\]

Because the data generates the joint distribution of \((\bar{p}, q)\) conditional on \((\pi, s)\), the base price of a variable-price contract, \( p \), is identified as a mapping of \( \pi \) up to \( \psi(\cdot) \).
\[
p(\pi) = p(q(\pi, s), s), \tag{17}
\]

where the mapping of \( \bar{p} \) from \( q \) and \( s \), \( p(q, s) \), is observed in the data.

Exploiting Lemma 4.1 that \(|q(\pi, s)|\) is declining in \( \pi \) for any given \( s \), we show that the density of \( \pi \) for exclusive, variable-price contracts, \( f_\pi(\pi|d = 0, r = 1) \) can be expressed as a mapping of identified functions up to \( \psi(\cdot) \). Similarly, the identification of \( f_\pi(\pi|d = 1, r = 1) \) follows from the monotonicity property of \( p(\pi, n) \) in \( \pi \) for any given \( n \). Furthermore, \( p(\pi, n) \) can be expressed as a function of the cumulative density of fixed price contracts.

Lemma 4.2 Suppose Assumptions A1–A4 hold. For all \((\pi, s) \in \Pi \times S\) and any \( n \), the following equations hold:
\[
\begin{align*}
f_\pi(\pi|d = 0, n, r = 1) &= g_q [ |q(\pi, s)| |s, n, r = 1] \frac{\partial |q(\pi, s)|}{\partial \pi}, \tag{18} \\
f_\pi(\pi|d = 1, n, r = 1) &= \frac{1 - (1 - \pi)^n}{\phi_n (1 - \pi)^n} f_\pi(\pi|d = 0, n, r = 1), \tag{19} \\
\text{and} \\
p(\pi, n) &= G_p^{-1} \left( \int_{\pi}^{\pi_{\text{max}}} f_\pi(v|d = 1, n, r = 1) dv |n, r = 1 \right). \tag{20}
\end{align*}
\]
Note that Lemma 4.2 implies the identification of \( f_\pi(\pi|n, r = 1) \) for any \( n \) up to \( \psi \). Given the conditional distribution, we identify the joint distribution, \( f_\pi, n(\pi, n|r = 1) \), and accordingly the bid arrival rate as a function of \( \pi, \lambda_r(\pi) \). The identification of \( \kappa_r(\pi) \) follows from the first order condition that determines the choice of \( \lambda_r(\pi) \).

Lemma 4.3 Suppose Assumptions A1, A3, and A4 hold. Then, for all \( \pi \in \Pi \),

\[
\lambda_r(\pi) = \sum_{n=0}^{\infty} n f_\pi, n(\pi, n|r = 1) f_\pi(\pi|r = 1),
\]

(21)

and

\[
\kappa_r(\pi) = \sum_{n=0}^{\infty} \left( \frac{[1 - (1 - \pi)^{n+1}] p(\pi, n + 1) + (1 - \pi)^{n+1}}{\bar{p}(\pi) + \int q(s, \pi) \bar{f}(s) ds} \right) \frac{(\lambda_r(\pi) - n) \lambda_r(\pi)^{n-1} e^{-\lambda_r(\pi)}}{n!}.
\]

(22)

Lemma 4.4 shows that \( c(\pi) \) and \( \beta(\pi) \) are identified up to \( \psi \), from the characterization of the fixed-price contracts and the base price of the variable-price contracts.

Lemma 4.4 For \( n > 1 \), \( c(\pi) \) and \( \beta(\pi) \) can be written as:

\[
c(\pi) = \frac{1 - (1 - \pi)^n}{1 - (1 - \pi)^{n-1}} p_\pi(n, n - 1) - \frac{\pi(1 - \pi)^{n-1}}{1 - (1 - \pi)^{n-1}} \left[ \bar{p}(\pi) + \int \psi \left( h \left[ \frac{1 - \pi}{1 - \pi l(s)} \right] \right) f(s) ds \right],
\]

(23)

and

\[
\beta(\pi) = \bar{p}(\pi) + \int \psi \left( h \left[ \frac{1 - \pi}{1 - \pi l(s)} \right] \right) \frac{\bar{f}(s) ds}{\bar{f}(s) ds - c(\pi)}.
\]

Combining Lemmas 4.2 to 4.4, we establish the following theorem that the distribution of \( \pi \), project costs, and per-bidder bid processing cost for exclusive competition, as well as the conditional signal distributions, are identified up to \( \psi(\cdot) \).

Theorem 4.1 Suppose Assumptions A1–A4 hold and \( \psi(\cdot) \) is known. Then \( \bar{F}(s) \) and \( F(s) \) for all \( s \in S \), \( f_\pi(\pi|r = 1) \), \( c(\pi) \), \( \beta(\pi) \), and \( \kappa_r(\pi) \) for all \( \pi \in \Pi \) and \( n \) are identified.

Previous discussion does not exhaust all over-identifying restrictions. One set of restrictions is that equation (17) hold for all \( s \), for any given \( \pi \). Another set of restrictions is that equation (23) holds for all \( n \), for any given \( \pi \). Lastly, when \( n = 1 \), the following identity holds regardless of \( c(\pi) \) and \( \beta(\pi) \):

\[
p(\pi, 1) = \bar{p}(\pi) + \int \psi \left( h \left[ \frac{1 - \pi}{1 - \pi l(s)} \right] \right) \frac{\bar{f}(s) ds}{\bar{f}(s) ds - c(\pi)}.
\]

(24)

Therefore, the variation of signal and the number of bids given unobserved type \( \pi \) may help (partially) identify \( \psi(\cdot) \) function.

Parametrization on \( \psi(\cdot) \) In our empirical work, we assume \( \psi(\cdot) \) takes the parametric form:

\[
\psi(q) = -\psi_0 \exp(-q/\psi_0) + \psi_0.
\]
Note that $\psi(0) = 0$ and $\psi'(0) = 1$. Also, $h(z) = -\psi_0 \log z$, which helps us simplify the ex-post price adjustment for variable contracts as:

$$q(\pi, s; \psi_0) = -\psi_0 \log \left[ \frac{1 - \pi}{1 - \pi l(s)} \right], \quad (25)$$

and by exploiting the identity $p(\pi) \equiv p(q(\pi, s), s)$ for all $(\pi, s)$ and the parameterization of $\psi(\cdot)$, we obtain for all $s$:

$$p(\pi; \psi_0) = p \left( -\psi_0 \log \left[ \frac{1 - \pi}{1 - \pi l(s)} \right], s \right). \quad (26)$$

By equation (24), the price of a fixed-price contract with one bidder, $p(\pi, 1)$, can be written as:

$$p(\pi, 1; \psi_0) = p \left( -\psi_0 \log \left[ \frac{1 - \pi}{1 - \pi l(s)} \right], s \right) + \psi_0 \int \left[ \frac{\pi - \pi l(s)}{1 - \pi l(s)} \right] f(s) ds. \quad (27)$$

Lastly, following equations (18) and (19), the (endogenous) distribution of $\pi$ for restricted, fixed-price contracts with one bidder can be written as:

$$f_\pi(d = 1, n = 1, r = 1; \psi_0) = \frac{\psi_0 |1 - l(s)| \pi}{\varphi_1 [1 - \pi l(s)] (1 - \pi)^2 g_q \left[ \psi_0 \log \left( \frac{1 - \pi l(s)}{1 - \pi} \right) \right] |s, n = 1, r = 1|. \quad (28)$$

Note that for equations (26), (27), and (28), the right hand side depends on $s$, while the left hand side doesn’t. Under the model, they must hold regardless of the value of the signal, $s$.

Now let us define a function, $D(\tilde{\psi}_0, \pi, \pi')$, for any $\tilde{\psi}_0 > 0$, and $\pi, \pi' \in \Pi$ by:

$$D(\tilde{\psi}_0, \pi, \pi') = G_p \left[ p(\pi, 1; \tilde{\psi}_0)|n = 1, r = 1 \right] - G_p \left[ p(\pi', 1; \tilde{\psi}_0)|n = 1, r = 1 \right] \quad (29)$$

Appealing equation (20) with $n = 1$, we have $G_p \left[ p(\pi, 1)|n = 1, r = 1 \right] = 1 - F_\pi(d = 1, n = 1, r = 1)$. Therefore, we can see that $D(\cdot, \pi, \pi')$ must be zero at $\psi_0$ for any $\pi$ and $\pi'$ in $\Pi$. To estimate $\psi_0$, we use this equality evaluated at two $\pi$ values that are in the support of $\Pi$. One difficulty lies in the fact that without observing $\pi$, we do not know the support. However, we show that for any $n, \pi_n$ as defined below is always in the support of $\pi$.

$$\pi_n \equiv 1 - \{1 - E[d = 1 | n, r = 1]\}^{\frac{1}{n}}.$$

The arguments so far are summarized in the following lemma.

**Lemma 4.5** For any $n$ and $n'$, $D(\tilde{\psi}_0, \pi_n, \pi_{n'}) = 0$.

### 4.2 Identification of Remaining Primitives of the Model from Nonexclusive Contracts

We turn to the identification of the remaining components of the model that only affect contracts without entry restrictions, namely the probability of drawing a favored contractor,
\( \rho, \) the efficiency differential parameter, \( \zeta \) in (16), the extra per-bidder cost of soliciting and processing bids in non-exclusive contracts, \( \kappa, \) the probability distribution of \( \pi \) conditional on no entry restrictions, \( f_{\pi}(\cdot| r = 0), \) and the entry restriction net cost distribution, \( F_{\Pi}(\cdot). \)

With the assumption that the distribution of bid arrivals is a mixture of Poisson distributions, we identify the distribution of \( \lambda \) for nonexclusive contracts from the observe distribution of the number of bids. Then, exploiting the monotone mapping between \( \lambda \) and \( \pi \) under certain conditions, we identify \( f_{\pi}(\cdot| n, r = 0). \) Given \( f_{\pi}(\cdot| n, r = 0), \) the observed contract distribution can be written as a function of the remaining parameters of the model, \( \rho, \zeta, \) and \( \kappa. \) Finally, \( F_{\Pi} \) is identified from the procurement officer’s preferences for imposing entry restrictions.

**Lemma 4.6** Suppose Assumptions A1–A4 hold. Then if \( \lambda(\pi) \) is monotone in \( \pi, f_{\pi}(\pi| n, r = 0) \) for all \( \pi \in \Pi \) and any \( n \) can be written as:

\[
 f_{\pi}(\pi| n, r = 0) = -\frac{\lambda(\pi)n \exp(-\lambda(\pi)) g_{\lambda}(\lambda(\pi)) \lambda'(\pi)}{n! g_n(n|r = 0)}, \tag{30}
\]

where the mapping between \( \lambda \) and \( \pi, \lambda(\pi), \) is characterized by equation (15). The distribution of \( \lambda, g_{\lambda}(\cdot), \) and the moment generating function for \( \lambda, m_{\lambda}(\cdot) \) are identified from the moment generating function for the number of bids, \( m_{\lambda}(\cdot|r = 0) \):

\[
m_{\lambda}(t) = m_n[\log(1 + t)|r = 0],
\]

for all \( t > -1. \)

Note that \( \lambda(\pi) \) is a function of \( (\kappa, \rho, \zeta) \) as well as the model primitives that have been identified so far. Therefore, the above lemma implies that \( f_{\pi}(\pi| n, r = 0) \) for all \( \pi \in \Pi \) is identified up to the three remaining parameters. Once \( f_{\pi}(\pi| n, r = 0) \) is identified, the exogenous distribution of \( \pi, f_{\pi}(\pi) \) for all \( \pi \in \Pi \) is identified from \( f_{\pi}(\pi| n, r = 0) \) and \( \Pr(n, r) \), both of which are already identified.

We observe the probability of a contract resulting in a fixed price contract not exceeding \( p^* \) conditional on \( n \) bids, \( \Pr(d = 1, p \leq p^*|n, r = 0) \). As for variable price contracts, note that all differences in these contracts can be attributed to differences in \( \pi \) for favored and inefficient contractors or \( \pi_u(\pi) \) for unfavored and inefficient contractors.\(^{15}\) Let \( \tilde{\pi}(q, s) \) be defined by:

\[
 \tilde{\pi}(q, s) = \frac{1 + \psi'(q)}{1 - \psi'(q)}.
\]

Given that \( (q, s) \) is observed and \( l(s) \) is identified from the signals conditional on the efficiency type, \( \tilde{\pi}(\cdot, \cdot) \) is identified up to \( \psi(\cdot). \) Therefore, \( \Pr(d = 0, \tilde{\pi}(q, s) \leq \pi^*|n, r = 0) \), the probability of a contract resulting in a variable price contract with \( \tilde{\pi} \) being less than \( \pi^* \) conditional on \( n \) bids is identified up to \( \psi(\cdot). \)

The following lemma shows that \( \Pr(d = 1, p \leq p^*|n, r = 0) \) and \( \Pr(d = 0, \tilde{\pi}(q, s) \leq \pi^*|n, r = 0) \) can be written as a function of \( (\kappa, \rho, \zeta) \) as well as the model primitives that are already identified.

\(^{15}\)It follows from their common support that the favorability status cannot be determined from the base or the variable components of a variable contract.
Lemma 4.7 For any \( n \) and \( p^* \in [c(\pi_{\text{max}}), b(\pi_{\text{min}}, 1)] \)

\[
\Pr \{ d = 1, p \leq p^* | n \} = (1 - \rho)^n \int_{\pi_u(p^*, n)}^{\pi_{\text{max}}} \left[ 1 - (1 - \pi_u(\pi))^n \right] f_\pi(\pi | n, r = 0) d\pi 
\]

\[
+ \sum_{n_f=1}^{n} \binom{n}{n_f} \rho^{n_f}(1 - \rho)^{n-n_f} \int_{\pi_u(p^*, n, n_f)}^{\pi_{\text{max}}} \left[ 1 - (1 - \pi)^n \right] f_\pi(\pi | n, r = 0) d\pi, 
\]

\[
+ \sum_{n_f=1}^{n-1} \binom{n}{n_f} \rho^{n_f}(1 - \rho)^{n-n_f} \int_{\pi_{\text{min}}}^{\pi^*} \left[ 1 - (1 - \pi)^n \right] f_\pi(\pi | n, r = 0) d\pi,
\]

where \( \pi_u(p, n) \) is the inverse function in \( \pi \) of \( p(\pi_u(\pi), n) \), as defined in equation (10), and \( \pi_f(p, n, n_f) \) is the inverse function in \( \pi \) of \( p_f(\pi, n, n_f) \) as defined in equation (11). For any \( n \) and \( \pi^* \in \Pi \),

\[
\Pr \{ d = 0, \pi (q, s) \leq \pi^* | n, r = 0 \} = (1 - \rho)^n \int_{\pi_{\text{min}}}^{\pi_u^{-1}(\pi^*)} [1 - \pi_u(\pi)]^n f_\pi(\pi | n, r = 0) d\pi
\]

\[
+ \sum_{n_f=1}^{n} \binom{n}{n_f} \rho^{n_f}(1 - \rho)^{n-n_f} \int_{\pi_{\text{min}}}^{\pi^*} \left[ 1 - (1 - \pi)^n \right] f_\pi(\pi | n, r = 0) d\pi. 
\]

Note that the right hand side of equations (31) and (32) is identified up to \( \kappa, \rho, \) and \( \zeta \). With enough variations in the number of bids independent of unobserved type, these three parameters can be identified.

Lastly, the procurer chooses to restrict entry if the expected total cost of doing so is less than or equal to the alternative. Therefore, the entry restrictions are imposed if and only if \( \eta \leq \Omega(\pi) \) where \( \Omega(\pi) \) is the difference in the total expected costs, other than \( \eta \), with and without entry restrictions:

\[
\Omega(\pi) = \sum_{n=0}^{\infty} \lambda(\pi) n e^{-\lambda(\pi)} \sum_{n_f=0}^{n+1} \binom{n+1}{n_f} \rho^{n_f}(1 - \rho)^{n+1-n_f} T(\pi, n + 1, n_f) + \kappa(\pi) \lambda(\pi)
\]

\[- \sum_{n=0}^{\infty} \lambda_f(\pi) n e^{-\lambda_f(\pi)} T_f(\pi, n + 1) - \kappa_f(\pi) \lambda_f(\pi). \]

The probability that entry restrictions are imposed conditional on \( \pi \), \( \Pr(r = 1 | \pi) \), is \( F_\eta[\Omega(\pi)] \), which can be written as \( F_\eta[\Omega(\pi)] = f_\pi(\pi | r = 1) \Pr(r = 1) / f_\pi(\pi) \). Since we have identified all components of the model except the distribution of \( \eta \), we identify \( \Omega(\pi) \) for all \( \pi \in \Pi \) are identified. Because \( \eta \) and \( \pi \) are independently distributed by Assumption A2, \( F_\eta(\eta) \) is identified for the range of \( \Omega(\pi) \) over \( \pi \in \Pi \). Summarizing our arguments, we establish the following theorem.

Theorem 4.2 Suppose Assumptions A1–A4 hold, \( \lambda \) is monotone in \( \pi \), and \( \psi(\cdot) \) is known. Then \( f_\pi(\pi) \) for all \( \pi \in \Pi \) and \( F_\eta(\eta) \) for all \( \eta \) in the range of \( \Omega(\pi) \) over \( \pi \in \Pi \) are identified up to \( \kappa, \rho, \) and \( \zeta \). These three parameters satisfy equations (31) and (32) for any \( n, p \in [c(\pi_{\text{max}}), b(\pi_{\text{min}}, 1)] \), and \( \pi^* \in \Pi \).
5 Results

5.1 Nonparametric Estimator

We closely follow the identification arguments to construct the nonparametric estimator. We assume that all components of the model to be estimated depend on the observed characteristics of a procurement project. These characteristics include (i) the number of unique bidders that won any of the procurement contracts of the industry in the state during the period of study, (ii) the average base price of the contracts of the industry in the state during the same period, and (iii) the base duration of the project.

Before we proceed, we estimate the conditional signal probability distributions using a kernel density estimator.

\[
\hat{f}(s|x) = \frac{\sum_i d_i k_s \left( \frac{s - x}{h_s} \right) k_x \left( \frac{x_i - x}{h_x} \right)}{h_s \sum_i d_i k_s \left( \frac{x_i - x}{h_x} \right)}; \quad \hat{g}(s|x) = \frac{\sum_i (1 - d_i) k_s \left( \frac{s - x}{h_s} \right) k_x \left( \frac{x_i - x}{h_x} \right)}{h_s \sum_i (1 - d_i) k_x \left( \frac{x_i - x}{h_x} \right)},
\]

where \( k_s(\cdot) \) and \( k_x(\cdot) \) are the Gaussian kernel functions and \( (h_s, h_x) > 0 \) are bandwidths based on the “normal reference rule-of-the-thumb” approach as suggested by Silverman (1986). We denote the estimated likelihood ratio by \( \hat{l}(s|x) \):

\[
\hat{l}(s|x) = \hat{f}(s|x)/\hat{g}(s|x).
\]

Using restricted contracts only, we also estimate the cumulative density of the price of fixed-price contracts conditional on the number of bids, \( G_p(\cdot|n, r = 1, x) \), and the probability density of the absolute value of the ex-post price adjustment of variable-price contracts conditional on the signal and the number of bids, \( g_{|q}(\cdot|s, n, r = 1, x) \).

\[
\hat{G}_p(p|n, r = 1, x) = \frac{\sum_i r_i d_i 1\{n_i = n\} K_p \left( \frac{p - p_i}{h_p} \right) k_x \left( \frac{x_i - x}{h_x} \right)}{\sum_i r_i d_i 1\{n_i = n\} k_x \left( \frac{x_i - x}{h_x} \right)},
\]

\[
\hat{g}_{|q}(q|s, n, r = 1, x) = \frac{\sum_i r_i (1 - d_i) 1\{n_i = n\} k_q \left( \frac{|q_i - q|}{h_q} \right) k_s \left( \frac{s_i - s}{h_s} \right) k_x \left( \frac{x_i - x}{h_x} \right)}{h_q \sum_i r_i (1 - d_i) 1\{n_i = n\} k_s \left( \frac{s_i - s}{h_s} \right) k_x \left( \frac{x_i - x}{h_x} \right)},
\]

where \( K_p(\cdot) \) is the standard normal cumulative density function, \( k_q(\cdot) \) is the Gaussian kernel function, and \( (h_p, h_q) > 0 \) are bandwidths.

We also estimate the mapping between the base price of fixed-price, restrictive contracts, \( \bar{p} \), and the realized signal and the resulting price adjustment, \( (q, s) \), when the entry is restricted. Let the mapping denoted by \( p(\cdot, \cdot|x) \), and its estimator is

\[
\hat{p}(q, s|x) = \frac{\sum_i \bar{p}_i r_i (1 - d_i) k_q \left( \frac{|q_i - q|}{h_q} \right) k_s \left( \frac{s_i - s}{h_s} \right) k_x \left( \frac{x_i - x}{h_x} \right)}{\sum_i r_i (1 - d_i) k_q \left( \frac{|q_i - q|}{h_q} \right) k_s \left( \frac{s_i - s}{h_s} \right) k_x \left( \frac{x_i - x}{h_x} \right)}.
\]

The probability that a contract is fixed-price conditioning on the number of bids under
entry restrictions is estimated using a sample analogue. For any $n$,

$$
\Pr(d = 1|n, r = 1, x) = \frac{\sum_i r_i d_i 1\{n_i = n\} k_x \left( \frac{x_i - x}{h_x} \right)}{\sum_i r_i 1\{n_i = n\} k_x \left( \frac{x_i - x}{h_x} \right)}.
$$

Lastly, the distribution of the number of bids conditional on entry restrictions, $\Pr(n|r, x)$ is estimated using a sample analogue:

$$
\Pr(n|r, x) = \frac{\sum_i 1\{n_i = n\} 1\{r_i = r\} k_x \left( \frac{x_i - x}{h_x} \right)}{\sum_i 1\{r_i = r\} k_x \left( \frac{x_i - x}{h_x} \right)},
$$

for those with entry restrictions $(r = 1)$ or without $(r = 0)$. All parameters and functions that we estimate are conditional on observed contract characteristics, $x$. For notational brevity, $x$ is omitted in the remainder.

**Step 1: Estimating $\psi_0$ from restrictive contracts** We have shown that $\psi_0$ satisfies equation (29) for all $\pi$ and $\pi'$ in the support. Therefore, an estimator of $\psi_0(x)$ minimizes a weighted sum of squared distances as follows:

$$
\hat{\psi}_0 = \arg \min_{\psi_0 > 0} \sum_i D(\hat{\psi}_0, \hat{\pi}_1, \hat{\pi}_2)^2,
$$

where $D$ is a sample analogue of $D$ as defined in equation (29) and $\hat{\pi}_n$ is $\hat{\pi}_n = 1 - [1 - \Pr(d = 1|n, r = 1)]^{1/n}$ for any $n$.

**Step 2: Estimating the distribution of $\pi$ for restrictive contracts** From equation (25), we can solve $\pi_i$ as a function of $q_i$ and $s_i$ for each variable-price contact $i$:

$$
\hat{\pi}_i = (e^{q_i/\hat{\psi}_0} - 1)/[e^{q_i/\hat{\psi}_0} - \hat{l}(s_i)],
$$

for any $s_i$ where $l(s_i) \neq 1$. Given the estimated $\pi$'s for variable-price contracts, we estimate the support of $\pi$ by $\hat{\pi}_{\text{min}} = \min_i \hat{\pi}_i$ and $\hat{\pi}_{\text{max}} = \max_i \hat{\pi}_i$, and obtain an estimator of $f_\pi(\pi|d = 0, n, r = 1)$ for any $\pi \in \hat{\Pi} \equiv [\hat{\pi}_{\text{min}}, \hat{\pi}_{\text{max}}]$ and $n$ by:

$$
\hat{f}_\pi(\pi|d = 0, n, r = 1) = \frac{\sum_i r_i (1 - d_i) 1\{n_i = n\} 1\{\hat{l}(s_i) \notin \mathcal{N}\} k_\pi \left( \frac{x_i - x}{h_\pi} \right) k_x \left( \frac{x_i - x}{h_x} \right)}{h_x \sum_i r_i (1 - d_i) 1\{n_i = n\} 1\{\hat{l}(s_i) \notin \mathcal{N}\} k_x \left( \frac{x_i - x}{h_x} \right)},
$$

where $\mathcal{N} \subset S$ is a neighborhood of 1. In the above equation, $k_\pi(\cdot)$ is the Gaussian kernel function and $h_\pi > 0$ is a bandwidth. Then, using a sample analogue of equation (19) in Lemma 4.2, we obtain an estimator for $f_\pi(\pi|d = 1, n, r = 1)$ by:

$$
\hat{f}_\pi(\pi|d = 1, n, r = 1) = (1 - (1 - \pi)^n)(\hat{\varphi}_n (1 - \pi)^n)^{-1}\hat{f}_\pi(\pi|d = 0, n, r = 1),
$$

25
for any $\pi \in \Pi$ and $n$. An estimator for $\hat{f}_\pi(\pi|n, r = 1)$ is a weighted average of the above two conditional densities:

$$\hat{f}_\pi(\pi|n, r = 1) = \hat{f}_\pi(\pi|d = 1, n, r = 1) \Pr(d = 1|n, r = 1) + \hat{f}_\pi(\pi|d = 0, n, r = 1) \Pr(d = 0|n, r = 1).$$

The joint distribution of $(\pi, n)$ conditional on the number of bids, $\hat{f}_{\pi,n}(\pi, n|r = 1)$, is estimated by multiplying the conditional distribution $\hat{f}_\pi(\pi|r = 1)$ and the marginal bid distribution $\Pr(n|r = 1)$.

**Step 3: Estimating the restrictive contracts as a function of $\pi$**  For any $\pi \in \Pi$, $q(\pi, s)$ is estimated using a sample analogue of equation (8):

$$\hat{q}(\pi, s) = \hat{\psi}_0 \log \left[ (1 - \pi \hat{l}(s))/(1 - \pi) \right].$$

Given $\hat{\pi}_i$ for each variable contract $i$, we obtain a Nadaraya-Watson estimator of $\hat{p}(\pi)$:

$$\hat{p}(\pi) = \sum_i \hat{p}_i r_i(1 - d_i)1\{n_i = n\}1\{\hat{l}(s_i) \not\in \mathcal{N}\} k_{\pi} \left( \frac{\hat{\pi}_i - \pi}{h_n} \right) h_x \left( \frac{x_i - x}{h_x} \right).$$

Using a sample analogue of equation (20) in Lemma 4.2, we provide an estimator for $\hat{p}(\pi, n)$:

$$\hat{p}(\pi, n) = \hat{C}_{p_{\max}} \left( \int_{\pi}^{\hat{\pi}_{\max}} \hat{f}_\pi(v|d = 1, n, r = 1) dv | n, r = 1 \right).$$

**Step 4: Estimating $\kappa_r$, $c$, and $\beta$ as a function of $\pi$**  To estimate $\kappa_r(\pi)$, we first estimate $\lambda_r(\pi)$. A sample analogue of equation (21) in Lemma 4.3 is:

$$\hat{\lambda}_r(\pi) = \frac{\sum_{n=1}^{\infty} n \hat{f}_{\pi,n}(\pi, n|r = 1)}{\sum_{n=1}^{\infty} \hat{f}_\pi(\pi, n|r = 1)} - 1.$$

An estimator for $\kappa_r(\pi)$ is now defined by:

$$\hat{\kappa}_r(\pi) = \sum_{n=0}^{\infty} \left[ (1 - (1 - \pi)^{n+1}) \hat{p}(\pi, n+1) + (1 - \pi)^{n+1} \left[ \hat{p}(\pi) + \int \hat{q}(s, \pi) \hat{f}(s) ds \right] \right] \times \frac{(\hat{\lambda}_r(\pi) - n) \hat{\lambda}_r(\pi)^{n-1} e^{-\hat{\lambda}_r(\pi)}}{n!},$$

which is a sample analogue of equation (22).

To estimate $\hat{c}(\pi)$ and $\hat{\beta}(\pi)$, we incorporate the parametric form of $\psi(\cdot)$ in the equations in Lemma 4.4, and use their sample analogues:

$$\hat{c}(\pi) = \frac{1 - (1 - \pi)^n}{1 - (1 - \pi)^{n-1}} \hat{p}(\pi, n) - \frac{\pi (1 - \pi)^{n-1}}{1 - (1 - \pi)^{n-1}} \left[ \hat{p}(\pi) + \hat{\psi}_0 \int \frac{\pi - \pi \hat{l}(s)}{1 - \pi \hat{l}(s)} \hat{f}(s) ds \right],$$

$$\hat{\beta}(\pi) = \int \frac{\pi - \pi \hat{l}(s)}{1 - \pi \hat{l}(s)} \hat{f}(s) ds.$$
and
\[ \hat{\beta}(\pi) = \hat{\rho}(\pi) + \hat{\psi}_0 \int \left( \frac{[\pi - \pi \hat{l}(s)]/[1 - \pi \hat{l}(s)]}{1} \right) \hat{f}(s) ds - \hat{c}(\pi). \]

The remaining steps are on the estimation of three parameters, \( \kappa_u, \rho, \) and \( \zeta, \) as well as the endogenous distribution of \( \pi \) conditional on the number of bids for nonexclusive contracts, \( f_\pi(\pi|n, r = 0). \) This will be included in the next version of the draft.

### 5.2 Estimation Results

The estimation results in this section are based on our identification argument in the previous version of the paper. The set of assumptions used in the previous identification are slightly different from those in the current identification argument; in particular, we allow the distribution of \( \pi \) to be continuous and we allow that favored contracts may not participate in the non-exclusive competition. The updated results will be included in the next version of the draft.

We provide the estimation results conditional on the median value of the observed characteristics of a procurement project. The median values of these characteristics are (i) 5 unique bidders that won any of the procurement contracts of the industry in the state during the period of study, (ii) $0.7 million of the average base prices of the contracts of the industry during the same period, and (iii) 364 days of base duration. The average size of the total payment of such contracts is $2.7 million. This can be divided into two parts: one is the sum of the base price and the ex-post price adjustment related to signal, and the other is the ex-post adjustment related to cost shocks. The average size of the former is $2.2 million and that of the latter is $0.5 million. Note that the findings here are confined to these contracts. The results are preliminary and the standard errors will be provided in the next version of this draft.

The distribution of the delay divided by the base duration varies by type, as shown in Figure 2. The estimated liquidity cost function, \( \psi(\cdot) \), can be found in Figure 3. The liquidity cost function is estimated to be increasing and concave. Our estimates indicate that absent cost shocks, it takes $0.8 million for efficient contractors to complete a procurement project, while it takes $1.8 million for inefficient contractors, more than twice of the cost of efficient contractors.

We find that the per-bidder bid processing and solicitation cost is $30,237, and the resulting average bid processing cost per a contract is $50,327. This is about 2% of the average size of the total payment. The average number of bidders for exclusively competed contracts is 3, while that for non-exclusively competed ones is 7.3. This implies a relatively large administrative cost of holding a full and open competition.

### 5.3 Counterfactual Analyses

The estimation results point to a conclusion that imposing eligibility restrictions can be cost-effective to the government. To see this, we consider a counter-factual scenario where only favored contractors are allowed to bid. We find that by imposing eligibility restrictions to the non-exclusively competed contracts, the government can reduce the expected payment to contractors by 29%. By limiting the entry of non-favored contractors, who are ex-ante slightly less likely to be efficient than the favored counterparts, the government can induce a lower
Figure 2
Signal Distributions by Type

Figure 3
Liquidity Cost Function, $\psi(\cdot)$
price from the efficient, favored contractors than otherwise. At the same time, the government forgoes a lower price from the efficient, non-favored contractors. Which of these opposing effects dominates is partially determined by how likely it is for a non-favored contractor to win a full and open competition. We find that although the ratio of favored contractors is estimated to be 71%, the probability that a favored contractor wins a non-exclusively competed contract is 97%. Therefore, the potential cost reduction from hiring an efficient, non-favored contractor does not realize very often, which further supports the use of entry restrictions. Although the procurement cost can decrease by imposing entry restrictions, the government holds a full and open competition because of various administrative or political costs that are associated with limiting competition, which is captured by $\eta$ in the model. The nature of such costs is to be further investigated.

We study the value of discretion by contracting officers by comparing the government cost of the current regime with two alternative regimes where competition for contracts is open to all contractors. In one regime (Alternative 1 in Figure 4), the contracting officers are not allowed to offer different menus of contracts depending on the favor type but they are allowed to choose the bid arrival rate and the menu of contracts based on the unobserved type, $\pi$. In the other regime (Alternative 1 in Figure 4), no discretion is allowed to contracting officers so that the menu of contracts and the bid arrival rate are invariant to type $\pi$. We find that the current regime where the contracting officers exploit their expertise of $\pi$ and practice favoritism yields the lowest procurement cost to the government.

6 Conclusion

In this paper, we study the determinants of eligibility requirements and the number of participating bidders in government procurement auctions. To understand the effects of the restrictions of competition on the total cost of government procurement, we develop, identify, and estimate a principal-contractor model in which the government selects a contractor
to undertake a project. We consider three reasons why restricting entry could be beneficial to the government: by decreasing bid processing and solicitation costs, by increasing the chance of selecting a favored contractor and consequently reaping benefits from the favored contractor, and by decreasing the expected amount of price to the winning contractor. Using our estimates, we decompose the effects of these three sources of entry restrictions, and quantify the effects of the eligibility restrictions on the total cost of procurement.

References


Appendix

A. Proofs

A.1. Proof of Theorem 3.1

The following five lemmas collectively prove Theorem 3.1. The first lemma shows that variable price contracts are only offered in conjunction with fixed price contracts, not by themselves.

**Lemma 6.1** The equilibrium contract menu includes a fixed price contract.

**Proof.** The proof is by a contradiction argument. Suppose to the contrary that every contract on the menu is a variable price contract. Denote by \{p, q(s)\} one of the contracts on the menu. There are three cases to consider.

First, suppose \( \mathbb{E}\{(q(s))\} = \int q(s)f(s)ds > \mathbb{E}\{\psi[q(s)]\} = \int \psi[q(s)]f(s)ds \). Then, the procurer can offer an additional, fixed price contract of \( p' = p + \mathbb{E}\{\psi[q(s)]\} \). The inefficient contractor would accept the contract, but the efficient contractor will not. By strict concavity of \( \psi(\cdot) \), we have \( \mathbb{E}\{\psi[q(s)]\} < \mathbb{E}\{q(s)\} \). Therefore, the expected payoff of the procurer increases when the inefficient contractor accept the fixed price contract with any positive probability.

Second, suppose \( \mathbb{E}\{\psi[q(s)]\} < \mathbb{E}\{\psi[q(s)]\} \). The procurer can offer an additional, fixed price contract of \( p' = p + \mathbb{E}\{\psi[q(s)]\} \). The efficient contractor would accept the contract, but the inefficient contractor will not. Since \( \mathbb{E}\{\psi[q(s)]\} < \mathbb{E}\{q(s)\} \), the expected payoff of the procurer increases when the efficient contractor to accept the new contract with any positive probability.

Lastly, suppose \( \mathbb{E}\{\psi[q(s)]\} = \mathbb{E}\{\psi[q(s)]\} \). The procurer can offer instead an fixed price contract of \( p' = p + \mathbb{E}\{\psi[q(s)]\} \). Both types of contractor would accept the contract. Since \( \mathbb{E}\{\psi[q(s)]\} < \mathbb{E}\{q(s)\} \) the expected payoff of the procurer increases when either or both contractor types to accept the new contract with any positive probability.

Given Lemma 6.1, an optimal menu of contracts includes at least one fixed price contract. We show that efficient contractors never selects a variable-price contract.
Lemma 6.2 It is optimal for the procurer to offer a menu of contracts that induces the efficient contractor to select a fixed-price contract with probability one.

Proof. Suppose not; i.e., the efficient contractors select a variable-price contract with positive probability. Then by Lemma 6.1, the menu must include a fixed-price contract that is selected by inefficient contractors. In that case, the fixed-price must be $c + \beta$ so that the individual rationality constraint for the inefficient contractor is satisfied. Notice that the individual rational constraint for the efficient is satisfied with strict inequality; otherwise, the efficient contractor will select the fixed-price contract instead. Given this, the procurer's problem boils down to choosing the terms of the variable-price contract, $p$ and $q(\cdot)$ to minimize expected total transfer:

$$\phi(n) \{ p + \mathbb{E}[q(s)] \} + (1 - \phi(n))(c + \beta),$$

where $\phi(n)$ is the probability that a contractor that chooses a variable-price contract becomes a winner, subject to the incentive compatibility constraint for the efficient contractor, which is:

$$\phi(n)(p - c + \mathbb{E}\{\psi[q(s)]\}) \geq \phi(n)\beta,$$

where $\phi$ and $\overline{\phi}$ denote the subjective probability that a contractor that chooses the variable-price contract (or the fixed-price contract) wins. Since the individual rationality constraint is satisfied with strict inequality, the incentive compatibility constraint must bind. Solving for $p$ when (34) holds with equality,

$$p = \frac{\phi(n)}{\overline{\phi}(n)} \beta + c - \mathbb{E}\{\psi[q(s)]\}$$

Substituting for $p$ in (33) and simplifying we obtain:

$$c + \phi(n)\mathbb{E}\{q(s) - \psi[q(s)]\} + \beta \left(1 - \phi(n) + \phi(n)\frac{\phi(n)}{\overline{\phi}(n)}\right),$$

which is minimized with respect to $q(s)$ for each $s \in S$. Since $q(s) \geq \psi[q(s)]$ when $q(s) \leq 0$ and $q(0) = \psi[q(0)]$, $q(s) = 0$ for all $s \in S$. This leads to a contradiction. 

Lemma 6.3 If two fixed-price contracts are offered, then it is optimal to offer $c + \beta$ and $c + \pi(1-\pi)^{n-1} \beta$, where the first priority going to contractors submitting the latter and the second priority to those who submit the former.

Proof. To ensure the project is undertaken, the procurer must meet the individual rationality (henceforth IR) constraint of the inefficient contractor, and the cheapest fixed price contract meeting this constraint is $c + \beta$. To meet the incentive compatibility (IC) constraint of an efficient contractor, the procurer must offer terms that are at least as profitable as $\overline{\phi}(n)\beta$, which are the expected profits to an efficient contractor from selecting $c + \beta$. Letting $p$ denote any price that solves the IC constraint:

$$\phi(n)(p - c) \geq \overline{\phi}(n)\beta$$
Appealing to (1) and (2), this inequality can be expressed as:

\[ p - c \geq \frac{\pi (1 - \pi)^{n-1}}{1 - (1 - \pi)^n} \beta \]

which is minimized by setting \( p = c + \frac{\pi(1-\pi)^{n-1}}{1-(1-\pi)^n} \beta \). \[ \blacksquare \]

This leaves us two generic possibilities on the optimal menu of contracts. Either two fixed price contracts comprise the optimal menu, or it consists of a fixed price contract designed for efficient contractors and one or more variable contracts designed for the inefficient contractors. If the signal was of very high quality and most of the contractors were efficient, we might expect the procurer to extract all the rent from efficient contractors, and limit his losses to the risk premium paid to inefficient contractors. As proved in Theorem 3.1, this is indeed the case.

In preparation for that theorem we now define the expression:

\[ H(\pi) \equiv \int_{l(s) < \tilde{l}(\pi, M)} \psi \left( h \left[ \frac{1 - \pi}{1 - \pi l(s)} \right] \right) [\tilde{f}(s) - f(s)] ds + \psi(M) \int_{l(s) \geq \tilde{l}(\pi, M)} [\tilde{f}(s) - f(s)] ds, \]

(35)

where \( l(s) \equiv f(s)/\tilde{f}(s) \) as the likelihood ratio at signal \( s \), and \( h(\cdot) \) the inverse function of \( \psi'(\cdot) \); that is \( h[\psi'(q)] = q \). The cutoff \( \tilde{l}(\pi, M) \) is defined as:

\[ \tilde{l}(\pi, M) \equiv \frac{1}{\pi} - \frac{1 - \pi}{\pi \psi'(M)}. \]

(36)

Because \( \psi'(q) > 1 \) for any \( q < 0 \), \( \tilde{l}(\pi, M) > 1 \) for any \( \pi \in (0, 1) \) and \( M < 0 \). Since \( \psi'(q) \) is monotone decreasing in \( q \) with \( \psi'(\infty) = \varsigma \) for some \( \varsigma > 0 \) and \( \psi'(-\infty) = \infty \), \( h(x) \) is well defined on \( x \in (\varsigma, \infty) \) and decreasing in \( x \), with \( h(\varsigma) = \infty \) and \( h(\infty) = -\infty \). Also \( h(1) = 0 \) because \( \psi'(0) = 1 \). Lemma 6.4 shows that if signals are informative, then the expression \( \beta - H(\pi) \) has a unique root, denoted by \( \pi \).

**Lemma 6.4** A unique probability denoted by \( \pi \in (0, 1) \) solves \( \beta = H(\pi) \) if there exists \( \epsilon > 0 \) such that \( \gamma \equiv \Pr \{ s | l(s) \leq 1 - \epsilon \} \) is positive.

**Proof.** Note from equation (35) that \( H(0) = 0 \). For \( \pi \in (0, 1) \), \( H(\pi) > 0 \). This holds because we have the following two inequalities. First,

\[ H_1(\pi, s) \equiv \psi \left( h \left[ \frac{1 - \pi}{1 - \pi l(s)} \right] \right) [\tilde{f}(s) - f(s)] > 0, \]

for any \( s \) such that \( l(s) < \tilde{l}(\pi, M) \). This is because \( f(s) > \tilde{f}(s) \) if and only if \( h \left[ \frac{1 - \pi}{1 - \pi l(s)} \right] < 0 \). Second,

\[ \psi(M) [\tilde{f}(s) - f(s)] ds > 0, \]

33
for any \( s \) such that \( l(s) \geq \tilde{l}(\pi, M) > 1 \) because \( \psi(M) < 0 \) and \( \underline{f}(s) > \bar{f}(s) \). Now, it can be seen that \( H(\cdot) \) is strictly increasing in \( \pi \). First, notice that

\[
\frac{\partial}{\partial \pi} \tilde{l}(\pi, M) = \frac{1}{\pi^2} \left( -1 + \frac{1}{\psi'(M)} \right) < 0.
\]

Second, the following inequality holds:

\[
\frac{\partial}{\partial \pi} H_1(\pi, s) = -\psi' \left( h \left[ \frac{1 - \pi}{1 - \pi l(s)} \right] \right) \cdot h' \left( \frac{1 - \pi}{1 - \pi l(s)} \right) \left( l(s) - 1 \right)^2 > 0.
\]

Therefore, as \( \pi \) increases, \( H(\pi) \) also increases, which guarantees that \( \beta - H(\pi) \) has at most one root.

Now, if there exists \( \epsilon > 0 \) such that \( \Pr \{ s \mid \underline{f}(s) \leq \bar{f}(s) (1 - \epsilon) \} = \gamma \), then we can bound \( H(\pi) \) as follows:

\[
H(\pi) \geq \int 1 \{ \underline{f}(s) \leq \bar{f}(s) (1 - \epsilon) \} \psi \left( h \left[ \frac{1 - \pi}{1 - \pi l(s)} \right] \right) \left( \bar{f}(s) - \underline{f}(s) \right) ds
\]

\[
\geq \int 1 \{ \underline{f}(s) \leq \bar{f}(s) (1 - \epsilon) \} \psi \left( h \left[ \frac{1 - \pi}{1 - \pi (1 - \epsilon)} \right] \right) \left( \bar{f}(s) - \underline{f}(s) \right) ds
\]

\[
= \psi \left( h \left[ \frac{1 - \pi}{1 - \pi (1 - \epsilon)} \right] \right) \int 1 \{ \underline{f}(s) \leq \bar{f}(s) (1 - \epsilon) \} \left( \bar{f}(s) - \underline{f}(s) \right) ds
\]

\[
\geq \psi \left( h \left[ \frac{1 - \pi}{1 - \pi (1 - \epsilon)} \right] \right) \epsilon \int 1 \{ \underline{f}(s) \leq \bar{f}(s) (1 - \epsilon) \} \bar{f}(s) ds
\]

\[
= \epsilon \gamma \psi \left( h \left[ \frac{1 - \pi}{1 - \pi (1 - \epsilon)} \right] \right) .
\]

Taking the limit as \( \pi \to (1 - \zeta) / [1 - \zeta (1 - \epsilon)] < 1 \) yields:

\[
\lim_{\pi \to (1 - \zeta) / [1 - \zeta (1 - \epsilon)]} \epsilon \gamma \psi \left( h \left[ \frac{1 - \pi}{1 - \pi (1 - \epsilon)} \right] \right) = \epsilon \gamma \psi (h [\gamma]) = \epsilon \gamma \psi (\infty) = \infty
\]

Since \( 0 < (1 - \zeta) / [1 - \zeta (1 - \epsilon)] < 1 \) and \( 0 < \beta < \infty \), the lemma is proved. \( \blacksquare \)

Now we characterize the optimal menu of contracts when it consists of one fixed-price contract and one variable-price contract.

**Lemma 6.5** Suppose the optimal menu of contracts consists of one fixed-price contract, denoted by \( \underline{p}(\cdot, \pi) \), and one variable-price contract, denoted by \( \{ \bar{p}(\pi), q(\cdot, \pi) \} \). If \( \pi < \bar{\pi} \), then the ex-post price adjustment schedule, \( q(\cdot, \pi) \), is:

\[
q(s, \pi) = \begin{cases} 
                h \left[ \frac{1 - \pi}{1 - \pi l(s)} \right] & \text{if } l(s) < \tilde{l}(\pi, M), \\
                M & \text{otherwise},
\end{cases}
\]

where \( \tilde{l}(\pi, M) \) is defined in equation (36). If \( \pi \geq \bar{\pi} \), then \( q(s, \pi) = q(s, \bar{\pi}) \) for all \( s \). The base
price of the variable-price contract is:
\[
\overline{p}(\pi) = c + \beta - \int \psi(q(s, \pi)) \overline{f}(s) ds.
\] (38)

The price of the fixed-price contract is:
\[
p(\pi, n) = c + \pi \left( 1 - \pi \right)^{n-1} \left[ \beta - \int \psi(q(s, \pi)) [\overline{f}(s) - f(s)] ds \right].
\] (39)

**Proof.** The principal designs a menu of two contracts that minimizes the expected transfer:
\[
\left[ 1 - (1 - \pi)^n \right] \overline{p} + (1 - \pi)^n \left[ \overline{p} + \mathbb{E}(\psi(q)) - c \right].
\] (40)

subject to the constraints that efficient contractors select the fixed price contract, inefficient contractors select the variable price contract, and the winning contractor never declares bankruptcy. A necessary condition of the optimal menu is that the IR constraint the inefficient contractors holds with equality (otherwise the base price \( \overline{p} \) could be further reduced, reducing the price and strengthening the IC constraint for efficient contractors). Solving for \( \overline{p} \) yields (38). The IC constraint for efficient contractors is:
\[
\phi(n) (p - c) \geq \overline{\phi}(n) \{ \overline{p} + \mathbb{E}(\psi(q)) - c \},
\]

Substituting for \( \overline{p} \) using equation (38), \( \overline{\phi}(n) \) using (2), and \( \phi(n) \) using (1), the IC inequality simplifies to:
\[
p \geq c + \pi (1 - \pi)^{n-1} \left( \beta - \int \psi(q(s)) [1 - l(s)] \overline{f}(s) ds \right)
\] (41)

Note that the IC for the inefficient will be satisfied with strict inequality at the optimum by Lemma 6.2. Therefore, at least one of the two remaining constraints, IR and IC for the efficient contractors, must bind. Otherwise, the price of the fixed-price contract could be reduced, earning the procurer higher revenue. This leads us to consider the following three cases separately.

**Case 1: IC binds but IR does not**  Solving for \( p \) from the IC constraint, and substituting the resulting expressions for \( p \) and \( \overline{p} \), obtained from equations and (38) and (39), into the expected total cost for the procurer, we obtain:
\[
\left[ 1 - (1 - \pi)^n \right] \left\{ c + \frac{\pi (1 - \pi)^{n-1}}{1 - (1 - \pi)^n} \left[ \beta - \int \psi(q(s)) [1 - l(s)] \overline{f}(s) ds \right] \right\}
\]
\[
+ (1 - \pi)^n \left\{ c + \beta + \int [q(s) - \psi(q(s))] \overline{f}(s) ds \right\}
\]
\[
= c + (1 - \pi)^{n-1} \left\{ \beta - \pi \int \psi(q(s)) [1 - l(s)] \overline{f}(s) ds + (1 - \pi) \left\{ \int [q(s) - \psi(q(s))] \overline{f}(s) ds \right\} \right\}.
\]
The (scaled) Lagrangian for the cost minimization problem can now be expressed as:

\[
L = -\pi \int \psi(q(s))[1-l(s)]f(s)ds + (1-\pi) \int [q(s) - \psi(q(s))]f(s)ds - \int \kappa_1(s)[q(s) - M]f(s)ds,
\]

where \(\kappa_1(s) \geq 0\) denotes the Kuhn Tucker multiplier for the linear constraint \(q(s) \geq M\). The first order condition for \(q(s)\) is:

\[
-\pi \psi'(q(s))[1-l(s)] + (1-\pi) [1-\psi'(q(s))] - \kappa_1(s) = 0.
\]

Rearranging terms we obtain:

\[
\psi'[q(s)] = \frac{1-\pi - \kappa_1(s)}{1-\pi l(s)}.
\]

Note that \(q(s, \pi)\) as defined in equation (37) satisfies the above first order condition. If \(l(s) < \bar{l}(\pi, M)\), then \(q(s) = h\left[\frac{1-\pi}{1-\pi l(s)}\right] > M\) and \(\kappa_1(s) = 0\) solve equation (42). If \(l(s) \geq \bar{l}(\pi, M)\), then \(\kappa_1(s) > 0\) and \(q(s) = M\) solve the equation.

Case 2: IR binds but IC does not When IR binds, \(p = c\). Substituting for \(p\) and \(\overline{p}\), using equation (38), the expected total transfer (40) simplifies to:

\[
c + (1-\pi)^n \left\{ \beta + \int [q(s) - \psi(q(s))]f(s)ds \right\}
\]

Substituting for \(p\) in inequality (41) yields:

\[
\beta \leq \int \psi(q(s))[1-l(s)]f(s)ds
\]

Notice the solution to this problem depends on neither \(\pi\) nor \(n\). If IR binds but IC does not, then the first order condition for the Kuhn Tucker formulation is:

\[
1 - \psi'(q(s)) = \kappa_1(s)
\]

If \(q(s) > M\), then the complementary slackness condition requires \(\kappa_1(s) = 0\), and hence \(1 = \psi'(q(s))\) or \(q(s) = 0\). Therefore, either \(q(s) = M\), and the marginal benefit of imposing a harsher signal would exceed its cost were it not for the bankruptcy constraint, or \(q(s) = 0\). Let us define \(S_M\) as the set of signals such that \(q(s) = M\) and let \(\mu\) denote \(\Pr(s \in S_M)\). Note that for any \(\mu \in [0, 1]\), both IR constraints and the IC constraint for the inefficient are satisfied. The total expected transfer can now be written as

\[
c + (1-\pi)^n \{\beta + [M-\psi(M)]\mu\}.
\]

Notice that the above transfer is increasing in \(\mu\), while \(\mu = 0\) does not satisfy the IC condition for the efficient, or inequality (44). This implies that when both IR constraints bind, the IC for the efficient must bind.
Case 3: Both IR and IC bind  If (44) holds with equality, the (scaled) Lagrangian for the minimization problem can be written as:

\[
L = \int (q(s) - \psi[q(s)]) \overline{f}(s)ds - \int \kappa_1 (s) [q(s) - M] \overline{f}(s)ds + \kappa_2 \left\{ \beta - \int \psi[q(s)] [1 - l(s)] \overline{f}(s)ds \right\}.
\]

The first order condition with respect to \(q(s)\) is:

\[
1 - \psi'(q(s)) - \kappa_1 (s) - \kappa_2 \psi'[q(s)] [1 - l(s)].
\]

This can be written as:

\[
\psi'[q(s)] = \frac{1 - \kappa_1 (s)}{1 + \kappa_2 [1 - l(s)]}
\]  \hspace{1cm} (45)

Substituting for \(\kappa_2 = \pi/(1 - \pi)\) in equation (45) follows that the solution for \(q(s)\) in this case can be obtained by setting \(\pi = \overline{\pi}\) in equation (37) as required.

We have ruled out the second case, implying that the IC for the efficient always binds at the optimum. The IR constraint for the efficient contractors does not always bind, i.e.

\[
\beta - \int \psi[q(s)] [1 - l(s)] \overline{f}(s)ds = \beta - H(\pi) \leq 0,
\]

where \(H(\pi)\) is defined in equation (35). As shown in Lemma 6.4, \(\beta - H(0) = \beta > 0, H(\cdot)\) is increasing in \(\pi\), and there always exists a unique root of \(\beta - H(\pi), \overline{\pi}\). Therefore, if \(\pi < \overline{\pi}\), then the IR does not bind; otherwise, it binds. This completes the proof.  \(\blacksquare\)

We now show that the menu of contracts characterized in Lemma 6.3 is always dominated by that of Lemma 6.5 if the signals are informative. In other words, the procurer is better off exploiting the signals.

**Lemma 6.6** Suppose \(\overline{F}(s) \neq F(s)\) for some signal \(s\) in the support. Then the menu of contracts characterized in Lemma 6.5 minimizes the total expected transfer.

**Proof.** Given that there are two efficiency types, the optimal menu includes two contracts. By Lemma 6.1, we have shown that at least one of them must be fixed-price, and it is optimal to induce the efficient contractors to choose a fixed-price contract in the menu, as shown in Lemma 6.2. There are two possibilities: one is to offer two fixed-price contracts, as characterized in Lemma 6.3, and the other is to offer one fixed-price contract for the efficient and one variable-price contract for the inefficient, as characterized in Lemma 6.5. We show that the latter is cheaper than the former.

The expected total cost of offering two fixed-price contracts of Lemma 6.3, denoted by \(T^F(\pi, n)\), is:

\[
T^F(\pi, n) = (1 - \pi)^n (c + \beta) + [1 - (1 - \pi)^n] c + \pi (1 - \pi)^{n-1} \beta = c + (1 - \pi)^{n-1} \beta.
\]
Denoting by $T^V(\pi, n)$, the total cost of offering the menu of contracts of Lemma 6.5 is:

$$T^V(\pi, n) = [1 - (1 - \pi)^n] \left[ c + \frac{\pi (1 - \pi)^{n-1}}{1 - (1 - \pi)^n} \left( \beta - \int \psi [q(s)] [\overline{f}(s) - \underline{f}(s)] ds \right) + (1 - \pi)^n \left[ c + \beta + \int \{q(s) - \psi [q(s)]\} \overline{f}(s) ds \right] \right]$$

$$= c + (1 - \pi)^{n-1} \left\{ \beta + \pi \int \psi [q(s)] \underline{f}(s) ds + (1 - \pi) \int q(s) \overline{f}(s) ds \right\}.$$

Thus $T^V(\pi, n) < T^F(\pi, n)$ if and only if:

$$\pi \int \psi [q(s)] \underline{f}(s) ds + (1 - \pi) \int q(s) \overline{f}(s) ds < 0$$

This condition is satisfied if and only if:

$$T^V(\pi, 1) < T^F(\pi, n) = c + \beta$$

We complete the proof by showing that the above inequality holds. When there is only one bidder, the procurer can offer one fixed-price contract, $c + \beta$, or two contracts, one fixed-price contract and one variable-price contract. The proof is done by construction that it is less profitable to offer one fixed-price contract than a menu of two contracts.

For some $\epsilon > 0$, we define $S \equiv \{s : \overline{f}(s) - \underline{f}(s) > \epsilon\}$. Let the probability that a signal is in $S$ conditional on that the contractor is efficient as $\gamma_1$ and that conditional on that the contractor is inefficient as $\gamma_2$. If $\overline{F}(s) \neq \underline{F}(s)$ for some signal $s$ in the support, there exists $\epsilon > 0$ such that $\gamma_1 \neq 0$ and $\gamma_2 \neq 0$. Note that $\gamma_2 > \gamma_1$. For any $\delta > 0$ choose $\mu(\delta)$ for a two-part variable contract in which $p = c + \beta$ and:

$$q(s) = \begin{cases} \delta & \text{if } s \in S, \\ \mu(\delta) & \text{if } s \notin S, \end{cases}$$

where

$$\gamma_2 \psi(\delta) + (1 - \gamma_2) \psi(\mu(\delta)) = 0.$$  

Note that the above equation implies that $\mu(\delta) < 0$. Because $\psi(\cdot)$ is strictly increasing, $\mu(\delta)$ is uniquely defined by the equation:

$$\mu(\delta) = \psi^{-1} \left[ -\frac{\gamma_2}{1 - \gamma_2} \psi(\delta) \right],$$

and is twice differentiable with:

$$\mu'(\delta) = \frac{-\gamma_2}{1 - \gamma_2} \psi'(\mu(\delta)).$$
where \( \mu(0) = 0 \). The fixed contract takes the form:

\[
p = c + \beta + \gamma_1 \psi(\delta) + (1 - \gamma_1) \psi(\mu(\delta))
\]

\[
= c + \beta + \gamma_1 \psi(\delta) - (1 - \gamma_1) \left( \frac{\gamma_2}{1 - \gamma_2} \right) \psi(\delta).
\]

Note that the incentive compatibility constraint is satisfied with equality by the efficient contractor and strict inequality by the inefficient contractor because \( \gamma_1 < \gamma_2 \). Similarly, the participation constraint is satisfied with equality by the inefficient contractor and strict inequality by the efficient contractor as long as \( \delta > 0 \) is small enough. The expected price to the procurer is:

\[
E(T|\delta) = c + \beta + \pi \left[ \gamma_1 \psi(\delta) + (1 - \gamma_1) \psi(\mu(\delta)) \right] + (1 - \pi) \left[ \gamma_2 \delta + (1 - \gamma_2) \mu(\delta) \right],
\]

\[
= c + \beta + \pi \left[ \gamma_1 \psi(\delta) - \left( \frac{1 - \gamma_1}{1 - \gamma_2} \right) \psi(\delta) \right] + (1 - \pi) \left[ \gamma_2 \delta + (1 - \gamma_2) \mu(\delta) \right].
\]

We now show this expression is decreasing in the neighborhood of \( \delta = 0 \). Differentiating with respect to \( \delta \) yields:

\[
\frac{\partial E(T|\delta)}{\partial \delta} = \pi \left[ \gamma_1 \psi'(\delta) - \left( \frac{1 - \gamma_1}{1 - \gamma_2} \right) \psi'(\delta) \right] + (1 - \pi) \left[ \gamma_2 - \gamma_2 \psi'(\mu(\delta)) \right].
\]

Evaluating \( \frac{\partial E(T|\delta)}{\partial \delta} \) at \( \delta = 0 \) gives us:

\[
\frac{\partial E(T|\delta = 0)}{\partial \delta} = \pi \frac{\gamma_1 - \gamma_2}{1 - \gamma_2} < 0,
\]

which shows that a fixed-price contract fails to meet a first order necessary condition.

**A.2. Proof of Theorem 3.2**

We now extend our model to the case in which bidders are treated differentially according to whether they are favored or not. The procurer minimizes the expected price to a winning contractor given \( n_f \) favored bidders and \( n_u \) unfavored bidders. Given that unfavored contractor will receive a fixed-price contract, \( c \), the procurer’s problem is to minimize the expected price to favored contractors. Note that this problem is identical to the procurer’s problem with symmetric bidders except that the probability that the winning contractor is inefficient and favored depends not only the average efficiency and the number of favored bidders, \( (\pi, n_f) \), but also those of unfavored bidders, \( (\pi_u, n_u) \). Therefore, the optimal menu of contracts to the favored contractors consists of one fixed-price contract, \( p_f \), and one variable-contract, \( \{\tilde{p}, q(\cdot)\} \). The expected transfer is:

\[
[1 - (1 - \pi)^{n_f}] p + (1 - \pi)^{n_f} (1 - \pi_u)^{n_u} \left( \tilde{p} + E[q(s)] \right).
\]

(46)

As shown in Lemma 6.5, the IR condition must be binding for inefficient contractors must
bind at the optimum, which characterizes $p$.

$$\bar{p} = c + \beta - \mathbb{E}[\psi(q(s))].$$

We also have shown that the IC condition for the efficient contractors binds at the optimum:

$$\phi(n_f)\{p - c\} = \bar{\phi}(n_f, n_u)\{p + \mathbb{E}(\psi(q)) - c\},$$

where $\bar{\phi}(n_f)$ (or $\phi(n_f, n_u)$) denotes the probability of winning for a favored contractor if he selects the fixed-price (or variable-price) contract.

Using these definitions of $\phi(n_f)$ and $\phi(n_f, n_u)$, the price of the fixed-price contract, $p_f$, is characterized as follows:

$$p_f = c + \frac{\pi(1 - \pi)^{n_f - 1}(1 - \pi_u)^{n_u}n}{1 - (1 - \pi)^{n_f}} \left[ \beta - \int \psi(q(s)) [\bar{f}(s) - f(s)] ds \right].$$

The objective function (46) can now be written as:

$$\left[1 - (1 - \pi)^{n_f}\right] c + \frac{\pi(1 - \pi)^{n_f - 1}(1 - \pi_u)^{n_u}n}{1 - (1 - \pi)^{n_f}} \left[ \beta - \int \psi(q(s)) [\bar{f}(s) - f(s)] ds \right] + (1 - \pi)^{n_f}(1 - \pi_u)^{n_u}\left\{c + \beta + \int \{q(s) - \psi[q(s)]\} \bar{f}(s) ds\right\}.$$}

The ex-post payment adjustment $q(\cdot)$ that minimizes the above also minimizes the following:

$$-\pi \int \psi(q(s))[1 - l(s)]\bar{f}(s) ds + (1 - \pi) \int [q(s) - \psi[q(s)]] \bar{f}(s) ds.$$

Note that $\pi_u$ does not enter the above expression, and it can be seen that the optimal $q(s)$ is $q(s, \pi)$ of Lemma 6.5 for all $s$.

A.3. Proof of Lemma 4.1

(i) $|q(s, \pi)|$ and $\pi$ Recall the first order condition is:

$$\psi' [q(s, \pi)] [1 - \pi l(s)] = 1 - \pi.$$

Note that $q(s, \pi) = 0$ if $l(s) = 1$ and $q(s, \pi) > 0$ if $l(s) < 1$. Similarly $q(s, \pi) < 0$ if $l(s) > 1$. Totally differentiating the first order condition with respect to $\pi$ yields:

$$\psi'' [q(s, \pi)] \frac{\partial q(s, \pi)}{\partial \pi} [1 - \pi l(s)] - \psi' [q(s, \pi)] l(s) = -1.$$
Rearranging to make $\frac{\partial q(s, \pi)}{\partial \pi}$ the subject of the equation gives:

$$\frac{\partial q(s, \pi)}{\partial \pi} = \frac{l(s) - 1}{\psi''[q(s, \pi)] [1 - \pi l(s)]^2}.$$  

Noting $\psi''(\cdot) < 0$ it follows that $\frac{\partial q(s, \pi)}{\partial \pi} > 0$ when $l(s) < 1$ and $\frac{\partial q(s, \pi)}{\partial \pi} < 0$ when $l(s) > 1$. Therefore,

$$\frac{\partial q(s, \pi)}{\partial \pi} > 0 \text{ if } q(s, \pi) > 0,$$
$$= 0 \text{ if } q(s, \pi) = 0,$$
$$< 0 \text{ if } q(s, \pi) < 0.$$

as was to be proved.

(ii) $p(\pi, n)$ and $\pi$  
Note that

$$p(\pi, n) = c(\pi) + \frac{\pi (1 - \pi)^{n-1}}{1 - (1 - \pi)^{n}} \left[ \beta(\pi) - \int \psi(q(s)) [\bar{f}(s) - f(s)] ds \right].$$  

To show that $p'(\pi) < 0$ we consider the two expressions involving $\pi$ separately. First:

$$\frac{\partial}{\partial \pi} \ln \left[ \frac{\pi (1 - \pi)^{n-1}}{1 - (1 - \pi)^{n}} \right] = \frac{1 - n\pi - (1 - \pi)^{n}}{\pi (1 - \pi) [1 - (1 - \pi)^{n}]}$$

Note that the derivative is zero at $n = 1$ and that at $n = 2$ is $-\pi^2$, which is negative. Now suppose it is negative for all $n \in \{2, \ldots, n_0\}$. Then for $n_0 + 1$ the denominator is clearly positive and the numerator is:

$$1 - (n_0 + 1) \pi - (1 - \pi) (1 - \pi)^{n_0} < \pi (1 - \pi)^{n_0} - \pi < 0.$$  

The first inequality follows from an induction hypothesis, and the second one from the inequalities $0 < \pi < 1$. Therefore $\pi (1 - \pi)^{n-1} / (1 - \pi)^{n-1}$ is decreasing in $\pi$ for all $n > 1$.

Second, we note that:

$$\frac{\partial}{\partial \pi} \int \psi [q(s, \pi)] [1 - l(s)] \bar{f}(s) ds = \int \psi' [q(s, \pi)] \frac{\partial q(s, \pi)}{\partial \pi} [1 - l(s)] \bar{f}(s) ds$$

$$= \int (1 - \pi) \frac{\partial q(s, \pi)}{\partial \pi} \left[ \frac{1 - l(s)}{1 - \pi l(s)} \right] \bar{f}(s) ds = \int \frac{\psi''[q(s, \pi)] (1 - l(s))^2}{\psi'[q(s, \pi)] [1 - \pi l(s)]^2} \bar{f}(s) ds > 0.$$  

The second equality follows from using the first order condition to substitute out $\psi' [q(s, \pi)]$, and the third equality uses the expression we derived for $\frac{\partial q(s, \pi)}{\partial \pi}$. The last inequality appeals to the concavity of $\psi(\cdot)$. Finally note that since the participation constraint is satisfied with an inequality for the efficient bidder under Assumption 4,

$$\beta(\pi) - \int \psi [q(s, \pi)] [1 - l(s)] \bar{f}(s) ds > 0$$
for all $\pi \in \Pi$. Hence, if Assumption 4 holds, the following inequality holds as claimed.

\[
\frac{\partial}{\partial \pi} p(\pi, n) = c' + \frac{\partial}{\partial \pi} \left[ \pi (1 - \pi)^{n-1} \right] \left\{ \beta(\pi) - \int \psi [q(s, \pi)] [1 - l(s)] \mathcal{F}(s) ds \right\} + \frac{\pi (1 - \pi)^n}{1 - (1 - \pi)^n} \left\{ \beta'(\pi) - \int \psi' [q(s, \pi)] [1 - l(s)]^2 \mathcal{F}(s) ds \right\} < 0.
\]

(iii) $p_f(\pi, n, n_f)$ and $\pi$  

Note that

\[
p_f(\pi, n, n_f) = c + \frac{\pi (1 - \pi)^{n_f-1} (1 - \pi_u)^{n-n_f}}{1 - (1 - \pi)^{n_f}} \left[ \beta - \int \psi [q(s, \pi)] \{ \mathcal{F}(s) - f(s) \} ds \right].
\]

To show that $\partial p_f(\pi, n, n_f)/\partial \pi < 0$, we rewrite the above equation as

\[
p_f(\pi, n, n_f) = c + (1 - \pi_u)^{n-n_f} w(\pi, n_f).
\]

where $w(\pi, n_f) \equiv \frac{\pi (1 - \pi)^{n_f-1} (1 - \pi_u)^{n-n_f}}{1 - (1 - \pi)^{n_f}} \left[ \beta - \int \psi [q(s, \pi)] \{ \mathcal{F}(s) - f(s) \} ds \right]$. Then:

\[
\frac{\partial}{\partial \pi} p_f(\pi, n, n_f) = (1 - \pi_u)^{n-n_f} \frac{\partial}{\partial \pi} w(\pi, n_f) - (n - n_f) \pi_u'(\pi) (1 - \pi_u(\pi))^{n-n_f-1} w(\pi, n_f).
\]

We have shown that $\partial w(\pi, n_f)/\partial \pi < 0$ by showing that $\partial p(\pi, n)/\partial \pi < 0$ for any $(\pi, n)$. Furthermore, $n \geq n_f$, $\pi_u'(\pi) \geq 0$ by assumption, and $w(\pi, n_f) > 0$. This completes the proof.

A.4. Proof of Lemma 4.2

First, by exploiting the monotone relationship between $|q|$ and $\pi$ given $s$ as proved in Lemma 4.1, we have

\[
F^r_\pi(\pi|D = 0, s, n) = G_{|q|} [q(\pi, s)|s, n].
\]

Given Assumption A1 that $s$ and $\pi$ are independent, $F^r_\pi(\pi|D = 0, n) = F^r_\pi(\pi|D = 0, s, n)$ for any $s$. Therefore, the following holds for any $(\pi, s) \in \Pi \times S$ and any $n$:

\[
F^r_\pi(\pi|D = 0, n) = G_{|q|} [q(\pi, s)|s, n].
\]

By taking the derivative with respect to $\pi$:

\[
f^r_\pi(\pi|D = 0, n) = g_{|q|} [q(\pi, s)|s, n] \frac{\partial q(\pi, s)}{\partial \pi}.
\]

(47)

Now, notice that there are two expressions for the joint probability that the contract type is variable and $\pi \leq \pi^*$:

\[
\Pr \{ \pi \leq \pi^*, D = 0 | n \} = F^r_\pi (\pi^* | D = 0, n) \Pr(D = 0 | n) = \int_{\pi = \pi^*}^{\pi^*} f^r_\pi (\pi | n) (1 - \pi)^n \, d\pi,
\]

42
for any $\pi^* \in \Pi$. By taking the derivative of both sides with respect to $\pi^*$:

$$f^r_\pi(\pi^* | D = 0, n) \Pr(D = 0 | n) = f^r_\pi(\pi^* | n) (1 - \pi^*)^n.$$

Plugging the right hand side of equation (47) for $f^r_\pi(\pi | D = 0, n)$ in the above equation and solving for $f^r_\pi(\pi^* | n)$, we have

$$f^r_\pi(\pi^* | n) = \frac{\Pr(D = 0 | n)}{(1 - \pi^*)^n} g_{|q|} [q(\pi^*, s)|s, n] \frac{\partial q(\pi^*, s)}{\partial \pi}. \quad (48)$$

As for the joint probability that the contract type is fixed and $\pi \leq \pi^*$ for any $\pi^* \in \Pi$:

$$\Pr\{\pi \leq \pi^*, D = 1 | n\} = F^r_\pi(\pi^* | D = 1, n) \Pr(D = 1 | n) = \int_{\pi=\pi^*}^{\pi^*} f^r_\pi(\pi | n) [1 - (1 - \pi)^n] d\pi.$$

By taking the first order derivative with respect to $\pi^*$:

$$f^r_\pi(\pi^* | D = 1, n) \Pr(D = 1 | n) = f^r_\pi^*(\pi^* | n) [1 - (1 - \pi^*)^n].$$

Plugging the right hand side of equation (48) for $f^r_\pi(\pi^* | n)$ in the above equation and solving for $f^r_\pi(\pi^* | D = 1, n)$, we have

$$f^r_\pi(\pi^* | D = 1, n) = \frac{1 - (1 - \pi^*)^n}{\varphi_n(1 - \pi^*)^n} g_{|q|} [q(\pi^*, s)|s, n] \frac{\partial q(\pi^*, s)}{\partial \pi}. \quad (49)$$

Recall that $\varphi_n$ is the odds ratio of fixed-price contracts as opposed to variable-price contracts conditional on the number of bids.

Lastly, since $p(\pi, n)$ is monotone declining in $\pi$ for each $n$, it follows that $\pi$ induces a probability distribution function on to $p$ denoted by $G_p(p | D = 1, n)$ for each $n$ such that for all $\pi^* \in \Pi$:

$$1 - G_p[p(\pi^*, n) | n] = F^r_\pi(\pi^* | D = 1, n). \quad (50)$$

Using the identified density of $\pi$ conditional on $D = 1$ in equation (49), the above equation can be written as:

$$1 - G_p[p(\pi^*, n) | n] = \int_{\Pi}^{\pi^*} \frac{1 - (1 - \pi)^n}{\varphi_n(1 - \pi)^n} g_{|q|} [q(\pi, s)|s, n] \frac{\partial q(\pi, s)}{\partial \pi} d\pi.$$

Because $\eta$ and $\pi$ are independent by Assumption A2 and the CDF of $\pi$ is strictly increasing by Assumption A4, $F^r_\pi(\pi | n)$ is also strictly increasing in $\pi$. Therefore, $G_p^{-1}$ is defined. Solving for $p(\pi, n)$ from the above equation, we have

$$p(\pi^*, n) = G_p^{-1} \left(1 - \int_{\inf \Pi}^{\pi^*} \frac{1 - (1 - \pi)^n}{\varphi_n(1 - \pi)^n} g_{|q|} [q(\pi, s)|s, n] \frac{\partial q(\pi, s)}{\partial \pi} d\pi | n\right).$$

Notice that the range of integration over $\pi$ is independent of $n$. This is because $n$ follows a Poisson distribution.
A.5. Proof of Lemma 4.3

We identify the joint distribution of $\pi$ and $n$ from the observed number of bids or $\Pr(n|D = 1)$ for any $n$ and the identified distribution of $\pi$ conditional on contract type, $f^r_\pi (\pi | D = 1, n)$.

$$f^r_{\pi,n}(\pi, n|D = 1) = f^r_\pi (\pi | D = 1, n) \Pr(n|D = 1).$$

Given the joint distribution of $\pi$ and $n$, $\lambda_r(\pi)$ can be written as follows:

$$\lambda_r(\pi) \equiv \mathbb{E}(n|\pi) = \sum_{n=0}^{\infty} n \Pr(n|\pi) = \sum_{n=0}^{\infty} n \frac{f^r_{\pi,n}(\pi, n)}{f^r_\pi(\pi)};$$

where

$$f^r_{\pi,n}(\pi, n) = f^r_{\pi,n}(\pi, n|D = 1) \Pr(D = 1) + f^r_{\pi,n}(\pi, n|D = 0) \Pr(D = 0).$$

Finally, substituting the expression for $T_r(n + 1)$ into the first order condition for $\lambda_r$, equation (14), and making $\kappa_r$ the subject of the resulting equation yields:

$$\kappa_r(\pi) = \sum_{n=0}^{\infty} \left\{ \left[ 1 - (1 - \pi)^{n+1} \right] p(\pi, n + 1) + (1 - \pi)^{n+1} \left[ \bar{p}(\pi) + \int q(s, \pi) f(s) ds \right] \right\}$$

$$\times \frac{(\lambda_r(\pi) - n) \lambda_r(\pi)^{n-1} e^{-\lambda_r(\pi)}}{n!}.$$  

By inspection $\kappa_r(\pi)$ is identified because all the elements on the right side of the equation are identified.

A.6. Proof of Lemma 4.4

For conciseness, define the functionals $\overline{\Psi}(\pi)$ and $\Psi(\pi)$ as:

$$\overline{\Psi}(\pi) = \int \psi \left( h \left[ \frac{1 - \pi}{1 - \pi l(s)} \right] \right) \overline{f}(s) ds,$$

and

$$\Psi(\pi) = \int \psi \left( h \left[ \frac{1 - \pi}{1 - \pi l(s)} \right] \right) f(s) ds.$$

From the formulas for $\overline{p}(\pi)$ and $p(\pi)$:

$$p(\pi, n) = c(\pi) + \pi \frac{(1 - \pi)^{n-1}}{1 - (1 - \pi)^n} \left[ \beta(\pi) - \overline{\Psi}(\pi) + \Psi(\pi) \right]$$  

(51)

and:

$$\overline{p}(\pi) = c(\pi) + \beta(\pi) - \overline{\Psi}(\pi)$$  

(52)

For $n \geq 1$ we obtain, upon substituting for $\beta(\pi) - \overline{\Psi}(\pi)$ in the equation for $p(\pi, n)$ using the formula for $\overline{p}(\pi)$:

$$p(\pi, n) = c(\pi) + \pi \frac{(1 - \pi)^{n-1}}{1 - (1 - \pi)^n} \left[ \overline{p}(\pi) - c(\pi) + \Psi(\pi) \right]$$
Solving for $c(\pi)$ gives:

$$p(\pi, n) = c(\pi) + \frac{\pi (1 - \pi)^{n-1}}{1 - (1 - \pi)^n} [\bar{p}(\pi) - c(\pi) + \Psi(\pi)]$$

$$= c(\pi) \left[ \frac{1 - (1 - \pi)^{n-1}}{1 - (1 - \pi)^n} \right] + \frac{\pi (1 - \pi)^{n-1}}{1 - (1 - \pi)^n} [\bar{p}(\pi) + \Psi(\pi)]$$

The result follows from making $c(\pi)$ the subject of the equation above. To derive (4.4) we subtract (51) from (52) to obtain:

$$p(\pi) - p(\pi, n) = \beta(\pi) - \Psi(\pi) - \frac{\pi (1 - \pi)^{n-1}}{1 - (1 - \pi)^n} [\beta(\pi) - \Psi(\pi) + \Psi(\pi)]$$

$$= \left[ \frac{1 - (1 - \pi)^{n-1}}{1 - (1 - \pi)^n} \right] \beta(\pi) - \Psi(\pi) - \frac{\pi (1 - \pi)^{n-1}}{1 - (1 - \pi)^n} \Psi(\pi)$$

Formula (4.4) follows from making $\beta(\pi)$ the subject of the equation above.

**A.8. Proof of Lemma 4.5**

We show that $\pi_n$ as defined as $1 - \{1 - \mathbb{E}[d = 1 | n, r = 1]\}^{\frac{1}{n}}$ for any $n$ is always in the support of $\pi$. Note that

$$\mathbb{E}[d = 1 | n, r = 1] = \int_{\pi_{\min}}^{\pi_{\max}} [1 - (1 - \pi)^n] f_{\pi}(\pi|d = 1, n, r = 1) d\pi.$$ 

Therefore, $\pi_n$ can be written as

$$\pi_n \equiv 1 - \{1 - \mathbb{E}[d = 1 | n, r = 1]\}^{\frac{1}{n}}$$

$$= 1 - \left\{ 1 - \int_{\pi_{\min}}^{\pi_{\max}} [1 - (1 - \pi)^n] f_{\pi}(\pi|d = 1, n, r = 1) d\pi \right\}^{\frac{1}{n}}$$

$$= 1 - \left\{ \int_{\pi_{\min}}^{\pi_{\max}} (1 - \pi)^n f_{\pi}(\pi|d = 1, n, r = 1) d\pi \right\}^{\frac{1}{n}}.$$ 

Arranging the above equation:

$$(1 - \pi_n)^n = \int_{\pi_{\min}}^{\pi_{\max}} (1 - \pi)^n f_{\pi}(\pi|d = 1, n, r = 1) d\pi.$$ 

Note that $1 - (1 - \pi)^n$ is increasing in $\pi$, and the integrand on the left is over $[\pi_{\min}, \pi_{\max}]$. Given that $f_{\pi}(\cdot|d = 1, n, r = 1)$ is a density function, it can be seen that $\pi_{\min} < \pi_n < \pi_{\max}$ for each $n \in \{1, 2, \ldots\}$. 

45
A.9. Proof of Lemma 4.6

Suppose $\lambda$ is decreasing in $\pi$. Then, it follows that $\Pr\{\pi \leq \pi^\ast\} = \Pr\{\lambda \geq \lambda(\pi^\ast)\}$, or $f_\pi(\pi|r = 0) = -g_\lambda[\lambda(\pi)] \lambda'(\pi)$. Now, $f_\pi(\pi|n, r = 0)$ can be written as follows:

$$f_\pi(\pi|n, r = 0) = \frac{g_n(n|\pi, r = 0) f_\pi(\pi|r = 0)}{g_n(n|r = 0)} = \frac{f_n[\lambda(\pi)] f_\pi(\pi|r = 0)}{g_n(n|r = 0)}$$

$$= \frac{\lambda(\pi)^n e^{-\lambda(\pi)} f_\pi(\pi|r = 0)}{n! g_n(n|r = 0)}.$$  

By plugging $f_\pi(\pi|r = 0) = -g_\lambda[\lambda(\pi)] \lambda'(\pi)$, we establish equation (30).

By exploiting our modeling assumption that the distribution of bid arrivals is a mixture of Poisson distributions, we establish the relationship between the moment generating function for $\lambda$, denoted by $m_\lambda(t)$, and the moment generating function for $n$, denoted by $m_n(t)$.

$$m_n(t) \equiv \mathbb{E}[e^{nt}|r = 0] = \mathbb{E}[\mathbb{E}[e^{nt}|\lambda, r = 0]|r = 0]$$

$$= \mathbb{E}[e^{\lambda(\epsilon - 1)}|r = 0] = m_\lambda(\epsilon - 1),$$

where the third equality follows from using the moment generating function of the Poisson distribution with mean $\lambda$. Using a change of variables, we have

$$m_\lambda(t) = m_n[\log(1 + t)],$$

for all $t > -1$. Note that $g_n(\cdot|r = 0)$ is directly identified from the data, and so is $m_n(\cdot)$. Given $m_n(\cdot)$, $m_\lambda(\cdot)$ is identified using the above equation, and hence $g_\lambda(\cdot|r = 0)$ is identified.

A.10. Proof of Lemma 4.7

Let $d_f \in \{0, 1\}$ denote an indicator function for whether a fixed price contract was awarded to favored contractor (by setting $d_f = 1$) or not, and define $d_u \in \{0, 1\}$ in a similar manner for unfavored contractors. To be more specific, letting $d = d_f + d_u$:

$$\Pr\{d = 1, p \leq p^\ast|n\} = \Pr\{d_f = 1, p \leq p^\ast|n\} + \Pr\{d_u = 1, p \leq p^\ast|n\} = \sum_{n_f=1}^{n} \binom{n}{n_f} \rho^{n_f}(1 - \rho)^{n-n_f} \Pr\{d_f = 1, p \leq p^\ast|n_f, n,r = 0\}$$

$$+ \sum_{n_f=0}^{n-1} \binom{n}{n_f} \rho^{n_f}(1 - \rho)^{n-n_f} \Pr\{d_u = 1, p \leq p^\ast|n_f, n,r = 0\}.$$
When \( d_f = 1 \), then \( p = p_f(\pi, n, n_f) \), where \( p_f(\cdot, \cdot, \cdot) \) is defined in equation (11). Let us define \( \pi_f(p, n, n_f) \) as the inverse function in \( \pi \) of \( p_f(\pi, n, n_f) \). Then,
\[
\Pr \{ d_f = 1, p \leq p^* | n_f, n, r = 0 \} = \Pr \{ d_f = 1, \pi \geq \pi_f(p^*, n, n_f) | n_f, n, r = 0 \}
\]
\[
= \int_{\pi_f(p^*, n, n_f)}^{\pi_{\max}} [1 - (1 - \pi)^{n_f}] f_\pi(\pi | n, r = 0) d\pi,
\]
where the first equality follows from Lemma 4.1.

When \( d_u = 1 \), there are two cases depending on whether or not at least one favored contractor compete. If no favored contractors compete, or \( n_f = 0 \), then \( p = p(\pi_u(\pi), n) \), where \( p(\cdot, \cdot) \) is defined in equation (10). Let \( \pi_u(p, n) \) be the inverse function of \( p(\pi_u(\pi), n) \) in \( \pi \). Then,
\[
\Pr \{ d_u = 1, p \leq p^* | n_f, n, r = 0 \} = \Pr \{ d_u = 1, \pi \geq \pi_u(p^*, n) | n_f = 0, n, r = 0 \}
\]
\[
= \int_{\pi_u(p^*, n)}^{\pi_{\max}} [1 - (1 - \pi_u(\pi))^n] f_\pi(\pi | n, r = 0) d\pi,
\]
where the first equality follows from Lemma 4.1. If at least one favored contractor compete or \( n_f > 0 \) and \( d_u = 1 \), then \( p = c(\pi) \). Then,
\[
\Pr \{ d_u = 1, p \leq p^* | n_f, n, r = 0 \} = \Pr \{ d_u = 1, \pi \geq c^{-1}(p^*) | n_f, n, r = 0 \}
\]
\[
= \int_{c^{-1}(p^*)}^{\pi_{\max}} (1 - \pi)^{n_f} [1 - (1 - \pi_u(\pi))^{n-n_f}] f_\pi(\pi | n, r = 0) d\pi,
\]
where the first equality follows from Assumption A3. Plugging the above three equations in equation (53), we establish equation (31).

The probability of a contract resulting in a variable price contract with \( \tilde{\pi}(q, s) \) not exceeding \( \pi^* \) can be written as:
\[
\Pr \{ d = 0, \tilde{\pi}(q, s) \leq \pi^* | n, r = 0 \} = \sum_{n_f=0}^{n} \binom{n}{n_f} \rho^{n_f} (1 - \rho)^{n-n_f} \Pr \{ d = 0, \tilde{\pi}(q, s) \leq \pi^* | n_f, n, r = 0 \}.
\]

Both favored and unfavored inefficient contractors win procurement auctions, the latter only if no favored contractors compete. Therefore, when \( n_f > 0 \), \( \tilde{\pi}(q, s; \pi) = \pi \), and hence we have the following equation for any \( n_f > 0 \):
\[
\Pr \{ d = 0, \tilde{\pi}(q, s) \leq \pi^* | n_f, n, r = 0 \}
\]
\[
= \Pr \{ d = 0, \pi \leq \pi^* | n_f, n, r = 0 \} = \int_{\pi_{\min}}^{\pi^*} (1 - \pi)^{n_f} [1 - \pi_u(\pi)]^{n-n_f} f_\pi(\pi | n, r = 0) d\pi.
\]
When $n_f = 0$, on the other hand, $\tilde{\pi}(q, s; \pi) = \pi_u(\pi)$, thereby yielding the following equation:

$$
\Pr\{d = 0, \tilde{\pi}(q, s) \leq \pi^* | n_f = 0, n, r = 0\} = \Pr\{d = 0, \pi_u(\pi) \leq \pi^* | n_f = 0, n, r = 0\} = \int_{\pi_{\min}}^{\pi_u^{-1}(\pi^*)} [1 - \pi_u(\pi)]^n f_\pi(\pi | n, r = 0) \, d\pi,
$$

where the inverse of $\pi_u(\cdot)$ exists by Assumption A3. Therefore, combining the above two equations, we establish equation (32).