Week 8: Roller Coaster Physics

Equations and Examples

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Equations

Capacity:

$$\text{riders per hour} = \left(\frac{3600}{\text{duration in seconds}}\right) * (\text{number of riders per train}) * (\text{number of trains})$$

Converting from meters to feet:

$$\text{meters} = 3.28 * (\text{feet})$$

Converting from miles per hour to meters per second:

$$\text{meters per second} = (\text{miles per hour}) * \left(\frac{5280 \text{ feet}}{1 \text{ mile}}\right) * \left(\frac{1 \text{ meter}}{3.28 \text{ feet}}\right) * \left(\frac{1 \text{ hour}}{3600 \text{ seconds}}\right)$$

$$= (\text{miles per hour}) * 0.4472$$

Kinetic Energy (m = mass, v = velocity):

$$KE = \frac{1}{2}mv^2$$

Potential Energy (m = mass, g = gravity accel, h = height):

$$PE = g * m * h = 9.8 * m * h$$

Final velocity (v_f) based on a initial velocity (v_o) and a height change (h):

$$v_f = \sqrt{v_o^2 + 2 * 9.8 * h}$$

Height change (h) based on a initial velocity (v_o) and a final velocity (v_f):

$$h = \frac{v_f^2 - v_o^2}{2 * 9.8}$$

Circular Acceleration (r = radius, v = velocity, a = acceleration, g = g-forces):

$$a = \frac{v^2}{r} \quad \text{or} \quad g * 9.8 = \frac{v^2}{r}$$

Ideal banking with no lateral forces (b = banking, p = positive G's, l = lateral G's):

$$b = \tan^{-1}\left(\frac{l}{p}\right)$$

Capacity estimation

Vortex at Kings Island, an Arrow Dynamics multi-looper, has a duration of 2 minutes 30 seconds (so 150 seconds), runs 3 trains, and each train holds 28 riders. We want to estimate the capacity.

Begin with the equation

$$\text{riders per hour} = \left(\frac{3600}{\text{duration in seconds}}\right) * (\text{number of riders per train}) * (\text{number of trains})$$

Substitute the values in and simplify

$$\text{riders per hour} = \left(\frac{3600}{150}\right) * (28) * (3) = 2016$$

So we estimate Vortex has a capacity of 2,016 riders per hour.

In reality, Vortex has an estimated capacity of 1,600 riders per hour. Refer to the notes for why there is a discrepancy in these figures.

Speed at bottom of a drop

Nitro at Six Flags Great Adventure, a B&M hyper coaster, has a first drop of 215 feet. We will estimate that the initial speed at the top of the hill is 7 miles per hour. We want to calculate the speed at the bottom of the first drop.

First, convert units

$$215 \text{ feet} \rightarrow 65.5 \text{ meters}$$

$$7 \text{ miles per hour} \rightarrow 3.6 \text{ meters per second}$$

Now get the equation

$$v_f = \sqrt{v_o^2 + 2 * 9.8 * h}$$

Plug in our values and solve

$$v_f = \sqrt{(3.6)^2 + 2 * 9.8 * (65.5)} = 36.0$$

And finally convert units back

$$36.0 \text{ meters per second} \rightarrow 80 \text{ miles per hour}$$

So we calculate Nitro will have a speed of 80 miles per hour at the bottom of the first drop.

As it turns out, this is the reported top speed, so the calculation was quite accurate.

Height of hill after launch

Let's imagine we created a roller coaster that launches riders to 100 miles per hour. We then want the train to go up a hill that is just high enough so the train goes over the hill at 30 miles per hour (that way the train is still going relatively fast rather than crawling over the top). We want to find the height of the hill. First, convert units

$$100 \text{ miles per hour} \rightarrow 44.7 \text{ meters per second}$$

$$30 \text{ miles per hour} \rightarrow 13.4 \text{ meters per second}$$

Now get the equation

$$h = \frac{v_f^2 - v_o^2}{2 * 9.8}$$

Plug in our values and solve

$$h = \frac{(44.7)^2 - (13.4)^2}{2 * 9.8} = 92.8$$

And finally convert units back

$$92.8 \text{ meters} \rightarrow 304 \text{ feet}$$

So we calculate that the hill should be 304 feet high.

Radius of bottom of hill

Let's imagine we are designing a roller coaster which is going at 65 miles per hour at the bottom of the first drop. We have decided that we want a maximum force of 3.5 G's at the bottom of the first drop. We want to find the radius of the curve that makes up the first drop's pull up.

First, convert units

$$65 \text{ miles per hour} \rightarrow 29.1 \text{ meters per second}$$

Now get the equation

$$g * 9.8 = \frac{v^2}{r}$$

Plug in our values and solve (NOTE: Make sure that you use 2.5 for g, not 3.5, since we want to account for the 1 G supplied by gravity)

$$(2.5) * 9.8 = \frac{(29.1)^2}{r} \implies r = 34.5$$

So we calculate that our curve should have a radius of 34.5 meters (or 113 feet).

Turn radius and banking

Let's imagine we are designing a roller coaster which is going 65 miles per hour at a point in the ride where we want to have a turn. We have decided we want the lateral force in the turn (if the turn was unbanked) to be 3 G's. We want to calculate the radius of the turn, as well as the banking of the turn so that all of the forces are vertical forces and the lateral forces are 0.

First, convert units

$$65 \text{ miles per hour} \rightarrow 29.1 \text{ meters per second}$$

Now get the equation

$$g * 9.8 = \frac{v^2}{r}$$

Plug in our values and solve (unlike the previous example, we DON'T subtract gravity)

$$(3) * 9.8 = \frac{(29.1)^2}{r} \implies r = 28.8$$

For the banking, we only need the vertical forces (which are just 1) and the lateral forces (3). Get the equation

$$b = \tan^{-1}\left(\frac{l}{p}\right)$$

Now plug in and solve

$$b = \tan^{-1}\left(\frac{3}{1}\right) = 71.6^o$$

So we calculate that our turn should have a radius of 28.8 meters (or 94 feet) and a bank of 71.6 degrees.