Roller Coaster Physics: Energy and Forces

Jordan Zink

Carnegie Mellon University

jzink@andrew.cmu.edu

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Outline

- Energy, and how to design with it
- Forces, and how to design with them
- Example Design in No Limits
- Some other miscellaneous calculations

Energy

Energy

- Many types of energy, but only three we care about
  - Potential Energy
  - Kinetic Energy
  - Thermal Energy (friction)
- Allows us to relate **height** and **speed**
- Equations derived from these

$$KE = \frac{1}{2}mv^2 \qquad PE = g * m * h = 9.8 * m * h$$

  - Mass ends up not mattering
- Our equations ignore friction (what does this mean?)

Velocity after Height Change

$$v_f = \sqrt{v_o^2 + 2 * 9.8 * h}$$

- v_f is the final velocity (in m/s)
- v_o is the initial velocity (in m/s)
- h is the change in height (in m) where positive means a decrease in height

Example - Nitro

Nitro at Six Flags Great Adventure, a B&M hyper coaster, has a first drop of 215 feet. We estimate that it is going 7 mph at the top of the hill. We want to calculate the speed at the bottom of the first drop.

Example - Nitro

First, convert units

215 feet / 3.28 = 65.5m
7 mph * 0.4472 = 3.1m/s

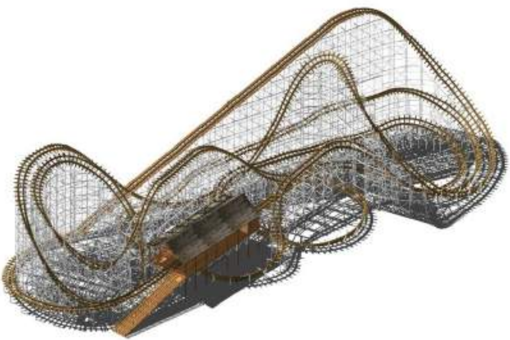
Plug in the values

$$v_f = \sqrt{(3.1)^2 + 2 * 9.8 * 65.5} = 36.0$$

Finally, convert units back

36.0m/s / 0.4472 = 80.5mph

Example - Crystal Beach Cyclone



The Crystal Beach Cyclone has a first drop of 96 feet. We estimate that it is going 6 mph at the top of the hill. We want to calculate the speed at the bottom of the first drop.

Example - Crystal Beach Cyclone

First, convert units

96 feet / 3.28 = 29.3m
6 mph * 0.4472 = 2.7m/s

Plug in the values

$$v_f = \sqrt{(2.7)^2 + 2 * 9.8 * 29.3} = 24.1$$

Finally, convert units back

24.1m/s / 0.4472 = 53.9mph

Height Change Based on Target Velocity

$$h = \frac{v_f^2 - v_o^2}{2 * 9.8}$$

- h is the change in height (in m) where positive means a decrease in height
- v_f is the final velocity (in m/s)
- v_o is the initial velocity (in m/s)

Example - Top Thrill Dragster

Top Thrill Dragster at Cedar Point, an Intamin Accelerator coaster, has a initial velocity of 120 mph. After the hill, we want the coaster to be going 30 mph. What should the change in height be?

Example - Top Thrill Dragster

First, convert units

$$120 \text{ mph * } 0.4472 = 53.7 \text{m/s}$$
$$30 \text{ mph * } 0.4472 = 13.4 \text{m/s}$$

Plug in the values

$$h = \frac{(13.4)^2 - (53.7)^2}{2 * 9.8} = -138.0$$

Finally, convert units back

$$\text{-138.0m * } 3.28 = \text{-453feet}$$

Example - Steel Dragon 2000

Steel Dragon 2000 at Nagashima Spa Land, a Morgan giga coaster, has a initial velocity of 8 mph. After the hill, we want the coaster to be going 95 mph. What should the change in height be?

Example - Steel Dragon 2000

First, convert units

$$8 \text{ mph * } 0.4472 = 3.6\text{m/s}$$
$$95 \text{ mph * } 0.4472 = 42.5\text{m/s}$$

Plug in the values

$$h = \frac{(42.5)^2 - (3.6)^2}{2 * 9.8} = 91.5$$

Finally, convert units back

$$91.5\text{m * } 3.28 = 300\text{feet}$$

Forces

Forces

- We say force but really mean acceleration (change in velocity)
- Unit of use is "G-Force", which is acceleration in terms of Earth's gravity
- Applies in all curves (turns, hills, loops, etc)
- They make coaster exciting; **very important**

G-Forces - Directions

- Positive G's (Eyes Down) - Pushes you down in your seat; bottom of a hill
- Negative G's (Eyes Up) - Lifts you out of your seat; top of a hill
- Lateral G's (Eyes Left/Right) - Pushes you to your side; turn with no banking
- Acceleration G's (Eyes Back) - Pushes you back in your seat; launch
- Deceleration G's (Eyes Front) - Train slows and you keep going forward; brakes

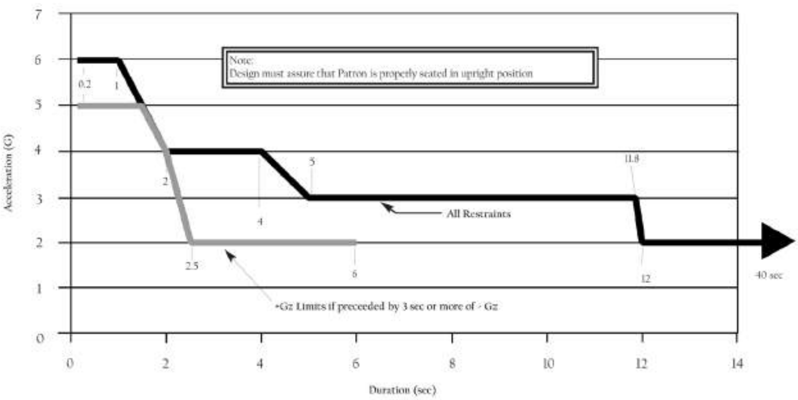
G-Forces - Safety

- Can't measure safety just by g-force; also need duration
  - 10Gs for a sustained time could easily kill you
  - When you sit down suddenly, you might temporarily feel 10Gs
- Too many positive G's cause blood to leave head
  - Tunnel vision, grey-out
  - Black out (G-LOC)
  - Brain cells start dying (lack of oxygen)
- Too many negative G's cause blood to enter head
  - "Red out"
  - Blood vessels in brain burst from pressure; stroke
  - Much worse
- Extreme G testing is actually pretty interesting

G-Force - Safety

Recommended Limits - Positive G's

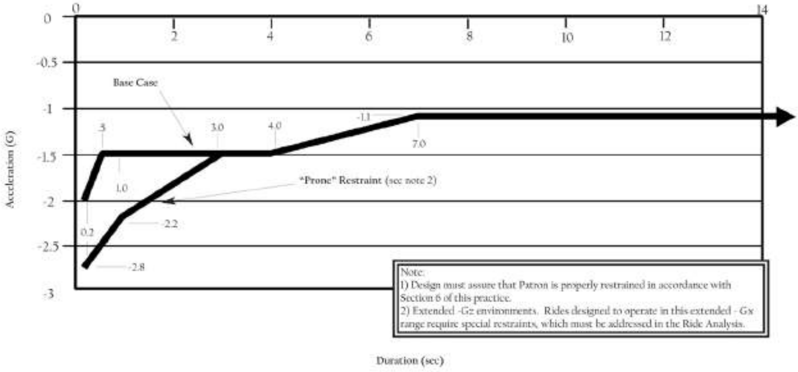
Figure 9 Time Duration Limits for +Gz (Eyes Down)



G-Force - Safety

Recommended Limits - Negative G's

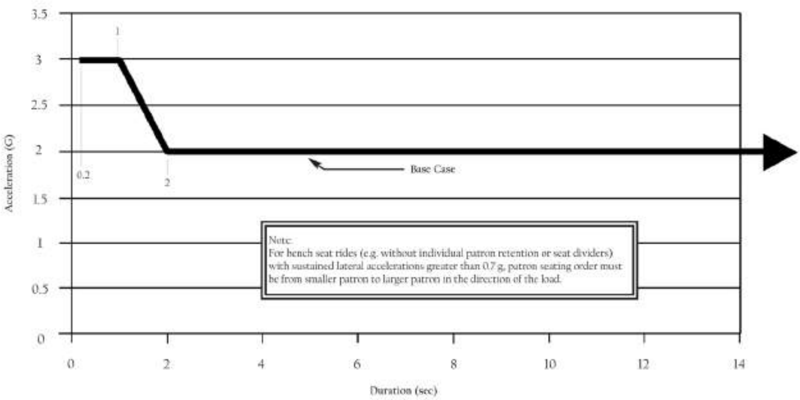
Figure 8 Time Duration Limits for -Gz (Eyes Up)



G-Force - Safety

Recommended Limits - Lateral G's

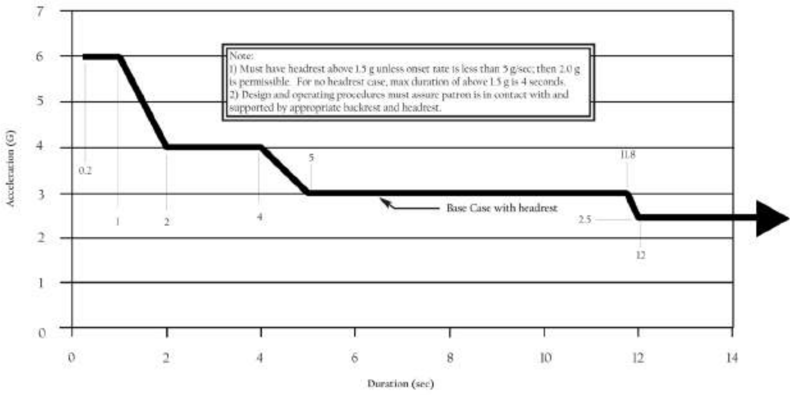
Figure 7 Time Duration Limits for +/-Gy (Eyes Left or Eyes Right)



G-Force - Safety

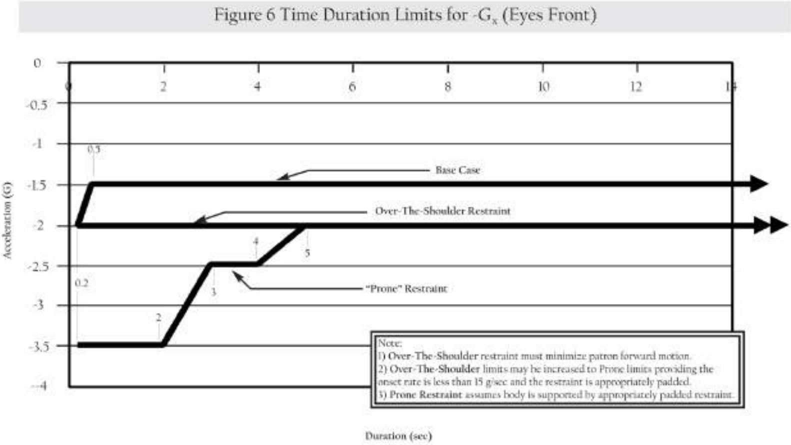
Recommended Limits - Acceleration G's

Figure 5 Time Duration Limits for $+G_x$ (Eyes Back)



G-Force - Safety

Recommended Limits - Deceleration G's

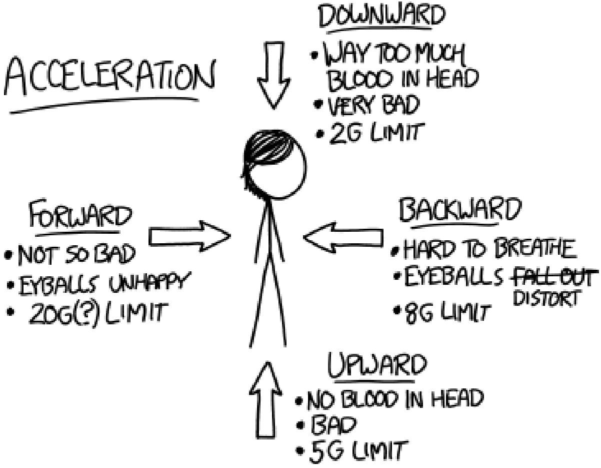


G-Forces - Recommended Limits by me

G-Force Type	Steel	Wood
Positive	3-4	2-3
Negative	-1 - -1.5	-1 - -1.5
Lateral	0	1 - 1.25
Acceleration	1	1
Deceleration	1.5	1.5

G-Force

Note that not everyone agrees on force terminology



Force in a Curve

$$a = \frac{v^2}{r} \quad \text{or} \quad g * 9.8 = \frac{v^2}{r}$$

- r is the radius of the curve (in m)
- v is the velocity around the curve (in m/s)
- a is the acceleration (in m/s$\hat{2}$)
- g is g-force

Ideal Banking (No Lateral Forces)

$$b = \tan^{-1}\left(\frac{l}{p}\right)$$

- b is the banking angle (where 0 is no banking)
- l is the lateral force
- p is the positive force

No Limits Design Example

Capacity

$$c = \left(\frac{3600}{d}\right) * r * n$$

- c is capacity (in rph, or riders per hour)
- d is duration (in seconds)
- r is the number of riders per train
- n is the number of trains