

An effective theory of initial conditions in inflation

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flat: [hep-th/0501158](#)

expanding: [hep-th/0507081](#)

energy-momentum: [hep-th/0605107](#)

Cosmology Seminar—UNC Chapel Hill

Friday, February 24, 2006

We begin with a simple question

Why are we able to explain what happens at long distances without knowing what happens at short distances?

In quantum field theory we have an answer: the details at short distances do not matter . . . at least not much!

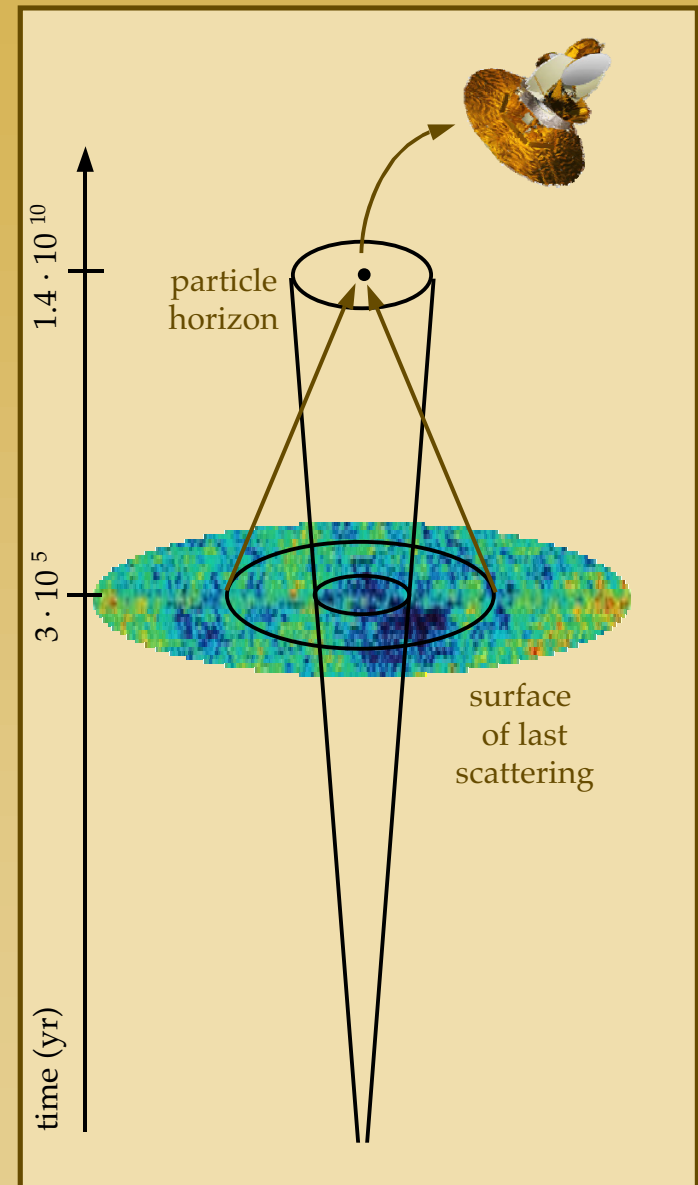
⇒ decoupling & effective field theory

Overview

- Inflation and structure formation
- The trans-Planckian problem of inflation
- An effective theory of initial conditions
- Boundary renormalization
- Observational outlook and conclusions

Inflation and structure

- Inflation began as a idea for addressing several fine-tunings that seemed to be needed in the standard cosmological picture
 - flatness problem Guth (1981), Linde (1982)
 - horizon problem
 - diluting topological relics
- Among these, the horizon problem is perhaps the most serious
- If we look back to the surface of last scattering, how large were the causally connected patches?
 - assuming a standard cosmology, radiation followed by matter domination, they are each about one degree of the sky
 - but they all look the same to about one part in 10^5



When a horizon is not the horizon

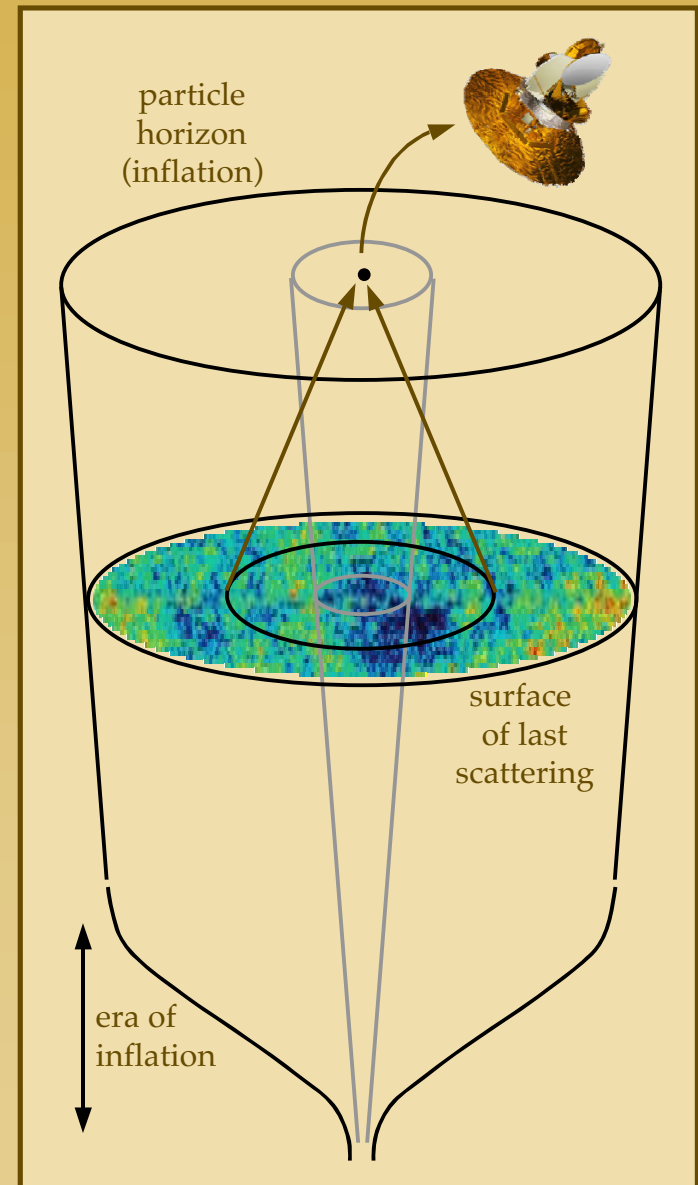
- Without inflation, this uniformity of the universe is difficult to understand
- The reason is that the distances a signal can travel from the beginning—the *particle horizon*—is too small

$$ds^2 = dt^2 - a^2(t) d\mathbf{x} \cdot d\mathbf{x} \quad r_{hor}(t) = \int_{t_0}^t \frac{dt'}{a(t')}$$

- During inflation the particle horizon and the Hubble horizon ($1/H$) can be dramatically different

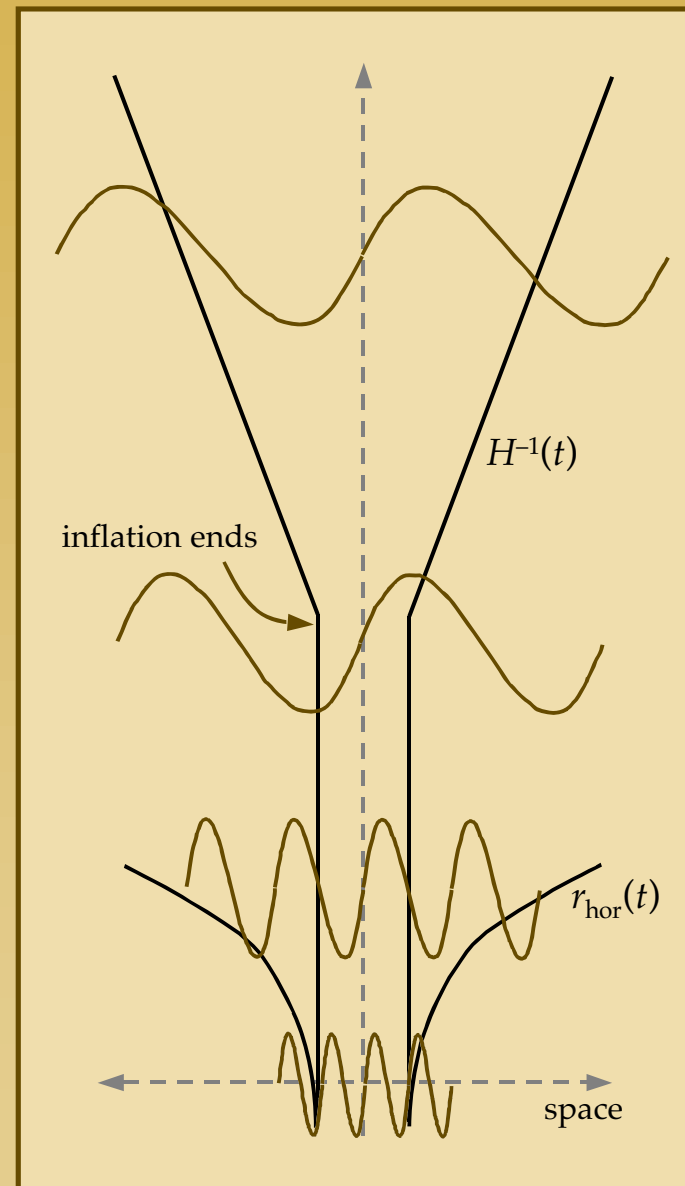
$$r_{hor}(t) = \int_{t_0}^t \frac{dt'}{a(t')} \approx \int_{t_0}^t e^{H(t-t')} dt' \approx \frac{e^{H(t-t_0)}}{H}$$

- Thus, with enough inflation, all that we see today could have grown from a single, causally connected region



Primordial perturbations

- Having an explanation of the horizon problem in inflation is interesting philosophically; but by itself, is it really testable?
- However, inflation does provide a definite prediction for the primordial perturbations
 - quantum fluctuation of a scalar field (tensor modes too)
 - rapidly stretched by expanding space-time
 - pattern essentially frozen after leaving the Hubble horizon
- This mechanism leaves a pattern of perturbations in the background that affects both matter (LSS) and radiation (CMB) in the universe
- Since its details are important for this talk, let us examine this mechanism more fully



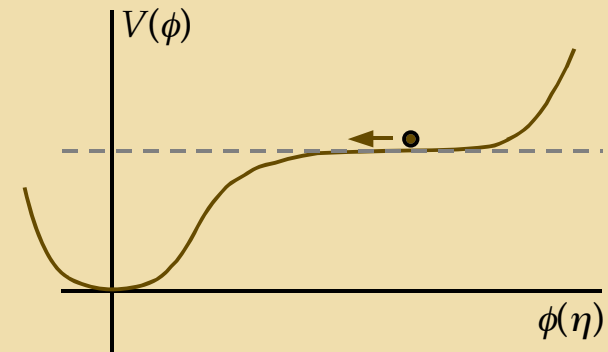
Primordial perturbations—de Sitter space

- Let us look at the simple case of a free, massless scalar field, φ , in de Sitter space
- We shall use conformally flat coordinates

$$ds^2 = \frac{d\eta^2 - d\mathbf{x} \cdot d\mathbf{x}}{H^2 \eta^2}$$

- In quantum field theory, there is always some inherent variation in the field. We shall divide φ into a term for its mean value, $\phi(\eta)$, and a term describing the fluctuations about that value, $\delta\varphi(\eta, \mathbf{x})$
- The pattern of fluctuations can be characterized by the variance of $\delta\varphi$

$$S = \int d^4x \sqrt{g} \left[\frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi \right]$$



$$\varphi(\eta, \mathbf{x}) = \phi(\eta) + \delta\varphi(\eta, \mathbf{x})$$

$$\langle 0 | \varphi(\eta, \mathbf{x}) | 0 \rangle = \phi(\eta)$$

$$\langle 0 | \delta\varphi(\eta, \mathbf{x}) \delta\varphi(\eta, \mathbf{y}) | 0 \rangle \neq 0$$

Primordial perturbations—de Sitter space

- The power spectrum is the Fourier transform of this two-point function
- Expand $\delta\varphi$ in its operator eigenmodes

$$\delta\varphi = \int \frac{d^3\mathbf{k}}{(2\pi)^3} \left[\delta\varphi_k e^{i\mathbf{k}\cdot\mathbf{x}} a_{\mathbf{k}} + \delta\varphi_k^* e^{-i\mathbf{k}\cdot\mathbf{x}} a_{\mathbf{k}}^\dagger \right]$$

where the time-dependent part satisfies the Klein-Gordon equation

- How then do we choose the mode functions, $\delta\varphi_k$?
 - It is made up of the two independent solutions to the Klein-Gordon equation
 - One further constraint comes from the canonical equal-time commutation relation
 - so the question is, how do we choose f_k ?

$$\begin{aligned} & \langle 0 | \delta\varphi(\eta, \mathbf{x}) \delta\varphi(\eta, \mathbf{y}) | 0 \rangle \\ &= \int \frac{d^3\mathbf{k}}{(2\pi)^3} e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})} \frac{2\pi^2}{k^3} P_k^{\delta\varphi}(\eta) \end{aligned}$$

$$\delta\varphi_k'' - \frac{2}{\eta} \delta\varphi_k' + k^2 \delta\varphi_k = 0$$

$$\delta\varphi_k = \frac{\delta\varphi_k^0 + f_k \delta\varphi_k^{0*}}{\sqrt{1 - f_k f_k^*}}$$

$$\delta\varphi_k^0(\eta) = \frac{iH}{k\sqrt{2k}} (1 + ik\eta) e^{-ik\eta}$$

Choosing the vacuum state

- At very short distances, $\ll 1/H$, the background curvature is not very apparent and space-time looks flat
- Therefore a natural choice is that state that matches with the flat space vacuum as $k \rightarrow \infty$ with η fixed
 - this choice fixes $f_k = 0$
- At some stage we might worry about some of our underlying assumptions
 - $H \ll k \ll M_{\text{pl}}$
 - sometimes η is taken to ∞
 - simple dynamics / other fields
- We have encountered the question posed at the very beginning:
 - how do we know what happens at very short length scales (or any scale $< 1/M_{\text{pl}}$)?

$$\delta\varphi_k = \delta\varphi_k^0 = \frac{iH}{k\sqrt{2k}} (1 + ik\eta)e^{-ik\eta}$$

$$P_k^{\delta\varphi}(\eta) = \frac{k^3}{2\pi^2} |\delta\varphi_k^0|^2$$

$$P_k^{\delta\varphi}(k\eta \rightarrow 0) = \frac{H^2}{4\pi^2}$$

If we assume that—to some degree—these details decouple, the leading result should be that given by this “vacuum”

Primordial perturbations from the vacuum

- What then are the leading predictions for the primordial spectrum of scalar and tensor perturbations?
- In the limit where the modes have been stretched outside the Hubble horizon, the power spectrum is flat
 - this sets the primordial pattern of perturbations in the background (using coordinate-invariant fields)
 - the power spectrum of the primordial gravity waves is similar
- Once we have a set of primordial perturbations, we can solve its evolution using well understood physics
 - Einstein & Boltzmann equations: matter (structure formation) and radiation (the CMB)

$$P_k^{\delta\varphi} \approx \frac{H^2}{4\pi^2}$$

$$P_k^h \approx 16\pi G \frac{H^2}{4\pi^2}$$

$$\Theta(\eta, \mathbf{x}) = \frac{\delta T_{\text{rad}}}{T_{\text{rad}}}$$

$$\delta(\eta, \mathbf{x}) = \frac{\delta\rho_{\text{dm}}}{\rho_{\text{dm}}}$$

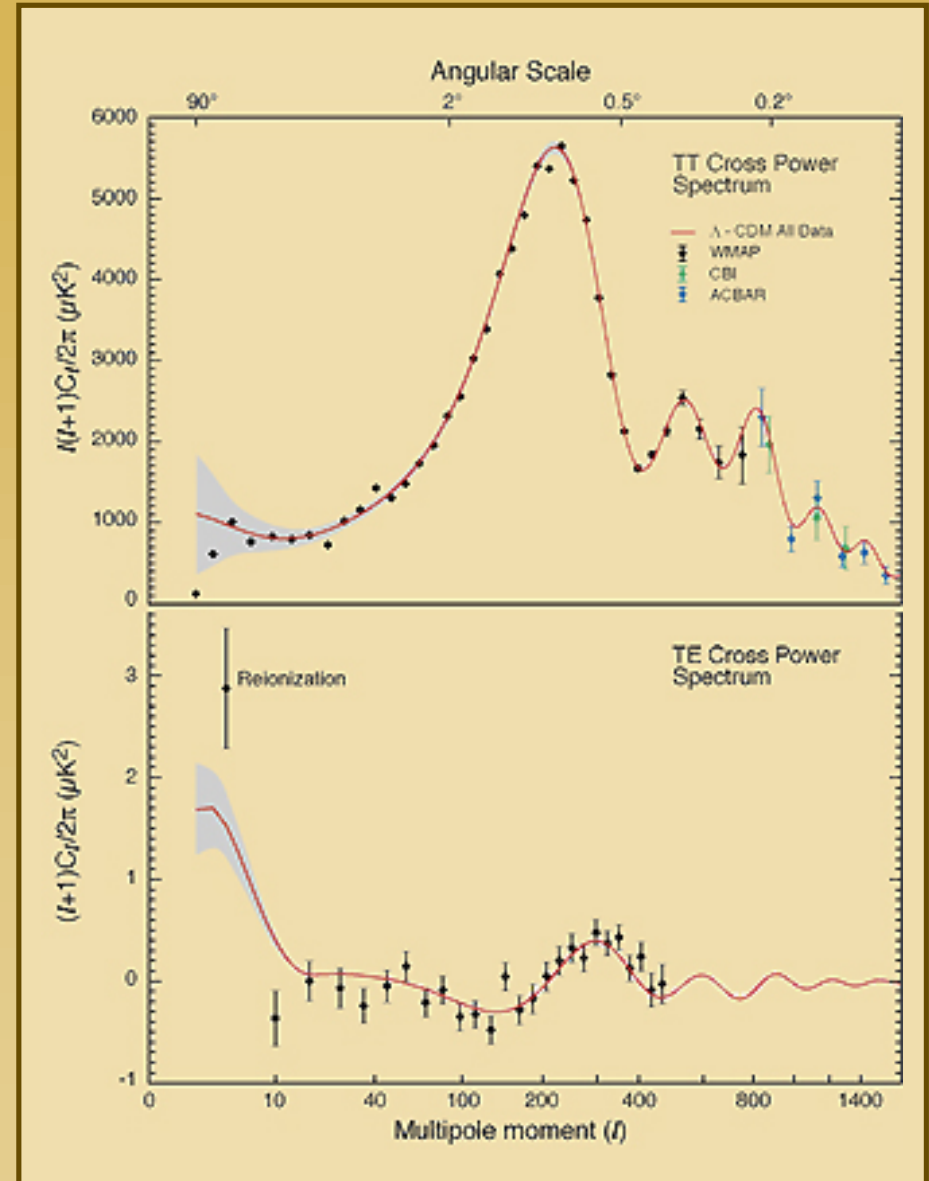
primordial

$$C_l = 4\pi \int_0^\infty \frac{dk}{k} P_k \left| \frac{\Theta_{l,k}}{\delta_k} \right|^2$$

evolution

The microwave background and inflation

- Of course, the power spectrum will not be perfectly flat, but will reflect some of the properties of the potential
- What is the leading form of the power spectrum of the CMB according to inflation?
 - nearly scale invariant (models)
 - nearly Gaussian
 - synchronized acoustic oscillations
 - correlations on super-Hubble horizon scales
- In particular, to leading order our choice of the standard vacuum seems to have been justified
 - but can we understand *why* this is so?



Conceptual problems of inflation

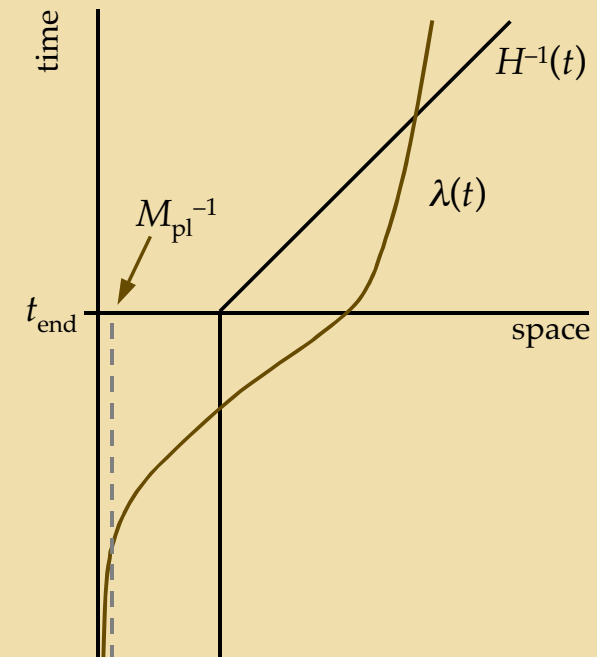
- So inflation has passed some very non-trivial tests
 - in particular the correlation of the polarization and the temperature on super-Hubble scales
- But as this subject moves into a more mature phase, we must begin to address some conceptual problems inherent within this picture
Brandenberger
 - hierarchy / amplitude problem (fine-tune)
 - trans-Planckian problem (an opportunity)
 - cosmological constant problem (fine-tune)
 - singularity problem (only as a final theory)
 - back-reaction

as well as to begin to determine the signatures from sub-leading effects

- inflation potential, new physics, . . .

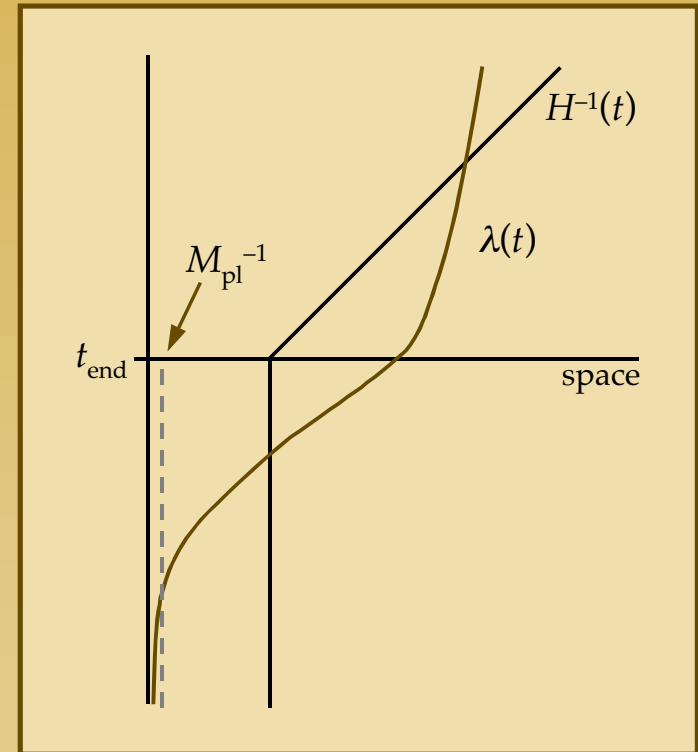
$$V(\phi)/\Delta\phi^4 \leq 10^{-12}$$

$$L_g = M_{\text{pl}}^4 \frac{\Lambda}{M_{\text{pl}}^2} + M_{\text{pl}}^2 R$$



The trans-Planckian problem

- In flat space as long as some sort of decoupling holds, we do not need usually to worry about details at the Planck length
- However, the expansion of the background means that what may be a large scale in the primordial background was smaller and smaller the earlier we follow it back during inflation
- So some perturbation that produces, for example, a feature in the CMB was much smaller when it arose during inflation
 - 60–70 e -folds to solve the horizon problem
 - a bit more and the wavelength of that mode would have been smaller than the Planck length at some time
- Perhaps the CMB/LSS could be used as a cosmic microscope



The trans-Planckian problem—two philosophies

- Suppose that you have a model for what happens above the Planck-scale
- Then you should be able to solve for the true vacuum state and see how the details alter the primordial spectrum
 - modified uncertainty relation
 - cut-off α -states
 - modified dispersion relations
 - minimal length scale
- Limitations
 - any particular model may not be right
 - how general are the conclusions of a particular case (amplitude and shape)
- The other approach is to find an effective theory description
 - for the evolution (H^2/M^2)
 - for the state (H/M)

- Brandenberger & J. Martin, 2001–2003
- Easter, Greene, Kinney, & Shiu, 2001–2002
- Mersini, 2001
- Niemeyer & Kempf, 2001
- Starobinsky, 2001
- Danielsson, 2002
- Goldstein & Lowe, 2003
- Collins & M. Martin, 2004
- and others

- Kaloper, Kleban, Lawrence, Shenker, & Susskind, 2002
- Burgess, Cline, Lemieux, & Holman, 2003

- Greene, Schalm, Shiu, & van der Schaar, 2004–2005
- Collins & Holman, 2005

An effective initial state—boundary conditions

- Let us return to the point where we chose a particular initial state
- We shall examine the case of flat space
 - the regime in which the new effects will appear should be at much shorter lengths than the Hubble horizon
 - so if an effective description is applicable to a cosmological background, it should also be able to be formulated for flat space
- Earlier we mentioned that a state is defined up to one k -dependent constant of integration
- For definiteness, let us define our state by imposing an initial condition at $t = t_0$ and evolve forward

$$S = \int d^4x \left[\frac{1}{2} \partial_\mu \varphi \partial_\nu \varphi - \frac{1}{2} m^2 \varphi^2 \right]$$

$$\varphi = \int \frac{d^3\mathbf{k}}{(2\pi)^3} \left[\varphi_k(t) e^{i\mathbf{k}\cdot\mathbf{x}} a_{\mathbf{k}} + \varphi_k^*(t) e^{-i\mathbf{k}\cdot\mathbf{x}} a_{\mathbf{k}}^\dagger \right]$$

$$\frac{d^2 \varphi_k}{dt^2} = -\omega_k^2 \varphi_k, \quad \omega_k = \sqrt{k^2 + m^2}$$

$$\varphi_k = \frac{\varphi_k^0 + f_k \varphi_k^{0*}}{\sqrt{1 - f_k f_k^*}}$$

$$\varphi_k^0(t) = \frac{e^{-ik(t-t_0)}}{\sqrt{2\omega_k}}$$

$$\frac{d\varphi_k}{dt}(t_0) = -i\omega_k \varphi_k(t_0)$$

An effective initial state—short-distance structure

- Notice that this initial condition includes the standard vacuum state, $\varpi_k = \omega_k$
- In an effective theory, there is always an inherent error between predictions based on our theory and those of a better description of nature
 - e.g. Feynman–Gell-Mann ($V - A$) theory compared with electroweak theory
- If we could solve for the “true vacuum” it might not be the same as our low energy idea of the vacuum; an effective state parameterizes this difference
 - non-localities?
 - strongly interacting gravity?
 - non-commutative space-time?
- to our “vacuum” this difference appears as new short-distance structure

$$\frac{d\varphi_k}{dt}(t_0) = -i\varpi_k\varphi_k(t_0)$$

$$\varpi_k = \omega_k \frac{1 - f_k}{1 + f_k}$$

for vacuum, 0

$f_k =$ “IR important”

+ “UV important”

possible for $k > M$

$$f_k = \sum_{n=1}^{\infty} c_n \frac{k^n}{M^n}$$

An effective initial state—propagation

- Next we must determine how this information in the initial state propagates
- The propagator, which is the Green's function associated with a point source, will include new terms representing the effect of the short-distance features of the state
 - Since the state is defined at an initial time, it is natural that we should include only a term for the forward propagating modes
 - avoids pinched singularities
 - both forward and backward propagating modes are included for the point source
- Note that this Green's function is also consistent with the initial condition, which is again defined for the modes

$$G_F(x, x') = \int \frac{d^3\mathbf{k}}{(2\pi)^3} e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}')} G_k(t, t')$$

$$\begin{aligned}
 -iG_k(t, t') = & \Theta(t - t') \varphi_k^0(t) \varphi_k^{0*}(t') \\
 & + \Theta(t' - t) \varphi_k^{0*}(t) \varphi_k^0(t') \\
 & + f_k^* \varphi_k^0(t) \varphi_k^0(t')
 \end{aligned}$$

point source
initial state "structure"

$$\begin{aligned}
 \partial_t G_k(t, t') \Big|_{t=t_0} &= i\varpi_k^* G_k(t_0, t') \\
 \partial_{t'} G_k(t, t') \Big|_{t'=t_0} &= i\varpi_k^* G_k(t, t_0)
 \end{aligned}$$

An aside on image sources

- The large degree of symmetry of Minkowski space permits a geometric interpretation for the extra term in the propagator
 - we define an image time, reflected across the initial time hypersurface
- Then the propagator can be defined as the Green's function associated with two sources, one of which is in the unphysical region before t_0
 - only one of the Θ functions contributes in the physical region
- A general expanding background may not have an analogous geometric interpretation, but otherwise the propagator is a simple generalization of what we had before

$$\text{image time: } t_I = 2t_0 - t$$

$$G_k(t, t') = G_k^0(t, t') + f_k^* G_k^0(t_I, t')$$

Vacuum ($f_k = 0$) propagator:

$$\begin{aligned} G_F^0(x, x') &= \int \frac{d^3 \mathbf{k}}{(2\pi)^3} G_k^0(t, t') \\ &= \int \frac{d^4 k}{(2\pi)^4} \frac{i}{k^2 - m^2 + i\epsilon} \end{aligned}$$

Renormalization—general properties

- Although our ultimate goal is to calculate the trans-Planckian signal in the CMB, this talk will focus on establishing the renormalizability of the effective state approach
- In ordinary field theory, loops appear in perturbative corrections to processes
 - these contain intermediate propagators of arbitrarily large momenta
 - in some cases, in summing over this short-distance behavior we encounter divergences
 - these are absorbed by rescaling the parameters of the theory (counterterms)
- For an general initial state, a loop will also introduce sums of over the short-distance structure of the state
 - new divergences & *boundary counterterms*

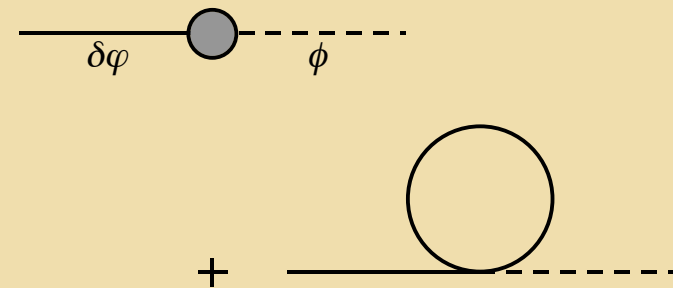
Renormalization—quartic theory

- Let us consider a specific example, a theory with a quartic interaction
- We shall consider a baby version of inflation
 - calculate in flat space
 - include an isotropic zero mode
 - the full Robertson-Walker case is considered in hep-th/0507081
- We impose the renormalization condition that the tadpole of the fluctuation should vanish
 - calculate to one loop order
 - isolate classes of divergences
 - determines renormalization scale dependence

$$L = \frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi - \frac{1}{2} m^2 \varphi^2 - \frac{1}{24} \lambda \varphi^4$$

$$\varphi(t, \mathbf{x}) = \phi(t) + \delta\varphi(t, \mathbf{x})$$

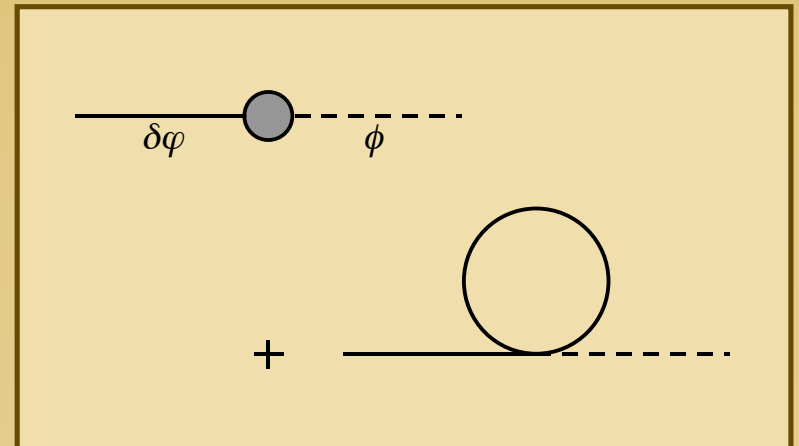
$$\langle 0_{\text{eff}} | \delta\varphi(t, \mathbf{x}) | 0_{\text{eff}} \rangle = 0$$



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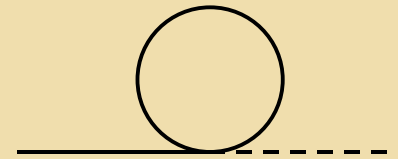
$$L = \frac{1}{2} \partial_\mu \delta\varphi \partial^\mu \delta\varphi - \frac{1}{2} \left[m^2 + \frac{1}{2} \lambda \phi^2 \right] \delta\varphi^2$$
$$- \left[\nabla^2 \phi + m^2 \phi + \frac{1}{6} \lambda \phi^3 \right] \delta\varphi$$
$$- \frac{1}{6} \lambda \phi \delta\varphi^3 - \frac{1}{24} \lambda \delta\varphi^4$$



Renormalization of the leading trans-Planckian effect

- The boundary divergences fall into two classes
 - from IR features \Rightarrow relevant/marginal *boundary* counterterms
 - from UV (trans-Planckian) features \Rightarrow irrelevant boundary counterterms
- The idea is that we determine the state by choosing the long distance features empirically and allowing a general set of short distance features
 - consider a leading example of the latter
 - use ω_k to simplify integrals
- At one loop order we find two new divergences which are confined to the initial surface
 - cancel these with boundary counterterms

$$f_k = \sum_{n=1}^{\infty} c_n \frac{k^n}{M^n} \rightarrow c_1 \frac{\omega_k}{M} + \dots$$



$$S_{\text{bnd}} = \int d^3 \mathbf{x} \left[\frac{z_1}{2} \frac{m^2}{M} \varphi^2 + \frac{z_3}{4} \frac{1}{M} \varphi^4 \right]$$

irrelevant boundary counterterm 

Boundary renormalization group running

- The coefficients of the boundary counterterms are determined by a renormalization prescription ($\overline{\text{MS}}$)
 - infinite, scale-independent part to cancel $1/\varepsilon$ pole (and $-\gamma$, $\ln 4\pi$, etc.)
 - finite, scale dependent part satisfies the Callan-Symanzik equation
- Note that the Callan-Symanzik equation applies to both bulk and boundary divergences
 - there is universal cut-off that applies to both the large spatial momenta and infinitesimal time intervals from t_0
 - the equivalence of bare and renormalized n -point functions means that the derivatives of both, with respect to the renormalization scale μ , vanish
- For the leading trans-Planckian effect:

$$S_{\text{bnd}} = \int d^3 \mathbf{x} \left[\frac{z_1}{2} \frac{m^2}{M} \varphi^2 + \frac{z_3}{4} \frac{1}{M} \varphi^4 \right]$$

$$\left[\mu \frac{\partial}{\partial \mu} + \beta \frac{\partial}{\partial \lambda_R} + m_R \gamma_m \frac{\partial}{\partial m_R} + \beta_1 \frac{\partial}{\partial z_1} + \beta_3 \frac{\partial}{\partial z_3} + \dots \right] G^1 = 0$$

$$\beta_1(\lambda_R) = \mu \frac{dz_1}{d\mu} = \frac{2ic_1^* \lambda_R}{\pi^2} + \dots$$

$$\beta_3(\lambda_R) = \mu \frac{dz_3}{d\mu} = \frac{ic_1^* \lambda_R^2}{\pi^2} + \dots$$

Renormalization of a state and its evolution

- Let us summarize generally what we have found, both for a flat and a completely general Robertson-Walker background

	IR/long distance			UV/short distance		
	structure	renormalization		structure	renormalization	
		operators	examples		operators	examples
bulk (evolution)	observed long distance degrees of freedom	relevant, marginal (dim ≤ 4)	$\nabla_\mu \varphi \nabla^\mu \varphi,$ $\varphi^2, \varphi^4,$ $R\varphi^2$	completely free, up to assumed symmetries of background	irrelevant (dim > 4)	$(\nabla_\mu \varphi \nabla^\mu \varphi)^p,$ $\varphi^6, \varphi^8,$ $R^2\varphi^2, \dots$
boundary (state)	appropriate state of long distance effective free theory	relevant, marginal (dim ≤ 3)	$\varphi^2,$ $\varphi \nabla_n \varphi, K\varphi^2$	completely free, up to assumed symmetries of state	irrelevant (dim > 3)	$\varphi^4, (\nabla_n \varphi)^2,$ $\nabla_i \varphi \nabla^i \varphi,$ $K^2\varphi^2, \dots$

- Here, $\nabla_n = n^\mu \nabla_\mu$ is a derivative normal to the initial surface and $K_{\mu\nu} = h_\mu^\lambda \nabla_\lambda n_\nu$ is the extrinsic curvature along the surface

Further work

- So we find an elegant correspondence between the long and short distance features of the initial state and the sorts of operators that appear in their renormalization
- This is still rather a young subject so there are many aspects which should be studied further
 - back-reaction (size of effect, types of operators that appear)
 - RG flow (de Sitter space?)
 - decoherence of quantum effects
 - generating effective states by integrating out excited heavy fields
 - calculation of the amplitude and the generic shape of the trans-Planckian correction to the power spectrum
 - ...

Back-reaction and naturalness:

- Porrati, 2004–2005
- Greene, Schalm, Shiu, & van der Schaar, 2004–2005

Somewhat related work on RG flows in de Sitter space:

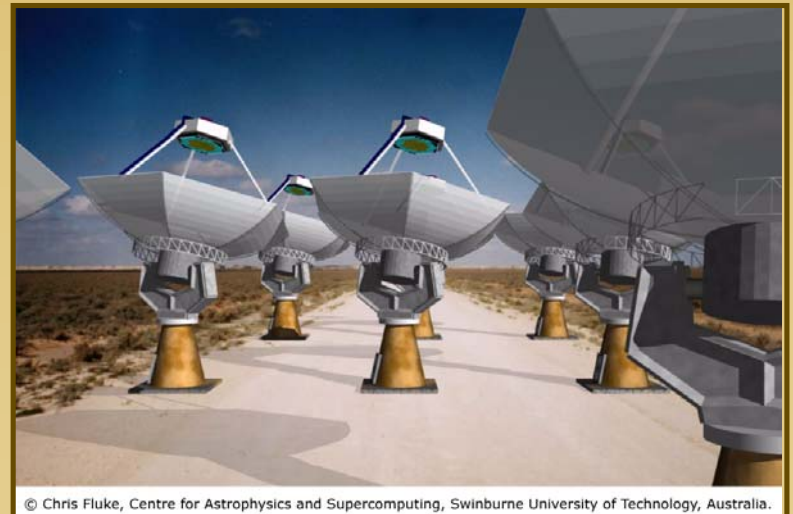
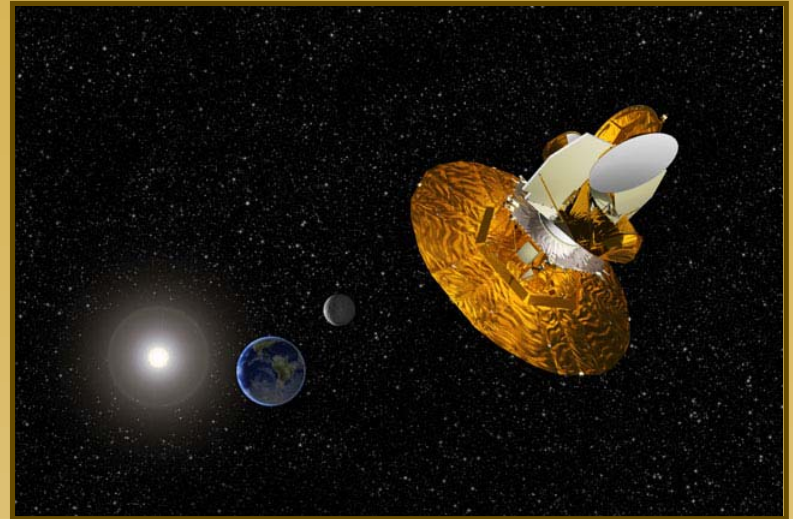
- Larsen & McNees, 2003–2004

Fits to the CMB data:

- Easter, Kinney & Peiris, 2004–2005

Observation of the primordial perturbations in the CMB

- What is the prospect for observing small effects in the primordial perturbations?
- Note that both matter, in the large scale structure, and radiation, in the CMB, trace these perturbations
- Radiation
 - WMAP (1 year): $l_{\max} = 300$
 - WMAP (6 year): $l_{\max} = 600$
 - Planck: $l_{\max} = 1500$
- Gravity waves
 - WMAP, Planck, balloons, CMBPOL
 - would help fix the value of H
- Large scale structure
 - Square kilometer array (10 x SDSS)
 - 21 cm high redshift gas
 - cosmic inflation probe (look to $z \approx 2$)



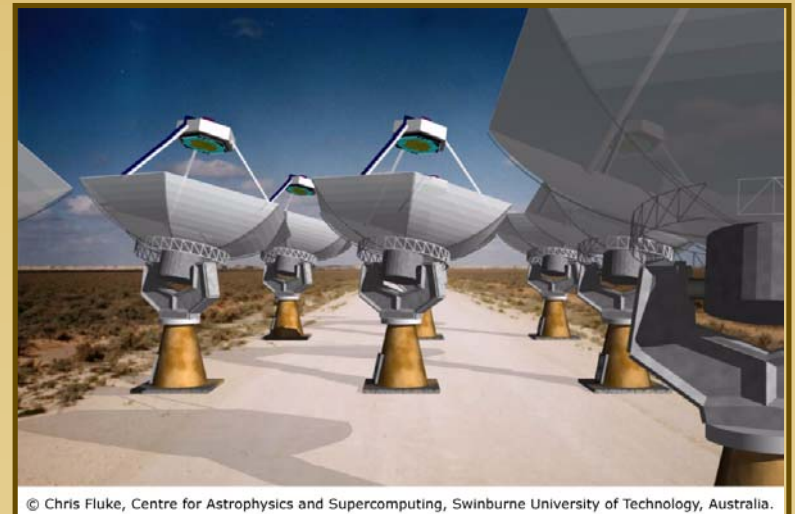
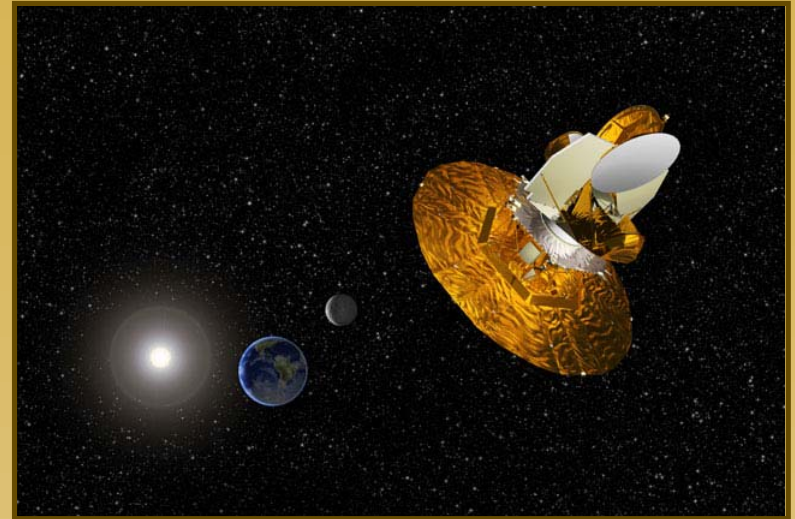
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Observation of the primordial perturbations in the LSS

- The same acoustic peaks also appear in the large scale structure (SDSS), before the non-linear growth of perturbations sets in
- Future experiments to measure the large scale structure are being developed that will map much larger volumes of the universe (21 cm line)
- So how accurately will the power spectrum be measured?

Spergel (ISCAP, 2005)

- | | |
|--------------------------|-----------|
| – today | 10^{-2} |
| – soon (WMAP / Planck) | 10^{-3} |
| – planned galaxy surveys | 10^{-4} |
| – future galaxy surveys | 10^{-5} |
| – theoretical limit | 10^{-6} |



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Conclusions

- What emerges is an effective theory with many familiar properties
 - the long distance features are fixed empirically and any divergences are cancelled by relevant or marginal counterterms with respect to the boundary action
 - we include a general set of short distance features consistent with the symmetries of the state; their divergences also require irrelevant boundary counterterms
 - note that for a long distance measurement ($1/E$), among the infinite set of possible boundary terms most will be too suppressed to affect measurements
- An effective theory of a state provides a model-independent description of the trans-Planckian effects
 - can match to particular models
 - typical effect scales as H/M
- Ultimately this approach provides an estimate of the amplitude and the shape of a generic trans-Planckian signal
 - distinguish from other small corrections (potentials, . . .)

$$\left(\frac{E}{M}\right)^n \geq \delta_{\text{exp}}$$

$$\frac{H}{M} > 10^{-5} \text{ (??)}$$