An effective theory of initial conditions in inflation

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flat: hep-th/0501158
expanding: hep-th/0507081
back-reaction: hep-th/0605107

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We begin with a simple question

Why are we able to explain what happens at long distances without knowing what happens at short distances?

In quantum field theory we have an answer: the details at short distances do not matter . . . at least not much!

$\Rightarrow$ decoupling & effective field theory

Overview

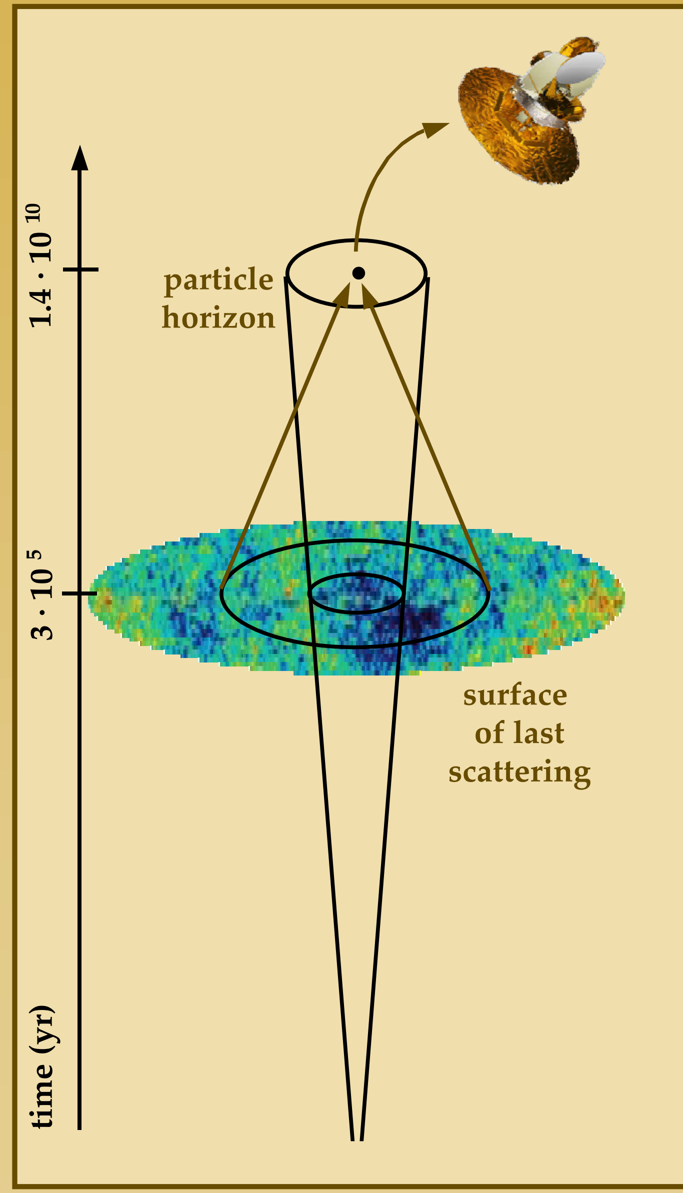
- Inflation and structure formation
- The trans-Planckian problem of inflation
- An effective theory of initial conditions
- Boundary renormalization
- Observational outlook and conclusions

Inflation and structure

- Inflation began as a idea for addressing several fine-tunings that seemed to be needed in the standard cosmological picture
  - flatness problem   Guth (1981), Linde (1982)
  - horizon problem
  - diluting topological relics

- Among these, the horizon problem is perhaps the most serious

- If we look back to the surface of last scattering, how large were the causally connected patches?
  - assuming a standard cosmology, radiation followed by matter domination, they are each about one degree of the sky
  - but they all look the same to about one part in 10^5



When a horizon is not the horizon

- Without inflation, this uniformity of the universe is difficult to understand

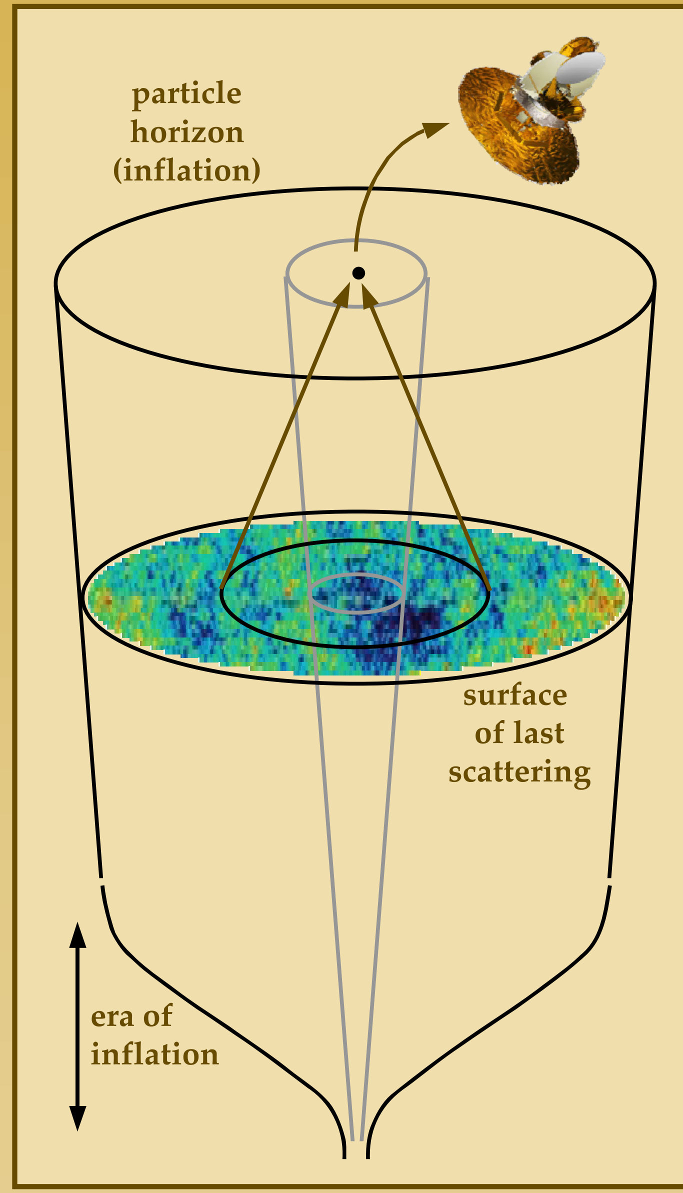
- The reason is that the distances a signal can travel from the beginning—the *particle horizon*—is too small

$$ds^2 = dt^2 - a^2(t)\,d\mathbf{x}\cdot d\mathbf{x} \qquad r_{hor}(t) = \int_{t_0}^{t} \frac{dt'}{a(t')}$$

- During inflation the particle horizon and the Hubble horizon ($1/H$) can be dramatically different

$$r_{hor}(t) = \int_{t_0}^{t} \frac{dt'}{a(t')} \approx \int_{t_0}^{t} e^{H(t-t')}\,dt' \approx \frac{e^{H(t-t_0)}}{H}$$

- Thus, with enough inflation, all that we see today could have grown from a single, causally connected region



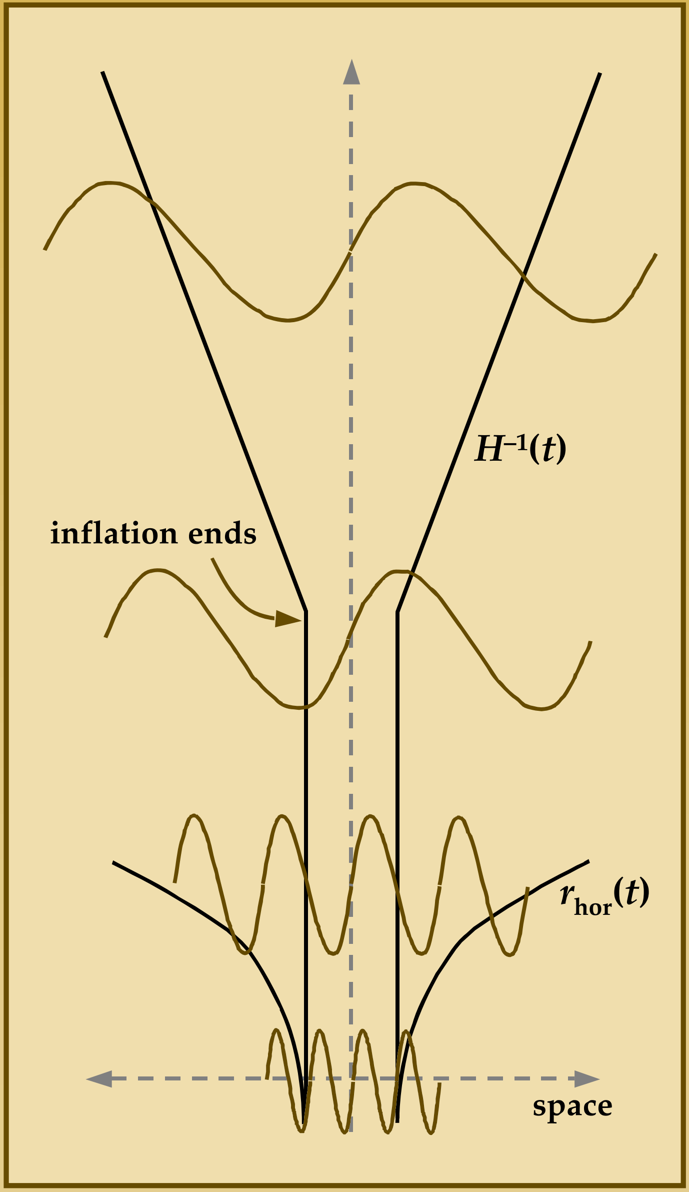
Primordial perturbations

- Having an explanation of the horizon problem in inflation is interesting philosophically; but by itself, is it really testable?

- However, inflation does provide a definite prediction for the primordial perturbations
  - quantum fluctuation of a scalar field (tensor modes too)
  - rapidly stretched by expanding space-time
  - pattern essentially frozen after leaving the Hubble horizon

- This mechanism leaves a pattern of perturbations in the background that affects both matter (LSS) and radiation (CMB) in the universe

- Since its details are important for this talk, let us examine this mechanism more fully



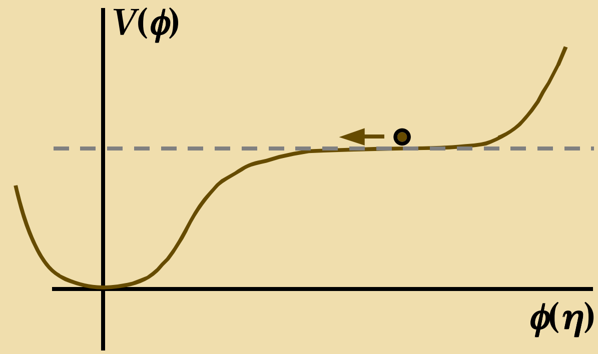
Primordial perturbations—de Sitter space

- Let us look at the simple case of a free, massless scalar field, φ, in de Sitter space

$$S=\int d^4x\sqrt{g}\left[\tfrac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi\right]$$

- We shall use conformally flat coordinates

$$ds^2=\frac{d\eta^2-d\mathbf{x}\cdot d\mathbf{x}}{H^2\eta^2}$$



- In quantum field theory, there is always some inherent variation in the field. We shall divide φ into a term for its mean value, $\phi(\eta)$, and a term describing the fluctuations about that value, $\delta\varphi(\eta,x)$

$$\varphi(\eta,\mathbf{x})=\phi(\eta)+\delta\varphi(\eta,\mathbf{x})$$

$$\langle 0|\varphi(\eta,\mathbf{x})|0\rangle=\phi(\eta)$$

$$\langle 0|\delta\varphi(\eta,\mathbf{x})\delta\varphi(\eta,\mathbf{y})|0\rangle\neq 0$$

- The pattern of fluctuations can be characterized by the variance of $\delta\varphi$

Primordial perturbations—de Sitter space

- The power spectrum is the Fourier transform of this two-point function

$$\langle 0|\delta\varphi(\eta,\mathbf{x})\delta\varphi(\eta,\mathbf{y})|0\rangle = \int \frac{d^3\mathbf{k}}{(2\pi)^3} e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})} \frac{2\pi^2}{k^3} P_k^{\delta\varphi}(\eta)$$

- Expand $\delta\varphi$ in its operator eigenmodes

$$\delta\varphi = \int \frac{d^3\mathbf{k}}{(2\pi)^3}\left[\delta\varphi_k e^{i\mathbf{k}\cdot\mathbf{x}} a_{\mathbf{k}} + \delta\varphi_k^* e^{-i\mathbf{k}\cdot\mathbf{x}} a_{\mathbf{k}}^{\leq}\right]$$

  where the time-dependent part satisfies the Klein-Gordon equation

$$\delta\varphi_k'' - \frac{2}{\eta}\delta\varphi_k' + k^2\delta\varphi_k = 0$$

- How then do we choose the mode functions, $\delta\varphi_k$?
  - It is made up of the two independent solutions to the Klein-Gordon equation
  - One further constraint comes from the canonical equal-time commutation relation
  - so the question is, how do we choose f_k?

$$\delta\varphi_k = \frac{\delta\varphi_k^0 + f_k\delta\varphi_k^{0\,*}}{\sqrt{1-f_k f_k^*}}$$

$$\delta\varphi_k^0(\eta) = \frac{iH}{k\sqrt{2k}}(1+ik\eta)e^{-ik\eta}$$

Choosing the vacuum state

- At very short distances, << $1/H$, the background curvature is not very apparent and space-time looks flat
- Therefore a natural choice is that state that matches with the flat space vacuum as $k \to \infty$ with η fixed
  - this choice fixes $f_k = 0$
- At some stage we might worry about some of our underlying assumptions
  - $H << k << M_{\mathrm{pl}}$
  - sometimes η is taken to ∞
  - simple dynamics/other fields
- We have encountered the question posed at the very beginning:
  - how do we know what happens at very short length scales (or any scale < $1/M_{\mathrm{pl}}$)?

$$\delta\varphi_k = \delta\varphi_k^0 = \frac{iH}{k\sqrt{2k}}(1 + ik\eta)e^{-ik\eta}$$

$$P_k^{\delta\varphi}(\eta) = \frac{k^3}{2\pi^2}\left|\delta\varphi_k^0\right|^2$$

$$P_k^{\delta\varphi}(k\eta \to 0) = \frac{H^2}{4\pi^2}$$

If we assume that—to some degree—these details decouple, the leading result should be that given by this "vacuum"

Primordial perturbations from the vacuum

- What then are the leading predictions for the primordial spectrum of scalar and tensor perturbations?

- In the limit where the modes have been stretched outside the Hubble horizon, the power spectrum is flat
  - this sets the primordial pattern of perturbations in the background (using coordinate-invariant fields)
  - the power spectrum of the primordial gravity waves is similar

- Once we have a set of primordial perturbations, we can solve its evolution using well understood physics
  - Einstein & Boltzmann equations: matter (structure formation) and radiation (the CMB)

$$P_k^{\delta\varphi} \approx \frac{H^2}{4\pi^2}$$

$$P_k^h \approx 16\pi G \frac{H^2}{4\pi^2}$$

$$\Theta(\eta, \mathbf{x}) = \frac{\delta T_{\text{rad}}}{T_{\text{rad}}}$$

$$\delta(\eta, \mathbf{x}) = \frac{\delta\rho_{\text{dm}}}{\rho_{\text{dm}}}$$

primordial

$$C_l = 4\pi \int_0^\infty \frac{dk}{k} P_k \left| \frac{\Theta_{l,k}}{\delta_k} \right|^2$$

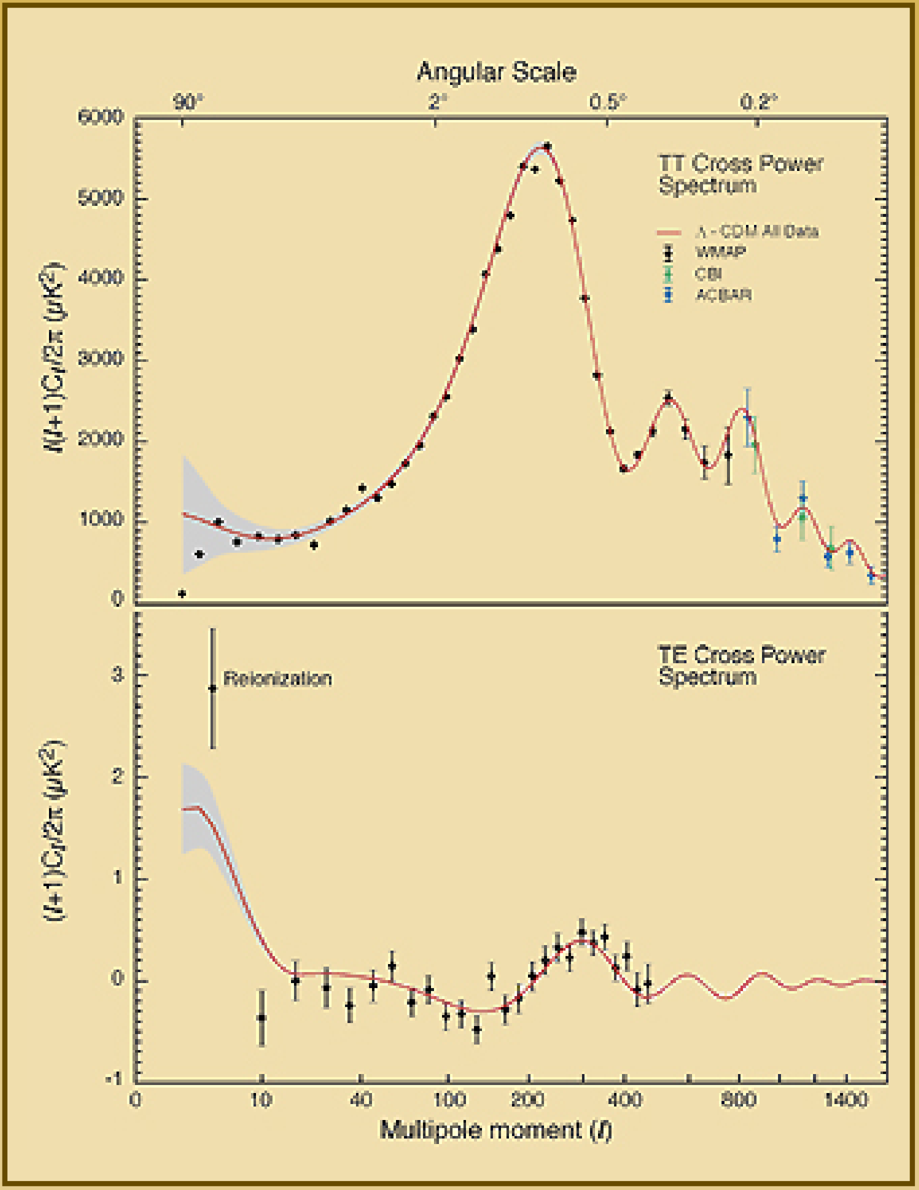
evolution

The microwave background and inflation

- Of course, the power spectrum will not be perfectly flat, but will reflect some of the properties of the potential

- What is the leading form of the power spectrum of the CMB according to inflation?
  - nearly scale invariant (models)
  - nearly Gaussian
  - synchronized acoustic oscillations
  - correlations on super-Hubble horizon scales

- In particular, to leading order our choice of the standard vacuum seems to have been justified
  - but can we understand *why* this is so?



WMAP

Conceptual problems of inflation

- So inflation has passed some very non-trivial tests
  - in particular the correlation of the polarization and the temperature on super-Hubble scales

- But as this subject moves into a more mature phase, we must begin to address some conceptual problems inherent within this picture

Brandenberger

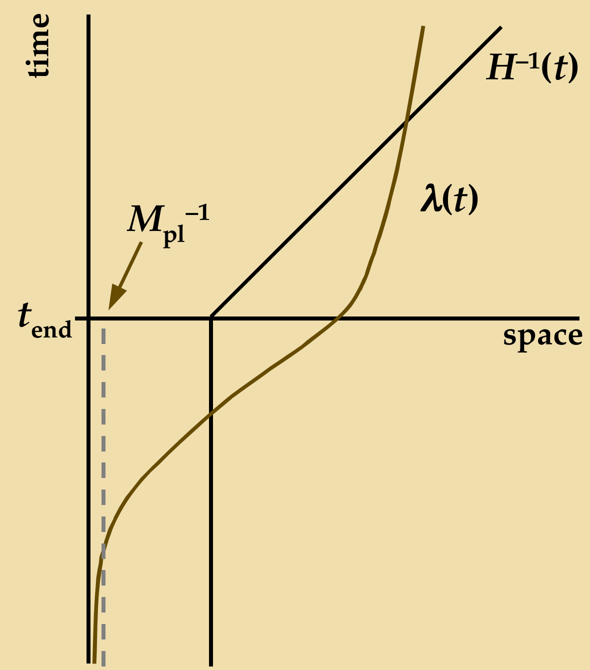
  - hierarchy / amplitude problem (fine-tune)
  - trans-Planckian problem (an opportunity)
  - cosmological constant problem (fine-tune)
  - singularity problem (only as a final theory)
  - back-reaction

as well as to begin to determine the signatures from sub-leading effects

  - inflation potential, new physics, . . .

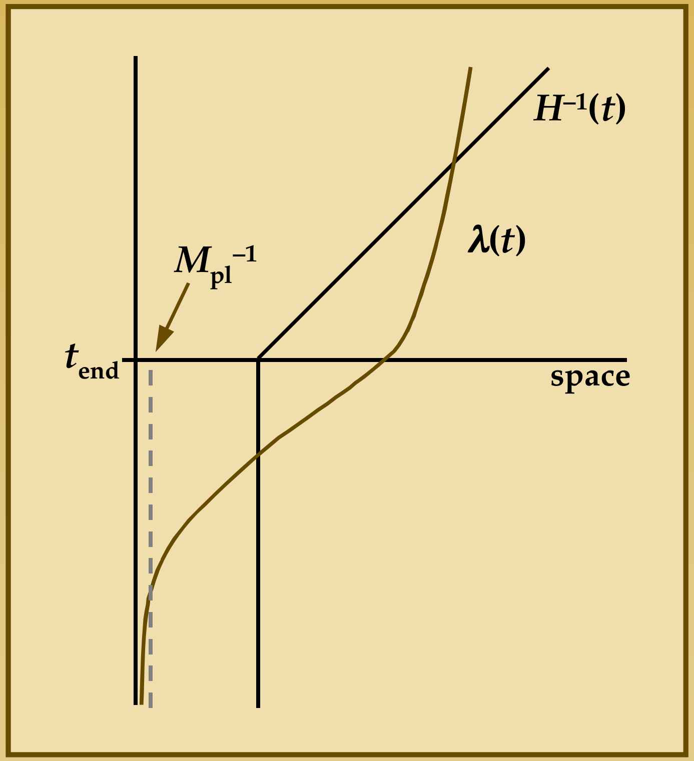
$$V(\phi)/\Delta\phi^4 \lesssim 10^{-12}$$

$$L_g = M_{\rm pl}^4 \frac{\Lambda}{M_{\rm pl}^2} + M_{\rm pl}^2 R$$



The trans-Planckian problem

- In flat space as long as some sort of decoupling holds, we do not need usually to worry about details at the Planck length
- However, the expansion of the background means that what may be a large scale in the primordial background was smaller and smaller the earlier we follow it back during inflation
- So some perturbation that produces, for example, a feature in the CMB was much smaller when it arose during inflation
  - 60–70 *e*-folds to solve the horizon problem
  - a bit more and the wavelength of that mode would have been smaller than the Planck length at some time
- Perhaps the CMB / LSS could be used as a cosmic microscope



The trans-Planckian problem—two philosophies

- Suppose that you have a model for what happens above the Planck-scale
- Then you should be able to solve for the true vacuum state and see how the details alter the primordial spectrum
  - modified uncertainty relation
  - cut-off α-states
  - modified dispersion relations
  - minimal length scale

- Brandenberger & J. Martin, 2001–2003
- Easther, Greene, Kinney, & Shiu, 2001–2002
- Niemeyer & Kempf, 2001
- Starobinsky, 2001
- Danielsson, 2002
- Goldstein & Lowe, 2003
- Collins & M. Martin, 2004
- and others

- Limitations
  - any particular model may not be right
  - how general are the conclusions of a particular case (amplitude and shape)

- Kaloper, Kleban, Lawrence, Shenker, & Susskind, 2002
- Burgess, Cline, Lemieux, & Holman, 2003

- The other approach is to find an effective theory description
  - for the evolution (H^2/M^2)
  - for the state (H/M)

- Greene, Schalm, Shiu, & van der Schaar, 2004–2005
- Collins & Holman, 2005

An effective initial state—boundary conditions

- Let us return to the point where we chose a particular initial state

- We shall examine the case of flat space
  - the regime in which the new effects will appear should be at much shorter lengths than the Hubble horizon
  - so if an effective description is applicable to a cosmological background, it should also be able to be formulated for flat space

- Earlier we mentioned that a state is defined up to one k-dependent constant of integration

- For definiteness, let us define our state by imposing an initial condition at $t = t_0$ and evolve forward

$$S = \int d^4x \left[\tfrac{1}{2} \partial_\mu \varphi \partial_\nu \varphi - \tfrac{1}{2} m^2 \varphi^2 \right]$$

$$\varphi = \int \frac{d^3\mathbf{k}}{(2\pi)^3} \Big[\varphi_k(t) e^{i\mathbf{k}\cdot\mathbf{x}} a_{\mathbf{k}} + \varphi_k^*(t) e^{-i\mathbf{k}\cdot\mathbf{x}} a_{\mathbf{k}}^{\leq} \Big]$$

$$\frac{d^2\varphi_k}{dt^2} = -\omega_k^2 \varphi_k, \quad \omega_k = \sqrt{k^2 + m^2}$$

$$\varphi_k = \frac{\varphi_k^0 + f_k \varphi_k^{0\,*}}{\sqrt{1 - f_k f_k^*}}$$

$$\varphi_k^0(t) = \frac{e^{-ik(t-t_0)}}{\sqrt{2\omega_k}}$$

$$\frac{d\varphi_k}{dt}(t_0) = -i\varpi_k \varphi_k(t_0)$$

An effective initial state—short-distance structure

- Notice that this initial condition includes the standard vacuum state, $\varpi_k = \omega_k$

- In an effective theory, there is always an inherent error between predictions based on our theory and those of a better description of nature
  - e.g. Feynman–Gell-Mann ($V - A$) theory compared with electroweak theory

- If we could solve for the "true vacuum" it might not be the same as our low energy idea of the vacuum; an effective state parameterizes this difference
  - non-localities?
  - strongly interacting gravity?
  - non-commutative space-time?

- to our "vacuum" this difference appears as new short-distance structure

$$\frac{d\varphi_k}{dt}(t_0) = -i\varpi_k \varphi_k(t_0)$$

$$\varpi_k = \omega_k \frac{1-f_k}{1+f_k}$$

for vacuum, 0

$f_k =$ ~~"IR important"~~

+ "UV important"

possible for $k > M$

$$f_k = \sum_{n=1}^{\infty} c_n \frac{k^n}{M^n}$$

An effective initial state—propagation

- Next we must determine how this information in the initial state propagates

- The propagator, which is the Green's function associated with a point source, will include new terms representing the effect of the short-distance features of the state
  - Since the state is defined at an initial time, it is natural that we should include only a term for the forward propagating modes
  - avoids pinched singularities
  - both forward and backward propagating modes are included for the point source

- Note that this Green's function is also consistent with the initial condition, which is again defined for the modes

$$G_F(x,x') = \int \frac{d^3\mathbf{k}}{(2\pi)^3} e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}')} G_k(t,t')$$

point source

$$\begin{aligned} -iG_k(t,t') = {} & \Theta(t-t')\varphi_k^0(t)\varphi_k^{0*}(t') \\ & + \Theta(t'-t)\varphi_k^{0*}(t)\varphi_k^0(t') \\ & + f_k^*\varphi_k^0(t)\varphi_k^0(t') \end{aligned}$$

initial state "structure"

$$\partial_t G_k(t,t')\big|_{t=t_0} = i\varpi_k^* G_k(t_0,t')$$

$$\partial_{t'} G_k(t,t')\big|_{t'=t_0} = i\varpi_k^* G_k(t,t_0)$$

An aside on image sources

- The large degree of symmetry of Minkowski space permits a geometric interpretation for the extra term in the propagator
  - we define an image time, reflected across the in initial time hypersurface

- Then the propagator can be defined as the Green's function associated with two sources, one of which is in the unphysical region before t_0
  - only one of the Θ functions contributes in the physical region

- A general expanding background may not have an analogous geometric interpretation, but otherwise the propagator is a simple generalization of what we had before

image time: $t_I = 2t_0 - t$

$$G_k(t,t') = G_k^0(t,t') + f_k^* G_k^0(t_I,t')$$

Vacuum ($f_k = 0$) propagator:

$$G_F^0(x,x') = \int \frac{d^3\mathbf{k}}{(2\pi)^3} G_k^0(t,t')$$
$$= \int \frac{d^4k}{(2\pi)^4} \frac{i}{k^2 - m^2 + i\varepsilon}$$

Renormalization—general properties

- Although our ultimate goal is to calculate the trans-Planckian signal in the CMB, this talk will focus on establishing the renormalizability of the effective state approach

- In ordinary field theory, loops appear in perturbative corrections to processes
  - these contain intermediate propagators of arbitrarily large momenta
  - in some cases, in summing over this short-distance behavior we encounter divergences
  - these are absorbed by rescaling the parameters of the theory (counterterms)

- For an general initial state, a loop will also introduce sums of over the short-distance structure of the state
  - new divergences & *boundary counterterms*

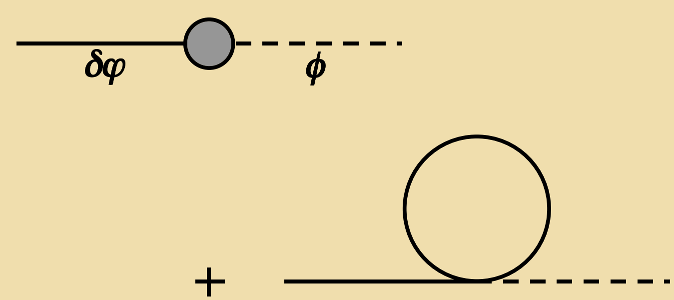
Renormalization—quartic theory

- Let us consider a specific example, a theory with a quartic interaction
- We shall consider a baby version of inflation
  - calculate in flat space
  - include an isotropic zero mode
  - the full Robertson-Walker case is considered in hep-th/0507081
- We impose the renormalization condition that the tadpole of the fluctuation should vanish
  - calculate to one loop order
  - isolate classes of divergences
  - determines renormalization scale dependence

$$L = \frac{1}{2}\partial_\mu \varphi \partial^\mu \varphi - \frac{1}{2} m^2 \varphi^2 - \frac{1}{24}\lambda \varphi^4$$

$$\varphi(t,\mathbf{x}) = \phi(t) + \delta\varphi(t,\mathbf{x})$$

$$\langle 0_{\mathrm{eff}} \,|\, \delta\varphi(t,\mathbf{x}) \,|\, 0_{\mathrm{eff}} \rangle = 0$$



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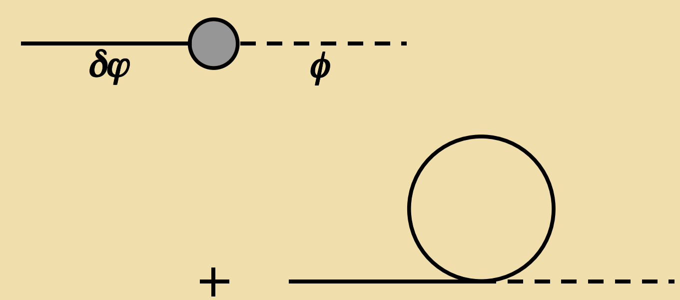
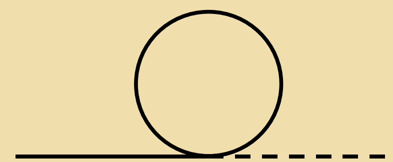
$$L = \frac{1}{2}\partial_\mu \delta\varphi \partial^\mu \delta\varphi - \frac{1}{2}\left[m^2 + \frac{1}{2}\lambda\phi^2\right]\delta\varphi^2$$
$$-\left[\nabla^2\phi + m^2\phi + \frac{1}{6}\lambda\phi^3\right]\delta\varphi$$
$$-\frac{1}{6}\lambda\phi\delta\varphi^3 - \frac{1}{24}\lambda\delta\varphi^4$$



Renormalization of the leading trans-Planckian effect

- The boundary divergences fall into two classes
  - from IR features ⇒ relevant/marginal *boundary* counterterms
  - from UV (trans-Planckian) features ⇒ irrelevant boundary counterterms
- The idea is that we determine the state by choosing the long distance features empirically and allowing a general set of short distance features
  - consider a leading example of the latter
  - use ω_k to simplify integrals
- At one loop order we find two new divergences which are confined to the initial surface
  - cancel these with boundary counterterms

$$f_k = \sum_{n=1}^{\infty} c_n \frac{k^n}{M^n} \longrightarrow c_1 \frac{\omega_k}{M} + \cdots$$



$$S_{\rm bnd} = \int d^3\mathbf{x} \left[\frac{z_1}{2} \frac{m^2}{M} \varphi^2 + \frac{z_3}{4} \frac{1}{M} \varphi^4 \right]$$

irrelevant boundary counterterm

Boundary renormalization group running

- The coefficients of the boundary counterterms are determined by a renormalization prescription (MS)
  - infinite, scale-independent part to cancel $1/\varepsilon$ pole (and $-\gamma$, ln 4π, etc.)
  - finite, scale dependent part satisfies the Callan-Symanzik equation
- Note that the Callan-Symanzik equation applies to both bulk and boundary divergences
  - there is universal cut-off that applies to both the large spatial momenta and infinitesimal time intervals from t_0
  - the equivalence of bare and renormalized n-point functions means that the derivatives of both, with respect to the renormalization scale μ, vanish
- For the leading trans-Planckian effect:

$$S_{\text{bnd}} = \int d^3\mathbf{x} \left[\frac{z_1}{2} \frac{m^2}{M} \varphi^2 + \frac{z_3}{4} \frac{1}{M} \varphi^4 \right]$$

$$\left[\mu \frac{\partial}{\partial \mu} + \beta \frac{\partial}{\partial \lambda_R} + m_R \gamma_m \frac{\partial}{\partial m_R} + \beta_1 \frac{\partial}{\partial z_1} + \beta_3 \frac{\partial}{\partial z_3} + \cdots \right] G^1 = 0$$

$$\beta_1(\lambda_R) = \mu \frac{dz_1}{d\mu} = \frac{2ic_1^* \lambda_R}{\pi^2} + \cdots$$

$$\beta_3(\lambda_R) = \mu \frac{dz_3}{d\mu} = \frac{ic_1^* \lambda_R^2}{\pi^2} + \cdots$$

Renormalization of a state and its evolution

- Let us summarize generally what we have found, both for a flat and a completely general Robertson-Walker background

<table>
<tr><th rowspan="3"></th><th colspan="3">IR/long distance</th><th colspan="3">UV/short distance</th></tr>
<tr><th rowspan="2">structure</th><th colspan="2">renormalization</th><th rowspan="2">structure</th><th colspan="2">renormalization</th></tr>
<tr><th>operators</th><th>examples</th><th>operators</th><th>examples</th></tr>
<tr><td>bulk (evolution)</td><td>observed long distance degrees of freedom</td><td>relevant, marginal (dim ≤ 4)</td><td>$\nabla_\mu\varphi\nabla^\mu\varphi$, φ^2, φ^4, $R\varphi^2$</td><td>completely free, up to assumed symmetries of background</td><td>irrelevant (dim > 4)</td><td>$(\nabla_\mu\varphi\nabla^\mu\varphi)^p$, φ^6, φ^8, $R^2\varphi^2$, …</td></tr>
<tr><td>boundary (state)</td><td>appropriate state of long distance effective free theory</td><td>relevant, marginal (dim ≤ 3)</td><td>φ^2, $\varphi\nabla_n\varphi$, $K\varphi^2$</td><td>completely free, up to assumed symmetries of state</td><td>irrelevant (dim > 3)</td><td>φ^4, $(\nabla_n\varphi)^2$, $\nabla_i\varphi\nabla^i\varphi$, $K^2\varphi^2$, …</td></tr>
</table>

- Here, $\nabla_n = n^\mu\nabla_\mu$ is a derivative normal to the initial surface and $K_{\mu\nu} = h_\mu{}^\lambda\nabla_\lambda n_\nu$ is the extrinsic curvature along the surface

Further work

- So we find an elegant correspondence between the long and short distance features of the initial state and the sorts of operators the appear in their renormalization

- This is still rather a young subject so there are many aspects which should be studied further
  - back-reaction (size of effect, types of operators that appear)
  - RG flow (de Sitter space?)
  - decoherence of quantum effects
  - generating effective states by integrating out excited heavy fields
  - calculation of the amplitude and the generic shape of the trans-Planckian correction to the power spectrum
  - . . .

Back-reaction and naturalness:

- Porrati, 2004–2005
- Greene, Schalm, Shiu, & van der Schaar, 2004–2005

Somewhat related work on RG flows in de Sitter space:

- Larsen & McNees, 2003–2004

Fits to the CMB data:

- Easther, Kinney & Peiris, 2004–2005

Observation of the primordial perturbations in the CMB

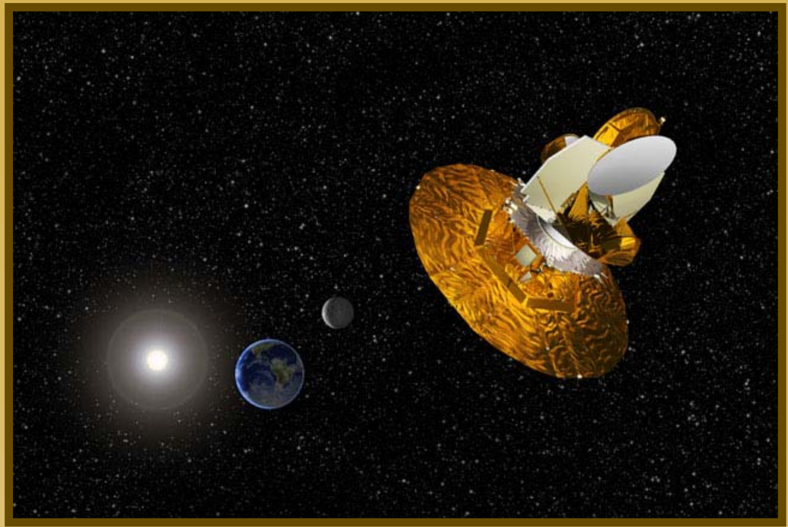
- What is the prospect for observing small effects in the primordial perturbations?

- Note that both matter, in the large scale structure, and radiation, in the CMB, trace these perturbations

- Radiation
  - WMAP (1 year): $l_{\text{max}} = 300$
  - WMAP (6 year): $l_{\text{max}} = 600$
  - Planck: $l_{\text{max}} = 1500$

- Gravity waves
  - WMAP, Planck, balloons, CMBPOL
  - would help fix the value of H

- Large scale structure
  - Square kilometer array (10 x SDSS)
  - 21 cm high redshift gas
  - cosmic inflation probe (look to $z \approx 2$)



Observation of the primordial perturbations in the LSS

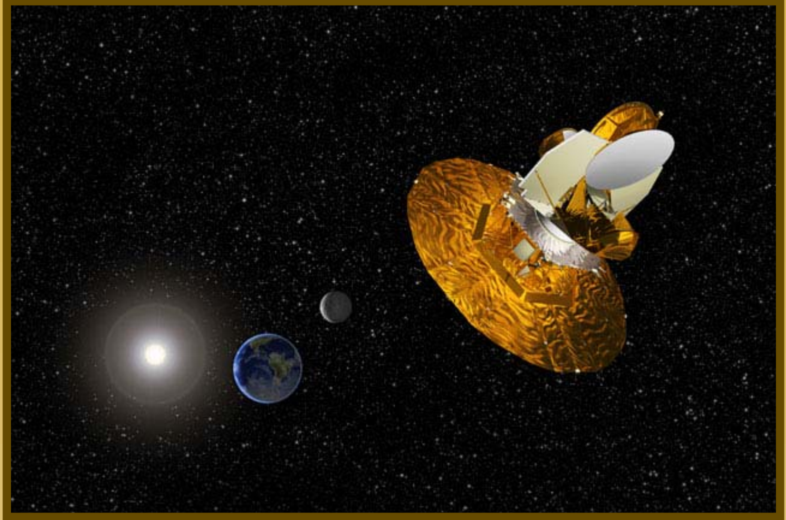
- The same acoustic peaks also appear in the large scale structure (SDSS), before the non-linear growth of perturbations sets in

- Future experiments to measure the large scale structure are being developed that will map much larger volumes of the universe (21 cm line)

- So how accurately will the power spectrum be measured?

Spergel (ISCAP, 2005)

– today	10^{-2}
– soon (WMAP/Planck)	10^{-3}
– planned galaxy surveys	10^{-4}
– future galaxy surveys	10^{-5}
– theoretical limit	10^{-6}

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Conclusions

- What emerges is an effective theory with many familiar properties
  - the long distance features are fixed empirically and any divergences are cancelled by relevant or marginal counterterms with respect to the boundary action
  - we include a general set of short distance features consistent with the symmetries of the state; their divergences also require irrelevant boundary counterterms
  - note that for a long distance measurement ($1/E$), among the infinite set of possible boundary terms most will be too suppressed to affect measurements

$$\boxed{\left(\frac{E}{M}\right)^n \geq \delta_{\mathrm{exp}}}$$

- An effective theory of a state provides a model-independent description of the trans-Planckian effects
  - can match to particular models
  - typical effect scales as H/M

$$\boxed{\frac{H}{M} > 10^{-5} \;\; (??)}$$

- Ultimately this approach provides an estimate of the <u>amplitude</u> and the <u>shape</u> of a generic trans-Planckian signal
  - distinguish from other small corrections (potentials, . . .)